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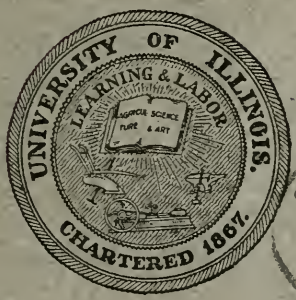
WIND STRESSES IN THE STEEL FRAMES OF OFFICE BUILDINGS

BY

W. M. WILSON

AND

G. A. MANEY



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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 80 JUNE, 1915

WIND STRESSES IN THE STEEL FRAMES OF OFFICE BUILDINGS.

BY W. M. WILSON, ASSISTANT PROFESSOR OF STRUCTURAL ENGINEERING, AND G. A. MANEY, FORMERLY RESEARCH FELLOW IN DEPARTMENT OF THEORETICAL AND APPLIED MECHANICS.*

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*While the bulletin is the result of the joint efforts of the two writers, certain parts are so distinctly the work of one that it seems desirable to make a statement relative to the part which should be credited to each. The method of making the analysis involving the use of the slope-deflection equations should be credited to G. A. Maney. The conception of the bulletin as a whole, the method of presenting the results, and the authorship of the text should be credited to W. M. Wilson. The numerical calculations of Tables 11 to 22, inclusive, were made by both writers.—The Editor.

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WIND STRESSES IN THE STEEL FRAMES OF OFFICE BUILDINGS

I. Introduction.

1. *Preliminary.*—The increase in the price of land in large cities has made it necessary to build high buildings in order to get a large rentable floor space on a small parcel of land. The type of building generally used is known as the steel-skeleton building. In this type of building the live and dead loads, including the weight of the walls, are carried by a system of beams and girders to columns and are carried by the columns to the footings.

In high buildings the horizontal shear due to the wind load is very large; and, since it is usually impracticable to put diagonal braces between the columns, it is customary to make the steel frame rigid enough to resist the horizontal shear by virtue of the stiffness of the columns and girders. The exact determination of the stresses in a steel frame due to a horizontal shear is one of the problems of structural engineering which remains to be solved. While the writers realize that the method of determining these stresses presented in this bulletin is based upon assumptions which are not exactly true, they believe that the method is more accurate than the methods ordinarily used.

2. *Acknowledgment.*—Messrs. Anderson, Becker, Gomez, and Richart, graduate students in the College of Engineering of the University of Illinois, calculated the moments in Table 23, and the moments as determined by methods I, II, and III in the keys to the diagrams in Figs. 10, 11, and 12, and assisted with the calculations necessary to determine the curves shown in these figures. Professor Ira O. Baker and Professor C. A. Ellis rendered valuable assistance, criticizing the bulletin during its prepartion. The writers gratefully acknowledge indebtedness to these men.

II. Present Methods of Calculating Wind Stresses in Office Buildings.

3. *Classification of Methods.*—Methods of calculating wind stresses in the steel frames of office buildings may be divided into two classes, viz.: (1), those used in the actual design of buildings and (2), those

which have resulted from an effort to make an exact analysis of the stresses. In the methods of the first class, accuracy has been sacrificed to shorten and simplify the calculations. In the methods of the second class, the aim has been to make an exact analysis rather than an analysis that can be used in the actual design of a building.

For the sake of convenience in reference, methods of the first class are designated as *approximate methods,* and those of the second class are designated as *exact methods.*

4. *Approximate Methods.*—(a) Fleming's Methods. Mr. R. Fleming, in an article in *Engineering News,* describes three methods, which are in current use.* These methods are designated as methods I, II, and III. The three methods, as applied to a building in which all columns of a story have the same section, are based upon the following assumptions:

Assumptions in Method I.

1. A bent of a frame acts as a cantilever.

2. The point of contra-flexure of each column is at mid-height of the story.

3. The point of contra-flexure of each girder is at its mid-length.

4. The direct stress in a column is directly proportional to the distance from the column to the neutral axis of the bent.

Assumptions in Method II.

1. A bent of a frame acts as a series of portals.

2. The point of contra-flexure of each column is at mid-height of the story.

3. The shear is the same on all columns of a story.

4. Each pair of adjacent columns of a bent acts as a portal, and each interior column is a member of two adjacent portals. The direct stress in an interior column, when the column is considered as a member of the portal on one side, is of opposite sign from the direct stress in the same column when considered as a member of the portal on the opposite side and the resultant direct stress is equal to zero.

*Wind Bracing without Diagonals for Steel-Frame Office Buildings, Engineering News, March 13, 1913.

Assumptions in Method III.

1. A bent of a frame acts as a continuous portal.

2. The point of contra-flexure of each column is at mid-height of the story.

3. The direct stress in a column is directly proportional to the distance from the column to the neutral axis of the bent.

4. The shear is the same on all columns of a story.

(b) Smith's Methods. Professor Albert Smith, in a paper before the Western Society of Engineers, describes a method which he has used in his classes in Structural Engineering at Purdue University.‡ This method is here designated as Method IV.

Assumptions in Method IV.

1. The point of contra-flexure of each column is at mid-height of the story.

2. The point of contra-flexure of each girder is at its mid-length.

3. The shears on the internal columns are equal and the shear on each external column is equal to one-half of the shear on an interior column.

If all of the assumptions of any of these methods are accepted, the stresses in a frame may be determined by applying the fundamental equations of static equilibrium. It is apparent from the assumptions that the results obtained by these methods are radically different.

5. *Exact Methods.*—(a) Melick's Method. Dr. Cyrus A. Melick* used a method which takes into account the form of the elastic curves and the deflections and changes in length of the members. The method is so long that, when applied to a building only four stories high, the amount of work required is almost prohibitive. To apply it to a building twenty stories high would be impracticable if not, in fact, impossible.

(b) Jonson's Method. Mr. Ernest F. Jonson suggested a method which takes into account the deflections of the columns and the changes in the slopes of the tangents to the elastic curves of the columns and girders at the points where they intersect.†

If the method which he suggests were used, it would give the stresses with a fair degree of accuracy; but his method involves so many unknowns that its use would not be practicable in the actual design of buildings.

‡Wind Stresses in the Frames of Office Buildings, by Albert Smith, Journal Western Society of Engineers, Vol. XX, No. 4, p. 341.

*Stresses in Tall Buildings, by Cyrus A. Melick. Bulletin No. 8, College of Engineering, University of Ohio.

†The Theory of Frameworks with Rectangular Panels and Its Application to Buildings which Have to Resist Wind, by Ernst F. Jonson, Tran. Am. Soc. C. E., Vol. 55, p. 413.

(c) Method of Least Work. Professor Albert Smith has deter-ined the wind stresses in symmetrical bents having two, three, and ur spans by the method of least work.* This method is exact, but, of urse, is extremely long.

III. Outline of the Proposed Analysis.

6. *Outline of the Method.*—In making an analysis of the stresses, e writers made certain assumptions and applied certain fundamental rinciples of mechanics and obtained equations from which the stresses a frame can be determined. The assumptions which have been made e stated in Section IV and the derivation of the equations is given in ections V and VI.

It can be proven that the moment at an end of a member of a ame is a function of the changes in the slopes of the tangents to the astic curve of the member at its ends and of the deflection of one end the member relative to the other end (see equation A, page 13).

In the strained position, all the columns and girders which intersect one point have been subjected to the same change in slope (see as-mption 1, Section IV); the vertical deflections of the ends of all rders are equal to zero; and the horizontal deflections of the tops of l columns of a story are equal (see assumption 2, Section IV).

Consider any story of a bent. Take the point of intersection of e neutral axes of a column and a girder as a free body. It is in equi-rium under the action of the moments at the extremities of the col-nns and girders which intersect at the point. Each of the moments ay be expressed in terms of the changes in the slopes at the extremities the member, and the deflection of one end of the member relative to e other. A moment equation can therefore be written for each point here the columns and girders intersect, and the only unknown quan-ties will be the changes in the slopes at the extremities of the columns nd the horizontal deflections of the columns in a story.

If all the columns of a story be taken together as a free body, the m of the moments at the two extremities of all the columns will be lanced by a couple whose moment is equal to the total shear on the ory multiplied by the story height. The shear and the height of the ory are known, and the moments in the columns can be expressed in rms of the slopes and the deflections at their extremities the same as in e previous equations. It is therefore possible to write as many equations r each story as there are columns in the story, plus one. As the only

*Wind Stresses in the Frames of Office Buildings, by Albert Smith, Journal Western ciety of Engineers, Vol. XX, No. 4, p. 341.

unknown quantities in these equations are the changes in the slopes at the extremities of the columns and the deflection in a story common to all columns, there are as many equations per story as there are unknowns. By solving these equations the slopes and the deflections can be determined. Knowing the slopes and the deflections, the moments can be computed.

The product of the shear on a member and the length of the member is equal to the algebraic sum of the moments at the extremities of the member. Since the moments and the length of the member are known the shear can be computed.

With the shears in the girders known, the direct stress in any column can be determined by taking the column as a free body and equating the sum of the vertical forces to zero.

The direct stress in a girder may be determined in a similar manner.

The method just described is based upon the proposition in mechanics used by Mr. Jonson, but the method which the writers have developed differs from the one used by him in that the changes in the slopes and the deflections have been used as the unknown quantities instead of the direct stresses and the moments. Four members, two columns and two girders, intersect in a point. Each member is subjected to a different direct stress and a different moment, whereas all of the members are subjected to the same change in slope, and all of the columns in a story are subjected to the same deflection. It is therefore apparent that there are fewer unknown slopes and deflections than moments and direct stresses. The large reduction in the number of unknowns very much simplifies the solution of the equations.

IV. Assumptions Upon Which the Analysis Is Based.

7. *Statement of Assumptions.*—The proposed analysis is based upon the following assumptions:

1. The connections between the columns and girders are perfectly rigid.

2. The change in the length of a member due to the direct stress is equal to zero.

3. The length of a girder is the distance between the neutral axes of the columns which it connects and the length of a column is the distance between the neutral axes of the girders which it connects.

4. The deflection of a member due to the internal shearing stresses is equal to zero.

5. The wind load is resisted entirely by the steel frame.

These assumptions will be discussed in Section XI.

V. Fundamental Equation.

8. *Fundamental Proposition.*—The fundamental equation used in this analysis is based upon the following proposition:

When a member is subjected to flexure, the deflection of any point in the neutral axis from the tangent to the elastic curve at any other point is equal to the moment of the area of the $\frac{M}{EI}$ *diagram for the portion of the member between the two points, about the point where the deflection is measured.*

9. *Proof of the Proposition.*—The line AB, Fig. 1, represents the neutral axis of a member subjected to flexure. The deflection of the member is very much exaggerated in the figure. The actual deflection is so small that the length of the curve may be considered equal to its horizontal projection. It is required to prove that the deflection of any point P from the tangent to the line AB at any other point Q is equal to $\int_Q^P \frac{M}{EI}\, dx{\cdot}x$.

Extend the tangents at the extremities of an element of the curve until they intersect the vertical line through P. The intercept on this vertical line between two consecutive tangents is equal to $x{\cdot}d\theta$. The total deflection of P from the tangent at Q is equal to the algebraic sum of these intercepts for the elements of the elastic curve between the points Q and P. That is, $y = \int_Q^P x d\theta$. The equation of the elastic curve may be written $\frac{d\theta}{dx} = \frac{M}{EI}$. Substituting the value of $d\theta$ from this equation in the expression for y, gives $y = \int_Q^P \frac{M}{EI}\, dx{\cdot}x$. The quantity $\frac{M}{EI}dx$ can be considered as an elementary area of the $\frac{M}{EI}$ diagram. The quantity $\int_Q^P \frac{M}{EI} dx{\cdot}x$ can therefore be considered as the moment of the $\frac{M}{EI}$ diagram about the point P.

10. *Derivation of Fundamental Equation.*—Consider a member which is not acted upon by any external forces or couples except at the ends. The line AB in Fig. 2 represents the neutral axis of such a member. The moment at A is represented by M_{AB} and at B by M_{BA}. The change in the slope of the elastic curve at A due to the external forces is represented by θ_A, and at B by θ_B. The deflection of A from its original position A', is d. The deflection of A from the tangent at B is represented by $(d - l\cdot\theta_B)$; and the deflection of B from the tangent

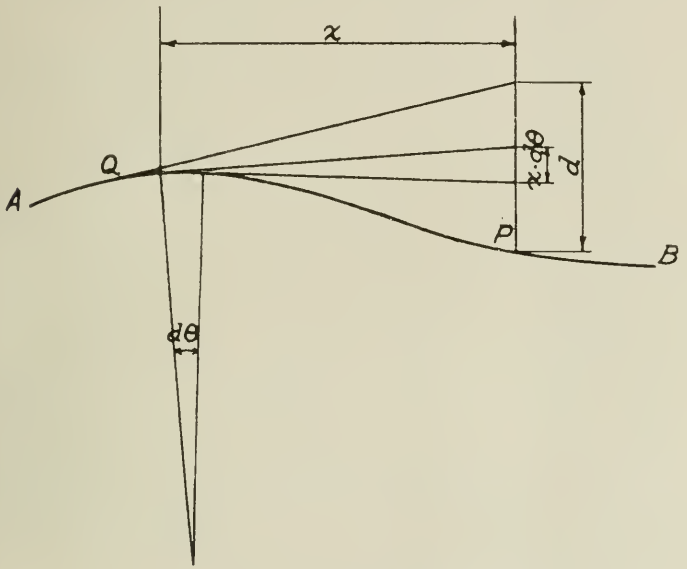

Fig. 1. Proof of Fundamental Equation.

at A is represented by $(d - l\cdot\theta_A)$. The $\frac{M}{EI}$ diagram is shown in Fig. 3.

The signs of the quantities are determined by the following rules:

The change in the slope, or the angular deformation, is positive (+) when the tangent to the elastic curve of the member is turned in a clockwise direction.

Distances and deflections are positive when they are measured in the same direction from the base line as are positive slopes.

The moment acting upon a member at the section where the deflection is measured is positive (+) when it produces a clockwise rotation.

Substituting $(d - l\cdot\theta_B)$, the deflection of A from the tangent at B,

for y in the equation $y = \int \frac{M}{EI}\, dx{\cdot}x$, gives

$$(d - l{\cdot}\theta_B) = \int_A^B \frac{M}{EI}\, dx{\cdot}x.$$

Substituting the value of M from the equation

$$M = M_{AB} + \left[\frac{M_{BA} - M_{AB}}{l}\right] x, \text{ gives}$$

$$(d - l{\cdot}\theta_B) = \int_A^B \frac{M_{AB}}{EI}\, x dx + \left[\frac{M_{BA} - M_{AB}}{EI{\cdot}l}\right] x^2{\cdot}dx.$$

If the material is homogeneous and the section uniform, E and I are constants. Performing the indicated integration, gives

$$(d - l{\cdot}\theta_B) = \frac{M_{AB}}{EI}\,\frac{l^2}{2} + \frac{M_{BA}}{EI}\,\frac{l^2}{3} - \frac{M_{AB}}{EI}\,\frac{l^2}{3}, \text{ or}$$

$$d = l{\cdot}\theta_B + \frac{l^2}{6EI}\left[2M_{BA} + M_{AB}\right] \quad \ldots\ldots\ldots\ldots (1)$$

Substituting $(d - l{\cdot}\theta_A)$, the deflection of B from the tangent at A, for y in the equation, $y = \int \frac{M}{EI}\, dx{\cdot}x$, gives

$$(d - l{\cdot}\theta_A) = \int_B^A \frac{M}{EI}\, dx{\cdot}x.$$

The moment at any section in the member, when considering the deflection at B, is of opposite sign from the moment at the same section when considering the deflection at A. (See preceding rule for determining the sign of the moment.)

Substituting the value of M from the equation

$$M = -M_{BA} - \left[\frac{M_{AB} - M_{BA}}{l}\right] x, \text{ gives}$$

$$(d - l{\cdot}\theta_A) = \int -\frac{M_{BA}}{EI}\, x dx - \left[\frac{M_{AB} - M_{BA}}{l}\right] x^2\, dx \text{ or}$$

$$d = l{\cdot}\theta_A - \frac{M_{BA}\, l^2}{EI\ 2} - \frac{M_{AB}\, l^2}{EI\ 3} + \frac{M_{BA}\, l^2}{EI\ 3}$$

$$d = l{\cdot}\theta_A + \frac{l^2}{6EI}\left[-M_{BA} - 2M_{AB}\right] \quad \ldots\ldots\ldots\ldots (2)$$

Multiplying equation (2) by 2, gives

$$2d = 2l{\cdot}\theta_A + \frac{l^2}{6EI}\left[-2M_{BA} - 4M_{AB}\right] \quad \ldots\ldots\ldots\ldots (3)$$

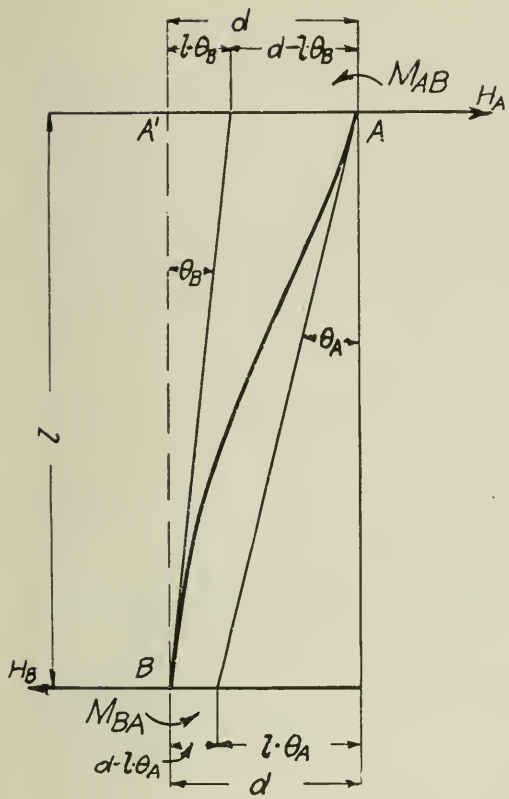

FIG. 2. DERIVATION OF FUNDAMENTAL EQUATION.

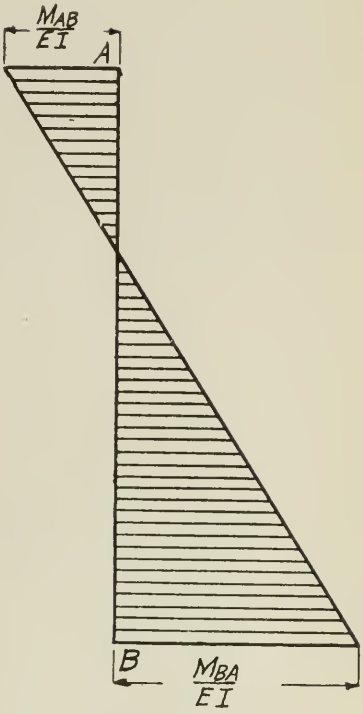

FIG. 3. $\frac{M}{EI}$ DIAGRAM.

Adding equations (1) and (3) gives

$$3d = 2l\cdot\theta_A + l\cdot\theta_B + \frac{l^2}{6EI}\left[-3M_{AB}\right].$$

Substituting K for $\frac{I}{l}$ and R for $\frac{d}{l}$ and solving for M_{AB}, gives,

$$M_{AB} = 2\,EK\ [2\,\theta_A + \theta_B - 3R] \ldots\ldots\ldots\ldots\ldots\ldots (A)$$

When $d = 0$, equation (A) becomes

$$M_{AB} = 2\,EK\ (2\,\theta_A + \theta_B) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (B)$$

Equation (A) is general and may be applied to any length of any member in bending provided the length considered has no intermediate external force applied to it. That is, one or more of the quantities

θ_A, θ_B, and d may be negative and equation (A) will still give the moment at the point A in both magnitude and sign. Equation (A) is the fundamental equation upon which the analysis which follows is based. Equation (B) is a special form of equation (A). Equation (A) may be expressed as follows:

The moment at the end of any member is equal to 2EK times the quantity: Two times the change in the slope at the near end plus the change in the slope at the far end minus three times the deflection divided by the length. E is the modulus of elasticity of the material and K is the ratio of the moment of inertia to the length of the member.

VI. Derivation of General Equations.

11. *Notation.*—The following notation has been used:

A, B, C, etc. = the columns of a bent, beginning at the right and reading toward the left.

a, b, c, etc.= the bays of a bent, beginning at the right and reading toward the left. The girder in bay a is designated as girder a, in bay b as girder b, in bay c as girder c, etc.

A_1, A_2, A_3, etc. = the intersections of the neutral axes of the girders at the tops of the first, second, third, etc., stories with the neutral axis of column A.

B_1, B_2, B_3, etc. = the intersections of the neutral axes of the girders at the tops of the first, second, third, etc., stories with the neutral axis of column B.

d = deflection of the columns in a story height.

E = modulus of elasticity of the material.

h = length of a column measured from neutral axis to neutral axis of the girders.

I = moment of inertia of the girder and column sections.

$J = 2\sum\left(\frac{I}{l}+\frac{I}{h}\right)$ for all columns and girders which intersect at a point.

$K = \frac{I}{l}$ for girders and $\frac{I}{h}$ for columns.

l = length of a girder measured from neutral axis to neutral axis of the columns.

M = bending moment.

$N = 2\sum\left(\frac{I}{h}\right)$ for all columns in a story.

$R = \frac{d}{h}$

W = total horizontal shear in a bent at any story.

w = increment of the horizontal shear in a bent in a story height.

θ = change in the slope of the tangent to the elastic curve.

Subscripts are added to the letters to indicate the particular part of a bent to which a given symbol applies. For example, referring to Fig. 4, girder b_1 is the girder in bay b at the top of the first story, A_1 is the intersection of the girder at the top of the first story with

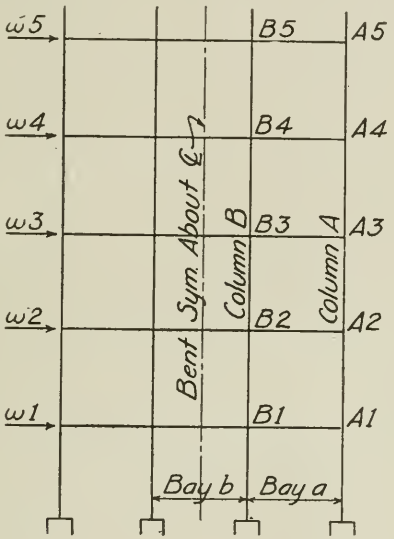

Fig. 4. Bottom Five Stories of a Symmetrical Three-Span Bent.

column A, θ_{A_1} is the slope of the tangent to the elastic curve at the point A_1; d_3 is the deflection of the columns in the third story; J_{B_4} is the J at the point B_4; K_{A_2} is the K of column A in the second story. The moment at the right end of the girder in bay a at the top of the second story is designated as M_2^{AB}, and the moment at the left end of the same girder is designated as M_2^{BA}. The moment at the top of the column B in the third story is designated as M_B^{32}, and the moment at the bottom of the same column is designated as M_B^{23}.

12. *Derivation of Equations.*—Fig. 4 shows the bottom five stories of a symmetrical three-span bent. It is required to find the stresses in all of the members.

Consider the columns of the third story acting together as a free body. (Any other story could have been used.) The algebraic sum of all of the moments at the tops and the bottoms of all the columns plus the product of the total shear in the story and the story height, is equal to zero. That is, $2\,(M_A{}^{32} + M_A{}^{23} + M_B{}^{32} + M_B{}^{23}) + W_3 h_3 = 0$

Substituting the values of the moments as given by equations A and B of Section V gives

$$2\left[2EK_{A3}\,(2\theta_{A3} + \theta_{A2} - 3R_3) + 2EK_{A3}\,(2\theta_{A2} + \theta_{A3} - 3R_3) + 2EK_{B3}\,(2\theta_{B3} + \theta_{B2} - 3R_3) + 2EK_{B3}\,(2\theta_{B2} + \theta_{B3} - 3R_3)\right] + W_3 h_3 = 0.$$

Letting $N = 2\sum\left(\frac{I}{h}\right)$ for all of the columns in a story, and collecting, gives

$$2K_{A3}\,\theta_{A2} + 2K_{B3}\,\theta_{B2} - N_3 R_3 + 2K_{A3}\,\theta_{A3} + 2K_{B3}\,\theta_{B3} = -\frac{W_3 h_3}{6E} \quad \ldots\ldots (1)$$

Consider the point A_3 as a free body. Taking $\sum M = 0$, gives $M_A{}^{34} + M_A{}^{32} + M_3{}^{AB} = 0$. Substituting the values of these moments as given by equations A and B gives,

$$2EK_{A4}\,(2\theta_{A3} + \theta_{A4} - 3R_4) + 2EK_{A3}\,(2\theta_{A3} + \theta_{A2} - 3R_3) + 2EK_{a3}\,(2\theta_{A3} + \theta_{B3}) = 0.$$

Combining the coefficients of the unknowns and cancelling the common factor $2E$ gives,

$$\theta_{A2}K_{A3} + \theta_{A3}\,(2K_{A4} + 2K_{A3} + 2K_{a3}) + \theta_{A4}\,K_{A4} + \theta_{B3}K_{a3} - R_3 3K_{A3} - R_4 3K_{A4} = 0.$$

Substituting J_{A3} for $2\sum\left(\frac{I}{h} + \frac{I}{l}\right)$ of all the members that intersect at A_3, the equation becomes:

$$K_{A3}\,\theta_{A2} - 3K_{A3}\,R_3 + J_{A3}\,\theta_{A3} + K_{a3}\,\theta_{B3} - 3K_{A4}R_4 + K_{A4}\theta_{A4} = 0 \ldots (2)$$

The point B_3 is in equilibrium under the action of the four moments $M_3{}^{BA}$, $M_3{}^{BB}$, $M_B{}^{34}$, and $M_B{}^{32}$. Equating the sum of these four moments to zero gives,

$$2EK_{a3}\,(2\theta_{B3} + \theta_{A3}) + 2EK_{b3}\,(2\theta_{B3} + \theta_{B3}) + 2EK_{B4}\,(2\theta_{B3} + \theta_{B4} - 3R_4) + 2EK_{B3}\,(2\theta_{B3} + \theta_{B2} - 3R_3) = 0.$$

Combining the coefficients for the unknowns and cancelling the common factor $2E$, gives,

$$2\theta_{B3}\,(K_{a3} + K_{b3} + K_{B4} + K_{B3}) + \theta_{A3}K_{a3} + \theta_{B3}K_{b3} + \theta_{B4}K_{B4} + \theta_{B2}K_{B3} - 3R_4K_{B4} - 3R_3K_{B3} = 0.$$

Substituting J_{B3} for $2\sum\left(\frac{I}{h} + \frac{I}{l}\right)$ of all the members intersecting at B_3, the equation becomes,

$$K_{B3}\,\theta_{B2} - 3K_{B3}R_3 + K_{a3}\,\theta_{A3} + (K_{b3} + J_{B3})\,\theta_{B3} - 3K_{B4}R_4 + K_{B4}\theta_{B4} = 0 \quad \ldots\ldots (3)$$

The three equations, 1, 2, and 3, can be written for any story by making the proper changes in the subscripts. As there are only three unknowns, θ_A, θ_B, and R for each story, as many equations can be written for a bent as there are unknowns to be determined. It is possible to solve these equations for the unknown quantities algebraically, but the large number of equations involved makes the work very difficult. It is simpler to substitute the numerical values of the coefficients in the equations and solve for the numerical values of the unknown quantities by the process of elimination explained in Section VII.

For convenience in the application of this process of elimination, the equations are written in tabular form as shown in Table 3 in which the unknown changes in the slopes and the ratios of deflection to story height are written at the tops of the columns and the coefficients of these unknowns are written below. For example, in writing equation A of Table 3, which is $-N_1R_1 + 2K_{A1}\theta_{A1} + 2K_{B1}\theta_{B1} = -\frac{W_1h_1}{6E}$ $-N_1$, the coefficient of R_1, is placed in the column under R_1; $2K_{A1}$, the coefficient of θ_{A1}, is placed in the column under θ_{A1}; $2K_{B1}$, the coefficient of θ_{B1}, is placed in the column under θ_{B1}; and $-\frac{W_1h_1}{6E}$ is placed in the column headed "Right-Hand Member of Equation." Having the equations written in this form, it is unnecessary to repeat the unknown quantities when eliminating them from the equations by the method used in Table 14.

Table 3 contains the general equations to be used in determining the slopes and the deflections in a symmetrical three-span bent any number of stories high. The subscripts 1, 2, and 3 refer to the first, second, and third stories respectively, and the subscripts *x*, *y*, and *z* refer to the second from the top, the next to the top, and the top stories respectively. The equations for the intervening stories are of the same form, but have different subscripts.

By using the general method outlined above, that is, by writing an equation similar to equation 1 for each story and an equation similar to equations 2 and 3 for each intersection of a column with a girder for each story, as many equations as there are unknowns can be written for a bent containing any number of spans and any number of stories. General equations for *symmetrical* bents of from one to five spans are given in Tables 1 to 5 and similar equations for *unsymmetrical* bents of from one to five spans are given in Tables 6 to 10.

In order to check the equations of Tables 1 to 10, a model of a

Table 1.

General Equations for a Symmetrical Single-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation														Right-Hand Member of Equation
	Story No. 1		Story No. 2		Story No. 3		Intervening Stories	Story No. X (Second from Top)		Story No. Y (Next to Top)		Story No. Z (Top)		
	R_1	θ_{A1}	R_2	θ_{A2}	R_3	θ_{A3}		R_X	θ_{AX}	R_Y	θ_{AY}	R_Z	θ_{AZ}		
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height														
A	$-N_1$	$2K_{A1}$													$-W_1h_1\div 6E$
B	$-3K_{A1}$	$J_{A1}+K_{a1}$	$-3K_{A2}$	K_{A2}											0
C		$2K_{A2}$	$-N_2$	$2K_{A2}$											$-W_2h_2\div 6E$
D		K_{A2}	$-3K_{A2}$	$J_{A2}+K_{a2}$	$-3K_{A3}$	K_{A3}									0
	Similar equations for intervening stories														
W									$2K_{AY}$	$-N_Y$	$2K_{AY}$				$-W_Yh_Y\div 6E$
X									K_{AY}	$-3K_{AY}$	$J_{AY}+K_{aY}$	$-3K_{AZ}$	K_{AZ}		0
Y											$2K_{AZ}$	$-N_Z$	$2K_{AZ}$		$-W_Zh_Z\div 6E$
Z											K_{AZ}	$-3K_{AZ}$	$J_{AZ}+K_{aZ}$		0

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Equation	Story No. X (...rom Top)			Story No. Y (Next to Top)			Story No. Z (Top)			Right-Hand Member of Equation
	R_1	...AX	θ_{AY}	R_Y	θ_{AY}	θ_{BY}	R_Z	θ_{AZ}	θ_{BZ}	
	... to Story Height									
A	$-N_1$									$-W_1h_1 \div 6E$
B	$-3K_{A1}$									O
C	$-3K_{B1}$									O
D										$-W_2h_2 \div 6E$
E										O
F										O
	...tories									
U		K_{AY}	K_{BY}	$-N_Y$	$2K_{AY}$	K_{BY}				$-W_Yh_Y \div 6E$
V		...AY		$-3K_{AY}$	J_{AY}	K_{aY}	$-3K_{AZ}$	K_{AZ}		O
W			K_{BY}	$-3K_{BY}$	$2K_{aY}$	J_{BY}	$-3K_{BZ}$		K_{BZ}	O
X					$2K_{AZ}$	K_{BZ}	$-N_Z$	$2K_{AZ}$	K_{BZ}	$-W_Zh_Z \div 6E$
Y					K_{AZ}		$-3K_{AZ}$	J_{AZ}	K_{aZ}	O
Z						K_{BZ}	$-3K_{BZ}$	$2K_{aZ}$	J_{BZ}	O

Table 2.

General Equations for a Symmetrical Two-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation																				Right-Hand Member of Equation
	Story No. 1			Story No 2			Story No. 3			Intervening Stories	Story No. X (Second from Top)			Story No. Y (Next to Top)			Story No. Z (Top)				
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	θ_{A3}	θ_{B3}		R_X	θ_{AX}	θ_{AY}	R_Y	θ_{AY}	θ_{BY}	R_Z	θ_{AZ}	θ_{BZ}		
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																				
A	$-N_1$	$2K_{A1}$	K_{B1}																		$-W_1h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}	$-3K_{A2}$	K_{A2}																0
C	$-3K_{B1}$	$2K_{a1}$	J_{B1}	$-3K_{B2}$		K_{B2}															0
D		$2K_{A2}$	K_{B2}	$-N_2$	$2K_{A2}$	K_{B2}															$-W_2h_2 \div 6E$
E		K_{A2}		$-3K_{A2}$	J_{A2}	K_{a2}	$-3K_{A3}$	K_{A3}													0
F			K_{B2}	$-3K_{B2}$	$2K_{a2}$	J_{B2}	$-3K_{B3}$		K_{B3}												0
	Similar equations for intervening stories																				
U												$2K_{AY}$	K_{BY}	$-N_Y$	$2K_{AY}$	K_{BY}					$-W_Yh_Y \div 6E$
V												K_{AY}		$-3K_{AY}$	J_{AY}	K_{aY}	$-3K_{AZ}$	K_{AZ}			0
W													K_{BY}	$-3K_{BY}$	$2K_{aY}$	J_{BY}	$-3K_{BZ}$		K_{BZ}		0
X															$2K_{AZ}$	K_{BZ}	$-N_Z$	$2K_{AZ}$	K_{BZ}		$-W_Zh_Z \div 6E$
Y															K_{AZ}		$-3K_{AZ}$	J_{AZ}	K_{aZ}		0
Z																K_{BZ}	$-3K_{BZ}$	$2K_{aZ}$	J_{BZ}		0

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Equation	Story No. X (Ston Top)		Story No. Y (Next to Top)			Story No. Z (Top)			Right-Hand Member of Equation
	R_1	θ_A	θ_{BX}	R_Y	θ_{AY}	θ_{BY}	R_Z	θ_{AZ}	θ_{BZ}	
	Story Height									
A	$-N_1$	$2K$								$-W_1 h_1 \div 6E$
B	$-3K_{A1}$	J								0
C	$-3K_{B1}$	K								0
D		$2K$								$-W_2 h_2 \div 6E$
E		K								0
F										0
	es									
U			$2K_{BY}$	$-N_Y$	$2K_{AY}$	$2K_{BY}$				$-W_Y h_Y \div 6E$
V				$-3K_{AY}$	J_{AY}	K_{aY}	$-3K_{AZ}$	K_{AZ}		0
W			K_{BY}	$-3K_{BY}$	K_{aY}	$J_{BY}+K_{bY}$	$-3K_{BZ}$		K_{BZ}	0
X					$2K_{AZ}$	$2K_{BZ}$	$-N_Z$	$2K_{AZ}$	$2K_{BZ}$	$-W_Z h_Z \div 6E$
Y					K_{AZ}		$-3K_{AZ}$	J_A	K_{aZ}	0
Z						K_{BZ}	$-3K_{BZ}$	K_{aZ}	$J_{BZ}+K_{bZ}$	0

Table 3.

General Equations for a Symmetrical Three-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation																			Right-Hand Member of Equation
	Story No. 1			Story No. 2			Story No. 3			Intervening Stories	Story No. X (Second from Top)			Story No. Y (Next to Top)			Story No. Z (Top)			
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	θ_{A3}	θ_{B3}		R_X	θ_{AX}	θ_{BX}	R_Y	θ_{AY}	θ_{BY}	R_Z	θ_{AZ}	θ_{BZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																			
A	$-N_1$	$2K_{A1}$	$2K_{B1}$																	$-W_1h_1+6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}	$-3K_{A2}$	K_{A2}															0
C	$-3K_{B1}$	K_{a1}	$J_{B1}+K_{b1}$	$-3K_{B2}$		K_{B2}														0
D		$2K_{A2}$	$2K_{B2}$	$-N_2$	$2K_{A2}$	$2K_{B2}$														$-W_2h_2+6E$
E		K_{A2}		$-3K_{A2}$	J_{A2}	K_{a2}	$-3K_{A3}$	K_{A3}												0
F			K_{B2}	$-3K_{B2}$	K_{a2}	$J_{B2}+K_{b2}$	$-3K_{B3}$		K_{B3}											0
	Similar equations for intervening stories																			
U												$2K_{AY}$	$2K_{BY}$	$-N_Y$	$2K_{AY}$	$2K_{BY}$				$-W_Yh_Y+6E$
V												K_{AY}		$-3K_{AY}$	J_{AY}	K_{aY}	$-3K_{AZ}$	K_{AZ}		0
W													K_{BY}	$-3K_{BY}$	K_{aY}	$J_{BY}+K_{bY}$	$-3K_{BZ}$		K_{BZ}	0
X															$2K_{AZ}$	$2K_{BZ}$	$-N_Z$	$2K_{AZ}$	$2K_{BZ}$	$-W_Zh_Z+6E$
Y															K_{AZ}		$-3K_{AZ}$	J_A	K_{aZ}	0
Z																K_{BZ}	$-3K_{BZ}$	K_{aZ}	$J_{BZ}+K_{bZ}$	0

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Equation	Story		Story No. Y (Next to Top)			Story No. Z (Top)				Right-Hand Member of Equation
	R_1	θ_{A1}	θ_{AY}	θ_{BY}	θ_{CY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	
A	$-N_1$	$2K_{A1}$								$-W_1 h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}								O
C	$-3K_{B1}$	K_{a1}								O
D	$-3K_{C1}$									O
E		$2K_{A2}$								$-W_2 h_2 \div 6E$
F		K_{A2}								O
G										O
H										O
S			$2K_{AY}$	$2K_{BY}$	K_{CY}					$-W_Y h_Y \div 6E$
T			J_{AY}	K_{aY}		$-3K_{AZ}$	K_{AZ}			O
U			K_{aY}	J_{BY}	K_{bY}	$-3K_{BZ}$		K_{BZ}		O
V				$2K_{bY}$	J_{CY}	$-3K_{CZ}$			K_{CZ}	O
W			$2K_{AZ}$	$2K_{BZ}$	K_{CZ}	$-N_Z$	$2K_{ZV}$	$2K_{BZ}$	K_{CZ}	$-W_Z h_Z \div 6E$
X			K_{AZ}			$-3K_{AZ}$	J_{AZ}	K_{aZ}		O
Y				K_{BZ}		$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}	O
Z					K_{CZ}	$-3K_{CZ}$		$2K_{bZ}$	J_{CZ}	O

Table 4.

General Equations for a Symmetrical Four-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation																								Right-Hand Member of Equation
	Story No. 1				Story No. 2				Story No. 3				Intervening Stories	Story No. X (Second from Top)				Story No. Y (Next to Top)				Story No. Z (Top)				
	R_1	θ_{A1}	θ_{B1}	θ_{C1}	R_2	θ_{A2}	θ_{B2}	θ_{C2}	R_3	θ_{A3}	θ_{B3}	θ_{C3}		R_X	θ_{AX}	θ_{BX}	θ_{CX}	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																								
A	$-N_1$	$2K_{A1}$	$2K_{B1}$	K_{C1}																					$-W_1 h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}		$-3K_{A2}$	K_{A2}																			0
C	$-3K_{B1}$	K_{a1}	J_{B1}	K_{b1}	$-3K_{B2}$		K_{B2}																		0
D	$-3K_{C1}$		$2K_{b1}$	J_{C1}	$-3K_{C2}$			K_{C2}																	0
E		$2K_{A2}$	$2K_{B2}$	K_{C2}	$-N_2$	$2K_{A2}$	$2K_{B2}$	K_{C2}																	$-W_2 h_2 \div 6E$
F		K_{A2}			$-3K_{A2}$	J_{A2}	K_{a2}		$-3K_{A3}$	K_{A3}															0
G			K_{B2}		$-3K_{B2}$	K_{a2}	J_{B2}	K_{b2}	$-3K_{B3}$		K_{B3}														0
H				K_{C2}	$-3K_{C2}$		$2K_{b2}$	J_{C2}	$-3K_{C3}$			K_{C3}													0
	Similar equations for intervening stories																								
S														$2K_{AY}$	$2K_{BY}$	K_{CY}	$-N_Y$	$2K_{AY}$	$2K_{BY}$	K_{CY}					$-W_Y h_Y \div 6E$
T														K_{AY}			$-3K_{AY}$	J_{AY}	K_{aY}		$-3K_{AZ}$	K_{AZ}			0
U															K_{BY}		$-3K_{BY}$	K_{aY}	J_{BY}	K_{bY}	$-3K_{BZ}$		K_{BZ}		0
V																K_{CY}	$-3K_{CY}$		$2K_{bY}$	J_{CY}	$-3K_{CZ}$			K_{CZ}	0
W																		$2K_{AZ}$	$2K_{BZ}$	K_{CZ}	$-N_Z$	$2K_{ZV}$	$2K_{BZ}$	K_{CZ}	$-W_Z h_Z \div 6E$
X																		K_{AZ}			$-3K_{AZ}$	J_{AZ}	K_{aZ}		0
Y																			K_{BZ}		$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}	0
Z																				K_{CZ}	$-3K_{CZ}$		$2K_{bZ}$	J_{CZ}	0

Equation	Story No. Y (Next to Top)			Story No. Z (Top)				Right-Hand Member of Equation
	θ_{AY}	θ_{BY}	θ_{CY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	
A								$-W_1 h_1 \div 6E$
B								0
C								0
D								0
E								$-W_2 h_2 \div 6E$
F								0
G								0
H								0
S	$2K_{AY}$	$2K_{BY}$	$2K_{CY}$					$-W_Y h_Y \div 6E$
T	J_{AY}	K_{aY}		$-3K_{AZ}$	K_{AZ}			0
U	K_{aY}	J_{BY}	K_{bY}	$-3K_{BZ}$		K_{BZ}		0
V		K_{bY}	$J_{CY}+K_{cY}$	$-3K_{CZ}$			K_{CZ}	0
W	$2K_{AZ}$	$2K_{BZ}$	$2K_{CZ}$	$-N_Z$	$2K_{AZ}$	$2K_{BZ}$	$2K_{CZ}$	$-W_Z h_Z \div 6E$
X	K_{AZ}			$-3K_{AZ}$	J_{AZ}	K_{aZ}		0
Y		K_{BZ}		$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}	0
Z			K_{CZ}	$-3K_{CZ}$		K_{bZ}	$J_{CZ}+K_{cZ}$	0

Table 5.

General Equations for a Symmetrical Five-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation																									Right-Hand Member of Equation
	Story No. 1				Story No. 2				Story No. 3				Intervening Stories	Story No. X (Second from Top)				Story No. Y (Next to Top)				Story No. Z (Top)				
	R_1	θ_{A1}	θ_{B1}	θ_{C1}	R_2	θ_{A2}	θ_{B2}	θ_{C2}	R_3	θ_{A3}	θ_{B3}	θ_{C3}		R_X	θ_{AX}	θ_{BX}	θ_{CX}	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																									
A	$-N_1$	$2K_{A1}$	$2K_{B1}$	$2K_{C1}$																						$-W_1h_1\div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}		$-3K_{A2}$	K_{A2}																				0
C	$-3K_{B1}$	K_{a1}	J_{B1}	K_{b1}	$-3K_{B2}$		K_{B2}																			0
D	$-3K_{C1}$		K_{b1}	$J_{b1}+K_{c1}$	$-3K_{C2}$			K_{c2}																		0
E		$2K_{A2}$	$2K_{B2}$	$2K_{C2}$	$-N_2$	$2K_{A2}$	$2K_{B2}$	$2K_{C2}$																		$-W_2h_2\div 6E$
F		K_{A2}			$-3K_{A2}$	J_{A2}	K_{a2}		$-3K_{A3}$	K_{A3}																0
G			K_{B2}		$-3K_{B2}$	K_{a2}	J_{B2}	K_{b2}	$-3K_{B3}$		K_{B3}															0
H				K_{C2}	$-3K_{C2}$		K_{b2}	$J_{C2}+K_{c2}$	$-3K_{C3}$			K_{C3}															0
	Similar equations for intervening stories																									
S														$2K_{AY}$	$2K_{BY}$	$2K_{CY}$	$-N_Y$	$2K_{AY}$	$2K_{BY}$	$2K_{CY}$						$-W_Yh_Y\div 6E$
T														K_{AY}			$-3K_{AY}$	J_{AY}	K_{aY}		$-3K_{AZ}$	K_{AZ}				0
U															K_{BY}		$-3K_{BY}$	K_{aY}	J_{BY}	K_{bY}	$-3K_{BZ}$		K_{BZ}			0
V																K_{CY}	$-3K_{CY}$		K_{bY}	$J_{CY}+K_{cY}$	$-3K_{CY}$			K_{CZ}		0
W																		$2K_{AZ}$	$2K_{BZ}$	$2K_{CZ}$	$-N_Z$	$2K_{AZ}$	$2K_{BZ}$	$2K_{CZ}$		$-W_Zh_Z\div 6E$
X																		K_{AZ}			$-3K_{AZ}$	J_{AZ}	K_{aZ}			0
Y																			K_{BZ}		$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}		0
Z																				K_{CZ}	$-3K_{CZ}$		K_{bZ}	$J_{CZ}+K_{cZ}$		0

CAL SINGLE-SPAN BENT
HIGH.

Equation	Story No. X (1 from Top)			Story No. Y (Next to Top)			Story No. Z (Top)			Right-Hand Member of Equation
	R_1	θ_{AX}	θ_{BX}	R_Y	θ_{AY}	θ_{BY}	R_Z	θ_{AZ}	θ_{BZ}	
	n to Story Height									
A	$-N_1$									$-W_1h_1 \div 6E$
B	$-3K_{A1}$									O
C	$-3K_{B1}$									O
D										$-W_2h_2 \div 6E$
E										O
F										O
	stories									
U		K_{AY}	K_{BY}	$-N_Y$	K_{AY}	K_{BY}				$-W_Yh_Y \div 6E$
V		K_{AY}		$-3K_{AY}$	J_{AY}	K_{aY}	$-3K_{AZ}$	K_{AZ}		O
W			K_{BY}	$-3K_{BY}$	K_{aY}	J_{BY}	$-3K_{BZ}$		K_{BZ}	O
X					K_{AZ}	K_{BZ}	$-N_Z$	K_{AZ}	K_{BZ}	$-W_Zh_Z \div 6E$
Y					K_{AZ}		$-3K_{AZ}$	J_{AZ}	K_{aZ}	O
Z						K_{BZ}	$-3K_{BZ}$	K_{aZ}	J_{BZ}	O

Table 6.

General Equations for an Unsymmetrical Single-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation																				Right-Hand Member of Equation
	Story No. 1			Story No. 2			Story No. 3			Intervening Stories	Story No. X (Second from Top)			Story No. Y (Next to Top)			Story No. Z (Top)				
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	θ_{A3}	θ_{B3}		R_X	θ_{AX}	θ_{BX}	R_Y	θ_{AY}	θ_{BY}	R_Z	θ_{AZ}	θ_{BZ}		
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																				
A	$-N_1$	K_{A1}	K_{B1}																		$-W_1h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}	$-3K_{A2}$	K_{A2}																0
C	$-3K_{B1}$	K_{a1}	J_{B1}	$-3K_{B2}$		K_{B2}															0
D		K_{A2}	K_{B2}	$-N_2$	K_{A2}	K_{B2}															$-W_2h_2 \div 6E$
E		K_{A2}		$-3K_{A2}$	J_{A2}	K_{a2}	$-3K_{A3}$	K_{A3}													0
F			K_{B2}	$-3K_{B2}$	K_{a2}	J_{B2}	$-3K_{B3}$		K_{B3}												0
	Similar equations for intervening stories																				
U												K_{AY}	K_{BY}	$-N_Y$	K_{AY}	K_{BY}					$-W_Yh_Y+6E$
V											K_{AY}			$-3K_{AY}$	J_{AY}	K_{aY}	$-3K_{AZ}$	K_{AZ}			0
W												K_{BY}	$-3K_{BY}$	K_{aY}	J_{BY}	$-3K_{BZ}$			K_{BZ}		0
X														K_{AZ}	K_{BZ}	$-N_Z$	K_{AZ}	K_{BZ}	$-W_Zh_Z \div 6E$		
Y														K_{AZ}		$-3K_{AZ}$	J_{AZ}	K_{aZ}	0		
Z															K_{BZ}	$-3K_{BZ}$	K_{aZ}	J_{BZ}	0		

Equation			Story No. Y (Next to Top)				Story No Z (Top)				Right-Hand Member of Equation
	R_1	X	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	
	Height										
A	$-N_1$										$-W_1 h_1 \div 6E$
B	$-3K_{A1}$										O
C	$-3K_{B1}$										O
D	$-3K_{C1}$										O
E											$-W_2 h_2 \div 6E$
F											O
G											O
H											O
S		Y	$-N_Y$	K_{AY}	K_{BY}	K_{CY}					$-W_Y h_Y \div 6E$
T			$-3K_{AY}$	J_{AY}	K_{aY}		$-3K_{AZ}$	K_{AZ}			O
U			$-3K_{BY}$	K_{aY}	K_{BY}	K_{bY}	$-3K_{BZ}$		K_{BZ}		O
V		Y	$-3K_{CY}$		K_{bY}	J_{CY}	$-3K_{CZ}$			K_{CZ}	O
W				K_{AZ}	K_{BZ}	K_{CZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	$-W_Z h_Z \div 6E$
X				K_{AZ}			$-3K_{AZ}$	J_{AZ}	K_{aZ}		O
Y					K_{BZ}		$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}	O
Z						K_{CZ}	$-3K_{CZ}$		K_{bZ}	J_{CZ}	O

Table 7.

General Equations for an Unsymmetrical Two-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation																								Right-Hand Member of Equation
	Story No. 1				Story No. 2				Story No. 3				Intervening Stories	Story No. X (Second from Top)				Story No. Y (Next to Top)				Story No Z (Top)				
	R_1	θ_{A1}	θ_{B1}	θ_{C1}	R_2	θ_{A2}	θ_{B2}	θ_{C2}	R_3	θ_{A3}	θ_{B3}	θ_{C3}		R_X	θ_{AX}	θ_{BX}	θ_{CX}	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																								
A	$-N_1$	K_{A1}	K_{B1}	K_{C1}																					$-W_1 h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}		$-3K_{A2}$	K_{A2}																			0
C	$-3K_{B1}$	K_{a1}	J_{B1}	K_{b1}	$-3K_{B2}$		K_{B2}																		0
D	$-3K_{C1}$		K_{b1}	J_{C1}	$-3K_{C2}$			K_{C2}																	0
E		K_{A2}	K_{B2}	K_{C2}	$-N_2$	K_{A2}	K_{B2}	K_{C2}																	$-W_2 h_2 \div 6E$
F		K_{A2}			$-3K_{A2}$	J_{A2}	K_{a2}		$-3K_{A3}$	K_{A3}															0
G			K_{B2}		$-3K_{B2}$	K_{a2}	J_{B2}	K_{b2}	$-3K_{B3}$		K_{B3}														0
H				K_{C2}	$-3K_{C2}$		K_{B2}	J_{C2}	$-3K_{C3}$			K_{C3}													0
	Similar equations for intervening stories																								
S														K_{AY}	K_{BY}	K_{CY}	$-N_Y$	K_{AY}	K_{BY}	K_{CY}					$-W_Y h_Y \div 6E$
T														K_{AY}			$-3K_{AY}$	J_{AY}	K_{aY}		$-3K_{AZ}$	K_{AZ}			0
U															K_{BY}		$-3K_{BY}$	K_{aY}	K_{bY}	K_{bY}	$-3K_{BZ}$		K_{BZ}		0
V																K_{CY}	$-3K_{CY}$		K_{bY}	J_{CY}	$-3K_{CZ}$			K_{CZ}	0
W																		K_{AZ}	K_{BZ}	K_{CZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	$-W_Z h_Z \div 6E$
X																		K_{AZ}			$-3K_{AZ}$	J_{AZ}	K_{aZ}		0
Y																			K_{BZ}		$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}	0
Z																				K_{CZ}	$-3K_{CZ}$		K_{bZ}	J_{CZ}	0

Equation	Story No. Y (Next to Top)				Story No. Z (Top)					Right-Hand Member of Equation
	θ_{AY}	θ_{BY}	θ_{CY}	θ_{DY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	θ_{DZ}	
A										$-W_1 h_1 \div 6E$
B										0
C										0
D										0
E										0
F										$-W_2 h_2 \div 6E$
G										0
H										0
I										0
J										0
Q	Y	K_{AY}	K_{BY}	K_{CY}	K_{DY}					$-W_Y h_Y \div 6E$
R	AY	J_{AY}	K_{aY}			$-3K_{AZ}$	K_{AZ}			0
S	BY	K_{aY}	J_{BY}	K_{bY}		$-3K_{BZ}$		K_{BZ}		0
T	CY		K_{bY}	J_{CY}	K_{cY}	$-3K_{CZ}$			K_{CZ}	0
U	DY			K_{cY}	J_{DY}	$-3K_{DZ}$				K_{DZ} 0
V		K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ} $-W_Z h_Z \div 6E$
W		K_{AZ}				$-3K_{AZ}$	J_{AZ}	K_{aZ}		0
X			K_{BZ}			$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}	0
Y				K_{CZ}		$-3K_{CZ}$		K_{bZ}	J_{CZ}	K_{cZ} 0
Z					K_{DZ}	$-3K_{DZ}$			K_{cZ}	J_{DZ} 0

Table 8.

General Equations for an Unsymmetrical Three-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation																														Right-Hand Member of Equation
	Story No. 1					Story No. 2					Story No. 3					Intervening Stories	Story No. X (Second from Top)					Story No. Y (Next to Top)					Story No. Z (Top)					
	R_1	θ_{A1}	θ_{B1}	θ_{C1}	θ_{D1}	R_2	θ_{A2}	θ_{B2}	θ_{C2}	θ_{D2}	R_3	θ_{A3}	θ_{B3}	θ_{C3}	θ_{D3}		R_X	θ_{AX}	θ_{BX}	θ_{CX}	θ_{DX}	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	θ_{DY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	θ_{DZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																														
A	$-N_1$	K_{A1}	K_{B1}	K_{C1}	K_{D1}																										$-W_1 h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}			$-3K_{A2}$	K_{A2}																								0
C	$-3K_{B1}$	K_{a1}	J_{B1}	K_{b1}		$-3K_{B2}$		K_{B2}																							0
D	$-3K_{C1}$		K_{b1}	J_{C1}	K_{c1}	$-3K_{C2}$			K_{C2}																						0
E	$-3K_{D1}$			K_{c1}	J_{D1}	$-3K_{D2}$				K_{D2}																						0
F		K_{A2}	K_{B2}	K_{C2}	K_{D2}	$-N_2$	K_{A2}	K_{B2}	K_{C2}	K_{D2}																						$-W_2 h_2 \div 6E$
G		K_{A2}				$-3K_{A2}$	J_{A2}	K_{a2}			$-3K_{A3}$	K_{A3}																				0
H			K_{B2}			$-3K_{B2}$	K_{a2}	J_{B2}	K_{b2}		$-3K_{B3}$		K_{B3}																			0
I				K_{C2}		$-3K_{C2}$		K_{b2}	J_{C2}	K_{c2}	$-3K_{C3}$			K_{C3}																			0
J					K_{D2}	$-3K_{D2}$			K_{c2}	J_{D2}	$-3K_{D3}$				K_{D3}																			0
	Similar equations for intervening stories																														
Q																	K_{AY}	K_{BY}	K_{CY}	K_{DY}	$-N_Y$	K_{AY}	K_{BY}	K_{CY}	K_{DY}						$-W_Y h_Y \div 6E$
R																	K_{AY}				$-3K_{AY}$	J_{AY}	K_{aY}			$-3K_{AZ}$	K_{AZ}				0
S																		K_{BY}			$-3K_{BY}$	K_{aY}	J_{BY}	K_{bY}		$-3K_{BZ}$		K_{BZ}			0
T																			K_{CY}		$-3K_{CY}$		K_{bY}	J_{CY}	K_{cY}	$-3K_{CZ}$			K_{CZ}		0
U																				K_{DY}	$-3K_{DY}$			K_{cY}	J_{DY}	$-3K_{DZ}$				K_{DZ}	0
V																						K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	$-W_Z h_Z \div 6E$
W																						K_{AZ}				$-3K_{AZ}$	J_{AZ}	K_{aZ}			0
X																							K_{BZ}			$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}		0
Y																								K_{CZ}		$-3K_{CZ}$		K_{bZ}	J_{CZ}	K_{cZ}	0
Z																									K_{DZ}	$-3K_{DZ}$			K_{cZ}	J_{DZ}	0

ABLE 9.

UNSYMMETRICAL
OF STORIES HIG

Equation	Member of Equation										Right-Hand Member of Equation
	ening ries		(		Story No. Z (Top)						
		R_X	θ_{AX}	EY	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	θ_{DZ}	θ_{EZ}	
	Ratios of Deflection to										
A											$-W_1 h_1 \div 6E$
B											0
C											0
D											0
E											0
F											0
G											$-W_2 h_2 \div 6E$
H											0
I											0
J											0
K											0
L											0
	s for intervening stories										
C			K_{AY}	EY							$-W_Y h_Y \div 6E$
E			K_{AY}		$-3K_{AZ}$	K_{AZ}					0
G					$-3K_{BZ}$		K_{BZ}				0
I					$-3K_{CZ}$			K_{CZ}			0
S				dY	$-3K_{DZ}$				K_{DZ}		0
T				EY	$-3K_{EZ}$					K_{EZ}	0
U				EZ	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	K_{EZ}	$-W_Z h_Z \div 6E$
V					$-3K_{AZ}$	J_{AZ}	K_{aZ}				0
W					$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}			0
X					$-3K_{CZ}$		K_{bZ}	J_{CZ}	K_{cZ}		0
Y					$-3K_{DZ}$			K_{cZ}	J_{DZ}	K_{dZ}	0
Z				EZ	$-3K_{EZ}$				K_{dZ}	J_{EZ}	0

Table 9.

General Equations for an Unsymmetrical Four-Span Bent
Any Number of Stories High.

Equation	Left-Hand Member of Equation																																		Right-Hand Member of Equation
	Story No. 1						Story No. 2						Story No. 3						Intervening Stories	Story No. X (Second from Top)						Story No. Y (Next to Top)						Story No. Z (Top)						
	R_1	θ_{A1}	θ_{B1}	θ_{C1}	θ_{D1}	θ_{F1}	R_2	θ_{A2}	θ_{B2}	θ_{C2}	θ_{D2}	θ_{E2}	R_3	θ_{A3}	θ_{B3}	θ_{C3}	θ_{D3}	θ_{E3}		R_X	θ_{AX}	θ_{BX}	θ_{CX}	θ_{DX}	θ_{EX}	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	θ_{DY}	θ_{EY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	θ_{DZ}	θ_{EZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height																																		
A	$-N_1$	K_{A1}	K_{A1}	K_{C1}	K_{D1}	K_{F1}																													$-W_1h_1+6E$
B	$-3K_{A1}$	J_{A1}	K_{a1}				$-3K_{A2}$	K_{A2}																											0
C	$-3K_{B1}$	K_{a1}	J_{B1}	K_{B1}			$-3K_{B2}$		K_{B2}																											0
D	$-3K_{C1}$		K_{b1}	J_{C1}	K_{c1}		$-3K_{C2}$			K_{C2}																										0
E	$-3K_{D1}$			K_{c1}	J_{D1}	K_{d1}	$-3K_{D2}$				K_{D2}																										0
F	$-3K_{E1}$				K_{d1}	J_{E1}	$-3K_{E2}$					K_{E2}																										0
G		K_{A2}	K_{B2}	K_{C2}	K_{D2}	K_{E2}	$-N_2$	K_{A2}	K_{B2}	K_{C2}	K_{D2}	K_{E2}																								$-W_2h_2+6E$
H		K_{A2}					$-3K_{A2}$	J_{A2}	K_{a2}				$-3K_{A3}$	K_{A3}																							0
I			K_{B2}				$-3K_{B2}$	K_{b2}	J_{B2}	K_{B2}			$-3K_{B3}$		K_{B3}																						0
J				K_{C2}			$-3K_{C2}$		K_{b2}	J_{C2}	K_{c2}		$-3K_{C3}$			K_{C3}																					0
K					K_{D2}		$-3K_{D2}$			K_{c2}	J_{D2}	K_{d2}	$-3K_{D3}$				K_{D3}																					0
L						K_{F2}	$-3K_{E2}$				K_{d2}	J_{E2}	$-3K_{E3}$					K_{E3}																					0
	Similar equations for intervening stories																																		
O																				K_{AY}	K_{BY}	K_{CY}	K_{DY}	K_{EY}	$-N_Y$	K_{AY}	K_{BY}	K_{CY}	K_{DY}	K_{EY}							$-W_Yh_Y+6E$
P																				K_{AY}					$-3K_{AY}$	J_{AY}	K_{aY}				$-3K_{AZ}$	K_{AZ}					0
Q																					K_{BY}				$-3K_{BY}$	K_{aY}	J_{BY}	K_{bY}			$-3K_{BZ}$		K_{BZ}				0
R																						K_{CY}			$-3K_{CY}$		K_{bY}	J_{CY}	K_{cY}		$-3K_{CZ}$			K_{CZ}			0
S																							K_{DY}		$-3K_{DY}$			K_{cY}	J_{DY}	K_{dY}	$-3K_{DZ}$				K_{DZ}		0
T																								K_{EY}	$-3K_{EY}$				K_{dY}	J_{EY}	$-3K_{EZ}$					K_{EZ}	0
U																										K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	K_{EZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	K_{EZ}	$-W_Zh_Z+6E$
V																										K_{AZ}					$-3K_{AZ}$	J_{AZ}	K_{aZ}				0
W																											K_{BZ}				$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}			0
X																												K_{CZ}			$-3K_{CZ}$		K_{bZ}	J_{CZ}	K_{cZ}		0
Y																													K_{DZ}		$-3K_{DZ}$			K_{cZ}	J_{DZ}	K_{dZ}	0
Z																														K_{EZ}	$-3K_{EZ}$				K_{dZ}	J_{EZ}	0

ONS FOR
NY NUM

Equation	Left Han			Story No. Z (Top)							Right-Hand Member of Equation
	R_1	θ_{F3}	θ_{FY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	θ_{DZ}	θ_{EZ}	θ_{FZ}	
	own Slop										
A	$-N$										$-W_1 h_1 \div 6E$
B	$-3K$										0
C	$-3K$										0
D	$-3K$										0
E	$-3K$										0
F	$-3K$										0
G	$-3K$										0
H											$-W_2 h_2 \div 6E$
I											0
J											0
K											0
L											0
M											0
N		K_{F3}									0
	lar equati										
M			K_{FY}								$-W_Y h_Y \div 6E$
N				$-3K_{AZ}$	K_{AZ}						0
O				$-3K_{BZ}$		K_{BZ}					0
P				$-3K_{CZ}$			K_{CZ}				0
Q				$-3K_{DZ}$				K_{DZ}			0
R			K_{eY}	$-3K_{EZ}$					K_{EZ}		0
S			J_{FY}	$-3K_{FZ}$						K_{FZ}	0
T			K_{FZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	K_{EZ}	K_{FZ}	$-W_Z h_Z \div 6E$
U				$-3K_{AZ}$	J_{AZ}	K_{aZ}					0
V				$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}				0
W				$-3K_{CZ}$		K_{bZ}	J_{CZ}	K_{cZ}			0
X				$-3K_{DZ}$			K_{cZ}	J_{DZ}	K_{dZ}		0
Y				$-3K_{EZ}$				K_{dZ}	J_{EZ}	K_{eZ}	0
Z			K_{FZ}	$-3K_{FZ}$					K_{eZ}	J_{FZ}	0

Table 10.

General Equations for an Unsymmetrical Five-Span Bent Any Number of Stories High.

| Equation | Left Hand Member of Equation | | |

bent was tested and the measured deflections and changes in the slopes were compared with the same quantities calculated by the above equations. As the entire model was cut from a sheet of celluloid the joints were perfectly rigid. The results of the tests are given in Fig. 15. The fact that the measured and computed deflections and changes in the slopes agree very closely indicates that the above analysis is correct.

A solution of a numerical problem illustrating the use of the equations in Table 3 is given in Section VII.

VII. Numerical Problem.

13. *Determination of the Stresses in a Symmetrical Three-Span Twenty-Story Bent.*—Fig. 5 shows a symmetrical three-span bent twenty stories high. The bent resists a horizontal wind load of 30 lb. per sq. ft. on a vertical strip one foot wide. It is required to find the moment and the shear in the columns and girders and the direct stress in the columns.

The properties of the girder and column sections are shown in Table 11, page 48. The equations of Table 3 are applicable. The numerical values of the constants in these equations are given in Table 12. Table 13 contains the equations of Table 3 with the substitution of the numerical values of the constants given in Table 12. The figures given in the right-hand column are coefficients of .0001 as indicated at the head of the column; that is, the right-hand members of the equations are equal to .0001 times the numbers in the right-hand column. This method of writing the right-hand members of the equations obviates the necessity of repeating a large number of ciphers.

The unknown quantities in the equations of Table 13 are eliminated in Table 14. The quantity R_1 appears in the first three equations of Table 13 only. Dividing each of these equations by the coefficient of R_1 gives the three equations A, B, and C of Table 14, in all of which the coefficient of R_1 is equal to plus unity. Combining these latter equations as indicated, that is, subtracting equation B from A to get equation (A − B) and subtracting equation C from B to get equation (B − C), gives two new equations from which R_1 has been eliminated. Reducing the coefficients of the left-hand terms of equations (A − B) and (B − C) to plus unity gives equations 1 and 2 of Table 14. Equations D and E of Table 13 have also been reduced so that the coefficients of the left-hand term of each are equal to plus unity. They are rewritten in their new form as equations D and E in Table 14. Combining the four equations 1, 2, D, and E, as indicated in Table 14, eliminates

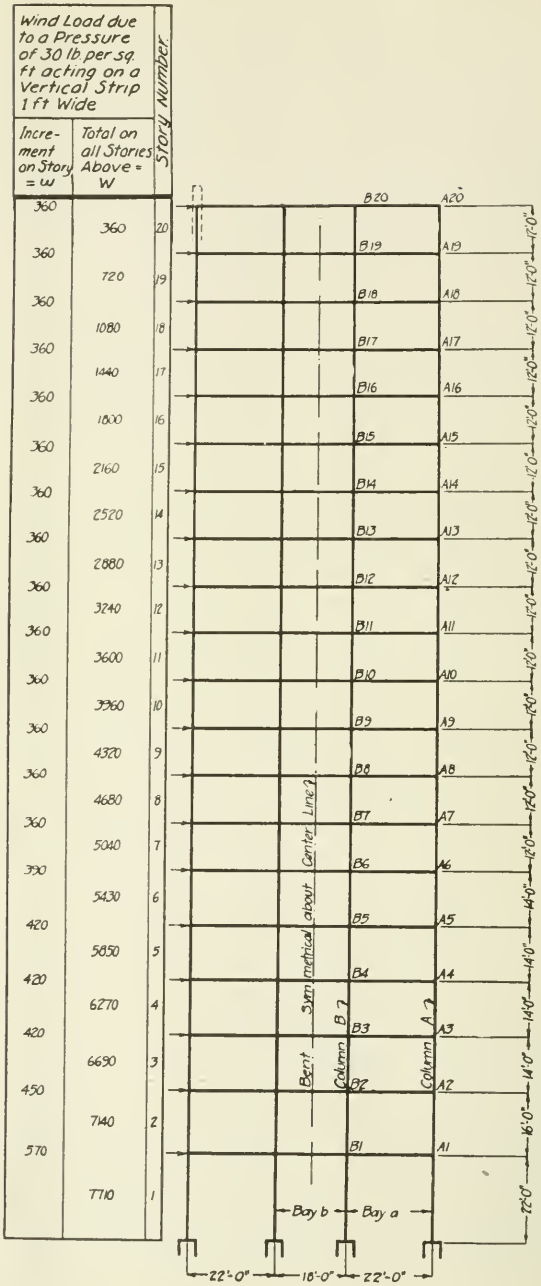


FIG. 5. SYMMETRICAL THREE-SPAN BENT, TWENTY STORIES HIGH.

the left-hand term, θ_{A1}. By continuing this process all the unknown quantities are eliminated except the last, or θ_{B20}. Its value thus becomes known and is found to be .0338 × .0001 radians. With the value of θ_{B20} known, the value of θ_{A20} can be determined from equation 153. With θ_{B20} and θ_{A20} both known, the value of R_{20} can be determined from equation 150. The other values of θ and R can be determined in a similar manner.

The process of determining the values of θ and R is given in Table 15. The equations used are taken from Table 14.

As in Tables 13 and 14, the right-hand member is equal to the number in the column at the right multiplied by .0001. To illustrate, equation 150 is

$$R_{20} - .2702\ \theta_{A20} - .2993\ \theta_{B20} = .0735 \times .0001.$$

The left-hand term of the left-hand member of each equation is the unknown which is to be determined. The first line in each group is the algebraic form of the equation; and the successive lines are the numerical values of the corresponding terms. For example, equation 155 is

$$\theta_{B20} = .0338 \times .0001 \text{ or } .00000338.$$

Again, equation 153 is:

$$\theta_{A20} - .0443\ \theta_{B20} = .0502 \times .0001.$$

From the preceding equation

$$.0443\ \theta_{B20} = .0443 \times .00000338 = .0015 \times .0001.$$

The sign of this quantity as a term of the left-hand member was negative, and the term will, therefore, be positive after it has been transferred.

Equation 153 now becomes

$$\theta_{A20} = .0502 \times .0001 + .0015 \times .0001 = .0517 \times .0001,$$

as indicated in the last line of the group containing equation 153. Equation 150 contains R_{20}, θ_{A20}, and θ_{B20}; but θ_{A20} and θ_{B20} have been determined by equations 153 and 155, respectively, therefore R_{20} can be determined by equation 150. Similarly, equation 147, containing θ_{B19} and the three known quantities, R_{20}, θ_{A20}, and θ_{B20} can be used to determine θ_{B19}. Equation 147 differs from the preceding equations in that it contains both positive and negative terms in the left-hand member. The second term of the left-hand member, being negative, when transferred, is added to the right-hand member. The sum of the two quantities is .0890 × .0001. Since the third and fourth terms of the left-hand member are positive, their sum, when transferred, is

subtracted from the quantity above. The final form of the equation is: $\theta_{B19} = .0835 \times .0001$. In a similar manner the other slopes, θ, and the ratios of the deflection to the story height, R, can be determined. θ is expressed in radians, and R is an abstract quantity.

The tabulation of the calculations as shown in Tables 14 and 15 facilitates the solution of a large number of equations. There are however a number of practical considerations which should be borne in mind when solving a problem by this method.

An error at any point affects all of the calculations which follow. It is therefore desirable to have two computers carry on the work simultaneously and compare results at frequent intervals in order to avoid the loss of time due to errors.

A number of combinations can be made from each group of equations. It is desirable to combine the equations so as to make the coefficient of the left-hand term of the left-hand member of the resulting equation as large as possible compared with the coefficients of the other terms in the equation. To illustrate this point, consider equations 25, 26, and M of Table 14. If equation M had been subtracted from equation 26, the coefficient of θ_{B4} in the resulting equation would have been .1463 and the coefficient of R_5 would have been 2.9027. The latter is about twenty times as great as the former. Any small error in the actual value of the former coefficient would be a large percentage of error. This same percentage of error would be introduced into the latter coefficient and produce a large actual error. With the combination of equations 25 and M as used, the coefficient of θ_{B4} is 1.0695 and of R_5, 3.4313. The latter coefficient is only about three times as large as the former and hence the effect of an inaccuracy in the value of the former upon the latter is correspondingly smaller than in the first case.

It is possible in some cases to combine the equations in such a way as to give equations which are algebraically independent, but which are numerically nearly identical. Combining such equations gives results which are likely to be inaccurate. Any tendency of the equations to become identities can be avoided by changing the order in which they are combined.

In getting the values of the remaining unknown quantities after one has been determined, each unknown is expressed in terms of the known quantities. By referring to Table 14 it is seen that in the first equation of a group of equations, any one of which could be used to get the value of an unknown, the coefficient of the quantity which is to be determined is larger than the coefficients of the known quantities. If

this equation is used, any inaccuracies in the known quantities will affect the accuracy of the results less than they would if an equation had been used in which the coefficient of the unknown is less than the coefficient of the known quantities. The various equations available should be examined and the one used which causes the least accumulation of inaccuracies.

In order to determine the extent to which inaccuracies in the calculations accumulate, two independent sets of calculations were made; one set was made on a 20-inch slide rule and the other on a Fuller's cylindrical slide rule. The two sets of calculations were compared at frequent intervals, and mistakes were corrected; but no adjustment of inaccuracies was made. The maximum variation in the slopes and deflections as determined in the two sets of calculations was very small. This indicates that the calculations can be made with a slide rule without greater inaccuracies than are permissible.

The moments at the ends of the columns and girders can be determined by substituting the values of the deflections and the slopes, given in Table 15, in equations A and B of Section V. To facilitate this work, the quantities which occur in these equations are given in Tables 16 and 17. The values of R and θ taken from Table 15 are given in the second, third, and fourth columns of Table 16; and the functions of the values of R and θ which occur in the equations are given in the remaining columns of the same table. The values of K taken from Table 12 are given in the second, third, fourth, and fifth columns of Table 17; and the functions of the values of K which occur in the equations are given in the remaining columns of the table. The moments at the ends of the columns and girders given by the fundamental equation, $M_{AB} = 2EK\ (2\theta_A + \theta_B - 3R)$, are the products of two factors, one of which is given in Table 16 and the other in Table 17. The values of the moments are given in Table 18.

The shear in any member is equal to the algebraic sum of the moments at the two ends of the member divided by its length. The direct stress at any section in column A is equal to the algebraic sum of the shears in the girders in bay a above the section. The direct stress at any section in column B is equal to the algebraic sum of the shears in the girders in bay a plus the algebraic sum of the shears in the girders in bay b above the section. The shears in the columns and girders and the direct stresses in the columns are given in Table 19.

The sum of all of the moments at the top and the bottom of all the columns in a story is equal to the total shear on the story multiplied

by the story height. The algebraic sum of the moments in the girders on the two sides of a column is equal to the algebraic sum of the moments in the columns on the two sides of the girder.

The total shear on a story multiplied by the story height and the algebraic sum of all of the moments at the top and the bottom of all columns of a story, as given in Table 18, are given in the second and third columns of Table 20. The algebraic sum of the moments in column *A* at sections immediately above and below the girders and the moments in the girders in bay *a* at sections adjacent to column *A*, are given in the fourth and fifth columns of Table 20. The algebraic sum of the moments in column *B* at sections immediately above and below the girders and the algebraic sum of the moments in the girders in bays *a* and *b* at sections adjacent to column *B*, are given in the sixth and seventh columns of Table 20. The accuracy of the computations can be checked by comparing the two values for the same quantity as given in adjacent columns of Table 20. To illustrate, for the sixth story, the total shear in the story multiplied by the story height is 913,000 inch-pounds, and the sum of the moments at the tops and bottoms of all columns in the story is 914,400 inch-pounds. For a perfect check, the two quantities would be equal. An inspection of the table shows that the values check very closely.

The moments given in Table 18 are due to a horizontal wind load of 30 lbs. per sq. ft. acting on a vertical strip one foot wide. To obtain the moments due to the wind load on any portion of the building, multiply the moments in Table 18 by the width in feet of the portion of the building on which the wind load acts.

VIII. Approximate Methods.

14. *Nomenclature of Methods.*—The method used in Section VII to determine the stresses in a symmetrical three-span twenty-story bent can be used in the actual design of a building, but a shorter method is desirable. The writers propose an approximate method which is much shorter than the method used in Section VII and which they believe is sufficiently accurate to be used in the actual design of buildings. For the sake of convenience in reference, this approximate method will be designated as the "Proposed Approximate Method" as distinguished from the method used in Section VII, which will be designated as the "Slope-Deflection Method." The slope-deflection method can be modified to advantage under certain special conditions. Such modifications

will be designated as "Modifications of the Slope-Deflection Method."

15. *Proposed Approximate Method.*—The proposed approximate method is based upon the following assumptions in addition to those used in the slope-deflection method.

1. The changes in the slope at the top of a column in the story above and in the story below the one in which the stresses are to be determined, are equal to the change in the slope at the top of the corresponding column in the latter story.

2. The ratio of the deflection to the length of the columns in the story above the one in which the stresses are to be determined, is equal to the ratio of the deflection to the length of the columns in the latter story.

In other words, in determining the stresses in the second story, θ_{A_1} and θ_{A_3} are assumed to be equal to θ_{A_2}; θ_{B_1} and θ_{B_3} are assumed to be equal to θ_{B_2}; and R_3 is assumed to be equal to R_2. Also, in figuring the stresses in the third story θ_{A_2} and θ_{A_4} are assumed to be equal to θ_{A_3}; θ_{B_2} and θ_{B_4} are assumed to be equal to θ_{B_3}; and R_4 is assumed to be equal to R_3. This does not mean, however, that the values of θ_{A_2}, θ_{B_2}, and R_2 used in determining the stresses in the second story are equal to the values of θ_{A_3}, θ_{B_3}, and R_3 respectively, used in determining the stresses in the third story.

An examination of the equations in Table 3 shows that, if assumptions 1 and 2 were true, three equations containing only three unknown quantities could be written for each story of the bent. To illustrate, consider the equations for the second story. In accordance with assumptions 1 and 2,

$$\theta_{A_1} = \theta_{A_2} = \theta_{A_3}$$
$$\theta_{B_1} = \theta_{B_2} = \theta_{B_3}$$
$$R_2 = R_3$$

Substituting θ_{A_2} for θ_{A_1} and θ_{A_3}, θ_{B_2} for θ_{B_1} and θ_{B_3}, and R_2 for R_3 in equations D, E, and F of Table 3 gives:

$$-N_2R_2 + 4K_{A_2}\theta_{A_2} + 4K_{B_2}\theta_{B_2} = -\frac{W_2h_2}{6E} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$-3(K_{A_2} + K_{A_3})R_2 + (K_{A_2} + J_{A_2} + K_{A_3})\theta_{A_2} + K_{a_2}\theta_{B_2} = 0 \ldots\ldots\ldots (2)$$

$$-3(K_{B_2} + K_{B_3})R_2 + K_{a_2}\theta_{A_2} + (K_{B_2} + J_{B_2} + K_{b_2} + K_{B_3})\theta_{B_2} = 0 \ldots (3)$$

These equations have been written for the second story. Similar equations can be written for any other story by making the proper changes in the subscripts.

Equations 1, 2, and 3, obtained from the equations in Table 3, can be used in determining the stresses in a symmetrical three-span bent

any number of stories high. If similar changes are made in the equations of Tables 1 to 10 inclusive, groups of equations can be obtained, one for each bent, which can be used to determine the stresses in any story of *symmetrical* bents of from one to five spans and also of *unsymmetrical* bents of from one to five spans.

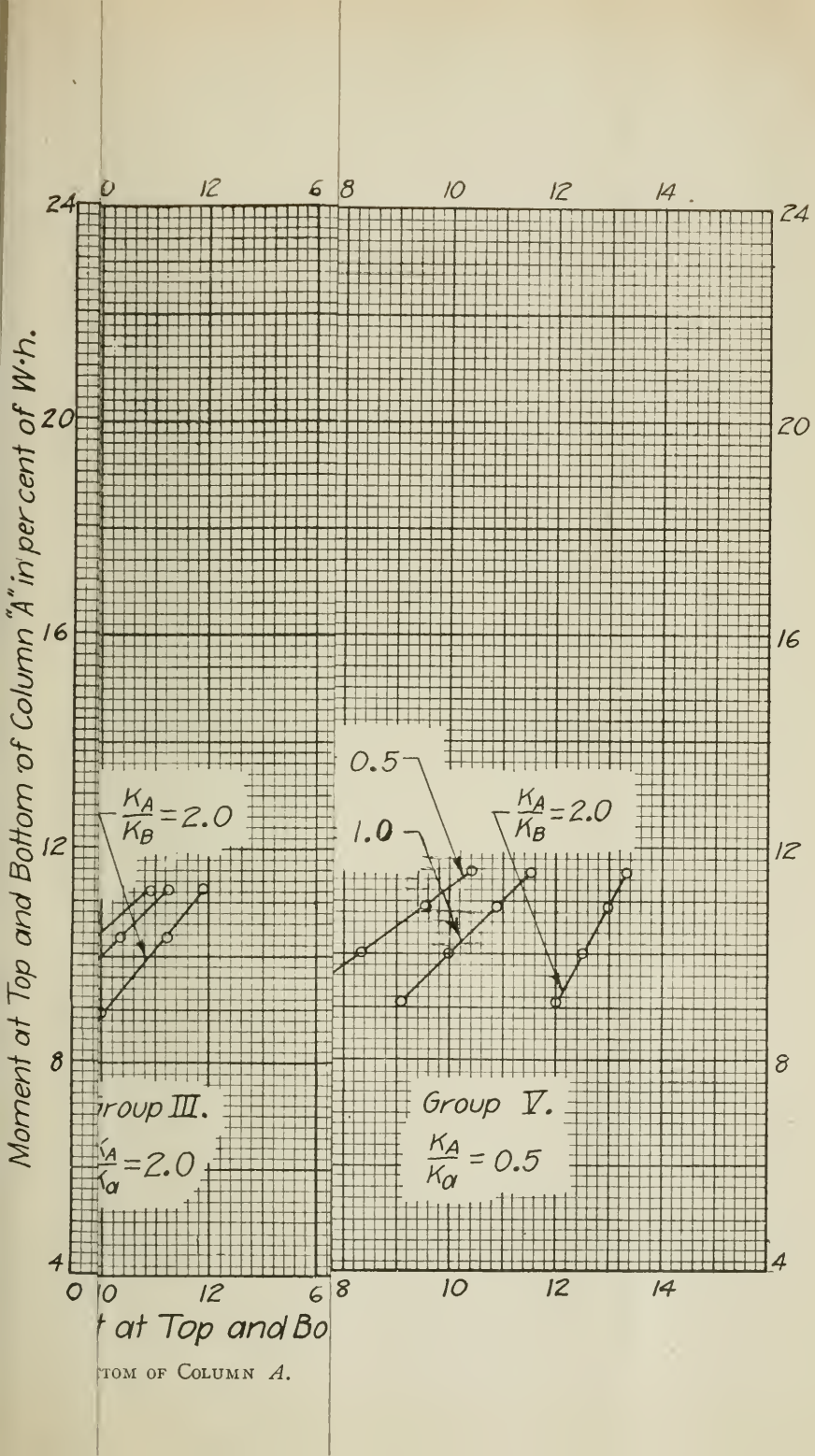
The sum of the moments at the tops and bottoms of all columns of a story is equal to the total shear in the story multiplied by the story height. The distribution of this moment to the ends of the columns depends upon: first, the ratio of the K of column A to the K of column B; second, the ratio of the K of column A to the K of girder a; and third, the ratio of the K of girder a to the K of girder b.

The distribution of the moment was determined in a number of bents for which the ratios of the K of column A to the K of column B, the K of column A to the K of girder a, and the K of girder a to the K of girder b had different values. Diagrams showing the distribution of the moment in these bents also give the distribution of the moment in other bents. The curves in Figs. 6, 7, and 8 show the distribution of the moment in symmetrical three-span bents. In Fig. 6, the curves in group I show the moment at the top and the bottom of column A in bents in which the K of column A and the K of column B are equal. The abscissae represent the ratios of the K of girder a to the K of girder b; and the ordinates represent the moment at the top and the bottom of column A in per cent of $W \times h$. Beginning at the top and reading down, the curves are for bents for which the ratio of the K of column A to the K of girder a equals 0.5, 1, 2, and 4 respectively. The moment in column A of a bent for which the ratio of the K of column A to the K of girder a has any intermediate value can be determined by interpolation.

The curves in groups II, III, IV, and V, of Fig. 6, when used with the curves in group I, give the moment at the top and the bottom of column A in bents in which the K of column A and the K of column B are not equal. The curves in groups II, III, IV, and V are for bents for which the ratio of the K for column A to the K for girder a equals 4, 2, 1, and 0.5, respectively. Beginning at the left of each group and reading to the right, the curves are for bents for which the ratio of the K for column A to the K for column B equals 0.5, 1, and 2, respectively.

The moment at the top and the bottom of column A can be obtained from the diagram in Fig. 6 in the following manner:

First consider the moment in a bent in which the K of column A is


Moment at Top and Bottom of Column "A" in per cent of W·h.
0
12
6
8
10
12
14
24
20
16
12
8
4
$\frac{K_A}{K_B}=2.0$
0.5
1.0
$\frac{K_A}{K_B}=2.0$
roup III.
$\frac{K_A}{K_a}=2.0$
Group V.
$\frac{K_A}{K_a}=0.5$
t at Top and Bo
TOM OF COLUMN A.

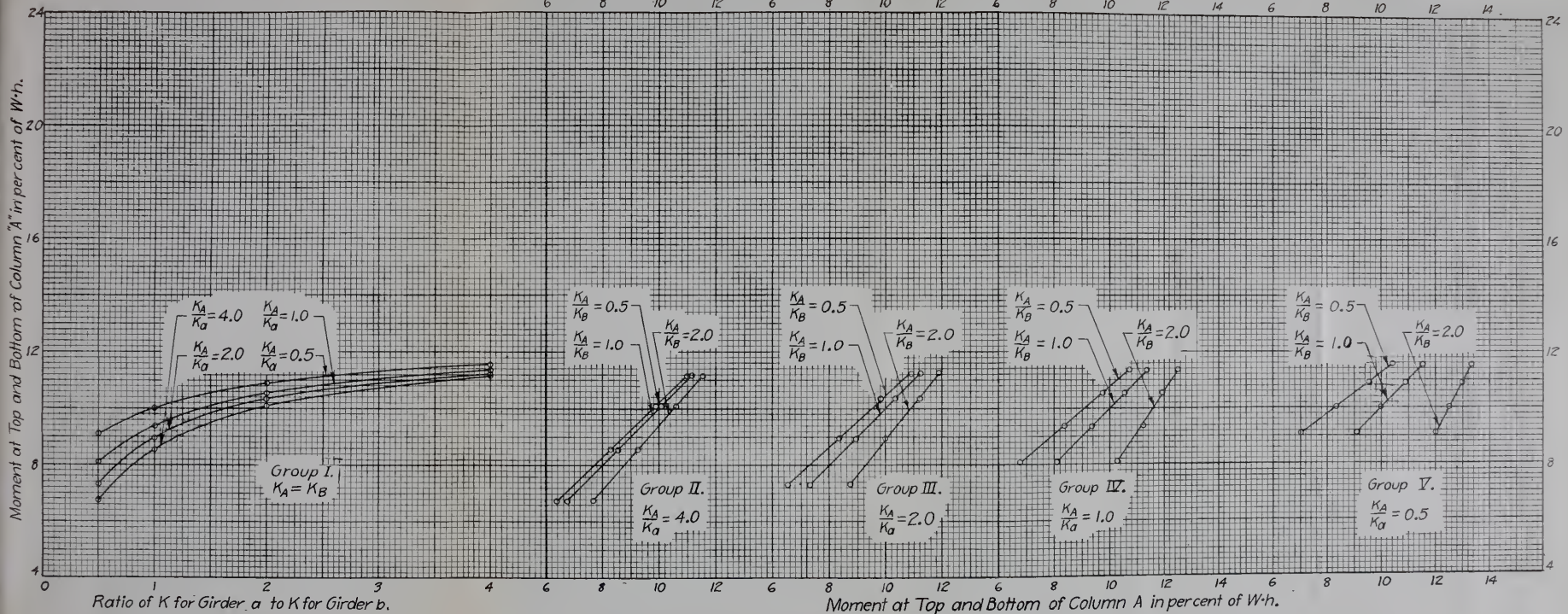
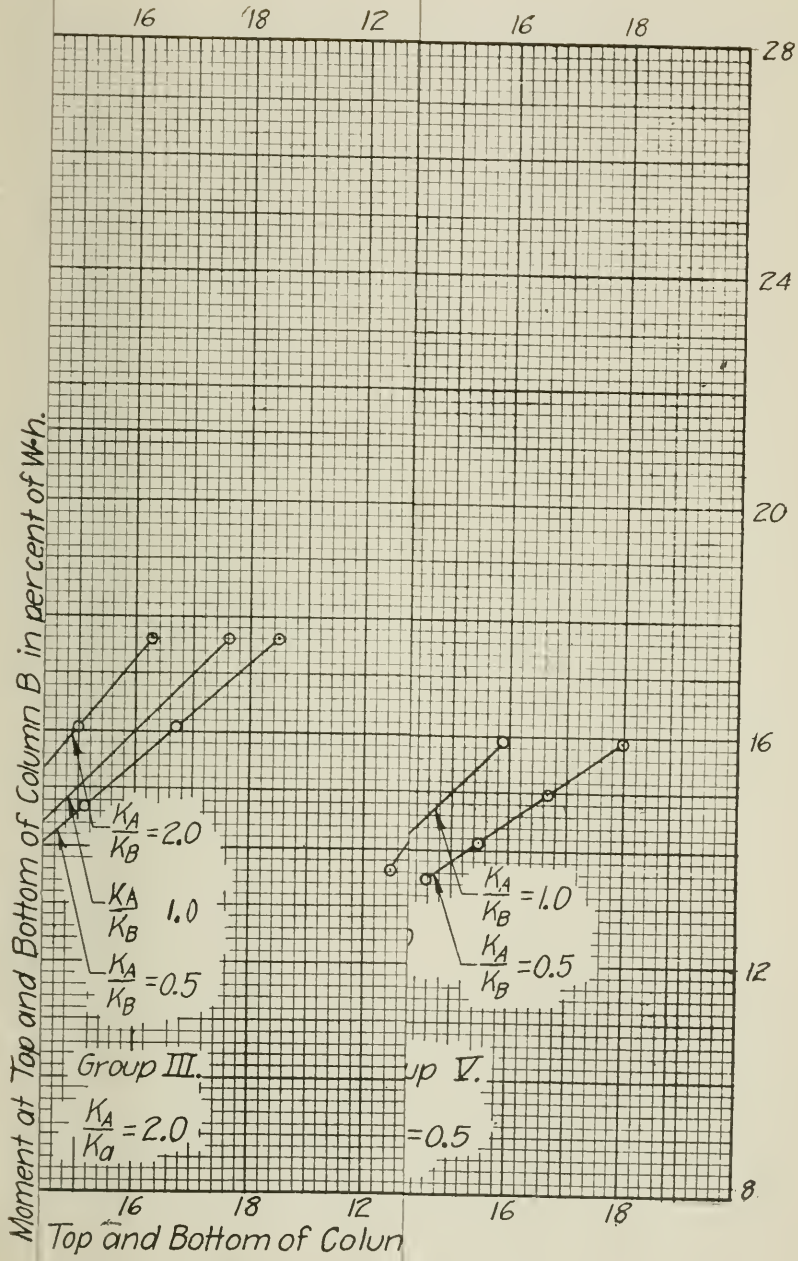


Fig. 6. Approximate Moment at the Top and the Bottom of Column *A*.



Bottom of Column *B*.

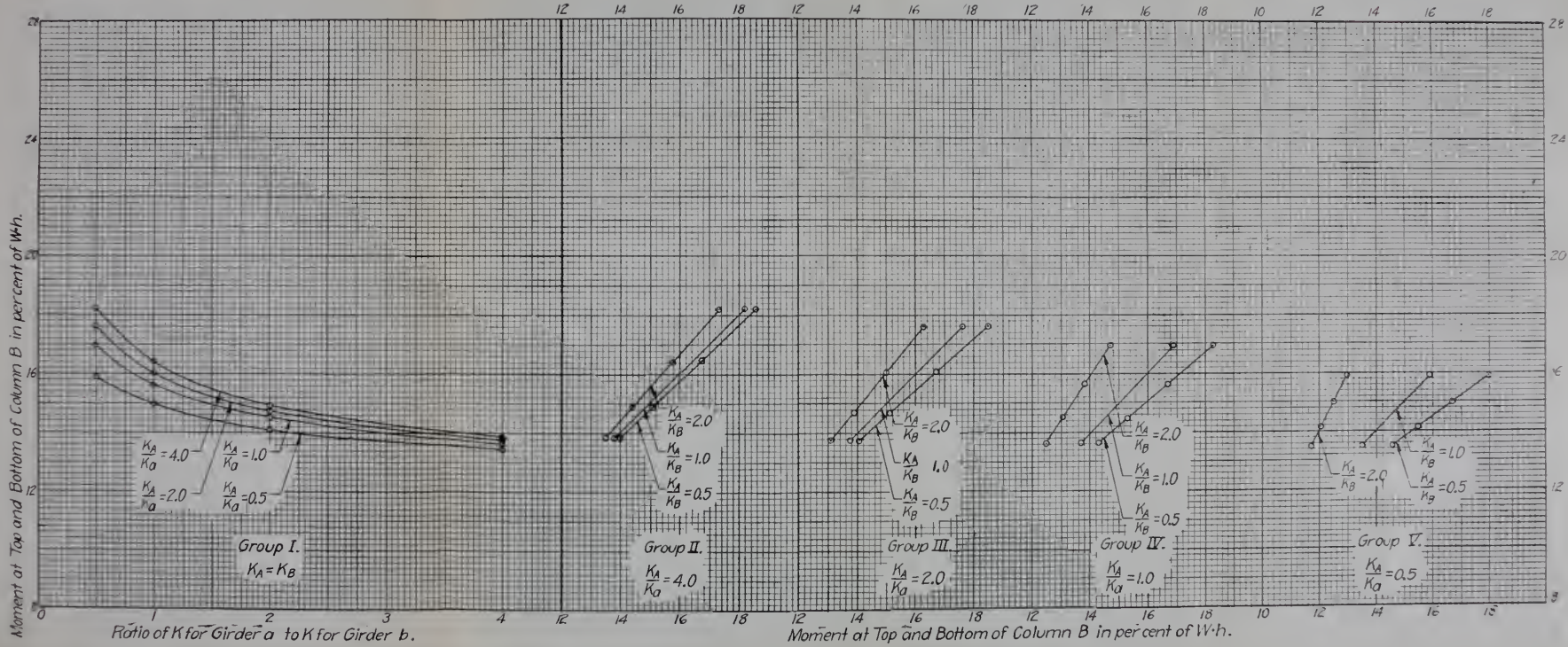
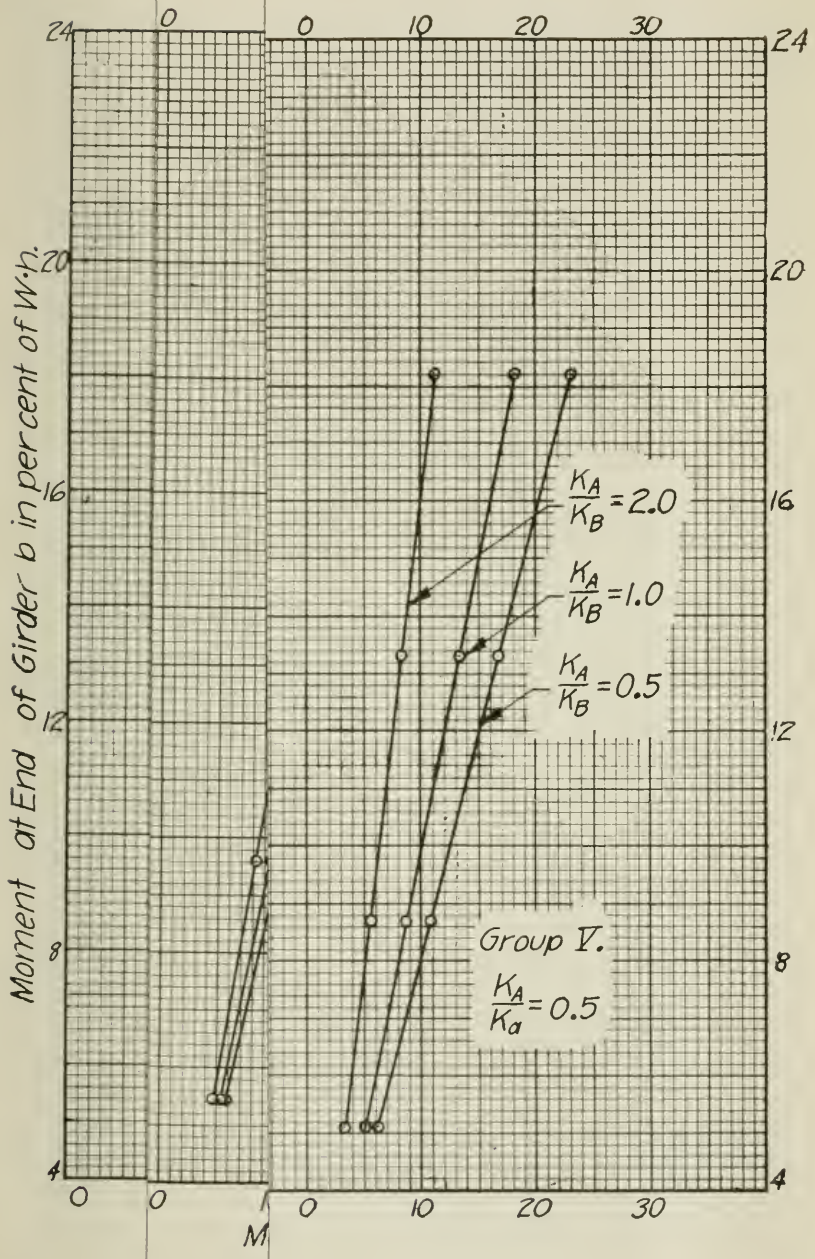


Fig. 7. Approximate Moment at the Top and the Bottom of Column *B*.


Moment at End of Girder b in per cent of W·n.
24
20
16
12
8
4
0
10
20
30
$\frac{K_A}{K_B}$ = 2.0
$\frac{K_A}{K_B}$ = 1.0
$\frac{K_A}{K_B}$ = 0.5
Group V.
$\frac{K_A}{K_a}$ = 0.5

THE ENDS

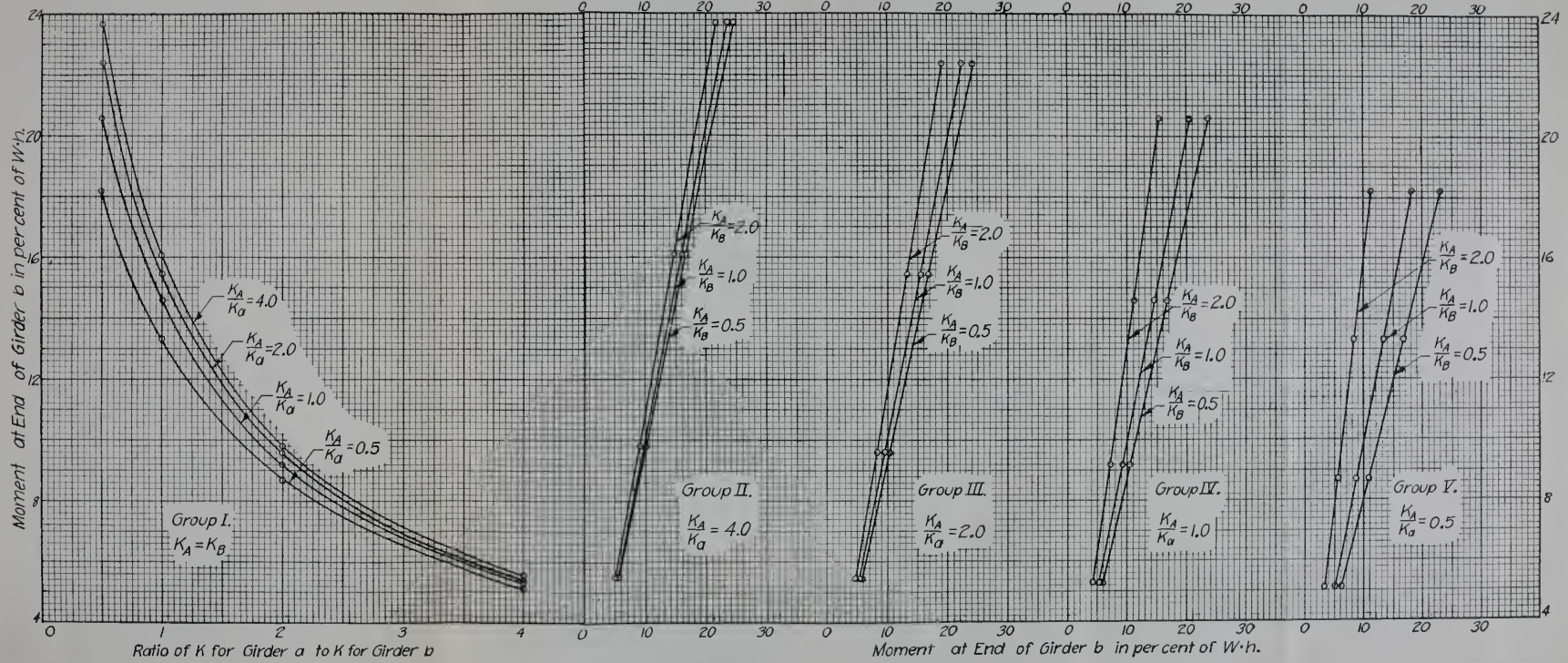


Fig. 8. Approximate Moment at the Ends of Girder *b*.

equal to the K of column B. Determine the value of $\frac{K_a}{K_b}$ and of $\frac{K_A}{K_a}$. Use the curves in group I. Trace a vertical line whose abscissa equals $\frac{K_a}{K_b}$ until it intersects the curve corresponding to the value of $\frac{K_A}{K_a}$. Project the point of intersection horizontally to the left and read the moment in per cent of $W \times h$ from the vertical scale. As W and h are known quantities the moment is determined.

Next consider the moment in a bent in which the K of column A and the K of column B are not equal. Determine the value of $\frac{K_a}{K_b}$, $\frac{K_A}{K_a}$, and $\frac{K_A}{K_B}$. First use the curves of group I. Trace a vertical line whose abscissa equals $\frac{K_a}{K_b}$ until it intersects the curve corresponding to the value of $\frac{K_A}{K_a}$. Project the point of intersection horizontally to the right to the group of curves corresponding to the value of $\frac{K_A}{K_a}$ until it intersects the particular curve of this group corresponding to the value of $\frac{K_A}{K_B}$. Project this intersection point vertically downward and read the moment in per cent of $W \times h$ from the horizontal scale.

Similarly, the moments at the top and the bottom of column B can be obtained from Fig. 7, and the moment at the end of girder b can be obtained from Fig. 8. It should be noted, however, that the moment in girder b depends equally upon the $W \times h$ in the story above and in the story below the girder. This being true, in getting the moment in girder b, the average of the $W \times h$ for the two stories should be used.

The moment at the right end of girder a balances the sum of the moments in column A just above and below the girder and is equal to their algebraic sum. The moment at the left end of girder a balances the sum of the moments at the right end of girder b together with the moments in column B just above and below girder a and is equal to their algebraic sum. It is therefore possible to obtain the moment at the ends of all members in a bent from the curves in Figs. 6, 7, and 8, subject, of course, to the error due to the use of assumptions 1 and 2 of this Section.

The curves in Figs. 6, 7, and 8 show that a large change in the ratio of the K of one member to the K of another member causes a

relatively small change in the distribution of the moments in the bent.

16. *Numerical Problem.*—To illustrate the use of these curves the solution of a problem is presented.

The seventh story of a symmetrical three-span bent is 20 ft. high, and is subjected to a shear of 3,000 lbs. The eighth story of the same bent is 20 ft. high, and is subjected to a shear of 2,500 lbs. The properties of the members in the seventh and eighth stories are as follows: $K_A = 30$ in.^{3}; $K_B = 40$ in.^{3}; $K_a = 20$ in.^{3}; and $K_b = 16$ in.^{3}. It is required to find the moments at the ends of all members in the seventh story.

$W \times h = 3{,}000 \times 20 \times 12 = 720{,}000$ in. lb. in the seventh story.
$W \times h = 2{,}500 \times 20 \times 12 = 600{,}000$ in. lb. in the eighth story.

$$\frac{K_A}{K_B} = \frac{30}{40} = .75$$
$$\frac{K_A}{K_a} = \frac{30}{20} = 1.5$$
$$\frac{K_a}{K_b} = \frac{20}{16} = 1.25$$

To get the moment in column A use Fig. 6. At the left of the figure trace the ordinate whose abscissa is 1.25 to a point half way between the two middle curves ($\frac{K_a}{K_A} = 1.5$, which is half way between 1 and 2), and project this point horizontally to a point half way ($\frac{K_A}{K_B} = .75$, which is half way between .5 and 1.0) between the two left-hand curves of Group II, and also to a point half way between the two left-hand curves of Group III. The abscissa of the former point is 9.35 per cent, and of the latter point 9.15 per cent. As $\frac{K_A}{K_a}$ is equal to 1.5 or the average of 1 and 2, the moment at the top and the bottom of column A in the seventh story, M_{A_7}, is the average of 9.35 per cent and 9.15 per cent or 9.25 per cent of $W \times h$, that is, $M_{A_7} = .0925 \times 720{,}000 = 66{,}700$ in. lb. The moment, M_{A_8}, at the top and the bottom of column A in the eighth story is $.0925 \times 600{,}000$, or $M_{A_8} = 55{,}500$ in. lb. Similarly, the moment at the top and bottom of column B, M_{B_7}, is 15.75 per cent of $W \times h$, that is, $M_{B_7} = .1575 \times 720{,}000 = 113{,}500$ in. lb. in the seventh story; and $M_{B_8} = .1575 \times 600{,}000 = 94{,}400$ in. lb. in the eighth story. The moment M_{b_7}, at the end of girder b at the top of the seventh story is 13.85 per cent of $W \times h$, that is,

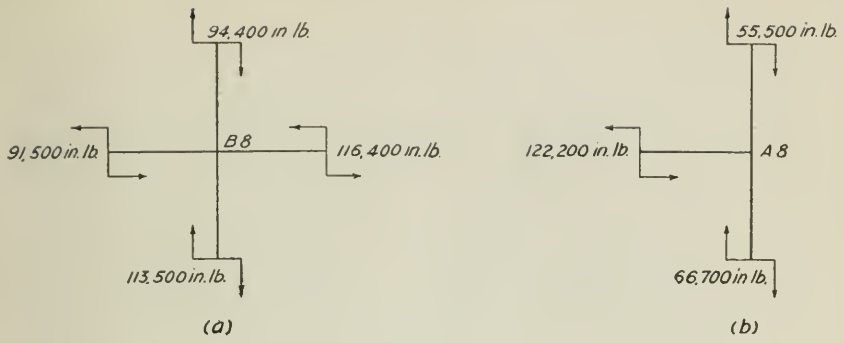


FIG. 9. MOMENTS ACTING AT POINTS *A8* AND *B8* OF A SYMMETRICAL THREE-SPAN BENT.

$$M_{b_7} = .1385 \left[\frac{720{,}000 + 600{,}000}{2} \right] = 91{,}500 \text{ in. lb.}$$

The moment at the right end of girder *a* at the top of the seventh story is equal to the sum of 66,700 in. lb., the moment at the top of column *A* in the seventh story, and 55,500 in. lb., the moment at the bottom of column *A* in the eighth story, or 122,200 in. lb. (see Fig. 9b). The moment at the left end of girder *a* is equal to the sum of 113,500 in. lb., the moment at the top of column *B* in the seventh story, and 94,400 in. lb., the moment at the bottom of column *B* in the eighth story, less 91,500 in. lb., the moment in the end of girder *b;* that is, the moment at the left end of girder *a* is 116,400 in. lb. (see Fig. 9a).

A comparison of the moments in a bent as obtained by the proposed approximate method and by the slope-deflection method is given in Tables 23 to 26 inclusive.

17. *Modifications of the Slope-Deflection Method.*—The writers made a study to determine the effect of a change in the section of one member of a bent upon the moment in the other members. The moments were determined in a number of bents in which $K = 1$ for all members except the one whose effect upon the distribution of the moment was to be studied. The quantity $\frac{Wh}{6E}$ was taken equal to 1 for all stories and for all bents. The K of column *B* was given successively the values 0.5, 1, 2, and 4; and the corresponding moment at the ends of columns *A* and *B* and at the ends of girders *a* and *b* was calculated

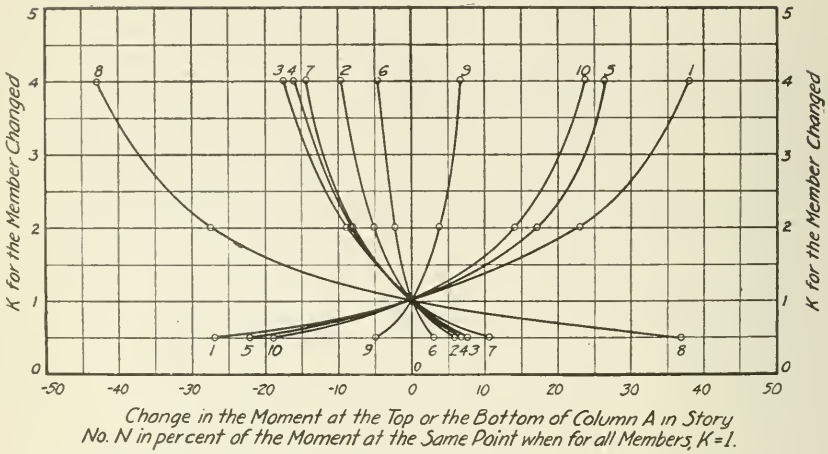


FIG. 10. DIAGRAM SHOWING CHANGE IN THE MOMENT AT THE TOP AND THE BOTTOM OF COLUMN *A* OF A SYMMETRICAL THREE-SPAN BENT DUE TO A CHANGE OF *K* IN THE OTHER MEMBERS.

KEY TO THE DIAGRAM IN FIG. 10.

Member Changed	Designation of the Curve Showing the Change in the Moment at	
	Top of Column *A*	Bottom of Column *A*
Girder *a* in Story No. N	1	2
" " No. (N—1)	2	1
Girder *b* " " No. N	3	4
" " No. (N—1)	4	3
Column *A* " " No. N	5	5
" " No. (N—1)	6	7
" " No. (N+1)	7	6
Column *B* " " No. N	8	8
" " No. (N—1)	9	10
" " No. (N+1)	10	9

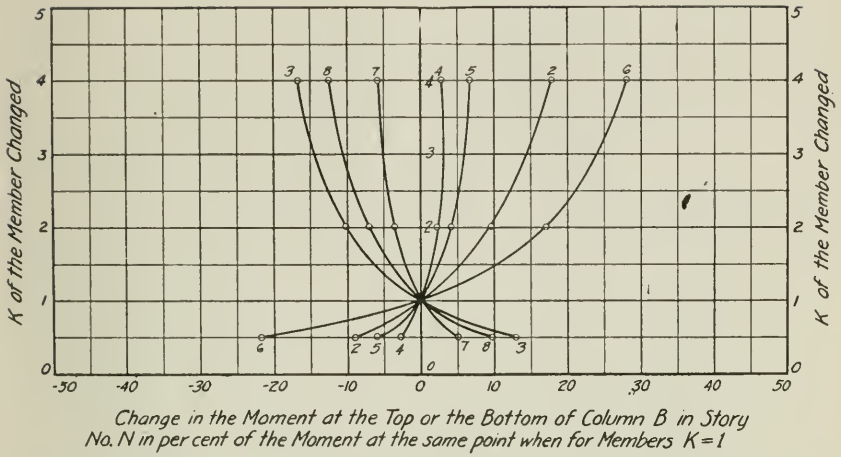


FIG. 11. DIAGRAM SHOWING CHANGE IN THE MOMENT AT THE TOP AND THE BOTTOM OF COLUMN *B* OF A SYMMETRICAL THREE-SPAN BENT DUE TO A CHANGE OF *K* IN THE OTHER MEMBERS.

KEY TO THE DIAGRAM IN FIG. 11.

Member Changed	Designation of the Curve Showing the Change in the Moment at	
	Top of Column *B*	Bottom of Column *B*
Girder *a* in Story No. N	—	1
" " No. (N—1)	1	—
Girder *b* " " No. N	2	—
" " No. (N—1)	—	2
Coulmn *A* " " No. N	3	3
" " No. (N—1)	4	5
" " No. (N+1)	5	4
Column *B* " " No. N	6	6
" " No. (N—1)	7	8
" " No. (N+1)	8	7

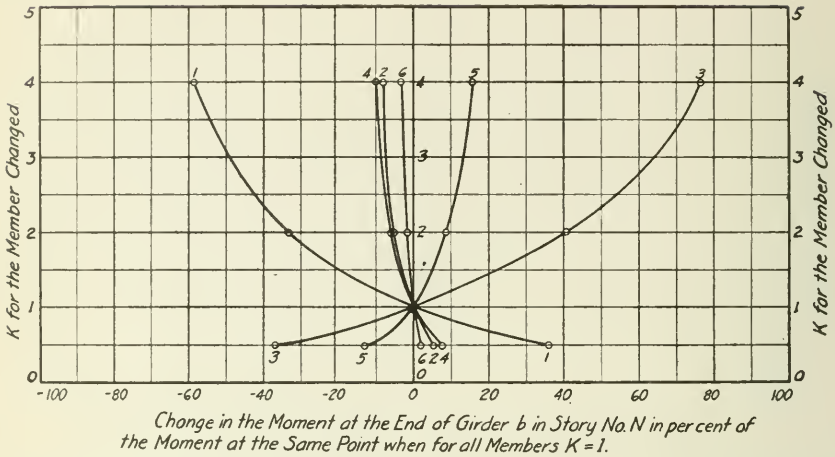


FIG. 12. DIAGRAM SHOWING CHANGE IN THE MOMENT AT THE END OF GIRDER *b* OF A SYMMETRICAL THREE-SPAN BENT DUE TO CHANGE OF *K* IN THE OTHER MEMBERS.

KEY TO THE DIAGRAM IN FIG. 12.

Member Changed	Designation of the Curve Showing the Change in the Moment at the End of Girder *b*
Girder *a* in Story No. N	1
" " No. (N—1)	2
" " No. (N+1)	2
Girder *b* " " No. N	3
Column *A* " " No. N	4
" " No. (N+1)	4
Column *B* " " No. N	
" " No. (N—1)	6
" " No. (N+1)	5

in the story in which the K of column B was changed, and also in the stories immediately above and below that story. Similarly, the K of column A, the K of girder a and the K of girder b were given successively the values 0.5, 1, 2, and 4, and the corresponding values of the moment at the ends of the girders and the columns were determined in the particular story and also in the stories immediately above and below that story. The effect of the changes in the values of the K of the members upon the moment is given in Figs. 10, 11, and 12.

Fig. 10 shows the changes in the moment at the top and the bottom of column A. The abscissae are changes in the moment expressed in per cent of the moment in a bent for which the K of all members is equal to 1. Increases in the moment are laid off to the right of the origin, and decreases are laid off to the left. The ordinates represent the values of the K in the member changed. The story in which the moment was calculated is designated as story No. N, the story above is designated as story No. $(N+1)$, and the story below No. $(N-1)$. Each curve shows the change in the moment at the top or the bottom of column A due to a change in the value of the K of some member. The number of the curve which shows the change in the moment in column A due to a change in the K of any particular member can be obtained from the key to the diagram in Fig. 10. For example, curve No. 3 shows the change in the moment at the top of column A due to a change in the K of girder b in story No. N. If the K of girder b is made equal to 2, the moment at the top of column A is 8.9 per cent less than when the K of girder b is equal to 1. Curve No. 3 also shows the change in the moment at the bottom of column A due to a change in the K of girder b in story No. $(N-1)$. If the K for girder b in story No. $(N-1)$ is equal to 2, the moment at the bottom of column A is 8.9 per cent less than when the K for girder b is equal to 1.

Similarly, the changes in the moment in column B due to changes in the K of the members of the bent are given in Fig. 11 and the changes in the moment in girder b are given in Fig. 12.

Fig. 10 shows that the moment in column A of any story not only depends upon the value of the K of the members in the same story but depends also upon the value of the K of the members in the adjacent stories. Figs. 11 and 12 show that the same statement is true relative to the moment in column B and girder b. This being true, any approximate method which considers only the members in the story in which the stresses are to be determined can not give accurate results except

in bents which have no very sudden changes in the sections of the columns and girders.

Where there are sudden changes in the column and girder sections, the proposed approximate method is not accurate. In such cases it is sometimes possible to use a modification of the slope-deflection method and obtain quite accurate results with a comparatively small expenditure of labor. This modification of the slope-deflection method may be made in either of two ways, as follows: (a), the bent may be divided by a horizontal plane between any two adjacent floors, and the lower part be treated independently of the upper part; or (b), the bent may be divided by two horizontal planes any number of stories apart, and the portion between these planes may be treated independently of the other portions of the building. These modifications will be considered in detail.

(a) The wind stresses in the top stories of a bent are comparatively small and their exact determination is not important. An examination of the equations of Table 15 shows that the coefficient of the quantity to be determined is greater than the coefficients of the quantities in which the unknown is expressed. This being true, errors in the slopes and the deflection of one story have but little effect upon the slopes and deflections of the succeeding stories. Further examination of the same table shows that there is a comparatively small difference between the change in the slope at the top of a column in one story and the change in the slope at the top of the same column in the story below. If the two changes in slope are assumed to be equal, the error introduced is small. Bearing these facts in mind, consider again the twenty-story bent of Section VII. If θ_{A12} and θ_{B12} are assumed equal to θ_{A11} and θ_{B11} respectively in equations f, g, and h of Table 13, the first 34 equations will contain 34 unknown quantities which can be determined by the method used in Tables 14 and 15. That is, assuming that the changes in the slopes of the columns in one story are equal to the changes in the slopes of the same columns in the story below, is equivalent to dividing the bent into two parts; and the equations for the lower part can be solved independently of those of the upper part. If the slopes which are assumed equal are not really equal, the calculated slopes and deflections in the top story of the lower part will not be exact, but the results of a number of calculations show that the error in the slopes and the deflection in the next to the top story is so small that it may be neglected. This being true, if, in the actual design of the twenty-story bent of Section VII, it is con-

sidered unnecessary to calculate the wind stresses in the top ten stories, the bottom eleven stories may be treated as a complete bent and the stresses determined by the method outlined above, thereby decreasing the work. The stresses in the bottom ten stories will be quite exact. The bent could have been divided at any other story without affecting the accuracy of the results.

If there are sudden changes in the members the bent can be divided above the story in which the change occurs, and the stresses in the lower part can be determined by the method outlined above.

Table 23 shows that the largest errors in the proposed approximate method are in the first story. The moments can be determined in the first story by the modification of the slope-deflection method as follows:

Assume that the slopes in the third story are equal to the corresponding slopes in the second story. The first seven equations of Table 13 will then contain only seven unknown quantities. The solution of these equations is given in Tables 21 and 22. The changes in the slopes and the deflection for the first story as given in Table 22 agree very closely with the values for the same quantities given in Table 15, and the moments, being functions of the slopes and deflection, will also agree very closely with the moments given in Table 18.

(b) The moments in any particular story may be determined if there is some story below the one in question in which it can be seen by inspection that the changes in the slopes at the tops and the bottoms of the columns are equal. Suppose the moments are to be determined in the tenth story of the bent shown in Fig. 5, and that it is apparent from inspection that $\theta_{A8} = \theta_{A9}$, and $\theta_{B8} = \theta_{B9}$. Then assume that $\theta_{A12} = \theta_{A11}$, and $\theta_{B12} = \theta_{B11}$. The ten equations, Y to h inclusive of Table 13, will contain ten unknown quantities. If the slopes in the eighth and ninth stories, which have been assumed to be equal, are equal then the slopes and the deflection, and therefore also the moments, in the tenth story will be quite accurate, but if the slopes *below* the story in question which are assumed to be equal, are not really equal, the required moments will not be exact. That is, a difference between the slopes *below* the story in question affects the results, whereas a difference between the slopes *above* the story does not materially affect the results.

18. *Application of the Proposed Approximate Method and Modification of the Slope-Deflection Method.*—To obtain the moment in a bent in which there are sudden changes in the sections, the following com-

bination of methods can sometimes be used to advantage. Use the proposed approximate method for obtaining the moment in the portion of the bent in which there are no sudden changes in the sections of the members. Use a modification of the slope-deflection method to get the moments in the bottom story and in any intermediate stories in which sudden changes in the members occur. The results thus obtained will be sufficiently accurate to be used in the actual designs of buildings, and the amount of work required will not be excessive.

IX. Comparison of the Approximate Methods With the Slope-Deflection Method.

19. *Symmetrical Three-Span Bent with Short Middle Span.*—For comparison, the moments in the symmetrical three-span twenty-story bent shown in Fig. 5 were calculated by the five following methods:

1. The slope-deflection method.
2. The proposed approximate method.
3. The three methods described by Mr. Fleming and known as methods I, II, and III.

The moments as determined by these methods are given in Tables 23 and 24. For each story the moments in the upper line are in 1,000 in. lb., and those in the lower line are in per cent of the moments as determined by the slope-deflection method. Tables 23 and 24 show in the case of this bent: first, that the moments determined by methods II and III are very seriously in error; second, that the moment determined by method I and by the proposed approximate method agree very closely with the moment determined by the slope-deflection method except at points where there are sudden changes in the members of the bent; third, that the errors in the moment determined by the proposed approximate method are less for the girders than for the columns.

20. *Symmetrical Three-Span Bent with Long Middle Span.*—The distribution of the moment determined by the slope-deflection method is affected by the ratio of the K of girder a to the K of girder b. As this ratio does not affect the distribution of the moment determined by methods I, II, and III, the accuracy of the latter methods will depend upon the relative values of K for the two girders.

In the bent shown in Fig. 5, the K is less for girder a than for girder b, since the girders have substantially the same sections and girder a is longer than girder b. In order to determine the effect of the relative values of the K of girders a and b upon the accuracy of the approximate methods, the moment in the bottom four stories of the

bent shown in Fig. 13, a bent like the one shown in Fig. 5 except that the long and short spans have been interchanged, was determined by each of the five methods mentioned in article 19. The results are given in Tables 25 and 26. The moments determined by the proposed approximate method are as accurate as those given in Tables 23 and 24; whereas the moments determined by method I are inaccurate, and those determined by methods II and III are very inaccurate.

21. *Effect of the Proportions of a Bent upon the Accuracy of Method I.*—In order to determine more fully the effect of the proportions of a bent upon the accuracy of method I, the moments were determined in a number of bents having different proportions, by the slope-deflection method and by method I. These moments are given in Table 27.

In bents 1 to 9 of this table all girders and columns have the same section. The ratio of the moment of inertia of the columns to the moment of inertia of the girders and also the ratio of the moment of inertia of the girder in bay *a* to the moment of inertia of the girder in bay *b*, affects the moment determined by the slope-deflection method but does not affect the moment as determined by method I. If the sections of the columns and girders are not the same, the difference between the moments determined by the two methods might vary even more than Table 27 indicates. Any errors in the moment due to sudden changes in the sections of the bent are in addition to the errors indicated in Table 27.

The difference between the two methods is due to the fact that in method I the direct unit stress in a column is proportional to the distance of the column from the neutral axis of the bent; whereas in the slope-deflection method the stress in a column depends upon the shears in the girders, and the shears in the girders depend upon the changes in the slopes at the ends of the girders and upon the moment of inertia of the girder sections.

22. *Accuracy of the Approximate Methods when the Moment of Inertia of the Girders is Proportional to the Bending Moment.*—In the comparison of the approximate methods with the slope-deflection method given in Tables 23 to 27, the sections of the girders were chosen without reference to the moment to which they were subjected, except the girders in the bent shown in Fig. 5. The girders in this bent were designed to resist the bending moment determined by method I. Another investigation was made to determine the accuracy of the approxi-

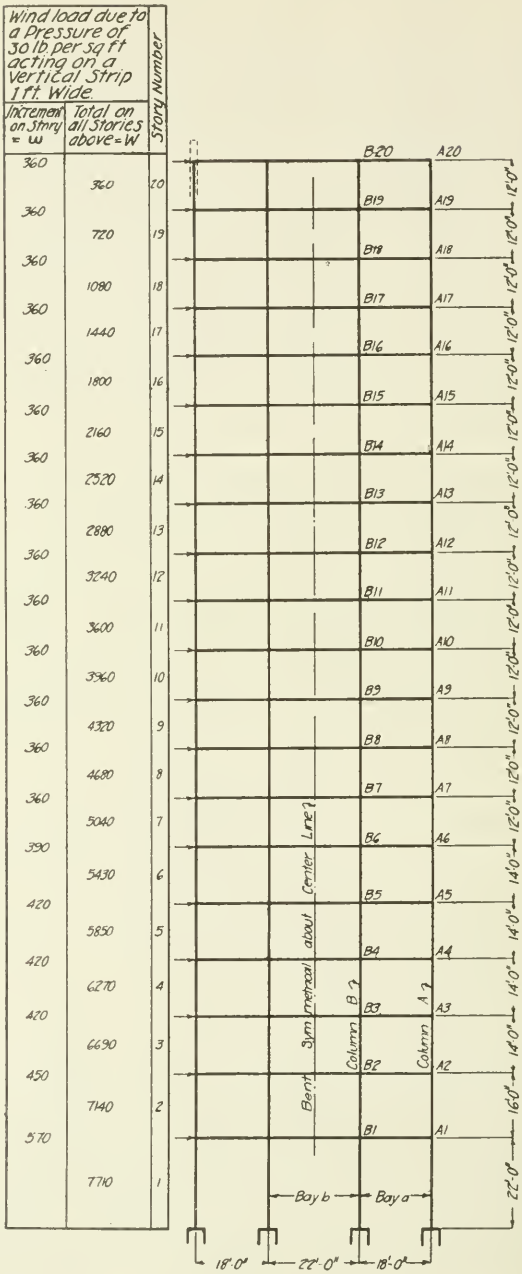


FIG. 13. SYMMETRICAL THREE-SPAN BENT TWENTY STORIES HIGH WITH LONG SPAN AT THE CENTER.

mate methods when applied to bents in which the girders were designed to resist the bending moment due to the wind load as determined by the method which is to be compared with the slope-deflection method. The results of this investigation are given in Tables 28 to 31. In Table 28 the moments of inertia of the sections of the girders are proportional to the bending moments determined by method I. The sections of the columns are equal, and the moment of inertia of the sections of the columns is equal to the moment of inertia of the section of girder *a*. Table 28 shows that method I gives the moment in the columns and girders quite accurately when the spans are equal and the story height does not exceed the length of the span. This statement is true, in general, only when the two column sections are equal and when the moments of inertia of the sections of column *A* and girder *a* are equal.

In considering the merits of the approximate methods, it should be noted that the moment which should be determined with the greatest accuracy is the moment to which the joint that connects the girder to the column is subjected. This moment is the moment at the ends of the girders. If the moments of inertia of the sections are not made proportional to the bending moment determined by method I, the method will not be as accurate as Table 28 indicates.

The moments of inertia of the girder sections of Table 29 are proportional to the bending moments in the respective girders as determined by method II. The moments of inertia of the girder sections of Table 30 are proportional to the bending moments in the respective girders determined by method III. It is apparent from Tables 29-30 and Tables 23-26 that methods II and III are so inaccurate that they should never be used.

The moments of inertia of the girder sections of Table 31 are proportional to the bending moments in the respective girders as determined by method IV. For bents having equal spans and equal column sections this method gives quite accurate results.

While methods I and IV may be quite accurate in some cases, their accuracy depends upon the proportions of the bent. For example, by comparing Tables 27 and 28 it is apparent that if the moments of inertia of the girder sections are not proportional to the bending moments determined by method I, that method will not be as accurate as when the girder sections are so proportioned. Again, in Tables 28 and 31 the sections of the columns are equal and the moments of inertia of the sections of column *A* and girder *a* are equal; but if these relations

do not exist, the methods will not in general be as accurate as Tables 28 and 31 seem to indicate.

Any inaccuracy in methods I and IV due to sudden changes in the members of the bent, are in addition to the inaccuracies shown in Tables 28 and 31.

For the bents shown in Tables 28 to 31, the moments determined by the proposed approximate method are exactly the same as the moments determined by the slope-deflection method.

X. Test of a Celluloid Model of a Bent.

23. *Description of Tests.*—In order to check the deflections and the changes in the slopes calculated by the slope-deflection method, a celluloid model of a bent was subjected to known shears, and the resulting deflections and changes in slopes were measured and compared with the calculated values. The model was made of celluloid ⅛ inch thick, and had the general dimensions shown in Fig. 14. A cord passing over a pulley and attached to a weight at one end and to the top of the model at the other, produced a uniform shear in all stories. Paper arms were fastened to the model at points where the members intersect. The movement of these arms indicated the changes in the slopes. One of these arms, A_2D, is shown in Fig. 14. The external force caused the point A_2 to move in approximately a horizontal line and at the same time to turn through a small angle, θ. The paper arm A_2D has the same motion as the point A_2; thus the vertical displacement of D measures the angle θ. The horizontal deflection of the model was obtained by measuring the displacement of a point at the middle of a girder. Paper arms were attached to all four columns at the tops of all stories simultaneously, and the changes in the slopes at all intersections were measured for each application of the load. Similarly, the horizontal deflection was measured at the middle of each of the three girders at the top of each story.

In the original model, known as model No. 1, members No. 1, 2, 3, and 4 were ½ inch wide; all other members were ¼ inch wide. After this model had been tested, member No. 1 was reduced to ¼ inch; and then the resulting model, now known as No. 2, was tested. Members No. 2, 3, and 4 were successively reduced to ¼ inch; and the resulting models, now known as models No. 3, 4, and 5, were also tested.

In testing model No. 1, observations were made when loads of 2½ lb., 5 lb., 7½ lb., and 10 lb. were applied. The readings for the two sides of the model agreed very closely in the bottom three stories;

but in the fourth story they did not agree. Thinking that this discrepancy might be due to excessive stresses, when testing model No. 2, a maximum load of only 7½ lb. was used; but there was still the same discrepancy in the readings. Thinking that the apparatus might be out of order, the loads were removed and then applied a second time but the second readings agreed very closely with the first. In testing models No. 3, 4, and 5, loads of 1½ lb., 3 lb., and 4½ lb. were applied. The loads were then removed and applied in the opposite direction. The readings for the two sides of the model continued to agree very closely in the bottom three stories; and in the fourth story they continued to

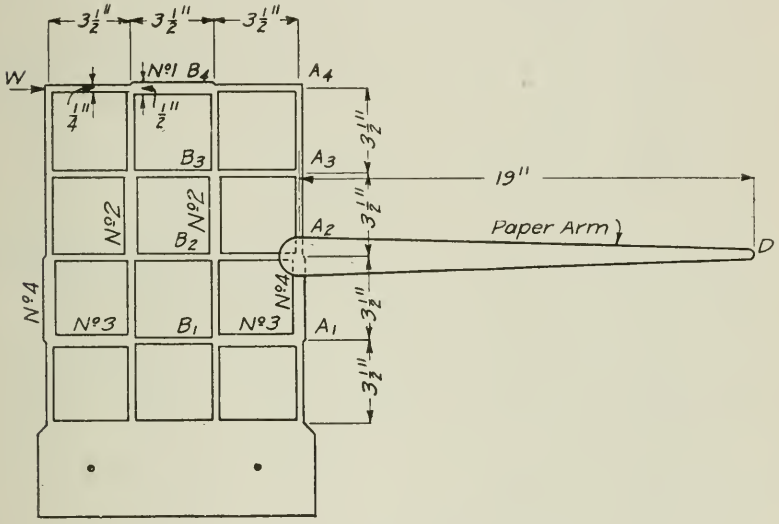


Fig. 14. Celluloid Model.

disagree. It is therefore probable that either the material was not homogeneous or that there were internal stresses or local weaknesses in the upper part of the model.

The observations made in the test of model No. 4 are given in Table 32.

24. *Results of Tests.*—The results of the tests of the models are given in Fig. 15. The models are shown by the sketches at the left of the figure. In these sketches members represented by heavy lines are ½ inch wide, and all other members are ¼ inch wide. The upper diagram of Fig. 15 shows the changes in the slopes and the lower diagram shows the deflections.

In the upper diagram the first group of lines at the left represents

the changes in the slope at the point A_1, the second group represents the changes at A_2, and similarly the third and fourth groups represent the changes at A_3 and A_4. The fifth group of lines from the left represents the changes in the slope at the point B_1, the sixth group at B_2, and similarly for the other groups. In the left-hand group of lines the change in the slope at the point A_1 of model No. 1 is laid off from the origin on a horizontal line opposite the sketch of model No. 1; the change in the slope at A_1 of model No. 2 is laid off from the origin on a horizontal line opposite model No. 2; the change in the slope at A_1 of model No. 3 is laid off from the origin on the horizontal line opposite model No. 3; and the change in the slope at A_1 of model No. 4 is laid off from the origin on the horizontal line opposite model No. 4. The full lines connect points which represent the quantities as measured; and the dotted lines connect points which represent the same quantities as computed by the slope-deflection method. The changes in the slopes at A_2, A_3, A_4, B_1, B_2, B_3, and B_4 are shown in a similar manner.

The lower diagram shows the ratios of the deflection to the story height in the different stories of the models. The method of representing the ratios of deflection to story height is similar to the method used to represent the changes in the slopes in the upper diagram.

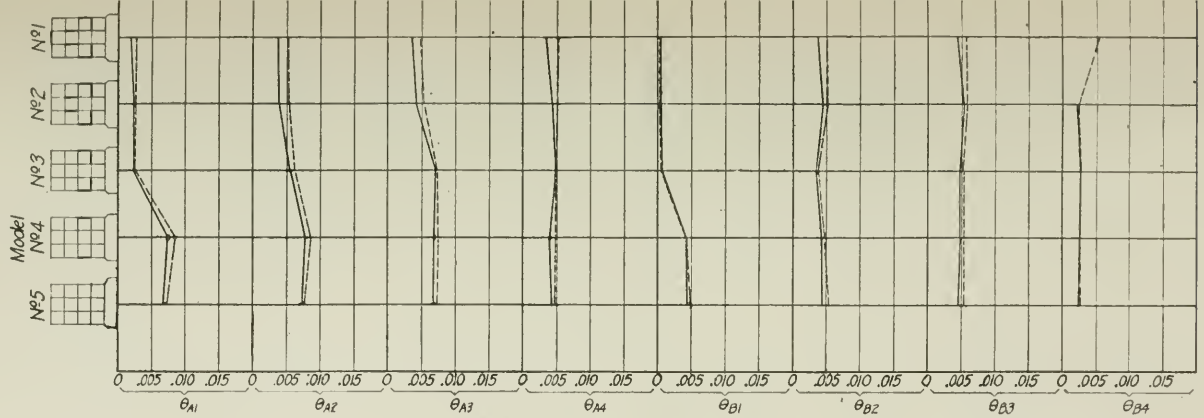
It will be noticed that the calculated quantities are in general greater than the observed quantities. The reason for this is: In the computations, the length of a member was taken equal to the distance between center lines, whereas the length that is actually free to bend is the distance from outside to outside of the members. This accounts for the difference between the observed and the computed values.

Fig. 15 shows that the changes in the slopes and the ratios of the deflection to the story height as observed and as computed agree closely. In other words, the tests support the theory upon which the slope-deflection method is based.

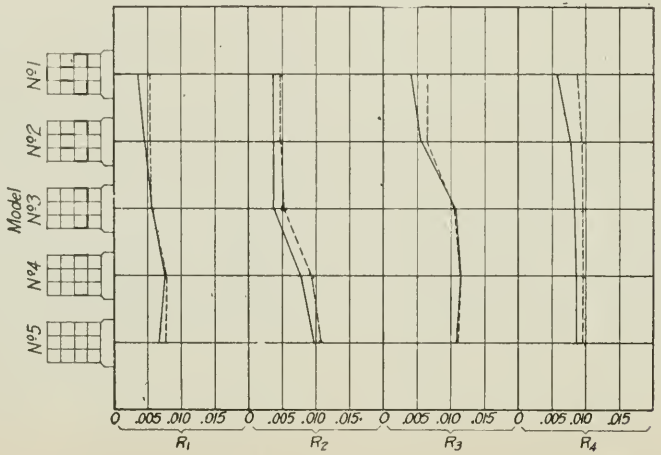
XI. Discussion of the Assumptions.

25. *Preliminary.*—In making the analysis of the stresses the writers made certain assumptions in regard to the action of the frame when stressed. The effect of inaccuracies in these assumptions will now be considered.

If all of the columns of a story are taken together as a free body, the algebraic sum of the moments at the tops and bottoms of all the columns is equal to the shear on the story multiplied by the story height. As the moments in the columns are balanced by the moments in the



Changes in Slope—in Radians.



Ratios of Deflection to Story Height—Abstract Numbers.

Note:—
In models N° 1 to 5 the heavy lines represent members $\frac{1}{2}$ inch wide. All other members are $\frac{1}{4}$ inch wide.

Measured quantities ———
Computed quantities - - - - -

girders, the algebraic sum of the moments at the ends of the girders is dependent upon the product of the shear in the story and the story height. These facts should be borne in mind in discussing the effect of inaccuracies in the assumptions upon the moments in a bent.

26. *Assumption of Perfect Rigidity.*—According to assumption No. 1, the connections between the columns and girders are perfectly rigid. The truth of this assumption can be determined only by tests. As far as the writers are aware no such tests have been made. While it is undesirable to make a mathematical analysis which is based upon an unverified assumption, some assumption relative to the rigidity of the connections must be made before the stresses in a frame can be determined. The distribution of the stresses depends more upon the relative stiffness than upon the actual stiffness of the connections. In view of these facts, it seems that the assumption of perfect rigidity of the joints is the most logical one that can be made.

27. *Assumption of the Unchanged Length.*—According to assumption No. 2, the change in the length of a member due to the direct stress is equal to zero.

If columns A and B change in length, the moment at the right-hand end of girder a will be given by the equation

$$M_{AB} = 2EK_a\,(2\theta_A + \theta_B - 3R_a),$$

in which R_a is equal to the difference between the changes in the lengths of columns A and B divided by the length of girder a. In the derivation of the general equations in Section VI, R_a was assumed to be equal to zero. If the changes in the lengths of the columns do not alter the values of θ_A and θ_B, the change in the moment ΔM_{AB}, at the right-hand end of girder a due to the changes in the lengths of the columns is:

$$\Delta M_{AB} = 2EK_a\,(-3R_a).$$

The change in the moment at the left-hand end of girder a is equal to the change at the right-hand end. The difference between the changes in length of columns A and B, or the deflection of one end of girder a relative to the other end, is given by the equation:

$d = \dfrac{Pl}{AE}$, in which

d = the deflection,

P = the difference between the stress in column A and the stress in column B,

l = the length of the columns,

A = the area of the column section,

E = the modulus of elasticity of steel.

From Table 19, page 75, the compressive stresses in the first story of columns A and B are 14,464 lb. and 4,587 lb. respectively.

Therefore $P = 14{,}464 - 4{,}587 = 9{,}877$ lb. From Table 11, page 48, $l = 264$ in., $K_a = 30.5$ in.^{3}, and $A = 112.8$ sq. in. $E = 29{,}000{,}000$ lb. per sq. in. Substituting these values in the above equation for d, gives:

$$d = .000798 \text{ in.}$$

By definition, $R_a = \frac{d}{l}$, therefore

$$R_a = \frac{.000798}{264} = .00000303, \text{ and}$$

$$\Delta M_{AB} = 2 \times 29{,}000{,}000 \times 30.5 \ (-3 \times .00000303) = -16{,}050 \text{ in. lb.}$$

This change in the moment is 5.6 per cent of the moment at the right-hand end of girder a as given in Table 18, page 74. Likewise the moment at the left-hand end of girder a is decreased by 16,050 in. lb., which is 6.6 per cent of the moment as given in Table 18.

The moment at the end of girder b is affected by the changes in the lengths of two columns, B, one on each side of the center line of the bent. One column is subjected to a tensile stress of 4,587 lb., and the other to a compressive stress of 4,587 lb. The deflection of girder b is given by the equation

$$d = \frac{2 \times 4{,}587 \times 264}{112.8 \times 29{,}000{,}000} = .00074 \text{ inches.}$$

$$R_b = \frac{.00074}{216} = .00000343.$$

$$\Delta M_{BB} = 2 \times 29{,}000{,}000 \times 37.3 \times 3 \times .00000343 = 22{,}300 \text{ in. lb.,}$$

which is 9.0 per cent of the moment at the end of girder b as given in Table 18.

The change in the length of a column is a function of the unit direct stress and also a function of the story height. The first story of the building considered is much higher (22 ft.) than the other stories, and the unit direct stress in the columns in the first story is greater than in the stories above; hence the change in the length of the columns is very much less in the other stories than in the first story. Therefore the changes in the moments for the other stories will be less than those in the first story computed above.

The changes in the moments in the columns have been deter-

mined and it remains to consider the effect of the direct stresses in the girders. The direct stress in a girder is very small in comparison with the direct stress in a column and therefore may be neglected.

If the changes in the slopes were not affected by the changes in the lengths of the columns, the moments at the ends of girders a and b would be decreased. As the sum of these moments is determined by the product of the shear in the story and the story height, they can not all be decreased simultaneously. Therefore all of the changes in the slopes must be increased until the sum of the moments at the ends of the girders will balance the moments in the columns. This will make the moments at the ends of the girders practically the same as they would have been if the columns had not changed in length. If the moments at the ends of the girders are not materially affected, the moments at the ends of the columns will not be materially affected. Therefore although the direct stresses in the columns do change the lengths of the columns, they do not affect the stresses in the frame to any great extent.

28. *Assumption as to Length of Members.*—In accordance with assumption No. 3, the length of a member was taken as the distance between the center lines of the members which it intersects. This makes the changes in the slopes and deflections, as calculated, greater than the actual values. The effect of the inaccuracy of this assumption upon the distribution of the moments is, however, not so apparent.

The curves in Fig. 6, 7, and 8 show that the distribution of the moment in a story depends upon the relative values of K of the members, but that it takes a comparatively large change in the K of a member to appreciably affect the distribution of the moments. The fact that the length of a member has been taken equal to the distance between center lines, has but little effect upon the relative values of K for the members; and therefore does not materially affect the distribution of the moments.

29. *Assumption as to Deflection Due to Shear.*—According to assumption No. 4, the internal shearing stresses in a member produce no deflection. The distribution of the stresses is dependent, not upon the actual deflection of the columns due to shear, but upon the differences between the deflections in the different columns. The shears on the columns are small and the differences between the shears are still smaller; and therefore the assumption that the deflection due to shear is equal to zero will not cause any appreciable error.

30. *Assumption as to Load.*—According to assumption No. 5, the

entire wind load is resisted by the steel frame. The walls of a building no doubt help to resist the wind loads, but the resistance which they exert is uncertain. If a portion of the load is considered as being resisted by the walls the stresses in the steel frame are correspondingly reduced; but the method of determining the stresses is not affected.

XII. Conclusions.

As a result of the investigation described in this bulletin the following conclusions can be made relative to the methods used to determine the wind stresses in the steel frames of office buildings.

a. Methods II and III of Section II are so inaccurate that they should never be used; I and IV are quite accurate in some cases, but they may give results which are seriously in error.

b. The method presented in Sections VI and VII, and designated as the slope-deflection method, contains no approximations except those in the assumptions. It has been shown that the inaccuracies in the assumptions do not materially affect the results. Therefore the method is very accurate.

c. While the slope-deflection method is long, it could be used in the actual design of a building; but it has its greatest value as a standard by means of which the accuracy of the approximate methods may be determined.

d. The proposed approximate method is short; and, except at points where there are very large changes in the size of the members, gives results which are accurate enough to be used in the actual design of a building.

Table 11.

Properties of the Columns and Girders in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

	Story Number	Section of Member: Web Plate	Section of Member: 4 Angles	Section of Member: Cover Plate	Area of Section Sq. In.	Moment of Inertia—(Inches)⁴: Web	Moment of Inertia—(Inches)⁴: 4 Angles: Primary	Moment of Inertia—(Inches)⁴: 4 Angles: Secondary	Moment of Inertia—(Inches)⁴: Cover Plate	Moment of Inertia—(Inches)⁴: Total	Length of Columns $=h$ of Girders $=l$ Inches	K for Column $=\frac{I}{h}$ for Girder $=\frac{I}{l}$ (Inches
Column *A*	1	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x1$\frac{1}{4}$	112.80	358	318	2190	3950	6816	264	25.8
	2	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x1$\frac{1}{4}$	112.80	358	318	2190	3950	6816	192	35.6
	3 and 4	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x1	103.80	358	318	2190	3080	5946	168	35.4
	5 and 6	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x$\frac{11}{16}$	92.55	358	318	2190	2060	4926	168	29.4
	7 and 8	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x$\frac{7}{16}$	83.55	358	318	2190	1266	4132	144	28.7
	9 and 10	17x$\frac{15}{16}$	8x8x$\frac{15}{16}$		72.42	384	337	2315		3036	144	21.1
	11 and 12	17x$\frac{7}{8}$	8x8x$\frac{13}{16}$		64.24	358	299	2050		2707	144	18.8
	13 and 14	17x$\frac{3}{4}$	8x6x$\frac{13}{16}$		53.79	308	131	2195		2634	144	18.3
	15 and 16	17x$\frac{1}{2}$	8x6x$\frac{5}{8}$		42.26	205	105	1745		2055	144	14.3
	17 and above	17x$\frac{1}{2}$	8x6x$\frac{9}{16}$		38.86	205	96	1590		1891	144	13.1
Column *B*	1	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x1$\frac{1}{4}$	112.80	358	318	2190	3950	6816	264	25.8
	2	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x1$\frac{1}{4}$	112.80	358	318	2190	3950	6816	192	35.6
	3 and 4	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x1	103.80	358	318	2190	3080	5946	168	35.5
	5 and 6	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x$\frac{3}{4}$	94.80	358	318	2190	2240	5106	168	30.4
	7 and 8	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x$\frac{1}{2}$	85.80	358	318	2190	1459	4325	144	30.0
	9 and 10	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$	2–18x$\frac{5}{16}$	79.05	358	318	2190	892	3758	144	26.1
	11 and 12	17x$\frac{7}{8}$	8x8x$\frac{7}{8}$		67.80	358	318	2190		2866	144	19.9
	13 and 14	17x$\frac{7}{8}$	8x6x$\frac{13}{16}$		57.92	358	131	2195		2684	144	18.6
	15 and 16	17x$\frac{5}{8}$	8x6x$\frac{5}{8}$		44.39	256	105	1745		2106	144	14.6
	17 and above	17x$\frac{5}{8}$	8x6x$\frac{9}{16}$		41.00	256	96	1544		1896	144	13.2
Girders in Bay *a*	1	42x$\frac{3}{8}$	6.3$\frac{1}{4}$x$\frac{3}{8}$		29.43	2315	13	5730		8058	264	30.5
	2 and 3	36x$\frac{3}{8}$	6x3$\frac{1}{4}$x$\frac{3}{8}$		27.18	1458	13	4170		5641	264	21.4
	4 and 5	36x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		25.70	1458	13	3690		5161	264	19.5
	6	30x$\frac{3}{8}$	6x3$\frac{1}{4}$x$\frac{3}{8}$		24.93	844	13	2860		3717	264	14.1
	7	30x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		23.45	844	13	2530		3387	264	12.8
	8 and above	24x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		21.20	432	13	1580		2025	264	7.7
Girders in Bay *b*	1	42x$\frac{3}{8}$	6x3$\frac{1}{4}$x$\frac{3}{8}$		29.43	2315	13	5730		8058	216	37.3
	2	36x$\frac{3}{8}$	6x3$\frac{1}{4}$x$\frac{7}{16}$		29.38	1458	15	4830		6303	216	29.2
	3 and 4	36x$\frac{3}{8}$	6x3$\frac{1}{4}$x$\frac{3}{8}$		27.18	1458	13	4170		5641	216	26.2
	5	36x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		25.70	1458	13	3690		5161	216	23.8
	6	30x$\frac{3}{8}$	6x3$\frac{1}{4}$x$\frac{3}{8}$		24.93	844	13	2860		3717	216	17.2
	7	30x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		23.45	844	13	2530		3387	216	15.7
	8 and above	24x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		21.20	432	13	1580		2025	216	9 4

Table 12.

Numerical Values of the Constants in the Equations of Table 3 for the Symmetrical Three-span Twenty-story Bent shown in Fig. 5.

All quantities are expressed in inches^3

Story No.	Values of $K\left(=\frac{I}{h}\right)$ for		Values of $K\left(=\frac{I}{l}\right)$ for		Values of $J\left[=2\Sigma\left(\frac{I}{h}+\frac{I}{l}\right)\right]$ for all members intersecting at the		Values of $N\left(=2\Sigma K\right)$ for all Columns in a Story
	Column A	Column B	Girders in Bay a at Top of Story	Girders in Bay b at Top of Story	Top of Column A	Top of Column B	
1	25.8	25.8	30.5	37.3	183.8	258.4	206.8
2	35.6	35.6	21.4	29.2	184.8	243.4	284.8
3	35.4	35.5	21.4	26.2	184.4	237.2	283.6
4	35.4	35.5	19.5	26.2	168.6	223.2	283.6
5	29.4	30.4	19.5	23.8	156.6	208.2	239.2
6	29.4	30.4	14.1	17.2	144.4	183.4	239.2
7	28.7	30.0	12.8	15.7	140.4	177.0	234.8
8	28.7	30.0	7.7	9.4	115.0	146.4	234.8
9	21.1	26.1	7.7	9.4	99.8	138.6	188.8
10	21.1	26.1	7.7	9.4	95.2	126.2	188.8
11	18.8	19.9	7.7	9.4	90.6	113.8	154.8
12	18.8	19.9	7.7	9.4	89.6	111.2	154.8
13	18.3	18.6	7.7	9.4	88.6	108.6	147.6
14	18.3	18.6	7.7	9.4	80.6	100.6	147.6
15	14.3	14.6	7.7	9.4	72.6	92.6	115.6
16	14.3	14.6	7.7	9.4	70.2	89.8	115.6
17	13.1	13.2	7.7	9.4	67.8	87.0	105.2
18	13.1	13.2	7.7	9.4	67.8	87.0	105.2
19	13.1	13.2	7.7	9.4	67.8	87.0	105.2
20	13.1	13.2	7.7	9.4	41.6	60.6	105.2

Table 13.

General Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 1			Story No. 2			Story No. 3			Story No. 4			Story No. 5			
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	θ_{A3}	θ_{B3}	R_4	θ_{A4}	θ_{B4}	R_5	θ_{A5}	θ_{B5}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															Coefficient of .0001
A	—206.8	51.6	51.6													—117.2
B	— 77.4	183.8	30.5	—106.8	35.6											0.0
C	— 77.4	30.5	295.7	—106.8		35.6										0.0
D		71.2	71.2	—284.8	71.2	71.2										—78.9
E		35.6		—106.8	184.8	21.4	—106.2	35.4								0.0
F			35.6	—106.8	21.4	272.6	—106.5		35.5							0.0
G					70.8	71.0	—283.6	70.8	71.0							—64.5
H					35.4		—106.2	184.4	21.4	—106.2	35.4					0.0
I						35.5	—106.5	21.4	263.4	—106.5		35.5				0.0
J								70.8	71.0	—283.6	70.8	71.0				—60.5
K								35.4		—106.2	168.6	19.5	—88.2	29.4		0.0
L									35.5	—106.5	19.5	249.4	—91.2		30.4	0.0

TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation.	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 4			Story No. 5			Story No. 6			Story No. 7			Story No. 8			
	R_4	θ_{A4}	θ_{B4}	R_5	θ_{A5}	θ_{B5}	R_6	θ_{A6}	θ_{B6}	R_7	θ_{A7}	θ_{B7}	R_8	θ_{A8}	θ_{B8}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															Co-efficient of .0001
M		58.8	60.8	—239.2	58.8	60.8										—56.5
N		29.4		— 88.2	156.6	19.5	— 88.2	29.4								0.0
O			30.4	— 91.2	19.5	232.0	— 91.2		—30.4							0.0
P					58.8	60.8	—239.2	58.8	60.8							—52.5
Q					29.4		— 88.2	144.4	14.1	— 86.1	28.7					0.0
R						30.4	— 91.2	14.1	200.6	— 90.0		30.0				0.0
S								57.4	60.0	—234.8	57.4	60.0				—41.8
T								28.7		— 86.1	140.4	12.8	—86.1	28.7		0.0
U									30.0	— 90.0	12.8	192.7	—90.0		30.0	0.0

Table 13.—(Continued).

General Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 7			Story No. 8			Story No. 9			Story No. 10			Story No. 11			
	R_7	θ_{A7}	θ_{B7}	R_8	θ_{A8}	θ_{B8}	R_9	θ_{A9}	θ_{B9}	R_{10}	θ_{A10}	θ_{B10}	R_{11}	θ_{A11}	θ_{B11}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															Co-efficient of .0001
V		57.4	60.0	—234.8	57.4	60.0										—38.7
W		28.7		— 86.1	115.0	7.7	— 63.3	21.1								0.0
X			30.0	— 90.0	7.7	155.8	— 78.3		26.1							0.0
Y					42.2	52.2	—188.8	42.2	52.2							—35.8
Z					21.1		— 63.3	99.8	7.7	— 63.3	21.1					0.0
a						26.1	— 78.3	7.7	148.0	— 78.3		26.1				0.0
b								42.2	52.2	—188.8	42.2	52.2				—32.8
c								21.1		— 63.3	95.2	7.7	—56.4	18.8		0.0
d									26.1	— 78.3	7.7	135.6	—59.7		19.9	0.0

TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 10			Story No. 11			Story No. 12			Story No. 13			Story No. 14			
	R_{10}	θ_{A10}	θ_{B10}	R_{11}	θ_{A11}	θ_{B11}	R_{12}	θ_{A12}	θ_{B12}	R_{13}	θ_{A13}	θ_{B13}	R_{14}	θ_{A14}	θ_{B14}	Coefficient of .0001
	Coefficient of Unknown Slopes and Ratios of Deflection to Story Height															
e		37.6	39.8	—154.8	37.6	39.8										—29.8
f		18.8		— 56.4	90.6	7.7	— 56.4	18.8								0.0
g			19.9	— 59.7	7.7	123.2	— 59.7		19.9							0.0
h					37.6	39.8	—154.8	37.6	39.8	— 54.9	18.3					—26.8
i					18.8		— 56.4	89.6	7.7	— 55.8		18.6				0.0
j						19.9	— 59.7	7.7	120.6							0.0
k										—147.6	36.6	37.2				—23.8
l										— 54.9	88.6	7.7	—54.9	18.3		0.0
m										— 55.8	7.7	118.0	—55.8		18.6	0.0

TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 13			Story No. 14			Story No. 15			Story No. 16			Story No. 17			
	R_{13}	θ_{A13}	θ_{B13}	R_{14}	θ_{A14}	θ_{B14}	R_{15}	θ_{A15}	θ_{B15}	R_{16}	θ_{A16}	θ_{B16}	R_{17}	θ_{A17}	θ_{B17}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															Coefficient of .0001
n		36.6	37.2	—147.6	36.6	37.2										—20.9
o		18.3		— 54.9	80.6	7.7	— 42.9	14.3								0.0
p			18.6	— 55.8	7.7	110.0	— 43.8		14.6							0.0
q					28.6	29.2	—115.6	28.6	29.2							—17.9
r					14.3		— 42.9	72.6	7.7	— 42.9	14.3					0.0
s						14.6	— 43.8	7.7	102.0	— 43.8		14.6				0.0
t								28.6	29.2	—115.6	28.6	29.2				—14.9
u								14.3		— 42.9	70.2	7.7	—39.3	13.1		0.0
v									14.6	— 43.8	7.7	99.2	—39.6		13.2	0.0

TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 16			Story No. 17			Story No. 18			Story No. 19			Story No. 20			
	R_{16}	θ_{A16}	θ_{B16}	R_{17}	θ_{A17}	θ_{B17}	R_{18}	θ_{A18}	θ_{B18}	R_{19}	θ_{A19}	θ_{B19}	R_{20}	θ_{A20}	θ_{B20}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															Co-efficient of .0001
w		26.2	26.4	—105.2	26.2	26.4										
x		13.1		— 39.3	67.8	7.7	— 39.3	13.1								
y			13.2	— 39.6	7.7	96.4	— 39.6		13.2							—11.9
z					26.2	26.4										0.0
A′					13.1		—105.2	26.2	26.4							0.0
B′						13.2	— 39.3	67.8	7.7	— 39.3	13.1					
C′							— 39.6	7.7	96.4	— 39.6		13.2				—8.94
D′								26.2	26.4							0.0
E′								13.1		—105.2	26.2	26.4				0.0
F′									13.2	— 39.3	67.8	7.7	— 39.3	13.1		—5.96
G′										— 39.6	7.7	96.4	— 39.6		13.2	0.0
H′											26.2	26.4	—105.2	26.2	26.4	0.0
											13.1		— 39.3	41.6	7.7	—2.98
												13.2	— 39.6	7.7	70.0	0.0
																0.0

Table 14.

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 1			Story No. 2			Story No. 3			Story No. 4			
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	θ_{A3}	θ_{B3}	R_4	θ_{A4}	θ_{B4}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Coefficient of .0001
A	1.0000	—0.2495	—0.2495										0.5667
B	1.0000	—2.3747	—0.3940	1.3798	—0.4599								0.0000
C	1.0000	—0.3940	—3.8204	1.3798		—0.4599							0.0000
A–B	0	2.1252	0.1445	—1.3798	0.4599								0.5667
B–C	0	—1.9806	3.4263	0	—0.4599	0.4599							0.0000
1		1.0000	0.0680	—0.6494	0.2164								0.2667
2		1.0000	—1.7300		0.2322	—0.2322							0.0000
D		1.0000	1.0000	—4.0000	1.0000	1.0000							—1.1082
E		1.0000		—3.0000	5.1912	0.6012	—2.9834	0.9944					0.0000
1–2		0	1.7980	—0.6494	—0.0158	0.2322							0.2667
2–D		0	—2.7300	4.0000	0.7678	—1.2322							1.1082
D–E		0	1.0000	—1.0000	—4.1912	0.3988	2.9834	—0.9944					—1.1082
3			1	—0.3611	—0.0088	0.1292							0.1483
4			1	—1.4652	0.2812	0.4513							—0.4059
5			1	—1.0000	—4.1912	0.3988	2.9834	—0.9941					—1.1082
F			1	—3.0000	0.6012	7.6575	—2.9915		0.9972				0.0000
3–4			0	1.1040	—0.2900	—0.3222							0.5543
4–5			0	—0.4652	4.4724	0.0525	—2.9834	0.9944					0.7023
5–F			0	2.0000	—4.7924	—7.2587	5.9749	—0.9944	—0.9972				—1.1082
6				1	—0.2627	—0.2918							0.5021
7				1	—9.6140	—0.1128	6.4127	—2.1378					—1.5095
8				1	—2.3962	—3.6293	2.9874	—0.4972	—0.4986				—0.5541
6–7				0	9.3513	—0.1790	—6.4127	2.1378					2.0116
7–8				0	—7.2178	3.5165	3.4252	—1.6406	0.4986				—0.9554
9					1	—0.0191	—0.6857	0.2286					0.2151
10					1	—0.4872	—0.4746	0.2273	—0.0691				0.1324
G					1	1.0028	—4.0055	1.0000	1.0028				—0.9110
H					1		—3.0000	5.2091	0.6045	—3.0000	1.0000		0.0000
9–10					0	0.4681	—0.2111	0.0013	0.0691				0.0827
10–G					0	—1.4900	3.5309	—0.7727	—1.0719				1.0434
G–H					0	1.0028	—1.0055	—4.2091	0.3983	3.0000	—1.0000		—0.9110
11						1	—0.4511	0.0027	0.1476				0.1768
12						1	—2.3700	0.5186	0.7194				—0.6990
13						1	—1.0028	—4.1971	0.3971	2.9915	—0.9972		—0.9084
I						1	—3.0000	0.6028	7.4200	—3.0000		1.0000	0.0000
11–12						0	1.0189	—0.5159	—0.5718				0.8758
12–13						0	—1.3672	4.7157	0.3223	—2.9915	0.9972		0.2094
13–I						0	1.9972	—4.7999	—7.0230	5.9915	—0.9972	—1.0000	—0.9084

Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 3			Story No. 4			Story No. 5			Story No. 6			
	R_3	θ_{A3}	θ_{B3}	R_4	θ_{A4}	θ_{B4}	R_5	θ_{A5}	θ_{B5}	R_6	θ_{A6}	θ_{B6}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												
14	1	—0.2689	—0.2980										0.4564
15	1	—3.4490	—0.2357	2.1881	—0.7294								—0.1532
16	1	—2.4033	—3.5179	2.9999	—0.4993	—0.5007							—0.4548
14-15	0	3.1802	—0.0623	—2.1881	0.7294								0.6096
14-16	0	2.1344	3.2199	—2.9999	0.4993	0.5007							0.9112
17		1	—0.0196	—0.6881	0.2294								0.1917
18		1	1.5086	—1.4057	0.2339	0.2346							0.4270
J		1	1.0028	—4.0057	1.0000	1.0028							—0.8546
K		1		—3.0000	4.7628	0.5508	—2.4914	0.8302					0 0000
17-18		0	—1.5282	0.7176	—0.0046	—0.2346							—0.2353
18-K		0	1.5086	1.5943	—4 5290	—0.3162	2.4914	—0.8302					0.4270
J-K		0	1.0028	—1.0057	—3.7628	0.4520	2.4914	—0.8302					—0.8546
19			1	—0.4696	0.0030	0.1535							0.1540
20			1	1.0568	—3.0019	—0.2096	1.6514	—0.5503					0 2830
21			1	—1.0028	—3.7520	0.4507	2.4842	—0.8279					—0.8521
L			1	—3.0000	0.5496	7.0255	—2.5691		0.8564				0.0000
19-20			0	—1.5264	3.0049	0.3631	—1.6514	0.5503					—0.1290
20-21			0	2.0596	0.7501	—0.6603	—0.8328	0.2776					1.1352
20-L			0	4.0568	—3.5515	—7.2351	4.2205	—0.5503	—0.8564				0.2830
22				1	—1.9687	—0.2379	1.0819	—0.3605					0.0846
23				1	0.3642	—0.3206	—0.4044	0.1348					0.5512
24				1	—0.8754	—1.7836	1.0404	—0.1357	—0.2111				0.0697
22-23				0	—2.3329	0.0827	1.4863	—0.4953					—0.4666
23-24				0	1.2397	1.4630	—1.4448	0.2705	0.2111				0.4815
25					1	—0.0355	—0.6371	0.2123					0.2000
26					1	1.1803	—1.1657	0.2182	0.1703				0.3884
M					1	1.0340	—4.0684	1.0000	1.0340				—0.9609
N					1		—3 0000	5.3260	0.6632	—3.0000	1.0000		0.0000
25-26					0	—1.2158	0.5286	—0.0059	—0.1703				—0.1884
25-M					0	—1.0695	3.4313	—0.7877	—1.0340				1.1609
M-N					0	1.0340	—1.0684	—4.3260	0.3708	3.0000	—1.0000		—0.9609
27						1	—0.4348	0.0048	0.1401				0.1550
28						1	—3.2083	0.7365	0.9668				—1.0855
29						1	—1.0333	—4.1842	0.3586	2.9016	—0.9672		—0.9294
O						1	—3 0000	—0.6415	7.6321	—3.0000		1.0000	0.0000
27-28						0	2.7735	—0.7316	—0.8267				1.2405
28-29						0	—2.1750	4.9207	0.6082	—2.9016	0.9672		—0.1561
29-O						0	1.9667	—4.8257	—7.2735	5.9016	—0.9672	—1.0000	—0.9294
30							1.	—0.2638	—0.2981				0.4473
31							1	—2.2623	—0.2796	1.3341	—0.4447		0.0718
32							1	—2.4540	—3.6986	3.0004	—0.4918	—0.5085	—0.4726
30-31							0	1.9985	—0.0185	—1.3341	0.4447		0.3755
30-32							0	2.1902	3.4005	—3.0004	0.4918	0.5085	0.9199

Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation.												Right-Hand Member of Equation
	Story No. 5			Story No. 6			Story No. 7			Story No. 8			
	R_5	θ_{A5}	θ_{B5}	R_6	θ_{A6}	θ_{B6}	R_7	θ_{A7}	θ_{B7}	R_8	θ_{A8}	θ_{B8}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height.												
33		1	—0.0092	—0.6676	0.2225								0.1879
34		1	1.5526	—1.3698	0.2246	0.2322							0.4200
P		1	1.0341	—4.0682	1.0000	1.0341							—0.8929
Q		1		—3.0000	4.9120	0.4796	—2.9285	0.9762					0.0000
33-34		0	—1.5618	0.7022	—0.0020	—0.2322							—0.2321
33-P		0	—1.0433	3.4006	—0 7775	—1.0341							1.0808
P-Q		0	1.0341	—1.0682	—3.9120	0.5545	2.9285	—0.9762					—0.8929
35			1	—0.4497	0.0013	0.1487							0.1486
36			1	—3.2593	0.7452	0.9911							—1.0359
37			1	—1.0330	—3.7818	0.5362	2.8320	—0.9440					—0.8634
R			1	—3.0000	0.4638	6.5993	—2.9606		0.9870				0.0000
35-36			0	2.8097	—0.7439	—0.8425							1.1845
36-37			0	—2.2263	4.5270	0.4549	—2.8320	0.9440					—0.1724
37-R			0	1.9670	—4.2456	—6.0631	5.7926	—0.9440	—0.9870				—0.8634
38				1	—0.2648	—0.2998							0.4216
39				1	—2.0335	—0.2044	1.2721	—0.4240					0.0775
40				1	—2.1586	—3.0827	2.9451	—0.4799	—0.5018				—0.4390
38-39				0	1.7687	—0.0955	—1.2721	0.4240					0.3441
38-40				0	1.8938	2.7828	—2.9451	0.4799	0.5018				0.8606
41					1	—0.0540	—0.7192	0.2397					0.1946
42					1	1.4695	—1.5553	0.2534	0.2650				0.4544
S					1	1.0453	—4.0909	1.0000	1.0453				—0.7281
T					1		—3.0000	4.8920	0.4460	—3.0000	1.0000		0.0000
41-42					0	—1.5235	0.8361	—0.0137	—0.2650				—0.2599
41-S					0	—1.0993	3.3717	—0.7603	—1.0453				0.9227
S-T					0	1.0453	—1.0909	—3.8920	0.5993	3.0000	—1.0000		—0.7281
43						1	—0.5489	0.0090	0.1739				0.1706
44						1	—3.0672	0.6916	0.9510				—0.8394
45						1	—1.0436	—3.7234	0.5733	2.8700	—0.9567		0.6964
U						1	—3.0000	0.4267	6.4233	—3.0000		1.0000	0.0000
43-44						0	2.5183	—0.6826	—0.7781				1.0100
44-45						0	—2.0236	4.4150	0.3777	—2.8700	0.9567		—0.1430
45-U						0	1.9564	—4.1501	—5.8500	5.8700	—0.9567	—1.0000	—0.6964
46							1	—0.2711	—0.3090				0.4011
47							1	—2.1818	—0.1867	1.4184	—0.4728		0.0706
48							1	—2.1213	—2.9903	3.0006	—0.4890	—0.5111	—0.3560
46-47							0	1.9108	—0.1223	—1.4184	0.4728		0.3304
46-48							0	1.8502	2.6813	—3.0006	0.4890	0.5111	0.7570

Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 7			Story No. 8			Story No. 9			Story No. 10			
	R_7	θ_{A7}	θ_{B7}	R_8	θ_{A8}	θ_{B8}	R_9	θ_{A9}	θ_{B9}	R_{10}	θ_{A10}	θ_{B10}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Coefficient of .0001
49		1	—0.0640	—0.7422	0.2474								0.1729
50		1	1.4491	—1.6219	0.2643	0.2763							0.4093
V		1	1.0453	—4.0909	1.0000	1.0453							—0.6741
W		1		—3.0000	4.0073	0.2683	—2.2056	0.7352					0.0000
49–50		0	—1.5131	0.8797	—0.0169	—0.2763							—0.2363
49–V		0	—1.1093	3.3487	—0.7526	—1.0453							0.8470
V–W		0	1.0453	—1.0909	—3.0073	—0.7770	2.2056	—0.7352					—0.6741
51			1	—0.5814	0.0112	0.1826							0.1562
52			1	—3.0186	0.6784	0.9423							—0.7636
53			1	—1.0436	—2.8769	0.7433	2.1100	—0.7036					—0.6449
X			1	—3.0000	0.2567	5.1933	—2.6100		0.8700				0.0000
51–52			0	2.4372	—0.6672	—0.7598							0.9198
52–53			0	—1.9750	3.5553	0.1990	—2.1100	0.7036					—0.1187
53–X			0	1.9564	—3.1336	—4.4500	4.7200	—0.7036	—0.8700				—0.6449
54				1	—0.2738	—0.3117							0.3774
55				1	—1.8002	—0.1008	1.0685	—0.3563					0.0601
56				1	—1 6019	—2.2747	2.4127	—0.3597	—0.4447				—0.3296
54–55				0	1.5265	—0.2110	—1.0685	0.3563					0.3173
54–56				0	1.3281	1.9630	—2.4127	0.3597	0.4447				0.7070
57					1	—0.1382	—0.7000	0.2334					0.2079
58					1	1.4779	—1.8165	0.2708	0.3348				0.5323
Y					1	1.2371	—4.4743	1.0000	1.2371				—0.8484
Z					1		—3.0000	4.7299	0.3649	—3.0000	1.0000		0.0000
57–58					0	—1.6161	1.1165	—0.0374	—0.3348				—0.3245
57–Y					0	—1.3753	3.7743	—0.7666	—1.2371				1.0562
Y–Z					0	1.2371	—1.4743	—3.7299	0.8722	3.0000	—1.0000		—0.8484
59						1	—0.6908	0.0231	0.2072				0.2008
60						1	—2.7443	0.5574	0.8995				—0.7681
61						1	—1.1918	—3.0151	0.7051	2.4253	—0.8084		—0.6859
a						1	—3.0000	0.2950	5.6705	—3.0000		1.0000	0.0000
59–60						0	2.0535	—0.5342	—0.6924				0.9689
60–61						0	—1.5525	3.5725	0.1945	—2.4253	0.8084		—0.0822
61–a						0	1.8082	—3.3102	—4.9654	5.4253	—0.8084	—1.0000	—0.6859
62							1	—0.2602	—0.3372				0.4719
63							1	—2.3013	—0.1253	1.5624	—0.5207		0.0529
64							1	—1.8308	—2.7462	3.0000	—0.4471	—0.5531	—0.3793
62–63							0	2.0411	—0.2119	—1.5624	0.5207		0.4189
62–64							0	1.5706	2.4090	—3.0000	0.4471	0.5531	0.8512

Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 9			Story No. 10			Story No. 11			Story No. 12			
	R_9	θ_{A9}	θ_{B9}	R_{10}	θ_{A10}	θ_{B10}	R_{11}	θ_{A11}	θ_{B11}	R_{12}	θ_{A12}	θ_{B12}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Coefficient of .0001
65		1	—0.1038	—0.7655	0.2551								0.2053
66		1	1.5340	—1.9103	0.2847	0.3522							0.5420
b		1	1.2371	—4.4743	1.0000	1.2371							—0.7771
c		1		—3.0000	4.5120	0.3649	—2.6730	0.8911					0.0000
65-66		0	—1.6378	1.1448	—0.0296	—0.3522							—0.3367
65-b		0	—1.3409	3.7088	—0.7449	—1.2371							0.9824
b-c		0	1.2371	—1.4743	—3.5120	0.8722	2.6730	—0.8911					—0.7771
67			1	—0.6991	0.0180	0.2151							0.2056
68			1	—2.7660	0.5555	0.9226							—0.7327
69			1	—1.1918	—2.8392	0.7050	2.1610	—0.7204					—0.6283
d			1	—3.0000	0.2950	5.1954	—2.2873		0.7624				0.0000
67-68			0	2.0669	—0.5375	—0.7075							0.9383
68-69			0	—1.5742	3.3947	0.2176	—2.1610	0.7204					—0.1044
69-d			0	1.8082	—3.1342	—4.4904	4.4483	—0.7204	—0.7624				—0.6283
70				1	—0.2601	—0.3423							0.4540
71				1	—2.1566	—0.1382	1.3728	—0.4576					0.0663
72				1	—1.7333	—2.4833	2.4602	—0.3984	—0.4216				—0.3474
70-71				0	1.8865	—0.2041	—1.3728	0.4576					0.3877
70-72				0	1.4732	2.1310	—2.4602	0.3984	0.4216				0.8014
73					1	—0.1082	—0.7277	0.2426					0.2055
74					1	1.4464	—1.6700	0.2705	0.2862				0.5441
e					1	1.0585	—4.1170	1.0000	1.0585				—0.7926
f					1		—3.0000	4.8195	0.4096	—3.0000	1.0000		0.0000
73-74					0	—1.5547	0.9422	—0.0278	—0.2862				—0.3385
73-e					0	—1.1668	3.3893	—0.7574	—1.0585				0.9981
e-f					0	1.0585	—1.1170	—3.8195	0.6499	3.0000	—1.0000		—0.7926
75						1	—0.6061	0.0179	0.1841				0.2178
76						1	—2.9051	0.6492	0.9076				—0.8555
77						1	—1.0553	—3.6083	0.6140	2.8343	—0.9449		—0.7488
g						1	—3.0000	0.3869	6.1913	—3.0000		1.0000	0.0000
75-76						0	2.2990	—0.6313	—0.7235				1.0733
76-77						0	—1.8498	4.2575	0.2936	—2.8343	0.9449		—0.1067
77-g						0	1.9447	—3.9952	—5.5773	5.8343	—0.9449	—1.0000	—0.7488
78							1	—0.2746	—0.3147				0.4669
79							1	—2.3018	—0.1587	1.5323	—0.5109		0.0577
80							1	—2.0547	—2.8682	3.0003	—0.4859	—0.5143	—0.3851
78-79							0	2.0272	—0.1560	—1.5323	0.5109		0.4092
78-80							0	1.7801	2.5535	—3.0000	0.4859	0.5143	0.8519

Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 11			Story No. 12			Story No. 13			Story No. 14			
	R_{11}	θ_{A11}	θ_{B11}	R_{12}	θ_{A12}	θ_{B12}	R_{13}	θ_{A13}	θ_{B13}	R_{14}	θ_{A14}	θ_{B14}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Coefficient of .0001
81		1	—0.0769	—0.7559	0.2520								0.2018
82		1	1.4344	—1.6854	0.2730	0.2889							0.4786
h		1	1.0585	—4.1170	1.0000	1.0585							—0.7128
i		1		—3.0000	4.7660	0.4096	—2.9205	0.9734					0.0000
81-82		0	—1.5113	0.9294	—0.0210	—0.2889							—0.2768
81-h		0	—1.1355	3.3611	—0.7480	—1.0585							0.9147
h-i		0	1.0585	—1.1170	—3.7660	0.6489	2.9205	—0.9734					—0.7128
83			1	—0.6150	0.0139	0.1911							0.1831
84			1	—2.9602	0.6587	0.9323							—0.8055
85			1	—1.0553	—3.5583	0.6131	2.7592	—0.9197					—0.6734
j			1	—3.0000	0.3869	6 0609	—2.8040		0.9347				0.0000
83-84			0	2.3452	—0.6449	—0.7412							0.9886
84-85			0	—1.9049	4.2170	0.3192	—2.7592	0.9197					—0.1321
85-j			0	1.9447	—3.9452	—5.4478	5.5632	—0.9197	—0.9347				—0.6734
86				1	—0.2750	—0.3160							0.4216
87				1	—2.2137	—0.1676	1.4484	—0.4828					0.0693
88				1	—2.0288	—2.8015	2.8609	—0.4730	—0.4807				—0.3463
86-87				0	1.9387	—0.1485	—1.4484	0.4828					0.3522
86-88				0	1.7539	2.4855	—2.8609	0.4730	0.4807				0.7679
89					1	—0.0766	—0.7471	0.2490					0.1816
90					1	1.4173	—1.6314	0.2697	0.2741				0.4378
k					1	1.0164	—4.0329	1.0000	1.0164				—0.6503
l					1		—3.0000	4.8417	0.4208	—3.0000	1.0000		0.0000
89-90					0	—1.4939	0.8842	—0.0207	—0.2741				—0.2562
89-k					0	—1.0930	3.2857	—0.7510	—1.0164				0.8320
k-l					0	1.0164	—1.0329	—3.8417	0.5956	3.0000	—1.0000		—0.6503
91						1	—0.5920	0.0138	0.1835				0.1715
92						1	—3.0063	0 6871	0.9299				—0.7612
93						1	—1.0163	—3.7802	0.5861	2.9520	—0.9840		—0.6389
m						1	—3.0000	0.4140	6.3443	—3.0000		1.0000	0.0000
91-92						0	2.4143	—0.6733	—0.7464				0.9327
92-93						0	—1.9900	4.4673	0.3438	—2.9520	0.9840		—0.1213
93-m						0	1.9837	—4.1942	—5.7582	5.9520	—0.9840	—1.0000	—0.6399
94							1	—0.2789	—0.3092				0.3863
95							1	—2.2413	—0.1728	1.4834	—0.4945		0.0610
96							1	—2.1143	—2.9030	3.0000	—0.4961	—0.5042	—0.3226
94-95							0	1.9624	—0.1364	—1.4834	0.4945		0.3254
94-96							0	1.8354	2.5938	—3.0000	0.4961	0.5042	0.7089

Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 13			Story No. 14			Story No. 15			Story No. 16			
	R_{13}	θ_{A13}	θ_{B13}	R_{14}	θ_{A14}	θ_{B14}	R_{15}	θ_{A15}	θ_{B15}	R_{16}	θ_{A16}	θ_{B16}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Coefficient of .0001
97		1	—0.0695	—0.7559	0.2520								0.1658
98		1	1.4135	—1.6349	0.2704	0.2747							0.3863
n		1	1.0164	—4.0329	1.0000	1.0164							—0.5711
o		1		—3.0000	4.4044	0.4208	—2.3443	0.7814					0.0000
97-98		0	—1.4830	0.8790	—0.0184	—0.2747							—0.2205
97-n		0	—1.0859	3.2769	—0.7480	—1.0164							0.7369
n-o		0	1.0164	—1.0329	—3.4044	0.5956	2.3443	—0.7814					—0.5711
99			1	—0.5928	0.0124	0.1853							0.1487
100			1	—3.0177	0.6888	0.9360							—0.6786
101			1	—1.0163	—3.3497	0.5860	2.3067	—0.7689					—0.5619
p			1	—3.0000	0.4140	5.9148	—2.3551		0.7850				0.0000
99-100			0	2.4249	—0.6765	—0.7507							0.8274
100-101			0	—2.0014	4.0385	0.3499	—2.3067	0.7689					—0.1167
101-p			0	1.9837	—3.7637	—5.3287	4.6618	—0.7689	—0.7850				—0.5619
102				1	—0.2790	—0.3096							0.3412
103				1	—2.0179	—0.1748	1.1525	—0.3842					0.0583
104				1	—1.8974	—2.6867	2.3505	—0.3877	—0.3953				—0.2833
102-103				0	1.7389	—0.1348	—1.1525	0.3842					0.2829
102-104				0	1.6184	2.3771	—2.3505	0.3877	0.3958				0.6246
105					1	—0.0775	—0.6624	0.2209					0.1627
106					1	1.4690	—1.4525	0.2396	0.2446				0.3859
q					1	1.0210	—4.0419	1.0000	1.0210				—0.6258
r					1		—3.0000	5.0771	0.5385	—3.0000	1.0000		0.0000
105-106					0	—1.5465	0.7901	—0.0186	—0.2446				—0.2232
105-q					0	—1.0985	3.3795	—0.7790	—1.0210				0.7886
q-r					0	1.0210	—1.0419	—4.0771	0.4825	3.0000	—1.0000		—0.6258
107						1	—0.5109	0.0120	0.1581				0.1444
108						1	—3.0762	0.7092	0.9294				—0.7178
109						1	—1.0206	—3.9933	0.4726	2.9383	—0.9795		—0.6130
s						1	—3.0000	0.5274	6.9832	—3.0000		1.0000	0.0000
107-108						0	2.5653	—0.6972	—0.7713				0.8622
108-109						0	—2.0556	4.7025	0.4568	—2.9383	0.9795		—0.1048
109-s						0	1.9794	—4.5207	—6.5106	5.9383	—0.9795	—1.0000	—0.6130
110							1	—0.2718	—0.3006				0.3361
111							1	—2.2875	—0.2222	1.4294	—0.4765		0.0510
112							1	—2.2836	—3.2888	3.0000	—0.4946	—0.5052	—0.3097
110-111							0	2.0157	—0.0784	—1.4294	0.4765		0.2851
111-112							0	2.0118	2.9882	—3.0000	0.4946	5052	0.6457

Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 15			Story No. 16			Story No. 17			Story No. 18			
	R_{15}	θ_{A15}	θ_{B15}	R_{16}	θ_{A16}	θ_{B16}	R_{17}	θ_{A17}	θ_{B17}	R_{18}	θ_{A18}	θ_{B18}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Coefficient of .0001
113		1	—0.0389	—0.7091	0.2364								0.1414
114		1	1.4854	—1.4913	0.2459	0.2511							0.3210
t		1	1.0210	—4.0419	1.0000	1.0210							—0.5209
u		1		—3.0000	4.9092	0.5385	—2.7480	0.9161					0.0000
113–114		0	—1.5243	0.7822	—0.0095	—0.2511							—0.1796
113–t		0	—1.0599	3.3328	—0.7636	—1.0210							0.6624
t–u		0	1.0210	—1.0419	—3.9092	0.4825	2.7480	—0.9161					—0.5209
115			1	—0.5132	0.0062	0.1647							0.1178
116			1	—3.1444	0.7204	0.9633							—0.6249
117			1	—1.0206	—3.8290	0.4726	2.6916	—0.8973					—0.5102
v			1	—3.0000	0.5281	6.8055	—2.7161		0.9054				0 0000
115–116			0	2.6312	—0.7142	—0.7986							0.7427
116–117			0	—2.1238	4.5494	0.4907	—2.6916	0.8973					—0.1147
117–v			0	1.9794	—4.3571	—6.3329	5.4077	—0.8973	—0.9054				—0.5102
118				1	—0.2714	—0.3035							0.2823
119				1	—2.1422	—0.2310	1.2673	—0.4225					0.0540
120				1	—2.2013	—3.1993	2.7318	—0.4533	—0.4574				—0.2579
118–119				0	1.8708	—0.0724	—1.2673	0.4225					0.2282
118–120				0	1.9299	2.8958	—2.7318	0.4533	0.4574				0.5399
121					1	—0.0387	—0.6774	0.2259					0.1220
122					1	1.5027	—1.4177	0.2353	0.2374				0.2802
w					1	1.0092	—4.0214	1.0000	1.0092				—0.4542
x					1		—3.0000	5.1753	0.5878	—3.0000	1.0000		0.0000
121–122					0	—1.5414	0.7403	—0.0094	—0.2374				—0.1582
121–w					0	—1.0479	3.3440	—0.7741	—1.0092				0.5762
w–x					0	1.0092	—1.0214	—4.1755	0.4214	3.0000	—1.0000		—0.4542
123						1	—0.4810	0.0061	0.1542				0.1028
124						1	—3.1958	0.7398	0.9630				—0.5507
125						1	—1.0136	—4.1437	0.4182	2.9773	—0.9924		—0.4507
y						1	—3.0000	0.5833	7.3021	—3.0000		1.0000	0.0000
123–124						0	2.7148	—0.7337	—0.8088				0.6535
124–125						0	—2.1822	4.8835	0.5448	—2.9773	0.9924		—0.0999
125–y						0	1.9864	—4.7270	—6.8839	5.9773	—0.9924	—1.0000	—0.4507
126							1	—0.2703	—0.2979				0.2407
127							1	—2.2380	—0.2496	1.3644	—0.4548		0.0458
128							1	—2.3797	—3.4655	3.0093	—0.4996	—0.5035	—0.2269
126–127							0	1.9677	—0.0483	—1.3644	0.4548		0.1949
126–128							0	2.1094	3.1676	—3.0093	0.4996	0.5035	0.4677

TABLE 14.—(Continued).

ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 17			Story No. 18			Story No. 19			Story No. 20			
	R_{17}	θ_{A17}	θ_{B17}	R_{18}	θ_{A18}	θ_{B18}	R_{19}	θ_{A19}	θ_{B19}	R_{20}	θ_{A20}	θ_{B20}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Coefficient of .0001
129		1	—0.0245	—0.6935	0.2311								0.0991
130		1	1.5017	—1.4268	0.2369	0.2387							0.2217
z		1	1.0092	—4.0214	1.0000	1.0092							—0.3412
A′		1		—3.0000	5.1755	0.5878	—3.0000	1.0000					0.0000
129–130		0	—1.5262	0.7333	—0.0057	—0.2387							—0.1226
129–z		0	—1.0337	3.3279	—0.7689	—1.0092							0.4403
z–A′		0	1.0092	—1.0214	—4.1755	0.4214	3.0000	—1.0000					—0.3412
131			1	—0.4805	0.0037	0.1564							0.0804
132			1	—3.2197	0.7438	0.9764							—0.4260
133			1	—1.0120	—4.1373	0.4176	2.9727	—0.9909					—0.3381
B′			1	—3.0000	0.5833	7.3021	—3.0000		1.0000				0.0000
131–132			0	2.7392	—0.7401	—0.8200							0.5063
132–133			0	—2.2077	4.8811	0.5588	—2.9727	0.9909					—0.0879
133–B′			0	1.9880	—4.7206	—6.8846	5.9727	—0.9909	—1.0000				—0.3381
134				1	—0.2702	—0.2993							0.1849
135				1	—2.2110	—0.2531	1.3465	—0.4485					0.0398
136				1	—2.3745	—3.4629	3.0042	—0.4980	—0.5030				—0.1701
134–135				0	1.9408	—0.0462	—1.3465	0.4485					0.1451
134–136				0	2.1043	3.1635	—3.0042	0.4980	0.5030				0.3549
137					1	—0.0238	—0.6938	0.2311					0.0747
138					1	1.5034	—1.4277	0.2367	0.2390				0.1687
C′					1	1.0092	—4.0214	1.0000	1.0092				—0.2275
D′					1		—3.0000	5.1755	0.5878	—3.0000	1.0000		0.0000
137–138					0	—1.5272	0.7339	—0.0056	—0.2390				—0.0939
137–C′					0	—1.0330	3.3276	—0.7689	—1.0092				0.3022
C′–D′					0	1.0092	—1.0214	—4.1755	0.4214	3.0000	—1.0000		—0.2275
139						1	—0.4806	0.0036	0.1565				0.0615
140						1	—3.2214	0.7444	0.9769				—0.2926
141						1	—1.0121	—4.1377	0.4176	2.9727	—0.9909		—0.2254
E′						1	—3.0000	0.5833	7.3021	—3.0000		1.0000	0.0000
139–140						0	2.7408	—0.7407	—0.8204				0.3541
140–141						0	—2.2093	4.8821	0.5594	—2.9727	0.9909		—0.0672
141–E′						0	1.9879	—4.7210	—6.8846	5.9727	—0.9909	—1.0000	—0.2254
142							1	—0 2703	—0.2993				0.1292
143							1	—2.2099	—0.2532	1.3455	—0.4485		0.0304
144							1	—2.3749	—3.4632	3.0046	—0.4985	—0.5031	—0.1134
142–143							0	1.9396	—0.0461	—1.3455	0.4485		0.0988
142–144							0	2.1047	3.1639	—3.0046	0.4985	0.5031	0.2426
145								1	—0.0238	—0.6938	0.2313		0.0509
146								1	1.5034	—1.4278	0.2369	0.2390	0.1153
F′								1	1.0092	—4.0214	1.0000	1.0092	—0.1137
G′								1		—3.0000	3.1759	0.5878	0.0000
145–146								0	—1.5272	0.7340	—0.0056	—0.2390	—0.0643
145–F′								0	—1.0330	3.3276	—0.7687	—1.0092	0.1647
F′–G′								0	1.0092	—1.0214	—2.1759	0.4214	—0.1137
147									1	—0.4806	0.0037	0.1565	0.0421
148									1	—3.2217	0.7442	0.9770	—0.1594
149									1	—1.0122	—2.1563	0.4176	—0.1127
H′									1	—3.0000	0.5834	5.3033	0.0000

Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation							Right-Hand Member of Equation
	Story No. 19			Story No. 20				
	R_{19}	θ_{A19}	θ_{B19}	R_{20}	θ_{A20}	θ_{B20}		
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height							Coefficient of .0001
147–148			0	2.7411	—0.7406	—0.8205		0.2015
148–149			0	—2.2095	2.9005	0.5595		—0.0467
149–H′			0	1.9878	—2.7397	—4.8857		—0.1127
150				1	—0.2702	—0.2993		0.0735
151				1	—1.3129	—0.2531		0.0211
152				1	—1.3783	—2.4580		—0.0567
150–151				0	1.0427	—0.0462		0.0524
150–152				0	1.1081	2.1587		0.1302
153					1	—0.0443		0.0502
154					1	1.9480		0.1175
153–154						—1.9923		—0.0673
155						1.0000		0.0338

Table 15.

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

The equations are taken from Table 14.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
155		θ_{B20}	=		.0338
153	$\theta_{A20} - .0443\,\theta_{B20}$		=	.0502	
		.0443 x .00000338	=	.0015	
		θ_{A20}	=		.0517
150	$R_{20} - .2702\,\theta_{A20} - .2993\,\theta_{B20}$		=	.0735	
		.2702 x .00000517	=	.0140	
		.2993 x .00000338	=	.0101	
		R_{20}	=		.0976
147	$\theta_{B19} - .4806\,R_{20} + .0037\,\theta_{A20} + .1565\,\theta_{B20}$		=	.0421	
		.4806 x .00000976	=	0469	.0890
		.0037 x .00000517	=	.0002	
		1565 x .00000338	=	.0053	—.0055
		θ_{B19}	=		.0835
145	$\theta_{A19} - .0238\,\theta_{B19} - .6938\,R_{20} + .2313\,\theta_{A20}$		=	.0509	
		.0238 x .00000835	=	.0020	
		.6938 x .00000976	=	.0676	.1205
		.2313 x .00000517	=		—.0120
		θ_{A19}	=		.1085
142	$R_{19} - .2703\,\theta_{A19} - .2993\,\theta_{B19}$		=	.1292	
		.2703 x .00001085	=	.0294	
		.2993 x .00000835	=	.0250	
		R_{19}	=		.1836
139	$\theta_{B18} - .4806\,R_{19} + .0037\,\theta_{A19} + .1566\,\theta_{B19}$		=	.0615	
		.4806 x .00001836	=	.0881	.1496
		.0037 x .00001085	=	0004	
		.1565 x .00000835	=	.0131	—.0135
		θ_{B18}	=		.1361
137	$\theta_{A18} - .0238\,\theta_{B18} - .6938\,R_{19} + .2311\,\theta_{A19}$		=	.0748	
		.0238 x .00001361	=	.0032	
		.6938 x .00001836	=	.1272	.2052
		.2311 x .00001085	=		—.0251
		θ_{A18}	=		.1801
134	$R_{18} - .2702\,\theta_{A18} - .2994\,\theta_{B18}$		=	.1849	
		.2702 x .00001801	=	.0486	
		.2994 x .00001361	=	0407	
		R_{18}	=		.2742

Table 15.—(Continued).

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001
131	$\theta_{B17} - .4805 R_{18} + .0038 \theta_{A18} + .1564 \theta_{B18}$		=	.0804
	.4805 x .00002742		=	.1319 .2123
	.0038 x .00001801		=	.0007
	.1564 x .00001361		=	.0213 — .0220
		θ_{B17}	=	.1903
129	$\theta_{A17} - .0246 \theta_{B17} - .6935 R_{18} + .2311 \theta_{A18}$		=	.0991
	.0246 x .00001903		=	.0047
	.6935 x .00002442		=	.1905 .2943
	.2311 x .00001801		=	—.0416
		θ_{A17}	=	.2527
126	$R_{17} - .2703 \theta_{A17} - .2979 \theta_{B17}$		=	.2407
	.2703 x .00002527		=	.0682
	.2979 x .00001903		=	.0566
		R_{17}	=	.3655
123	$\theta_{B16} - .4810 R_{17} + .0061 \theta_{A17} + .1542 \theta_{B17}$		=	.1028
	.4810 x .00003655		=	.1758 .2786
	.0061 x .00002527		=	.0015
	.1542 x .00001903		=	.0294 —.0309
		θ_{B16}	=	.2477
121	$\theta_{A16} - .0387 \theta_{B16} - .6774 R_{17} + .2259 \theta_{A17}$		—	.1220
	.0387 x .00002477		=	.0096
	.6774 x .00003655		=	.2473 .3789
	.2259 x .00002527		=	—.0570
		θ_{A16}	—	.3219
118	$R_{16} - .2714 \theta_{A16} - .3035 \theta_{B16}$		=	.2823
	.2714 x .00003219		=	.0873
	.3035 x .00002477		=	.0752
		R_{16}	=	.4448
115	$\theta_{B15} - .5132 R_{16} + .0062 \theta_{A16} + .1647 \theta_{B16}$		=	.1178
	.5132 x .00004448		=	.2280 .3458
	.0062 x .00003219		=	.0020
	.1647 x .00002477		=	.0408 — .0428
		θ_{B15}	=	.3030
113	$\theta_{A15} - .0389 \theta_{B15} - .7091 R_{16} + .2364 \theta_{A16}$		=	.1414
	.0389 x .00003030		=	.0118
	.7091 x .00004448		=	.3180 .4712
	.2364 x .00003219		=	—.0762
		θ_{A15}	=	.3950
110	$R_{15} - .2718 \theta_{A15} - .3006 \theta_{B15}$		=	.3361
	.2718 x .00003950		=	.1072
	.3006 x .00003030		=	.0912
		R_{15}	=	.5345
107	$\theta_{B14} - .5109 R_{15} + .0120 \theta_{A15} + .1581 \theta_{B15}$		=	.1414
	.5109 x .00005345		=	.2730 .4174
	.0120 x .00003950		=	.0047
	.1581 x .00003030		=	.0480 — .0527
		θ_{B14}	=	.3647

Table 15.—(Continued).

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
105	$\theta_{A14} - .0775\,\theta_{B14} - .6624\,R_{15} + .2210\,\theta_{A15}$		=	.1627	
		.0775 x .00003647	=	.0283	
		.6624 x .00005345	=	.3541	.5451
		.2210 x .00003950	=		—.0873
		θ_{A14}	=		.4578
102	$R_{14} - .2790\,\theta_{A14} - .3096\,\theta_{B14}$		=	.3412	
		.2790 x .00004578	=	.1278	
		.3096 x .00003647	=	.1160	
		R_{14}	=		.5850
99	$\theta_{B13} - .5928\,R_{14} + .0124\,\theta_{A14} + .1853\,\theta_{B14}$		=	.1487	
		.5928 x .00005850	=	.3470	.4957
		.0124 x .00004578	=	.0057	
		.1853 x .00003647	=	.0676	—.0733
		θ_{B13}	=		.4224
97	$\theta_{A13} - .0695\,\theta_{B13} - .7560\,R_{14} + .2520\,\theta_{A14}$		=	.1658	
		.0695 x .00004224	=	.0294	
		.7560 x .00005850	=	.4420	.6372
		.2520 x .00004578	=		—.1154
		θ_{A13}	=		.5218
94	$R_{13} - .2789\,\theta_{A13} - .3092\,\theta_{B13}$		=	.3864	
		.2789 x .00005218	=	.1458	
		.3092 x .00004224	=	.1308	
		R_{13}	=		.6630
91	$\theta_{B12} - .5920\,R_{13} + .0138\,\theta_{A13} + .1835\,\theta_{B13}$		=	.1715	
		.5920 x .00006630	=	.3921	.5636
		.0138 x .00005218	=	.0072	
		.1835 x .00004224	=	.0775	—.0847
		θ_{B12}	=		.4789
89	$\theta_{A12} - .0766\,\theta_{B12} - .7472\,R_{13} + .2490\,\theta_{A13}$		=	.1816	
		.0766 x .00004789	=	.0366	
		.7472 x .00006630	=	.4960	.7142
		.2490 x .00005218	=		—.1300
		θ_{A12}	=		.5842
86	$R_{12} - .2750\,\theta_{A12} - .3160\,\theta_{B12}$		=	.4216	
		.2750 x .00005842	=	.1603	
		.3160 x .00004789	=	.1511	
		R_{12}	=		.7330
83	$\theta_{B11} - .6150\,R_{12} + .0139\,\theta_{A12} + .1912\,\theta_{B12}$		=	.1831	
		.6150 x .00007330	=	.4512	.6343
		.0139 x .00005842	=	.0081	
		.1912 x .00004789	=	.0916	—.0997
		θ_{B11}	=		.5346
81	$\theta_{A11} - .0770\,\theta_{B11} - .7560\,R_{12} + .2520\,\theta_{A12}$		=	.2018	
		.0770 x .00005346	=	.0411	
		.7560 x .00007330	=	.5550	.7979
		.2520 x .00005842	=		—.1471
		θ_{A11}	=		.6508
78	$R_{11} - .2746\,\theta_{A11} - .3147\,\theta_{B11}$		=	.4669	
		.2746 x .00006508	=	.1790	
		.3147 x .00005346	=	.1681	
		R_{11}	=		.8140

Table 15.—(Continued).

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
75	$\theta_{B10} - .6061\,R_{11} + .0179\,\theta_{A11} + .1841\,\theta_{B11}$		=	.2178	
		.6061 x .00008140	=	.4940	.7118
		.0179 x .00006508	=	.0116	
		.1841 x .00005346	=	.0984	—.1100
		θ_{B10}	=		.6018
73	$\theta_{A10} - .1082\,\theta_{B10} - .7278\,R_{11} + .2426\,\theta_{A11}$		=	.2055	
		.1082 x .00006018	=	.6651	
		.7278 x .00008140	=	.5925	.8631
		.2426 x .00006508	=		—.1580
		θ_{A10}	=		.7051
70	$R_{10} - .2601\,\theta_{A10} - .3423\,\theta_{B10}$		=	.4540	
		.2601 x .00007051	=	.1833	
		.3423 x .00006018	=	.2059	
		R_{10}	=		.8432
67	$\theta_{B9} - .6991\,R_{10} + .0181\,\theta_{A10} + .2151\,\theta_{B10}$		=	.2056	
		.6991 x .00008432	=	.5890	.7946
		.0181 x .00007051	=	.0127	
		.2151 x .00006018	=	.1293	—.1420
		θ_{B9}	=		.6526
65	$\theta_{A9} - .1038\,\theta_{B9} - .7655\,R_{10} + .2551\,\theta_{A10}$		=	.2053	
		.1038 x .00006526	=	.0677	
		.7655 x .00008432	=	.6460	.9190
		.2551 x .00007051	=		—.1800
		θ_{A9}	=		.7390
62	$R_9 - .2602\,\theta_{A9} - .3372\,\theta_{B9}$		=	.4719	
		.2602 x .00007390	=	.1922	
		.3372 x .00006526	=	.2200	
		R_9	=		.8841
59	$\theta_{B8} - .6908\,R_9 + .0231\,\theta_{A9} + .2072\,\theta_{B9}$		=	.2008	
		.6908 x .00008841	=	.6100	.8108
		.0231 x .00007390	=	.0171	
		.2072 x .00006526	=	.1351	—.1522
		θ_{B8}	=		.6586
57	$\theta_{A8} - .1382\,\theta_{B8} - .7000\,R_9 + .2334\,\theta_{A9}$		=	.2079	
		.1382 x .00006586	=	.0910	
		.7000 x .00008841	=	.6182	.9171
		.2334 x .00007390	=		—.1725
		θ_{A8}	=		.7446
54	$R_8 - .2738\,\theta_{A8} - .3118\,\theta_{B8}$		=	.3774	
		.2738 x .00007446	=	.2038	
		.3118 x .00006586	=	.2055	
		R_8	=		.7867
51	$\theta_{B7} - .5814\,R_8 + .0112\,\theta_{A8} + .1826\,\theta_{B8}$		=	.1562	
		.5814 x .00007867	=	.4570	.6132
		.0112 x .00007446	=	.0083	
		.1826 x .00006586	=	.1203	—.1286
		θ_{B7}	=		.4846

Table 15.—(Continued).

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
49	$\theta_{A7} - .0640\ \theta_{B7} - .7422\ R_8 + .2474\ \theta_{A8}$		=	.1729	
		.0640 x .00004846	=	.0310	
		.7422 x .00007867	=	.5845	.7884
		.2474 x .00007446	=		—.1840
			θ_{A7} =		.6044
46	$R_7 - .2711\ \theta_{A7} - .3090\ \theta_{B7}$		=	.4011	
		.2711 x .00006044	=	.1639	
		.3090 x .00004846	=	.1497	
			R_7 =		.7147
43	$\theta_{B6} - .5489\ R_7 + .0090\ \theta_{A7} + .1739\ \theta_{B7}$		=	.1706	
		.5489 x .00007147	=	.3920	.5626
		.0090 x .00006044	=	.0054	
		.1739 x .00004846	=	.0842	—.0896
			θ_{B6} =		.4730
41	$\theta_{A6} - .0540\ \theta_{B6} - .7192\ R_7 + .2397\ \theta_{A7}$		=	.1946	
		.0540 x .00004730	=	.0256	
		.7192 x .00007147	=	.5140	.7342
		.2397 x .00006044	=		—.1448
			θ_{A6} =		.5894
38	$R_6 - .2648\ \theta_{A6} - .3000\ \theta_{B6}$		=	.4216	
		.2648 x .00005894	=	.1561	
		.3000 x .00004730	=	.1420	
			R_6 =		.7197
35	$\theta_{B5} - .4496\ R_6 + .0013\ \theta_{A6} + .1487\ \theta_{B6}$		=	.1486	
		.4496 x .00007197	=	.3240	.4726
		.0013 x .00005894	=	.0008	
		.1487 x .00004730	=	.0704	—.0712
			θ_{B5} =		.4014
33	$\theta_{A5} - .0092\ \theta_{B5} - .6676\ R_6 + .2225\ \theta_{A6}$		=	.1879	
		.0092 x .00004014	=	.0037	
		.6676 x .00007197	=	.4802	.6718
		.2225 x .00005894	=		—.1310
			θ_{A5} =		.5408
30	$R_5 - .2638\ \theta_{A5} - .2981\ \theta_{B5}$		=	.4473	
		.2638 x .00005408	=	.1424	
		.2981 x .00004014	=	.1196	
			R_5 =		.7093
27	$\theta_{B4} - .4348\ R_5 + .0048\ \theta_{A5} + .1401\ \theta_{B5}$		=	.1550	
		.4348 x .00007093	=	.3080	.4630
		.0048 x .00005408	=	.0026	
		.1401 x .00004014	=	.0563	—.0589
			θ_{B4} =		.4041
25	$\theta_{A4} - .0355\ \theta_{B4} - .6371\ R_5 + .2123\ \theta_{A5}$		=	.2000	
		.0355 x .00004041	=	.0143	
		.6371 x .00007093	=	.4515	.6658
		.2123 x .00005408	=		—.1150
			θ_{A4} =		.5508

Table 15.—(Continued).

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation.	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.		Right-Hand Member of Equation Coefficient of .0001	
22	$R_4 + .3642\ \theta_{A4} - .3206\ \theta_{B4} - .4044\ R_5 + .1348\ \theta_{A5}$	=	.5512	
	.3206 x .00004041	=	.1295	
	.4044 x .00007093	=	.2862	.9669
	.3642 x .00005508	=	.2008	
	.1348 x .00005408	=	.0729	—.2737
	R_4	=		.6932
19	$\theta_{B3} - .4696\ R_4 + .0030\ \theta_{A4} + .1535\ \theta_{B4}$	=	.1540	
	.4696 x .00006932	=	.3257	.4797
	.0030 x .00005508	=	.0017	
	.1535 x .00004041	=	.0620	—.0637
	θ_{B3}	=		.4160
17	$\theta_{A3} - .0196\ \theta_{B3} - .6881\ R_4 + .2294\ \theta_{A4}$	=	.1917	
	.0196 x .00004160	=	.0082	
	.6881 x .00006932	=	.4770	.6769
	.2294 x .00005508	=		—.1263
	θ_{A3}	=		.5506
14	$R_3 - .2689\ \theta_{A3} - .2980\ \theta_{B3}$	=	.4564	
	.2689 x .00005506	=	.1481	
	.2980 x .00004160	=	.1240	
	R_3	=		.7285
11	$\theta_{B2} - .4511\ R_3 + .0027\ \theta_{A3} + .1476\ \theta_{B3}$	=	.1768	
	.4511 x .0000 285	=	.3285	.5053
	.0027 x .00005506	=	.0015	
	.1476 x .00004160	=	.0614	—.0629
	θ_{B2}	=		.4424
9	$\theta_{A2} - .0191\ \theta_{B2} - .6857\ R_3 + .2286\ \theta_{A3}$	=	.2151	
	.0191 x .00004424	=	.0085	
	.6857 x .00007285	=	.4990	.7226
	.2286 x .00005506	=		—.1259
	θ_{A2}	=		.5967
6	$R_2 - .2627\ \theta_{A2} - .2918\ \theta_{B2}$	=	.5021	
	.2627 x .00005967	=	.1565	
	.2918 x .00004424	=	.1291	
	R_2	=		.7877
3	$\theta_{B1} - .3612\ R_2 - .0088\ \theta_{A2} + .1292\ \theta_{B2}$	=	.1483	
	.3612 x .00007877	=	.2845	
	.0088 x .00005967	=	.0053	.4381
	.1292 x .00004424	=		—.0572
	θ_{B1}	=		.3809
1	$\theta_{A1} + .0680\ \theta_{B1} - .6494\ R_2 + .2164\ \theta_{A2}$	=	.2667	
	.6494 x .00007877	=	.5110	.7777
	.0680 x .00003809	=	.0259	
	.2164 x .00005967	=	.1291	—.1550
	θ_{A1}	=		.6227
A	$R_1 - .2495\ \theta_{A1} - .2495\ \theta_{B1}$	=	.5667	
	.2495 x .00006227	=	.1553	
	.2495 x .00003809	=	.0951	
	R_1	=		.8171

Table 16.

Values of R and θ for the Symmetrical Three-span Twenty-Story Bent Shown in Fig. 5, and the Functions of these Values that Occur in the Equations Used to Determine the Moments in the Columns and Girders.

Story No.	10,000 x R	10,000 x θ_A	10,000 x θ_B	10,000 x $3R$	10,000 x $2\theta_A$	10,000 x $2\theta_B$	10,000 x $(2\theta_A+\theta_B)$	10,000 x $(2\theta_B+\theta_A)$	10,000 x $3\theta_B$	For Column A* 10,000 x $(2\theta_{N-1}+\theta_N-3R)$	For Column A* 10,000 x $(2\theta_N+\theta_{N-1}-3R)$	For Column B 10,000 x $(2\theta_{N-1}+\theta_N-3R)$	For Column B 10,000 x $(2\theta_N+\theta_{N-1}-3R)$	Story No.
1	.8171	.6227	.3803	2.4413	1.2454	.7618	1.6263	1.3845	1.1427	1.8186	1.1959	2.0604	1.6795	1
2	.7877	.5967	.4424	2.3631	1.1934	.8848	1.6358	1.4815	1.3372	.5210	.5470	1.1589	1.0974	2
3	.7285	.5508	.4160	2.1855	1.1012	.8320	1.5172	1.3826	1.2480	.4415	.4876	.8847	.9111	3
4	.6932	.5508	.4011	2.0796	1.1016	.8082	1.5057	1.3590	1.2123	.4276	.4274	.8435	.8554	4
5	.7093	.5408	.4014	2.1279	1.0816	.8028	1.4830	1.3436	1.2042	.4855	.4955	.9183	.9210	5
6	.7197	.5894	.4730	2.1591	1.1788	.9460	1.6518	1.5354	1.4190	.4881	.4395	.8833	.8117	6
7	.7147	.6044	.4846	2 1441	1.2088	.9692	1.6934	1.5736	1.4538	.3609	.3459	.7135	.7019	7
8	.7867	.7446	.6586	2.3601	1.4892	1.3172	2.1478	2.0618	1.9758	.4067	.2665	.7323	.5583	8
9	.8841	.7390	.6526	2.6523	1.4780	1.3052	2.1306	2.0442	1.9578	.4241	.4297	.6825	.6885	9
10	.8432	.7051	.6018	2.5296	1 4102	1.2036	2.0120	1.9087	1.8054	.3465	.3804	.6226	.6734	10
11	.8140	.6508	.5346	2.4420	1.3016	1.0692	1 8362	1 7200	1.6038	.3810	.4353	.7038	.7710	11
12	.7330	.5842	.4789	2 1990	1.1684	.9578	1.6473	1.5420	1.4367	.3132	.3798	.6509	.7066	12
13	.6630	.5218	.4224	1.9890	1.0436	.8448	1.4660	1.3666	1.2682	.2988	.3612	.6088	.6653	13
14	.5850	.4578	.3647	1.7550	.9156	.7294	1.2803	1.1872	1.0941	.2536	.3176	.5455	.6032	14
15	.5345	.3950	.3030	1.6035	.7900	.6060	1.0930	1.0010	.9090	.2929	.3557	.5711	.6328	15
16	.4418	.3219	.2477	1 3344	.6438	.4954	.8915	.8173	.7431	.2225	.2956	.4807	.5360	16
17	.3655	.2527	.1903	1.0965	.5054	.3806	.6957	.6333	.5709	.2000	.2692	.4108	.4682	17
18	.2742	.1801	.1361	.8226	.3602	.2722	.4963	.4523	.4083	.1371	.2097	.3059	.3601	18
19	.1836	.1085	.0335	.5503	.2170	.1670	.3005	.2755	.2505	.0821	.1537	.1951	.2477	19
20	.0976	.0517	.0338	.2928	.1034	.0676	.1372	.1193	.1014	.0231	.0809	.0920	.1417	20

* N represents the number of the story in question, and (N—1) the story below.

Table 17.

Values of K for the Columns and the Girders of the Symmetrical Three-span Twenty-story Bent Shown in Fig. 5, and the Functions of these Values that Occur in the Equations Used to Determine the Moments in the Columns and Girders.

Story No.	K for Girder at Top of Story in		K for		.0001 x $2EK$ for Girder at Top of Story in		.0001 x $2EK$ for	
	Bay *a*	Bay *b*	Column *A*	Column *B*	Bay *a*	Bay *b*	Column *A*	Column *B*
1	30.5	37.3	25.8	25.8	177 000	216 200	149 600	149 600
2	21.4	29.2	35.6	35.6	124 100	169 400	206 300	206 300
3	21.4	26.2	35 4	35.5	124 100	152 000	205 500	205 800
4	19.5	26.2	35.4	35.5	113 050	152 000	205 500	205 800
5	19.5	23.8	29.4	30.4	113 050	138 050	170 500	176 300
6	14.1	17.2	29.4	30.4	81 750	99 700	170 500	176 300
7	12.8	15.7	28.7	30.0	74 250	91 100	166 500	174 000
8	7.7	9.4	28.7	30.0	44 600	54 500	166 500	174 000
9	7 7	9.4	21.1	26.1	44 600	54 500	122 400	151 300
10	7.7	9.4	21.1	26.1	44 600	54 500	122 400	151 300
11	7.7	9.4	18.8	19.9	44 600	54 500	109 000	115 400
12	7.7	9.4	18.8	19.9	44 600	54 500	109 000	115 400
13	7.7	9.4	18.3	18.6	44 600	54 500	106 100	107 900
14	7.7	9.4	18.3	18.6	44 600	54 500	106 100	107 900
15	7.7	9.4	14.3	14.6	44 600	54 500	82 900	84 600
16	7.7	9.4	14 3	14.6	44 600	54 500	82 900	84 600
17	7.7	9 4	13.1	13.2	44 600	54 500	76 000	76 500
18	7.7	9.4	13.1	13.2	44 600	54 500	76 000	76 500
19	7.7	9 4	13.1	13.2	44 600	54 500	76 000	76 500
20	7.7	9 4	13.1	13.2	44 600	54 500	76 000	76 500

Table 18.

Moments at the Ends of the Columns and Girders of the Symmetrical Three-span Twenty-story Bent shown in Fig. 5.

Moments are expressed in inch-pounds.

No. of Story	Moment in Column A*		Moment in Column B		Moment in Girder at Top of Story in Bay a		Moment in Girder at Top of Story in Bay b
	At Top $= 2EK\,(2\theta_N+\theta_{N-1}-3R)$	At Bottom $= 2EK\,(2\theta_{N-1}+\theta_N-3R)$	At Top $= 2EK\,(2\theta_N+\theta_{N-1}-3R)$	At Bottom $= 2EK\,(2\theta_{N-1}+\theta_N-3R)$	At Right-hand End $= 2EK\,(2\theta_A+\theta_B)$	At Left-hand End $= 2EK\,(2\theta_B+\theta_A)$	At Each End $= 2EK\,(3\theta_B)$
1	178 800	272 000	251 000	308 200	287 800	245 000	247 500
2	113 000	107 500	226 500	239 000	203 000	184 000	226 000
3	100 300	90 700	187 500	182 000	187 100	171 700	189 600
4	87 800	87 800	176 100	173 500	170 500	153 600	184 400
5	84 500	82 800	162 300	162 000	167 700	152 000	166 000
6	75 000	83 200	143 300	155 700	135 000	125 600	141 500
7	57 600	60 000	122 000	124 000	125 900	117 000	132 500
8	44 400	67 700	97 200	127 500	95 800	92 000	107 800
9	52 550	51 900	104 200	103 300	95 000	91 100	106 800
10	46 500	42 400	102 000	94 200	89 600	85 000	98 500
11	47 500	41 500	89 000	81 200	81 900	76 600	87 500
12	41 400	34 100	81 500	75 100	73 400	68 600	78 300
13	38 300	31 700	71 800	65 600	65 300	60 800	69 100
14	33 700	26 900	65 100	58 900	5 700	52 800	59 600
15	29 420	24 300	53 600	48 350	48 700	49 600	49 600
16	24 450	18 450	45 400	40 600	39 700	36 400	40 500
17	20 450	15 400	35 800	31 400	31 000	28 200	31 100
18	15 920	10 430	27 600	23 400	22 100	20 150	22 300
19	11 680	6 240	18 900	14 930	13 380	12 250	13 650
20	6 150	1 750	10 830	7 040	6 110	5 320	5 530

*N represents the number of the story in question, and (N—1) the story below.

Table 19.

Direct Stresses in the Columns, and the Shears in the Columns and Girders of the Symmetrical Three-span Twenty-story Bent shown in Fig. 5.

All quantities are in pounds.

Story No.	Shear in		Shear in Girder at Top of Story in		Direct Stress in	
	Column *A*	Column *B*	Bay *a*	Bay *b*	Column *A*	Column *B*
1	1 709	2 115	2 020	2 290	14 464	4 587
2	1 149	2 421	1 467	2 095	12 444	4 317
3	1 136	2 200	1 360	1 755	10 977	3 689
4	1 044	2 080	1 228	1 709	9 617	3 294
5	995	1 931	1 210	1 536	8 389	2 813
6	940	1 780	986	1 310	7 179	2 489
7	816	1 709	920	1 228	6 193	2 163
8	779	1 560	712	996	5 273	1 855
9	725	1 440	705	988	4 561	1 571
10	617	1 363	661	912	3 856	1 288
11	618	1 180	600	810	3 195	1 037
12	524	1 088	538	725	2 595	827
13	486	954	478	640	2 057	640
14	421	861	416	552	1 579	478
15	373	710	354	459	1 163	342
16	298	597	288	375	809	237
17	249	466	221	288	521	150
18	182	354	160	206	300	83
19	124	235	97	126	140	37
20	55	124	43	51	43	8

Table 20.

Check on the Numerical Values of the Moments at the Ends of the Columns and Girders of the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

Quantities are in inch-pounds.

For a perfect check the corresponding quantities in the two columns of each group of columns should be identical.

Story No.	Total Shear in a Story Multiplied by the Story Height	Sum of the Moments at the Tops and the Bottoms of all of the Columns in a Story	Sum of the Moments in Column *A* just above and below Girder *a* at the Top of a Story	Moment in Girder *a* at the Top of a Story, at the Section adjacent to Column *A*	Sum of the Moments in Column *B* just above and below Girders *a* and *b* at the Top of a Story	Sum of the Moments in Girders *a* and *b* at the Top of a Story at Sections adjacent to Column *B*
1	2 035 000	2 020 000	286 300	287 800	490 000	492 500
2	1 370 000	1 372 000	203 700	203 600	408 500	410 000
3	1 123 000	1 121 000	187 100	188 100	361 000	361 300
4	1 053 000	1 050 400	170 500	170 600	338 100	338 000
5	983 000	983 200	167 700	167 700	318 000	318 000
6	913 000	914 400	135 000	135 000	267 300	267 100
7	725 000	727 200	125 900	125 300	249 500	249 500
8	673 000	673 600	95 800	96 300	200 500	199 800
9	622 000	624 000	95 000	94 950	198 400	197 900
10	570 000	570 200	89 600	88 000	183 200	183 500
11	518 000	518 400	81 900	81 600	164 100	164 100
12	466 000	464 200	73 400	72 100	147 100	146 900
13	414 000	414 800	65 300	65 200	130 700	129 900
14	363 000	369 200	57 000	58 000	113 450	112 400
15	311 000	311 400	48 700	47 870	94 200	94 200
16	259 000	259 800	39 700	39 850	76 800	76 900
17	207 500	206 200	31 000	30 880	59 200	59 300
18	155 300	154 800	22 100	22 160	42 530	42 450
19	103 600	103 500	13 380	13 430	25 940	25 900
20	51 800	51 540	6 110	6 150	10 830	10 850

Table 21.

Elimination of the Unknown Quantities in the Equations Used to Determine the Slopes and the Deflections in the Bottom Story of the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5, by a Modification of the Slope-Deflection Method.

No. of Equation	Left-Hand Member of Equation							Right-Hand Member of Equation
	Story No. 1			Story No. 2			Story No. 3	
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height							Coefficient of .0001
A	−205.8	51.6	51.6					−117.2
B	− 77.4	183.8	30.5	−106.8	35.6			0.0
C	− 77.4	30.5	295.7	−106.8		35.6		0.0
D		71.2	71.2	−284.8	71.2	71.2		− 78.9
E		35.6		−106.8	220.2	21.4	−106.2	0.0
F			35.6	−106.8	21.4	308.1	−106.5	0.0
G					141.6	142.0	−283.6	− 64.5
A	1	−0.2495	−0.2495					0.5667
B	1	−2.347	−0.3941	1.3798	−0.4599			0.0000
C	1	−0.3941	−3.8204	1.3798		−0.4599		0.0000
A–B	0	2.1252	0.1445	−1.3798	0.4599			0.5667
B–C	0	−1.9807	3.4264		−0.4599	0.4599		0.0000
1		1	0.0680	−0.6494	0.2164			0.2667
2		1	−1.7300		0.2322	−0.2322		0.0000
D		1	1.0000	−4.0000	1.0000	1.0000		−1.1082
E		1		−3.0000	6.1856	0.6012	−2.9834	0.0000
1–2		0	1.7980	−0.6494	−0.0158	0.2322		0.2667
2–D		0	−2.7300	4.0000	−0.7678	−1.2322		1.1082
D–E		0	1.0000	−1.0000	−5.1856	0.3988	2.9834	−1.1082
3			1	−0.3612	−0.0088	0.1292		0.1483
4			1	−1.4652	0.2812	0.4513		−0.4059
5			1	−1.0000	−5.1856	0.3988	2.9834	−1.1082
F			1	−3.0000	0.6012	8.6547	−2.9915	0.0000
3–4			0	1.1041	−0.2900	−0.3222		0.5543
4–5			0	−0.4652	5.4668	0.0525	−2.9834	0.7023
5–F			0	2.0000	−5.7868	8.2559	5.9749	−1.1082
6				1	−0.2627	−0.2918		0.5021
7				1	−11.7518	−0.1128	6.4127	−1.5095
8				1	−2.8934	−4.1279	2.9875	−0.5541
6–7				0	11.4891	−0.1790	−6.4127	2.0116
7–8				0	−8.8534	4.0151	3.4252	−0.9554
9					1	−0.0156	−0.5600	0.1750
10					1	−0.4540	−0.3862	0.1079
G					1	1.0030	−2.0010	−0.4555
9–10					0	0.4384	−0.1738	0.0671
10–G					0	−1.4570	1.6148	0.5634
11						1	−0.3961	0.1530
12						1	−1.1090	−0.3865
11–12						0	0.7131	0.5395
13							1	0.7560

Table 22.

Determination of the Changes in the Slopes and the Ratio of the Deflection to Story Height in the Bottom Story of the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5, by a Modification of the Slope-Deflection Method.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001
13			R_3 =	.7560
11	$\theta_{B2} - .3961 R_3$		=	.1530
		.3961 x .00007560	=	.2995
			θ_{B2} =	.4525
9	$\theta_{A2} - 0156\ \theta_{B2} - .5600 R_3$		=	.1750
		.0156 x .00004525	=	.0071
		.5600 x .00007560	=	.4240
			θ_{A2} =	.6061
6	$R_2 - .2627\ \theta_{A2} - .2918\ \theta_{B2}$		=	.5021
		.2627 x .00006061	=	.1593
		.2918 x .00004525	=	.1320
			R_2 =	.7934
3	$\theta_{B1} - .3612 R_2 - .0088\ \theta_{A2} + .1292\ \theta_{B2}$		=	.1483
		.3612 x .00007934	=	.2865
		.0088 x .00006061	=	.0053 .4401
		.1292 x .00004525	=	—.0585
			θ_{B1} =	.3816
1	$\theta_{A1} + .0680\ \theta_{B1} - 6494 R_2 + .2164\ \theta_{A2}$		=	.2667
		.6494 x .00007934	=	.5150 .7817
		.0680 x .00003816	=	.0260
		.2164 x .00006061	=	.1313 —.1573
			θ_{A1} =	.6244
A	$R_1 - .2495\ \theta_{A1} - .2495\ \theta_{B1}$		=	.5667
		.2495 x .00006244	=	.1556
		.2495 x .00003816	=	.0951
			R_1 =	.8174

Table 23.

Comparison of the Approximate Methods with the Slope-Deflection Method When Applied to the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

Moments in the Columns.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Story No.	Moment at Top of Column *A*					Moment at Bottom of Column *A*					Moment at Top of Column *B*					Moment at Bottom of Column *B*					Story No.
	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			
			I	II	III			I	II	III			I	II	III			I	II	III	
1	178.8	191.5	166.9	254.5	254.5	272.0	191.5	166.9	254.5	254.5	251.0	319.0	343.2	254.5	254.5	308.2	319.0	343.2	254.2	254.5	1
	100.0	*106.8*	*93.7*	*145.0*	*145.0*	*100.0*	*70.3*	*61.3*	*93.6*	*93.6*	*100.0*	*126.8*	*136.9*	*101.4*	*101.4*	*100.0*	*103.5*	*111.3*	*82.5*	*82.5*	
2	113.0	113.8	111.0	171.4	171.4	107.5	113.8	111.0	171.4	171.4	226.5	227.5	228.6	171.4	171.4	239.0	227.5	228.6	171.4	171.4	2
	100.0	*100.7*	*98.3*	*152.0*	*152.0*	*100.0*	*106.0*	*103.4*	*159.4*	*159.4*	*100.0*	*100.4*	*101.0*	*75.7*	*75.7*	*100.0*	*95.3*	*95.8*	*71.7*	*71.7*	
3	100.3	96.8	91.5	140.5	140.5	90.7	96.8	91.5	140.5	140.5	187.5	184.1	187.6	140.5	140.5	182.0	184.1	187.6	140.5	140.5	3
	100.0	*96.5*	*91.2*	*140.3*	*140.3*	*100.0*	*106.8*	*101.0*	*155.0*	*155.0*	*100.0*	*98.3*	*100.0*	*75.0*	*75.0*	*100.0*	*101.2*	*103.0*	*77.2*	*77.2*	
4	87.8	87.1	87.1	131.7	131.7	87.8	87.1	87.1	131.7	131.7	176.1	175.0	175.3	131.7	131.7	173.5	175.0	175.3	131.7	131.7	4
	100.0	*99.3*	*99.2*	*150.0*	*150.0*	*100.0*	*99.3*	*99.2*	*150.0*	*150.0*	*100.0*	*99.4*	*99.5*	*74.6*	*74.6*	*100.0*	*101.0*	*101.2*	*75.9*	*75.9*	
5	84.5	85.6	80.9	120.4	120.4	82.8	85.6	80.9	120.4	120.4	162.3	160.0	163.1	125.3	125.3	162.0	160.0	163.1	125.3	125.3	5
	100.0	*101.3*	*95.6*	*142.3*	*142.3*	*100.0*	*103.3*	*97.6*	*145.3*	*145.3*	*100.0*	*98.5*	*100.6*	*77.2*	*77.2*	*100.0*	*98.8*	*100.8*	*77.5*	*77.5*	
6	75.0	76.8	74.3	111.7	111.7	83.2	76.8	74.3	111.7	111.7	143.3	151.8	153.0	116.3	116.3	155.7	151.8	153.0	116.3	116.3	6
	100.0	*102.0*	*99.0*	*149.0*	*149.0*	*100.0*	*92.2*	*89.5*	*134.2*	*134.2*	*100.0*	*106.0*	*106.8*	*81.1*	*81.1*	*100.0*	*97.5*	*98.3*	*74.8*	*74.8*	
7	57.6	60.3	59.2	88.6	88.6	60.0	60.3	59.2	88.6	88.6	122.0	120.7	122.0	92.8	92.8	124.0	120.7	122.0	92.8	92.8	7
	100.0	*105.0*	*102.8*	*154.0*	*154.0*	*100.0*	*100.5*	*98.5*	*147.6*	*147.6*	*100.0*	*98.8*	*100.0*	*76.1*	*76.1*	*100.0*	*97.3*	*98.3*	*74.8*	*74.8*	
8	44.4	54.7	55.9	82.3	82.3	67.7	54.7	55.9	82.3	82.3	97.2	114.0	113.0	86.2	86.2	127.5	114.0	113.0	86.2	86.2	8
	100.0	*125.0*	*126.0*	*185.5*	*185.5*	*100.0*	*80.7*	*82.5*	*121.8*	*121.8*	*100.0*	*117.2*	*116.2*	*88.7*	*88.7*	*100.0*	*89.5*	*88.6*	*67.6*	*67.6*	
9	52.6	49.8	51.4	69.5	69.5	51.9	49.8	51.4	69.5	69.5	104.2	105.5	104.5	86.0	86.0	103.3	105.5	104.5	86.0	86.0	9
	100.0	*94.8*	*97.7*	*132.0*	*132.0*	*100.0*	*96.1*	*99.1*	*134.0*	*134.0*	*100.0*	*100.8*	*100.3*	*82.5*	*82.5*	*100.0*	*102.1*	*101.0*	*83.2*	*83.2*	
10	46.5	45.6	46.0	63.7	63.7	42.4	45.6	46.0	63.7	63.7	102.2	96.3	96.8	78.8	78.8	94.2	96.3	96.8	78.8	78.8	10
	100.0	*98.0*	*99.0*	*137.0*	*137.0*	*100.0*	*107.8*	*108.5*	*150.4*	*150.4*	*100.0*	*94.3*	*94.9*	*77.3*	*77.3*	*100.0*	*102.1*	*102.7*	*83.6*	*83.6*	
11	47.5	43.0	41.9	62.9	62.9	41.5	43.0	41.9	62.9	62.9	89.0	86.4	88.1	66.7	66.7	81.2	86.4	88.1	66.7	66.7	11
	100.0	*90.6*	*88.1*	*132.3*	*132.3*	*100.0*	*103.8*	*101.0*	*152.8*	*152.8*	*100.0*	*97.1*	*99.0*	*74.9*	*74.9*	*100.0*	*106.5*	*108.5*	*82.0*	*82.0*	
12	41.4	38.7	38.2	56.7	56.7	34.1	38.7	38.2	56.7	56.7	81.5	77.9	79.1	60.0	60.0	75.1	77.9	79.1	60.0	60.0	12
	100.0	*93.6*	*92.4*	*137.0*	*137.0*	*100.0*	*113.7*	*112.0*	*166.4*	*166.4*	*100.0*	*95.6*	*97.0*	*73.6*	*73.6*	*100.0*	*103.5*	*105.4*	*79.9*	*79.9*	

Table 24.

Comparison of the Approximate Methods with the Slope-Deflection Method When Applied to the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

Moments in the Girders.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Story No.	Moment at Right End of Girder *a*					Moment at Left End of Girder *a*					Moment at End of Girder *b*					Story No.
	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			
			I	II	III			I	II	III			I	II	III	
1	287.8	305.3	279.3	425.9	425.9	245.0	274.5	279.3	143.7	130.9	247.5	272.0	295.0	282.2	295.0	1
	100.0	*106.1*	*97.1*	*148.0*	*148.0*	*100.0*	*112.0*	*114.0*	*58.6*	*53.4*	*100.0*	*110.0*	*119.0*	*114.0*	*119.0*	
2	203.0	210.6	199.8	311.9	311.9	184.0	185.6	199.8	104.4	100.7	226.0	226.0	211.2	207.5	211.2	2
	100.0	*103.5*	*98.5*	*153.6*	*153.6*	*100.0*	*100.8*	*108.5*	*56.7*	*54.8*	*100.0*	*100.0*	*93.4*	*91.9*	*93.4*	
3	187.1	183.9	178.2	272.2	272.2	171.7	173.1	178.2	90.7	84.3	189.6	186.0	187.9	181.5	187.9	3
	100.0	*98.2*	*95.2*	*145.8*	*145.8*	*100.0*	*100.8*	*104.0*	*52.9*	*49.1*	*100.0*	*98.0*	*99.0*	*95.8*	*99.0*	
4	170.5	172.7	169.2	252.1	252.1	153.6	152.0	169.2	87.3	84.9	184.4	183.0	172.1	169.7	172.1	4
	100.0	*101.0*	*99.5*	*148.0*	*148.0*	*100.0*	*99.0*	*110.3*	*56.9*	*55.3*	*100.0*	*99.0*	*93.5*	*92.0*	*93.5*	
5	167.7	162.4	151.8	232.1	232.1	152.0	151.8	154.8	83.7	77.3	166.0	160.0	164.4	157.9	164.4	5
	100.0	*97.0*	*92.4*	*138.7*	*138.7*	*100.0*	*99.9*	*101.8*	*55.0*	*50.9*	*100.0*	*96.5*	*99.0*	*95.0*	*99.0*	
6	135.0	137.1	133.2	200.3	200.3	125.6	128.5	133.2	73.0	68.3	141.5	144.0	140.8	136.1	140.8	6
	100.0	*101.2*	*98.8*	*148.2*	*148.2*	*100.0*	*102.6*	*112.2*	*58.1*	*54.4*	*100.0*	*102.0*	*99.5*	*96.3*	*99.5*	
7	125.9	115.0	114.5	170.9	170.9	117.0	110.9	114.5	62.4	57.4	132.5	123.8	121.6	116.6	121.6	7
	100.0	*91.5*	*91.0*	*135.9*	*135.9*	*100.0*	*94.9*	*98.0*	*53.3*	*49.1*	*100.0*	*93.0*	*91.8*	*88.0*	*91.8*	
8	95.8	104.5	108.7	151.8	151.8	92.0	103.5	108.7	64.2	56.0	107.8	116.0	106.2	108.0	106.2	8
	100.0	*109.3*	*113.3*	*158.4*	*158.4*	*100.0*	*112.5*	*118.0*	*69.8*	*60.9*	*100.0*	*108.0*	*95.8*	*100.3*	*95.8*	
9	95.0	95.4	96.8	133.2	133.2	91.1	95.8	96.8	65.5	60.5	106.8	105.0	104.3	99.3	104.3	9
	100.0	*100.4*	*101.9*	*140.3*	*140.3*	*100.0*	*105.0*	*112.4*	*71.9*	*66.4*	*100.0*	*99.4*	*97.6*	*93.0*	*97.6*	
10	89.6	88.6	87.1	126.6	126.6	85.0	85.7	87.1	54.8	48.0	98.5	97.0	97.5	90.7	97.5	10
	100.0	*98.9*	*97.3*	*141.2*	*141.2*	*100.0*	*100.8*	*102.4*	*64.5*	*56.5*	*100.0*	*98.5*	*99.0*	*92.1*	*99.0*	
11	81.9	81.7	80.3	119.6	119.6	76.6	77.0	80.3	44 6	40.8	87.5	87.3	85.9	82.1	85.9	11
	100.0	*99.8*	*98.2*	*146.2*	*146.2*	*100.0*	*100.4*	*104.8*	*58.2*	*53.3*	*100.0*	*99.8*	*98.1*	*94.0*	*98.1*	
12	73.4	73.1	72.3	108.0	108.0	68.6	68.5	72.3	38.9	36.4	78.3	78.1	75.9	73.4	75.9	12
	100.0	*99.6*	*98.7*	*147.2*	*147.2*	*100 0*	*100 0*	*105 5*	*56.8*	*53.0*	*100.0*	*99.8*	*96 9*	*93.8*	*96.9*	

Table 25.

Comparison of the Approximate Methods with the Slope-Deflection Method When Applied to the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 13.

Moments in the Columns.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Story No.	Moment at Top of Column A					Moment at Bottom of Column A					Moment at Top of Column B					Moment at Bottom of Column B					Story No.
	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			
			I	II	III			I	II	III			I	II	III			I	II	III	
1	193.2 *100.0*	203.5 *105.5*	138.0 *71.4*	254.4 *131.5*	254.4 *131.5*	275.9 *100.0*	203.5 *73.8*	138.0 *50.0*	254.4 *92.0*	254.4 *92.0*	248.0 *100.0*	305.0 *122.8*	370.5 *149.5*	254.4 *102.2*	254.4 *102.2*	302.8 *100.0*	305.0 *100.5*	370.5 *122.0*	254.4 *84.0*	254.4 *80.4*	1
2	125.9 *100.0*	127.5 *101.3*	92.5 *73.5*	171.4 *136.0*	171.4 *136.0*	123.8 *100.0*	127.5 *102.8*	92.5 *75.0*	171.4 *148.2*	171.4 *148.2*	212.7 *100.0*	214.0 *100.5*	248.5 *117.0*	171.4 *80.5*	171.4 *80.5*	223.7 *100.0*	214.0 *95.8*	248.5 *111.3*	171.4 *76.8*	171.4 *76.8*	2
3	110.6 *100.0*	108.0 *97.5*	75.9 *68.7*	140.5 *127.0*	140.5 *127.0*	101.6 *100.0*	108.0 *106.5*	75.9 *74.7*	140.5 *138.2*	140.5 *138.2*	177.1 *100.0*	174.0 *98.2*	203.9 *114.9*	140.5 *79.3*	140.5 *79.3*	171.6 *100.0*	174.0 *101.5*	203.9 *118.6*	140.5 *82.0*	140.5 *82.0*	3
4	99.4 *100.0*	97.8 *98.3*	71.5 *72.0*	131.7 *132.2*	131.7 *132.2*	99.4 *100.0*	97.8 *98.3*	71.5 *72.0*	131.7 *132.0*	131.7 *132.0*	165.3 *100.0*	164.1 *99.5*	191.9 *116.0*	131.7 *79.6*	131.7 *79.6*	163.7 *100.0*	164.1 *100.3*	191.9 *117.5*	131.7 *80.5*	131.7 *80.5*	4

Table 26

Comparison of the Approximate Methods with the Slope-Deflection Method When Applied to the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 13.

Moments in the Girders.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Story No.	Moment at Right End of Girder *a*					Moment at Left End of Girder *a*					Moment at End of Girder *b*					Story No.
	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			
			I	II	III			I	II	III			I	II	III	
1	316.9	331.0	230.9	425.8	425.8	277.4	309.0	230.9	138.5	36.0	194.5	210.0	389.2	287.3	389.8	1
	100.0	*104.5*	*73.0*	*135.2*	*135.2*	*100.0*	*111.5*	*83.3*	*50.0*	*13.0*	*100.0*	*108.0*	*200.0*	*148.0*	*200.0*	
2	227.6	235.5	167.7	311.9	311.9	209.4	212.2	167.7	103.0	23.6	174.6	175.8	282.8	208.8	288.2	2
	100.0	*103.8*	*73.5*	*137.0*	*137.0*	*100.0*	*101.9*	*80.0*	*49.4*	*11.3*	*100.0*	*100.5*	*162.0*	*119.6*	*165.0*	
3	210.1	205.8	147.7	272.2	272.2	194.7	194.1	147.7	90.7	23.2	146.4	144.0	249.1	181.4	249.0	3
	100.0	*98.0*	*70.0*	*129.5*	*129.5*	*100.0*	*99.7*	*75.8*	*46.6*	*11.9*	*100.0*	*98.2*	*170.0*	*124.0*	*170.0*	
4	191.0	192.3	138.1	254.5	254.5	175.6	173.3	138.1	84.8	21.7	143.3	143.0	232.9	169.7	232.8	4
	100.0	*100.8*	*72.3*	*133.0*	*133.0*	*100.0*	*98.8*	*79.0*	*48.3*	*12.4*	*100.0*	*98.8*	*162.2*	*118.1*	*162.2*	

Table 27.

Effect of the Proportions of a Bent Upon the Accuracy of Method I.

All Girder Sections Are the Same.

All stories of a bent are identical, and the shears on all stories are equal. All column sections are equal, all girder sections are equal, and the column sections are equal to the girder sections.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column *A*		Moment at Top and Bottom of Column *B*		Moment at Right End of Girder *a*		Moment at Left End of Girder *a*		Moment at End of Girder *b*	
	Ratio of Story Height to Width of Bay *a*	Ratio of Width of Bay *a* to Width of Bay *b*	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I
1	2	2	9.09 *100.0*	9.62 *105.9*	15.91 *100.0*	15.38 *96.5*	18.20 *100.0*	19.24 *105.9*	13.67 *100.0*	19.24 *140.7*	18.20 *100.0*	11 52 *63.2*
2	2	1	10.00 *100.0*	7.50 *75.0*	15.00 *100.0*	17.50 *116.5*	20.00 *100.0*	15.00 *75.0*	16.66 *100.0*	15.0 *90.0*	13.33 *100.0*	20.00 *150.0*
3	2	0.5	10.87 *100.0*	5.00 *46.0*	14.13 *100.0*	20.00 *141.5*	21.70 *100.0*	10.00 46.1	19.54 *100.0*	10.00 *51.2*	8.70 *100.0*	30.00 *345.0*
4	1	2	8.09 *100.0*	9.62 *119.00*	16.91 *100.0*	15.38 *90.9*	16.18 *100.0*	19.24 *119.3*	13.23 *100.0*	19.24 *145.5*	20.60 *100.0*	11.52 *55.9*
5	1	1	9.38 *100.0*	7.50 *79.9*	15.62 *100.0*	17.50 *111.9*	18.75 *100.0*	15.00 *80.0*	16.66 *100.0*	15.00 *90.0*	14.60 *100.0*	20.00 *137.0*
6	1	0.5	10.53 *100.0*	5.00 *47.5*	14.47 *100.0*	20.00 *138.0*	21.04 *100.0*	10.00 *47.5*	19.74 *100.0*	10.00 *50.6*	9.20 *100.0*	30.00 *326.0*
7	0.5	2	7.32 *100.0*	9.62 *131.3*	17.67 *100.0*	15.38 *87.0*	14.66 *100.0*	19.24 *131.0*	12.93 *100.0*	19.24 *149.0*	22.40 *100.0*	11.52 *51.3*
8	0.5	1	8.93 *100.0*	7.50 *83.9*	16.06 *100.0*	17.50 *108.7*	17.84 *100.0*	15.00 *83.9*	16.66 *100.0*	15.00 *90.00*	15.45 *100.0*	20.00 *129.5*
9	0.5	0.5	10.33 *100.0*	5.00 *48.4*	14.68 *100.0*	20.00 *136.1*	20.61 *100.0*	10.00 *48.5*	19.76 *100.0*	10.0 *50.6*	9.60 *100.0*	30 00 *312.0*

Table 28.

Effect of the Proportions of a Bent Upon the Accuracy of Method I.

All Girders Proportional to the Bending Moments.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column A and girder a are equal, and the ratio of the moment of inertia of girder a to the moment of inertia of girder b equals the ratio of the bending moment in girder a to the bending moment in girder b, as determined by method I.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column A		Moment at Top and Bottom of Column B		Moment at Right End of Girder a		Moment at Left End of Girder a		Moment at End of Girder b	
	Ratio of Story Height to Width of Bay a	Ratio of Width of Bay a to Width of Bay b	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I
1	2	2	9.75 *100.0*	9.62 *98.8*	15.25 *100.0*	15.38 *100.5*	19.50 *100.0*	19.24 *98.8*	16.05 *100.0*	19.24 *120.0*	14.45 *100.0*	11.52 *79.6*
2	2	1	9.60 *100.0*	7.50 *78.1*	15.40 *100.0*	17.50 *113.6*	19.2 *100.0*	15.00 *78.1*	15.60 *100.0*	15.00 *96.1*	15.20 *100.0*	20.00 *131.5*
3	2	0.5	9.45 *100.0*	5.00 *52.9*	15.55 *100.0*	20.00 *129.0*	18.9 *100.0*	10.00 *52.9*	15.10 *100.0*	10.00 *66.3*	16.00 *100.0*	30.00 *187.5*
4	1	2	9.05 *100.0*	9.62 *106.2*	15.95 *100.0*	15.38 *96.2*	18.10 *100.0*	19.24 *106.2*	15.70 *100.0*	19.24 *122.5*	16.20 *100.0*	11.52 *92.6*
5	1	1	8.85 *100.0*	7.50 *84.8*	16.15 *100.0*	17.50 *108.3*	17.70 *100.0*	15.00 *84.8*	15.20 *100.0*	15.00 *98.8*	17.10 *100.0*	20.00 *117.0*
6	1	0.5	8.60 *100.0*	5.00 *58.0*	16.40 *100.0*	20.00 *122.0*	17.20 *100.0*	10.00 *58.1*	14.7 *100.0*	10.00 *68.0*	18.1 *100.0*	30.00 *165.8*
7	0.5	2	8.55 *100.0*	9.62 *112.5*	16.45 *100.0*	15.38 *93.2*	17.10 *100.0*	19.24 *112.5*	15.80 *100.0*	19.24 *122.0*	17.1 *100.0*	11.52 *67.3*
8	0.5	1	8.30 *100.0*	7.50 *90.3*	16.70 *100.0*	17.50 *105.0*	16.60 *100.0*	15.00 *90.4*	15.20 *100.0*	15.00 *98.8*	18.2 *100.0*	20.00 *110.0*
9	0.5	0.5	8.00 *100.0*	5.00 *62.5*	17.00 *100.0*	20.00 *117.5*	16.00 *100.0*	10.00 *62.5*	14.60 *100.0*	10.00 *68.5*	19.4 *100.0*	30.00 *154.8*

Table 29.

Effect of the Proportions of a Bent Upon the Accuracy of Method II.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column A and girder a are equal, and the ratio of the moment of inertia of girder a to the moment of inertia of girder b equals the ratio of the bending moment in girder a to the bending moment in girder b, as determined by method II.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column A		Moment at Top and Bottom of Column B		Moment at Right End of Girder a		Moment at Left End of Girder a		Moment at End of Girder b	
	Ratio of Story Height to Width of Bay a	Ratio of Width of Bay a to Width of Bay b	Slope-Deflection Method	Method II	Slope-Deflection Method	Method II	Slope-Deflection Method	Method II	Slope-Deflection Method	Method II	Slope-Deflection Method	Method II
1	2	2	9.65 *100.0*	12.50 *129.5*	15.35 *100.0*	12.50 *81.5*	19.30 *100.0*	25.00 *129.5*	15.45 *100.0*	8.33 *53.9*	15.25 *100.0*	16.66 *109.3*
2	2	1	10.55 *100.0*	12.50 *118.5*	14.45 *100.0*	12.50 *86.5*	21.10 *100.0*	25.00 *118.5*	18.30 *100.0*	8.33 *45.6*	10.60 *100.0*	16.66 *157.1*
3	2	0.5	11.25 *100.0*	12.50 *111.0*	13.75 *100.0*	12.50 *91.0*	22.50 *100.0*	25.00 *111.1*	20.95 *100.0*	8.33 *39.8*	6.55 *100.0*	16.66 *255.0*
4	1	2	8.82 *100.0*	12.50 *141.8*	16.18 *100.0*	12.50 *77.3*	17.64 *100.0*	25.00 *141.8*	15.26 *100.0*	8.33 *54.6*	17.10 *100.0*	16.66 *97.5*
5	1	1	10.10 *100.0*	12.50 *123.8*	14.90 *100.0*	12.50 *83.9*	20.20 *100.0*	25.00 *124.0*	18.45 *100.0*	8.33 *45.20*	11.35 *100.0*	16.66 *146.8*
6	1	0.5	11.00 *100.0*	12.50 *113.5*	14.00 *100.0*	12.50 *89.3*	22.00 *100.0*	25.00 *113.7*	21.20 *100.0*	8.33 *39.3*	6.80 *100.0*	16.66 *245.0*
7	0.5	2	8.30 *100.0*	12.50 *150.6*	16.70 *100.0*	12.50 *74.9*	16.60 *100.0*	25.00 *150.7*	15.20 *100.0*	8.33 *54.9*	18.20 *100.0*	16.66 *91.6*
8	0.5	1	9.80 *100.0*	12.50 *127.5*	15.20 *100.0*	12.50 *82.3*	19.60 *100.0*	25.00 *127.0*	18.55 *100.0*	8.33 *44.9*	11.85 *100.0*	16.66 *140.5*
9	0.5	0.5	10.85 *100.0*	12.50 *115.2*	14.15 *100.0*	12.50 *88.4*	21.70 *100.0*	25.00 *115.0*	21.40 *100.0*	8.33 *38.9*	6.9 *100.0*	16.66 *241.5*

Table 30.

Effect of the Proportions of a Bent Upon the Accuracy of Method III.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column A and girder a are equal, and the ratio of the moment of inertia of girder a to the moment of inertia of girder b equals the ratio of the bending moment in girder a to the bending moment in girder b, as determined by method III.

For each bent the upper line is the moment in per cent of W x h, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column A		Moment at Top and Bottom of Column B		Moment at Right End of Girder a		Moment at Left End of Girder a		Moment at End of Girder b	
	Ratio of Story Height to Width of Bay a	Ratio of Width of Bay a to Width of Bay b	Slope-Deflection Method	Method III	Slope-Deflection Method	Method III	Slope-Deflection Method	Method III	Slope-Deflection Method	Method III	Slope-Deflection Method	Method III
1	2	2	10.12 *100.0*	12.50 *123.5*	14.88 *100.0*	12.50 *84.1*	20.24 *100.0*	25.00 *123.5*	17.06 *100.0*	13.48 *79.0*	12.70 *100.0*	11.52 *90.8*
2	2	1	10.30 *100.0*	12.50 *121.5*	14.70 *100.0*	12.50 *85.2*	20.60 *100.0*	25.00 *121.5*	17.60 *100.0*	5.0 *28.4*	11.80 *100.0*	20.00 *170.0*
3	2	0.5	10.67 *100.0*	12.50 *117.4*	14.33 *100.0*	12.50 *87.3*	21.34 *100.0*	25.00 *117.3*	18.79 *100.0*	—5.0	9.87 *100.0*	30.00 *304.5*
4	1	2	9.55 *100.0*	12.50 *131.0*	15.45 *100.0*	12.50 *80.9*	19.10 *100.0*	25.00 *131.0*	17.00 *100.0*	13.48 *79.3*	13.90 *100.0*	11.52 *83.0*
5	1	1	9.80 *100.0*	12.50 *127.7*	15.20 *100.0*	12.50 *82.3*	19.60 *100.0*	25.00 *127.6*	17.60 *100.0*	5.0 *28.4*	12.80 *100.0*	20.00 *156.3*
6	1	0.5	10.26 *100.0*	12.50 *122.0*	14.74 *100.0*	12.50 *84.9*	20.52 *100.0*	25.00 *122.0*	18.98 *100.0*	—5.0	10.50 *100.0*	30.00 *286.1*
7	0.5	2	9.15 *100.0*	12.50 *136.9*	15.85 *100.0*	12.50 *78.8*	18.30 *100.0*	25.00 *136.8*	17.10 *100.0*	13.48 *78.8*	14.60 *100.0*	11.52 *79.0*
8	0.5	1	9.46 *100.0*	12.50 *132.2*	15.54 *100.0*	12.50 *80.5*	18.92 *100.0*	25.00 *132.1*	17.63 *100.0*	5.0 *28.4*	13.45 *100.0*	20.00 *148.8*
9	0.5	0.5	10.05 *100.0*	12.50 *124.5*	14.95 *100.0*	12.50 *83.70*	20.10 *100.0*	25.00 *124.5*	18.95 *100.0*	—5.0	10.95 *100.0*	30.00 *274.5*

Table 31.

Effect of the Proportions of a Bent Upon the Accuracy of Method IV.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column A and girder a are equal, and the ratio of the moment of inertia of girder a to the moment of inertia of girder b equals the ratio of the bending moment in girder a to the bending moment in girder b, as determined by method IV.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column A		Moment at Top and Bottom of Column B		Moment at Right End of Girder a		Moment at Left End of Girder a		Moment at End of Girder b	
	Ratio of Story Height to Width of Bay a	Ratio of Width of Bay a to Width of Bay b	Slope-Deflection Method	Method IV	Slope-Deflection Method	Method IV	Slope-Deflection Method	Method IV	Slope-Deflection Method	Method IV	Slope-Deflection Method	Method IV
1	2	2	9.09 *100.0*	8.33 *91.8*	15.91 *100.0*	16.66 *104.5*	18.18 *100.0*	16.66 *91.7*	13.62 *100.0*	16.66 *122.1*	18.20 *100.0*	16.66 *91.5*
2	2	1	10.0 *100.0*	8.33 *83.3*	15.00 *100.0*	16.66 *111.1*	20.00 *100.0*	16.66 *83.3*	16.66 *100.0*	16.66 *100.0*	13.33 *100.0*	16.66 *125.0*
3	2	0.5	10.89 *100.0*	8.33 *76.8*	14.13 *100.0*	16.66 *117.8*	21.74 *100.0*	16.66 *76.6*	19.56 *100.0*	16.66 *85.20*	8.70 *100.0*	16.66 *191.5*
4	1	2	8.09 *100.0*	8.33 *103.2*	16.91 *100.0*	16.66 *98.3*	16.18 *100.0*	16.66 *103.0*	13.22 *100.0*	16.66 *126.0*	20.60 *100.0*	16.66 *80.9*
5	1	1	9.38 *100.0*	8.33 *88.8*	15.62 *100.0*	16.66 *106.4*	18.76 *100.0*	16.66 *88.9*	16.64 *100.0*	16.66 *100.0*	14.60 *100.0*	16.66 *114.0*
6	1	0.5	10.53 *100.0*	8.33 *79.2*	14.47 *100.0*	16.66 *115.0*	21.06 *100.0*	16.66 *79.1*	19.74 *100.0*	16.66 *84.4*	9.20 *100.0*	16.66 *181.0*
7	0.5	2	7.32 *100.0*	8.33 *113.9*	17.68 *100.0*	16.66 *94.2*	14.64 *100.0*	16.66 *113.8*	12.96 *100.0*	16.66 *128.7*	22.40 *100.0*	16.66 *74.4*
8	0.5	1	8.93 *100.0*	8.33 *93.3*	16.07 *100.0*	16.66 *103.7*	17.86 *100.0*	16.66 *93.3*	16.69 *100.0*	16.66 *100.0*	15.45 *100.0*	16.66 *107.8*
9	0.5	0.5	10.33 *100.0*	8.33 *80.7*	14.67 *100.0*	16.66 *113.5*	20.66 *100.0*	16.66 *80.6*	19.74 *100.0*	16.66 *84.4*	9.60 *100.0*	16.66 *173.5*

Table 32.

Log of the Test of Celluloid Model No. 4.

Story No.	Shear W in Lb.	Horizontal Deflection at Center of Girder at Top of Story in Inches			Change in Slope Measured on an Arc Having a 19-inch Radius, in Inches.							
					Force at Top of Model Acting Toward the Right				Force at Top of Model Acting Toward the Left			
					θ at Top of Column A		θ at Top of Column B		θ at Top of Column A		θ at Top of Column A	
		Right-Hand Span	Middle Span	Left-Hand Span	Right-Hand Side of Model	Left-Hand Side of Model	Right-Hand Side of Model	Left-Hand Side of Model	Right-Hand Side of Model	Left-Hand Side of Model	Right-Hand Side of Model	Left-Hand Side of Model
	1.5	.04	.04	.04	.20	.20	.09	.10	.19	.18	.09	.10
1	3.0	.08	.08	.08	.40	.40	.20	.21	.40	.40	.19	.23
	4.5	.11	.11	.11	.61	.59	.31	.33	.61	.63	.30	.38
	1.5	.08	.08	.08	.21	.21	.10	.10	.19	.19	.11	.10
2	3.0	.16	.16	.16	.43	.43	.21	.20	.41	.41	.24	.22
	4.5	.25	.24	.24	.66	.66	.34	.33	.63	.65	.37	.37
	1.5	.14	.14	.14	.18	.17	.11	.12	.17	.16	.13	.14
3	3.0	.28	.29	.28	.37	.35	.26	.26	.36	.35	.26	.29
	4.5	.42	.42	.42	.59	.54	.39	.40	.56	.55	.40	.42
	1.5	.18	.18	.18	.13	.10	.06	.07	.13	.09	.06	.07
4	3.0	.36	.36	.37	.25	.19	.13	.15	.26	.20	.13	.15
	4.5	.55	.55	.55	.40	.30	.20	.22	.39	.27	.18	.25

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904 *None available.*

Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

Circular No. 3. Fuel Tests with Illinois Coal (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907), by L. P. Breckenridge and Paul Diserens. 1909. *Thirty cents.*

Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

Bulletin No. 5. Resistance of Tubes to Collapse, by Albert P. Carman and M. L. Carr. 1906. *None available.*

Bulletin No. 6. Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *None available.*

Bulletin No. 7. Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr, and Henry B. Dirks. 1906. *None available.*

Bulletin No. 8. Tests of Concrete: I, Shear; II, Bond, by Arthur N. Talbot. 1906. *None available.*

Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition 1912. *Fifty cents.*

Bulletin No. 10. Tests of Concrete and Reinforced Concrete Columns. Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 11. The Effect of Scale on the Transmission of Heat Through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

Bulletin No. 12. Tests of Reinforced Concrete T-Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 13. An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *None available.*

Bulletin No. 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 15. How to Burn Illinois Coal Without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*

Bulletin No. 16. A Study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*

Bulletin No. 17. The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *None available.*

Bulletin No. 18. The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*

Bulletin No. 19. Comparative Tests of Carbon, Metallized Carbon and Tantalum Filament Lamps, by T. H. Amrine. 1908. *None available.*

Bulletin No. 20. Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908. *None available.*

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INFLUENCE OF TEMPERATURE ON THE STRENGTH OF CONCRETE

BY

A. B. McDANIEL

ONTARIO RESEARCH FOUNDATION
47 Queen's Park
NOV 16 1938
TORONTO

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 81 JULY, 1915

INFLUENCE OF TEMPERATURE ON THE STRENGTH OF CONCRETE

BY A. B. MCDANIEL, ASSISTANT PROFESSOR OF CIVIL ENGINEERING.

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INFLUENCE OF TEMPERATURE ON THE STRENGTH OF CONCRETE

I. Introduction.

1. *Preliminary.*—The general use of concrete in various kinds of construction and at all seasons of the year renders important a knowledge of the effect of temperature upon the strength of this material. It is of special economic importance to the contractor or the builder to be informed concerning the strength of concrete at early ages under different temperature conditions so that he may know when to remove forms and what loads may be safely applied to the different parts of a structure.

2. *Scope of Bulletin.*—It is the purpose of this bulletin to furnish some information concerning the influence of temperature on the attainment of strength in concrete.

Three groups of tests were made, viz.; forty-five 6 by 6-in. cylinders; fifty-one 6-in. cubes; and sixty 8 by 16-in. cylinders.

The temperature conditions were limited by available facilities.

3. *Acknowledgment.*—The tests reported herein were made in the Laboratory of Applied Mechanics of the University of Illinois. The work was done under the supervision of the writer. Special acknowledgment is due to the Department of Theoretical and Applied Mechanics for the use of material and apparatus. The writer is indebted to A. N. Talbot, Professor in Charge of Theoretical and Applied Mechanics, and to Ira O. Baker, Professor of Civil Engineering, for their co-operation in planning the tests and in interpreting the data.

The tests of Groups I and II—1913 Series—were made by J. Albert Anderson and W. J. Bublitz, senior civil engineering students of the class of 1914; and furnished the subject matter of their baccalaureate thesis. The tests of Group III—1914 Series—were made by J. Albert Anderson, a graduate student in the Department of Civil Engineering; and special credit is due Mr. Anderson for the preparation of the tables and diagrams. All the tests were made with painstaking care and faithful attention to uniformity and accuracy of manipulation.

II. Materials, Form of Test Pieces, and Methods of Storing and Testing.

4. *Materials.*—The materials were of the same character and quality as those used for other concrete and reinforced concrete specimens made and tested by the Engineering Experiment Station during the past five years. The quality of the materials may be taken as representative of that used in first-class concrete work in the Middle West.

Cement. All of the test specimens were made with Universal portland cement. Samples were taken at the beginning of each series and were tested for fineness, soundness, and tensile strength. The cement passed the requirements of the Standard Specifications of the American Society for Testing Materials. The tensile strength tests of neat and 1:3 mortar briquettes made of a sample of the cement used in Group III of the 1914 Series gave average values of 542 and 609 lb. per sq. in. for the neat cement at seven and twenty-eight days, respectively; and 174 and 295 lb. per sq. in. for the 1:3 mortar at seven and twenty-eight days, respectively.

Sand. The sand used came from a deposit of glacial drift near the Wabash River at Attica, Indiana. The sand was clean and well graded. The sand of the 1913 Series was somewhat coarser than that of the 1914 lot. The sand used in Group III—1914 Series—gave a density of 1.79, a specific gravity of 2.65, and contained 32 per cent voids.

Stone. The crushed limestone came from Kankakee, Illinois. The stone used in the 1913 Series contained 87 per cent material smaller than one-half inch and 46 per cent material smaller than one-fourth inch. The stone used in the 1914 series was well graded. It contained 49 per cent voids, and had a density of 1.35 and a specific gravity of 2.65. It was carefully screened over a ¼-in. screen before use, and contained 10 per cent of material smaller than one-fourth inch.

5. *Concrete.*—All the concrete was composed of one part cement, 2 parts sand, and 4 parts broken stone, by weight; corresponding to 1 part cement, 2.2 parts sand, and 3.6 parts broken stone, by volume. The materials for each specimen were weighed out separately and then mixed.

The mixing of the concrete for the 1913 series was done with a trowel in a large galvanized iron pan. The cement and sand were first mixed dry to a uniform color and spread out in a layer of uni-

TABLE 1.
DESCRIPTION OF TEST SPECIMENS

Series	Group	Set	Specimens		Number and Age of Specimens When Tested
			Number	Form	
1913	I	A	15	6 x 6-in. cylinders	5 specimens of each set; at 7, 14, and 28 days.
		B	15		
		C	15		
1913	II	D	15	6-in. cubes	3 specimens of each set; at 4, 7, 11, 14, and 28 days.
		E	18		
		F	18		
1914	III	G	15	8 x 16-in. cylinders	3 specimens of each set; at 3, 7, 10, 14, and 28 days.
		H	15		
		I	15		
		M	15		

form thickness over the bottom of the pan. The stone was then added, and the whole mass given four complete turnings, which secured thorough incorporation of the dry materials. Water was added, and the material turned until thoroughly mixed. The concrete was gathered together in a compact mass, in one end of the mixing pan, so as to reduce evaporation losses to a minimum. The time of mixing of each specimen was kept as nearly constant as possible.

The concrete used in the 1914 Series was mixed in similar manner to that of the 1913 Series, but was mixed on the concrete floor of the laboratory with shovels.

6. *Molding and Storage of Test Specimens.*—The specimens were classified according to the form of test specimen and storage conditions. Table 1 gives the details of the classification.

TABLE 2.
DATA CONCERNING MOLDING OF SPECIMENS

Type of Specimen	Set	Average Time of Molding, minutes	Average Temperature		Weights of Materials			Water, per cent*
			Air	Concrete	Cement, lb.	Sand, lb.	Stone, lb.	
6-in. cylinders	A	8.5	32°F.	70°F.	2.17	4.34	8.68	10.0
	B	8.5	65	71	2.17	4.34	8.68	10.0
	C	8.5	84	70	2.17	4.34	8.68	10.0
6-in. cubes	D	7.0	77	70	2.42	4.84	9.68	10.0
	E	7.0	75	70	2.42	4.84	9.68	11.0
	F	7.0	71	69	2.42	4.84	9.68	10.0
8 x 16-in. cylinders	G		68	69	10.2	20.4	40.8	9.3
	H							
	I							
	M							

*The concrete used in Groups I and II was of a medium or quaking consistency; while that used in Group III was wet, and was similar in consistency to that used in concrete building construction.

Molding. The specimens of Group I of the 1913 Series were molded in the storage rooms under the following temperatures: Set A at 32°F., Set B at 65°F., and Set C at 84°F. The specimens of Group II of the 1913 Series were molded in the cement laboratory at the following temperatures: Set D at 77°F., Set E at 75°F., and Set F at 71°F. The specimens of Group III—1914 Series—were molded in the concrete room of the Engineering Experiment Station at a temperature of 68°F. The specimens of Group II and Group III were moved to their respective storage rooms after a set of six hours.

The forms used for Group I were sheet-iron cylinders 6 in. in diameter and 6 in. high. The specimens of Group II were molded in three-gang cube forms made up of two 6-in. channels and plates placed 6 in. apart. The forms for the specimens of Group III were sections of standard 8-in. wrought iron pipe, 16 in. long. The forms were removed from the specimens after a storage of two days.

Table 2 shows the weight of the dry materials, the per cent of water in terms of the total dry materials, the temperature of the room and of the concrete, and the average time of molding.

Storage. The temperature of the storage room was determined by daily readings of the maximum and minimum thermometers. The temperatures for the several groups are shown in Fig. 1–10.

Set A was stored in the ice-storage room of the Smith Ice Company in Urbana, at an average temperature of 30°F. Set B was stored in the meat storage room of the Smith Ice Company in Urbana, at an average temperature of 48.5°F. Set C was stored in the cement laboratory of the University of Illinois at an average temperature of 72.8°F.

Set D was stored in the cement laboratory of the University of Illinois at an average temperature of 68°F. Set E was stored in the ice chest of the Dairy Department of the University of Illinois at an average temperature of 35.5°F. Set F was stored in the ice-storage room of the Twin City Ice and Cold Storage Company of Champaign, at an average temperature of 27.1°F.

Set G was stored in the ice-storage room of the Twin City Ice and Cold Storage Company at Champaign, at an average temperature of 26.5°F. Set H was stored in the ice chest of the Dairy Department of the University of Illinois at an average temperature of 34.7°F. Set I was stored in an interior heated room of the Twin City Ice and Cold Storage Company of Champaign, at an average temperature of 71.8°F. Set M was stored in a chamber of the conduit tunnel under the Floriculture building of the University of Illinois, at an average temperature of 95.6°F.

All the specimens while in storage were covered with several layers of moist sacking, which was sprinkled daily.

7. *Method of Testing.*—All the specimens of Group I were taken from their storage places to the Laboratory of Applied Mechanics of the University of Illinois the day before they were tested. They were measured and weighed, their bearing surfaces coated with plaster of paris, and then were left in the open air of the laboratory for about twenty hours under a temperature of about 70°F.

The specimens of Group II were tested after about one hour from the time of their removal from the storage rooms. Two specimens of Set F, designated as F_{17} and F_{18}, after being stored under an average mean daily temperature of 27.1°F. for forty-four days, were stored in the testing laboratory under an average mean daily temperature of 70°F., the former for seven days and the latter for twenty-one days.

The specimens of Group III were brought to the testing laboratory from their storage places, weighed, measured, plastered, and tested within one hour. The specimens of Set G, which were stored under freezing temperatures, were allowed to thaw out before being tested.

In the tests a spherical-seated bearing block was used.

III. The Data.

8. *Observed Results.*—The results of the tests are given in Tables 3 to 11, pages 8 to 16, and in Fig. 1–10.

9. *Standardized Strength.*—Since a cube or a cylinder having a height equal to its diameter, tested for compressive strength, may be expected to give a value which is higher than the representative compressive strength of the material, it seems desirable for the purposes of comparison to reduce the observed values for the cubes and short cylinders of Groups I and II to what may be considered as the equivalent values which would be obtained from cylinders of height equal to twice their diameter. To do this the values for the cubes and cylinders have been multiplied by 0.73, which is the ratio of strength of prisms to strength of cubes determined by the Committee on Specifications and Methods of Tests for Concrete Materials of the American Concrete Institute. The reduced values are designated as the standardized strengths in Table 3 to 11, and are shown by the lower curve in Fig. 1–10.

Table 3.

Compressive Strength—Age 7 Days

Group I. 6 x 6-in. Cylinders

Set	Average Diameter, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Standardized Strength, lb. per sq. in.	Remarks
A	6.0	29 240	1030	890	650	Cracked uniformly around circumferential area.
	6.18	27 670	930			
	6.06	24 890	870			
	6.06	22 450	780			
	6.06	24 450	850			
B	6.12	36 130	1230	1100	800	Cracked uniformly
	6.00	34 770	1230			" "
	6.00	31 660	1120			" "
	5.87	25 030	930			Skewed
	5.87	26 950	980			"
C	5.87	27 080	1000	1200	880	Visible voids
	5.81	35 830	1350			
	5.94	37 660	1370			
	5.87	31 140	1150			
	6.00	32 250	1140			

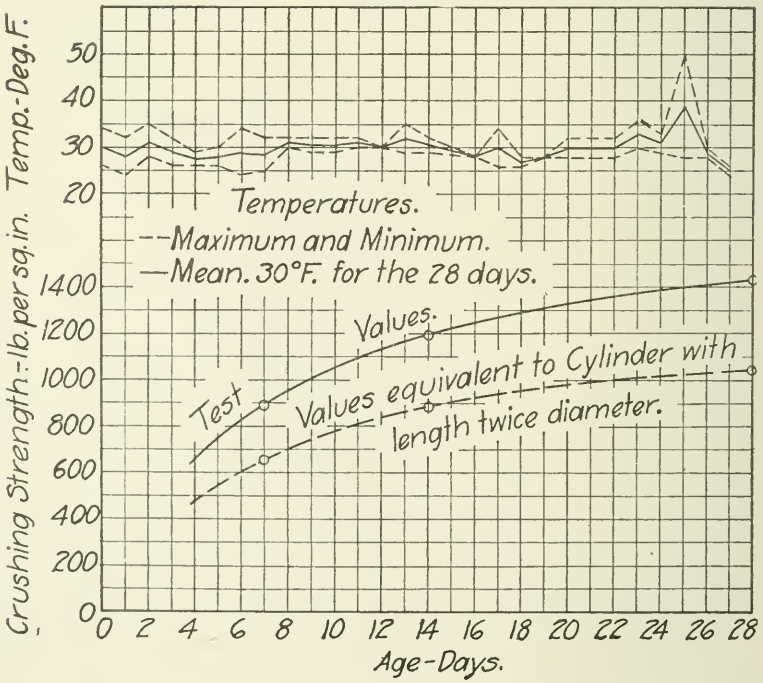

Fig. 1. Set A, Group I—1913 Series—6 x 6-in. Cylinders.

TABLE 4.

COMPRESSIVE STRENGTH—AGE 14 DAYS

Group I. 6 x 6-in. Cylinders

Set	Average Diameter, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Standardized Strength, lb. per sq. in.	Remarks
A	6.0	38 020	1340	1190	870	Cracked uniformly around circumferential area.
	6.12	33 340	1140			
	6.0	39 220	1390			
	6.0	31 390	1110			
	5.87	26 700	980			
B	6.0	47 090	1670	1540	1130	Cracked uniformly
	5.94	50 460	1820			" "
	5.97	45 850	1640			" "
	5.94	30 000	1090			Badly skewed
	5.87	40 640	1500			Slightly skewed
C	5.87	46 170	1700	1660	1210	Uniform throughout
	6.0	50 000	1770			
	6.12	44 190	1510			
	6.0	44 420	1570			
	5.87	47 100	1740			

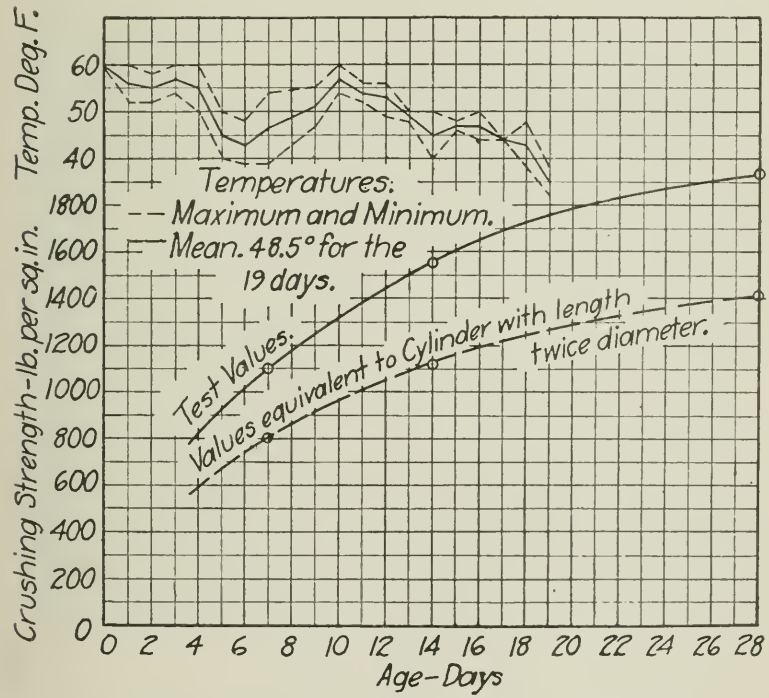

FIG. 2. SET B, GROUP I—1913 SERIES—6 x 6-IN. CYLINDERS.

TABLE 5.

COMPRESSIVE STRENGTH—AGE 28 DAYS

Group I. 6 x 6-in. Cylinders

Set	Average Diameter, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Standardized Strength, lb. per sq. in.	Remarks
A	6.06	39 340	1370	1430	1040	All uniform.
	6.06	37 730	1320			
	5.94	48 450	1750			
	6.12	37 660	1280			
	6.00	40 300	1430			
B	6.06	56 240	1960	1940	1410	Area reduced by visible voids.
	6.0	55 300	1950			
	6.06	54 600	1900			
	6.0	34 670	1230			
	6.0	40 000	1420			
C	5.97	55 720	2000	2090	1530	
	5.94	63 650	2310			
	5.87	60 260	2220			
	5.87	49 760	1840			Slightly skewed
	6.0	40 390	*1430			Badly skewed

*Not used in calculating average.

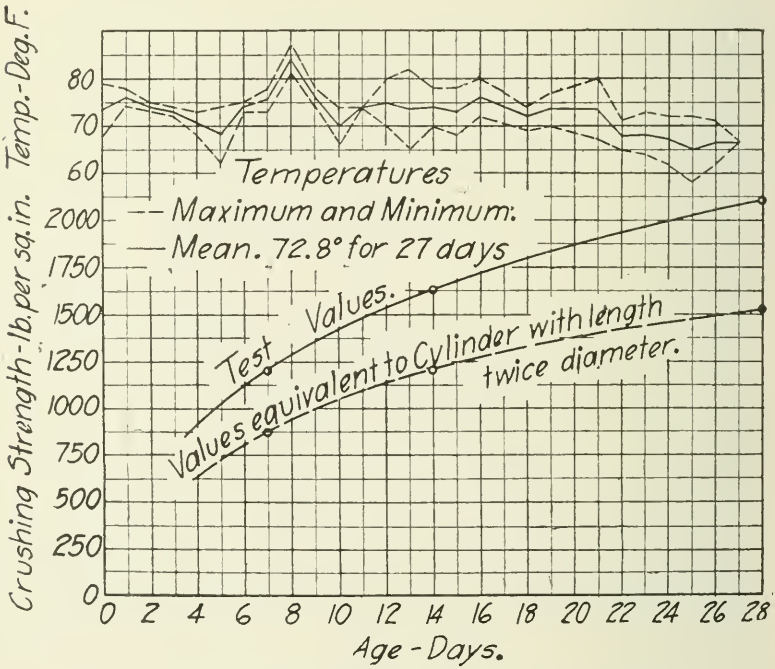

FIG. 3. SET C, GROUP I—1913 SERIES—6 x 6-IN. CYLINDERS.

TABLE 6.

COMPRESSIVE STRENGTH—AGE 4 DAYS

Group II. 6-in. Cubes

Set	Weight lb.	Size, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Standardized Strength, lb. per sq. in.	Remarks
D	18.50	6x6x6	28 400	790	780	570	
	18.50	"	22 450	620			
	18.50	"	33 340	920			
E	19.00	"	16 690	460	450	330	
	19.00	"	10 000	280			
	18.75	"	21 300	590			
F	18.75	"	15 680	440	390	280	Slight coating of frost, but all had uniform break.
	18.75	"	13 050	360			
	18.75	"	13 000	360			

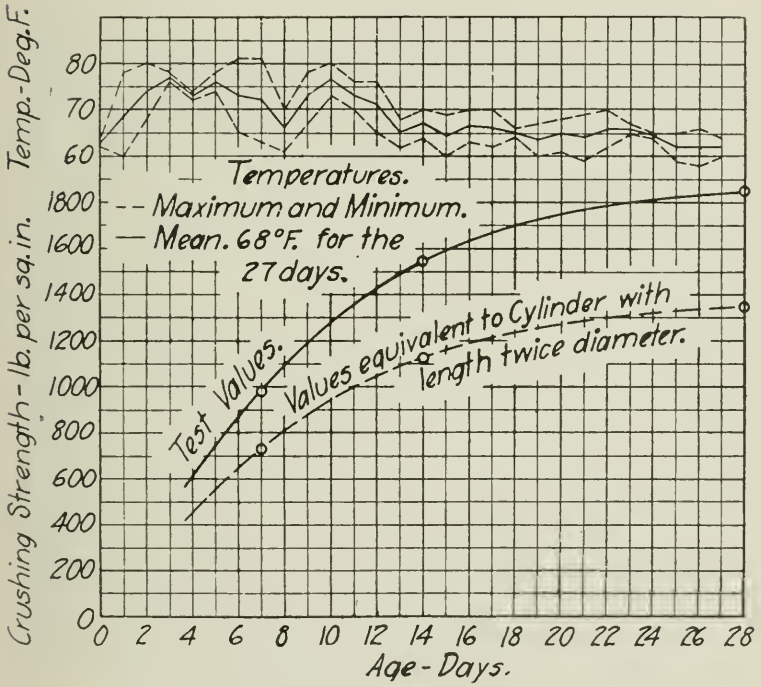

FIG. 4. SET D, GROUP II—1913 SERIES—6-IN. CUBES

TABLE 7.

COMPRESSIVE STRENGTH—AGE 7 DAYS

Group II. 6-in. Cubes

Set	Weight, lb.	Size, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Standardized Strength, lb. per sq. in.	Remarks
D	18.75	6x6x6	39 390	1090			
	19.00	"	35 930	1000	980	720	
	18.75	"	31 300	860			
E		"	17 100	470			
		"	19 820	550	470	340	Broke uniformly
		"	14 060	390			
F	18.50	"	20 880	580			
	18.75	"	19 230	530	560	410	
	18.75	"	20 760	580			

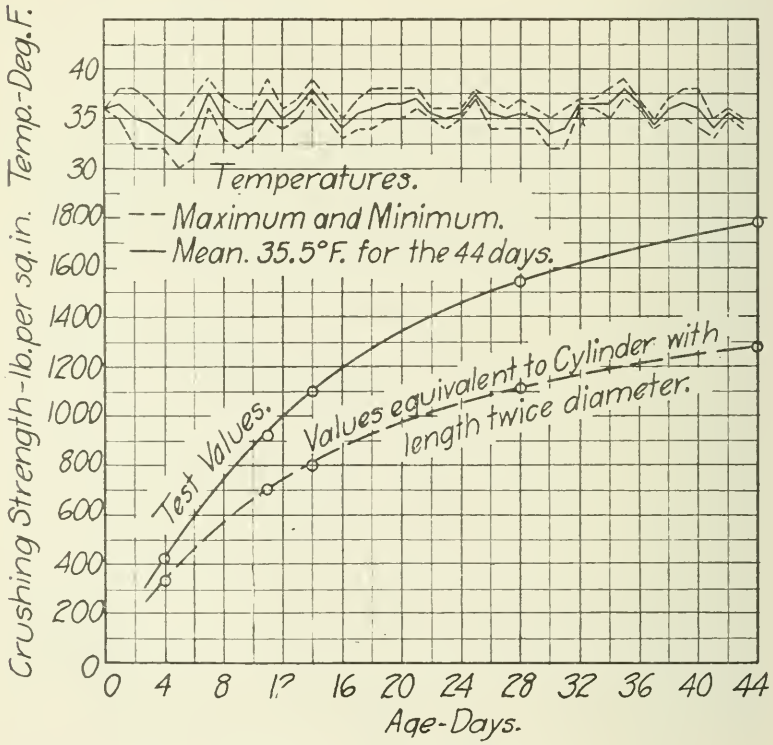

FIG. 5. SET E, GROUP II—1913 SERIES—6-IN. CUBES.

TABLE 8.

COMPRESSIVE STRENGTH—AGE 11 DAYS

Group II. 6-in. Cubes

Set	Weight lb.	Size, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Standardized Strength, lb. per sq. in.	Remarks
D	18.75	6x6x6	43 420	1200	1320	970	
	18.75	"	52 440	1460			
	18.75	"	47 300	1300			
E	19.00	"	42 310	1180	920	670	Visible voids
	19.00	"	31 060	860			Broke at one corner
	18.75	"	26 000	720			
F	18.75	"	15 080	420	500	370	
	18.75	"	17 760	490			
	18.75	"	21 820	610			

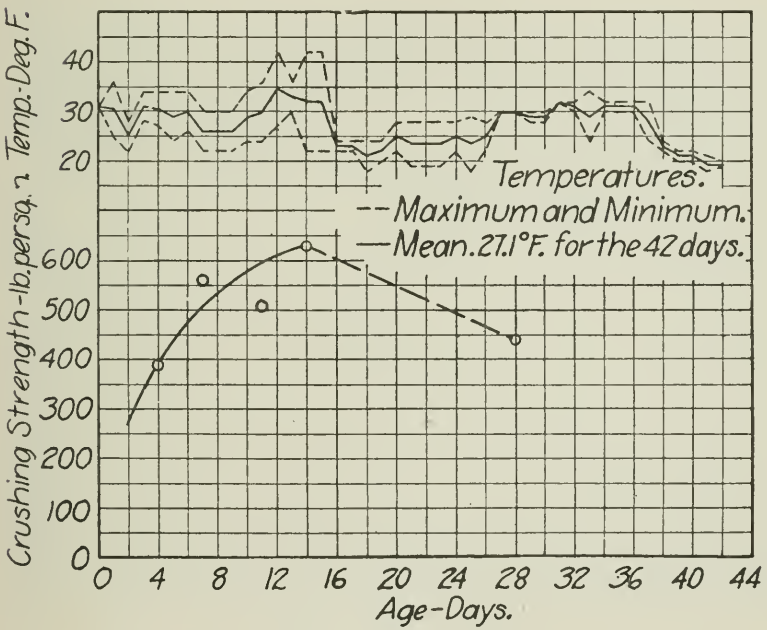

FIG. 6. SET F, GROUP II—1913 SERIES—6-IN. CUBES.

TABLE 9.

COMPRESSIVE STRENGTH—AGE 14 DAYS

Group II. 6-in. Cubes

Set	Weight, lb.	Size, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Standardized Strength, lb. per sq. in.	Remarks
D	19.00	6x6x6	58 710	1630	1540	1130	
	19.00	"	62 810	1740			
	18.75	"	45 530	1260			
E		"	38 630	1070	1100	800	
		"	40 300	1120			
		"	40 280	1120			
F	18.75	"	19 330	540	640	470	
	18.75	"	26 350	730			
	18.75	"	22 900	650			

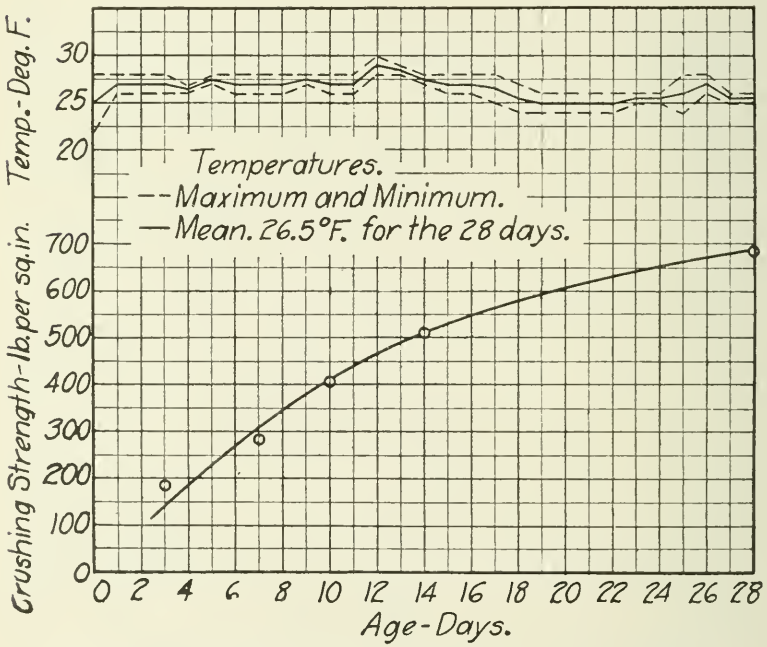

FIG. 7. SET G, GROUP III—1914 SERIES—8 x 16-IN. CYLINDERS.

TABLE 10.

COMPRESSIVE STRENGTH—AGE 28 DAYS

Group II. 6-in. Cubes

Set	Weight, lb.	Size, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Standardized Strength, lb. per sq. in.	Remarks
D	19.00	6x6x6	64 000	1780	1850	1350	
	19.00	"	67 460	1870			
	19.00	"	68 440	1900			
E	18.75	"	52 590	1460	1550	1130	
	18.75	"	58 270	1620			
	18.75	"	56 350	1560			
F	19.00	"	16 540	460	440	320	
	19.00	"	15 160	420			
	18.75	"	10 400	*290			

*Specimen in bad condition; not included in average.

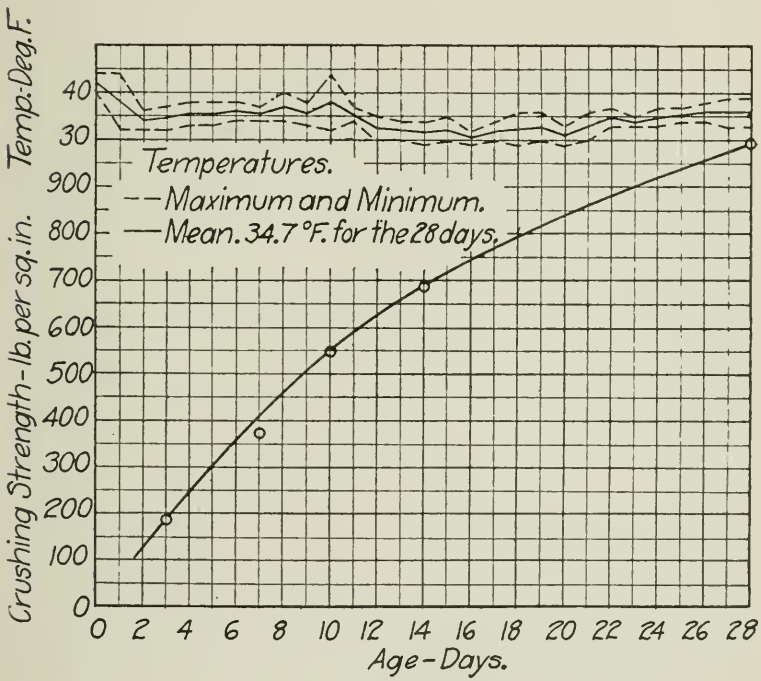

FIG. 8. SET H, GROUP III—1914 SERIES—8 x 16-IN. CYLINDERS.

TABLE 11.

COMPRESSIVE STRENGTH—AGE 42 DAYS

Group II. 6-in. Cubes.

Set	Weight, lb.	Size, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Standardized Strength, lb. per sq. in.	Remarks
E	18.75	6x6x6	66 710	1850	1780	1300	Broke uniformly
	18.75	''	62 880	1740	1780	1300	One corner broke
	18.75	''	62 240	1740			Slightly skewed
F	18.00	''	15 240	420			Specimens in a soft and crumbling condition
		5x5x6	31 720	1270*			
		4x5x6	8 040	400†			

*Age when tested, 49 days.
†Age when tested, 63 days.

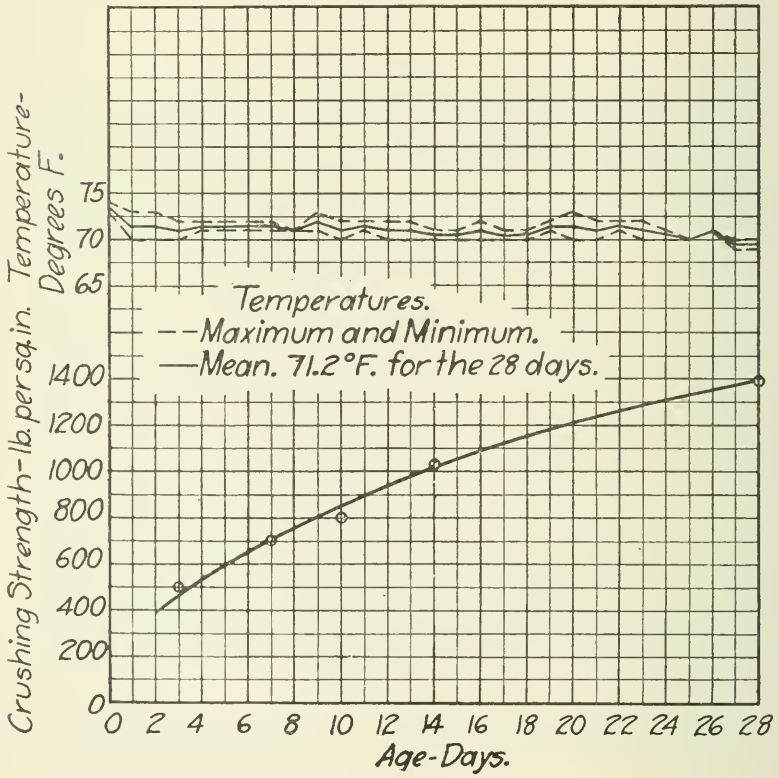

FIG. 9. SET I, GROUP III—1914 SERIES—8 x 16-IN. CYLINDERS.

TABLE 12.

COMPRESSIVE STRENGTH—AGE 3 DAYS

Group III—1914 Series—8 x 16-in. Cylinders

Set	Weight lb.	Average Diameter, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Remarks
G	67.75	7.94	8 880	180	190	Crumbled
	67.25	7.87	9 720	200		
	70.25	8.06	9 340	180		Crumbled badly
H	65.0	7.94	9 950	200	180	
	66.0	7.87	8 250	170		
	65.0	8.0	9 250	180		Plaster loose on one end
I	69.0	8.0	29 650	600	500	
	70.25	8.06	23 750	460		
	69.75	8.06	22 600	440		
M	64.0	8.0	24 000	480	500	
	65.0	7.94	30 850	620.		
	65.0	7.94	20 000	400		

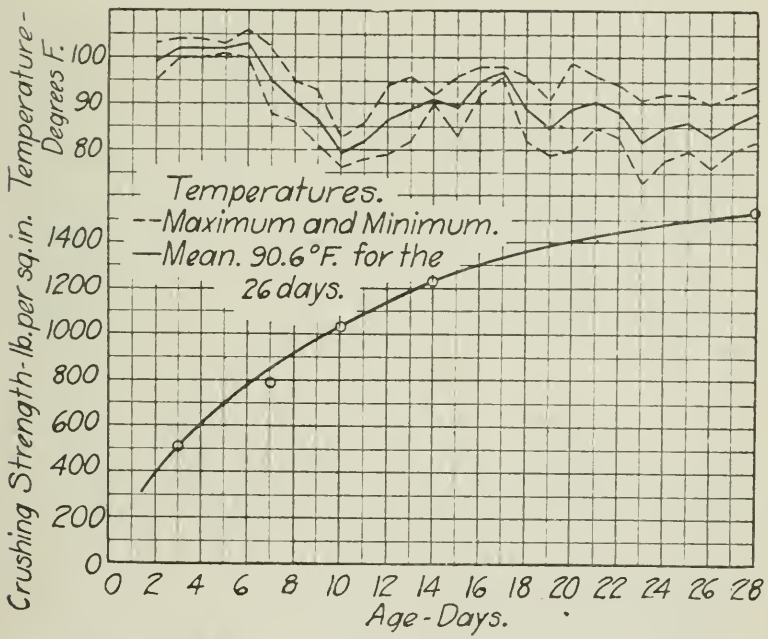

FIG. 10. SET M, GROUP III—1914 SERIES—8 x 16-IN. CYLINDERS.

Table 13.

Compressive Strength—Age 7 Days

Group III—1914 Series—8 x 16-in. Cylinders

Set	Weight, lb.	Average Diameter, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Remarks
G	67.75	8.0	17 200	340	280	
	67.25	8.0	14 750	290		
	65.50	8.0	10 700	210		
H	66.0	7.87	14 450	300*	370	Skewed one inch, horizontal crack
	69.5	8.06	18 920	370		
	65.5	7.94	18 530	370		
I	67.75	8.0	40 800	810	700	
	67.25	7.94	31 630	640		
		8.12	34 340	660		
M	68.0	8.0	44 500	890	790	
	69.0	8.06	40 250	790		
	69.0	8.06	35 150	690		

*Not used in calculating average strength.

Table 14.

Compressive Strength—Age 10 Days

Group III—1914 Series—8 x 16-in. Cylinders

Set	Weight, lb.	Average Diameter, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Remarks
G	66.5	8.0	18 670	370	400	
	67.5	7.87	18 280	370		
	70.5	8.0	23 630	470		
H	68.0	8.0	25 000	500	540	2—8x8 forms
	68.5	8.12	30 680	600		
	69.0	8.06	26 830	530		
I	67.5	8.06	44 700	880	800	Top crumbled
	68.0	8.0	36 050	720		Skewed
						Fractured in transit
M	68.0	7.94	59 620	1200	1030	
	69.0	8.06	46 700	920		
	69.5	8.06	49 540	970		2—8x8 forms

Table 15.

Compressive Strength—Age 14 Days

Group III—1914 Series—8 x 16-in. Cylinders

Set	Weight, lb.	Average Diameter, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Remarks
G	70.0	8.0	26 430	520	510	
	68.5	7.94	25 330	510		
	69.5	8.06	25 420	500		
H	67.5	7.94	30 550	620	690	
	64.5	8.06	35 100	690		
	66.5	8.0	37 750	750		
I	66.0	7.94	51 000	1030	1040	Skewed slightly
	69.5	8.0	63 230	1260		Bearing faces not parallel
	67.0	8.0	41 430	820		
M	67.5	8.0	58 000	1150	1220	
	66.5	7.94	40 650	820*		Visible voids
	66.5	8.0	65 000	1290		

*Not used in calculating average strength.

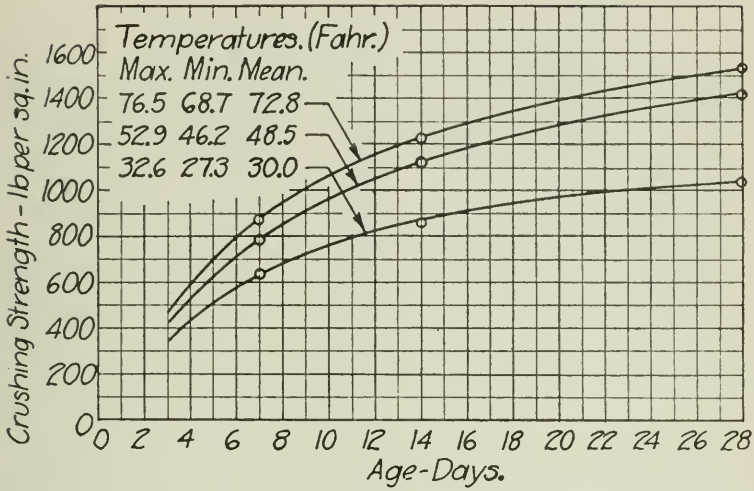

FIG. 11. GROUP I—1913 SERIES—6 x 6-IN. CYLINDERS. STANDARDIZED VALUES.

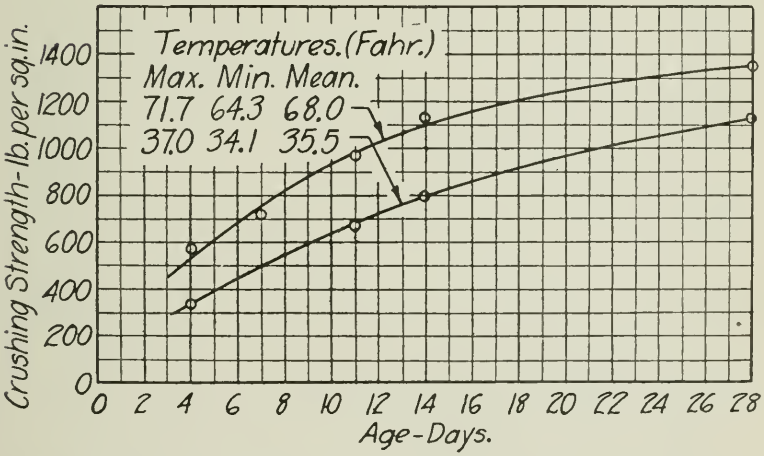

FIG. 12. GROUP II—1913 SERIES—6-IN. CUBES. STANDARDIZED VALUES.

TABLE 16.

COMPRESSIVE STRENGTH—AGE 28 DAYS

Group III—1914 Series—8 x 16-in. Cylinders

Set	Weight, lb.	Average Diameter, in.	Crushing Strength, lb.	Strength, lb. per sq. in.	Average Strength, lb. per sq. in.	Remarks
G	68.0	8.0	32 100	630	680	
	67.5	7.94	35 900	730		
	65.0	7.94	34 100	690		
H	65.0	8.06	45 900	900	990	
	65.0	8.0	51 600	1030		
	64.0	7.87	51 400	1050		
I	68.5	8.0	83 950	1670	1380	Odd fracture
	68.5	8.0	60 000	1190		
	66.0	7.87	63 900	1290		
M	66.0	8.0	63 500	1260	1530	
	66.0	8.12	101 900	1960		
	66.0	8.0	68 200	1360		

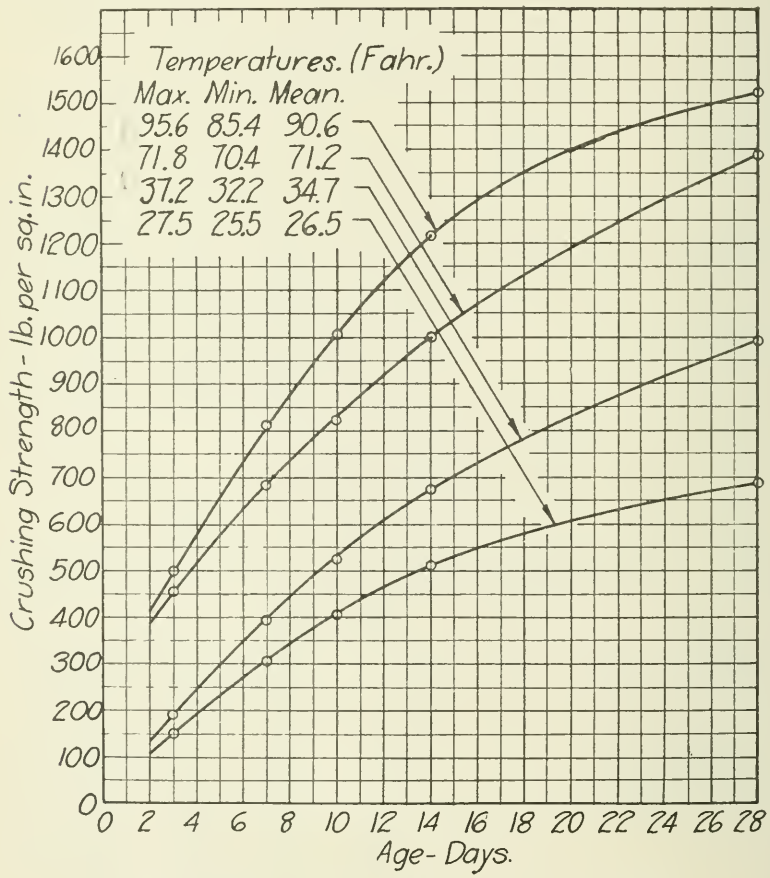

FIG. 13. GROUP III—1914 SERIES—8 x 16-IN. CYLINDERS.

10. *Group I.*—The results of the tests of Group I, 6 by 6-in. cylinders, are given in Table 3, 4, and 5; and the relation between strength and age is shown in Fig. 1, 2, and 3, pages 8, 9, and 10. The curves are drawn through the average values for each group of five specimens for 7, 14, and 28 days. At the top of each figure is shown the temperature conditions for that set; the maximum, the minimum, and the mean temperatures.

11. *Group II.*—The results of the tests of Group II, 6-in. cubes, are given in Tables 6–11; and the relation between strength and age is shown in Fig. 4, 5, and 6, pages 11, 12, and 13. The strength and temperature curves are drawn as stated for Group I.

The Sets D and E, Fig. 4 and 5, pages 11 and 12, were stored under substantially uniform temperature conditions, and give results of practically the same character as those of Group I.

The specimens of Set F were stored in a room where it was known the temperature would not be uniform. All of the specimens tested at 11 days were slightly disintegrated on the surface, and those tested at 28 days were badly disintegrated; while of those reserved to be tested at 42 days only one could be tested at that date, the remaining specimens, F_{17} and F_{18}, being very badly disintegrated. Specimen F_{17} was tested at 49 days, and F_{18} at 63 days. Since there was only one specimen at each of these ages, and since none of the other groups contained specimens at corresponding ages, the results of these two tests are not plotted in Fig. 6, and are not further considered.

The results of Set F, indicate that the low temperature retarded the hardening action of the concrete, and that the alternations above and below freezing caused a softening and crumbling of the material.

12. *Group III.*—The results of the tests of Group III, 8 by 16-in. cylinders, are given in Tables 12–16; and the relation between strength and age is shown graphically in Fig. 7–10. It is noteworthy that under a temperature slightly below freezing the concrete gained strength continuously, see Fig. 7, page 14. It is also interesting to note that the curve for a mean temperature of 26.5°F. is substantially of the same character as that for a mean temperature of 71.2°,—compare Fig. 7 and Fig. 9.

13. *Summary.*—The results for the three sets of Group I are presented in Fig. 11, page 19; and the corresponding values for Groups II and III are given in Fig. 12 and 13. Fig. 11–13 show the relation between strength and age for the several mean temperatures.

In Group I the test specimens were cylinders 6 inches in diameter and 6 inches high, and in Group II the specimens were 6-inch cubes;

and owing to the effect of the restraint of the pressing surfaces of the testing machine, the results of these tests are not further considered.

In Group III the test specimens were cylinders 8 inches in diameter and 16 inches high, and the interpolated results for these tests are presented in Fig. 14 to show the relation between strength and temperature for the several ages. Fig. 14 may be employed to determine (1) the strength which the concrete attained at different ages under a constant temperature, (2) the age at which a particular strength was gained under the different temperatures, and (3) the strength which may be expected at different ages under different temperatures. The relative strength attained by concrete at different temperatures during hardening and at different ages may be expected to vary somewhat with differences in cements, aggregates, and consistencies; but

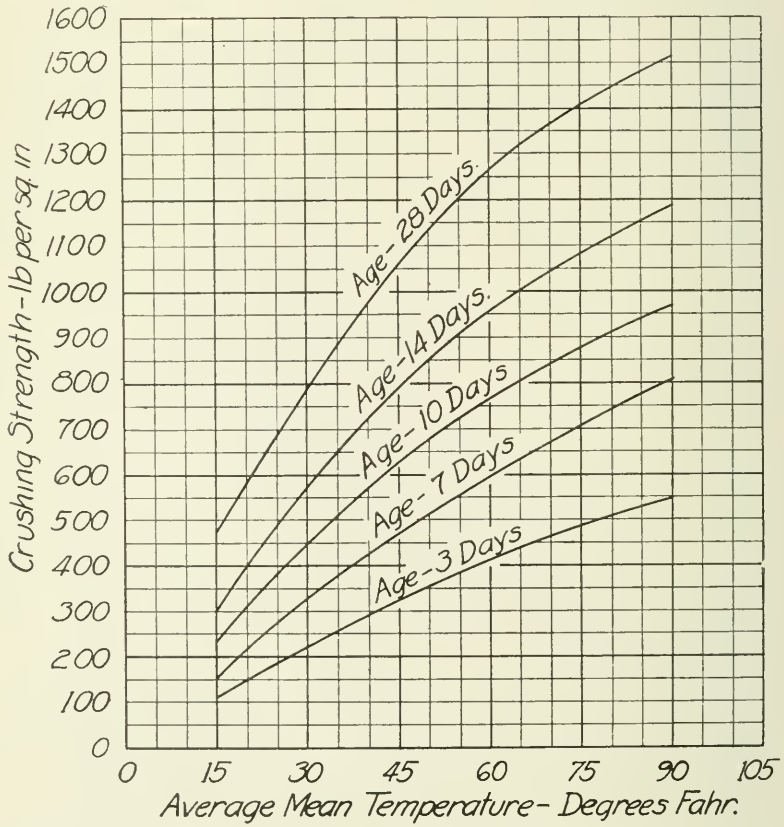

Fig. 14. Relation of Strength to Temperature for Different Ages.

it is thought that the values in Fig. 14 may be taken to represent the effect of the variation in the temperatures during hardening upon the strength.

Fig. 15 has been drawn by taking values from the curves in Fig. 14. It shows in a general way, the relation between the strength at 28 days under 70°F. and the strength attained at various ages under varying temperatures. Fig. 15 can be used in substantially the same way as Fig. 14.

The tests summarized in Fig. 14 and 15 cover a wide range of temperature conditions, the average temperature varying from 20.4°F. to 90.6°F., and are fairly consistent; and hence it is believed these values are sufficiently accurate to furnish suggestive information which may be useful in determining the time when forms may be removed and loads applied.

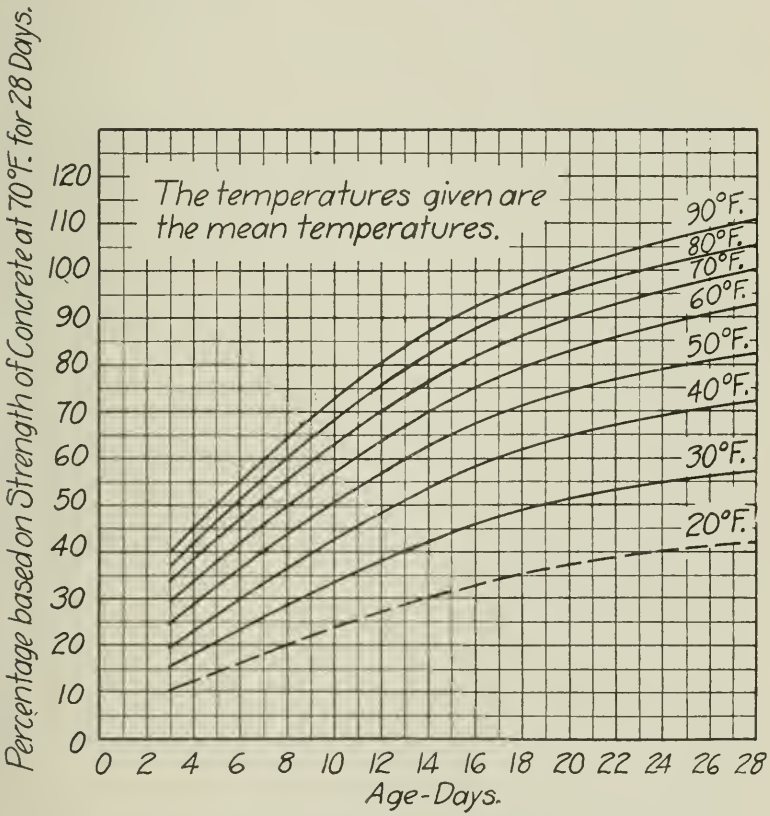

FIG. 15. PERCENTAGES OF STRENGTH FOR DIFFERENT TEMPERATURES.

IV. Conclusions

14. *General Conclusions.*—It is believed the following general conclusions are justifiable.

1. Under uniform temperature conditions, there was an increase of strength with age within the limits of the tests. For any temperature the rate of increase decreases with the age of the specimen; and this rate of increase is less correspondingly at the lower temperature conditions. For the specimens tested, under normal hardening temperature conditions of from 60 to 70°F., the compressive strength of the concrete subjected to a uniform temperature at the ages of 7, 14, and 21 days may be taken as approximately 50 per cent, 75 per cent, and 90 per cent of the strength at twenty-eight days, respectively. For lower temperatures the percentage values are less; and for higher temperatures the percentages are higher. The relation between the percentage values at the ages of 7, 14, 21, and 28 days is nearly the same for temperature conditions from 30° to 70° F. However, the values for the lower temperatures should be used with caution.

2. Concrete which is maintained at a temperature of 60° to 70° F. will at the age of one week have practically double the strength of the same material which is kept at a temperature of 32° to 40° F.

3. Fig. 14 and 15 may be used to determine the representative strength of concrete similar to that used in these tests, for various temperature conditions and for ages up to 28 days. These diagrams may be used with a fair degree of approximation to ascertain the relative strengths which concrete of ordinary practice may be expected to attain at the different temperatures. It should be noted that generally in this investigation the specimens were stored under temperatures which were nearly uniform during the whole storage period. In set F the variations in temperature include a number of alternations above and below the freezing point and the specimens were seriously injured. The results accord with the well-known effect of freezing and thawing upon green concrete.

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

Circular No. 3. Fuel Tests with Illinois Coal (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907), by L. P. Breckenridge and Paul Diserens. 1909. *Thirty cents.*

Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

Bulletin No. 5. Resistance of Tubes to Collapse, by Albert P. Carman and M. L. Carr. 1906. *None available.*

Bulletin No. 6. Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *None available.*

Bulletin No. 7. Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr, and Henry B. Dirks. 1906. *None available.*

Bulletin No. 8. Tests of Concrete: I, Shear; II, Bond, by Arthur N. Talbot. 1906 *None available.*

Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition 1912. *Fifty cents.*

Bulletin No. 10. Tests of Concrete and Reinforced Concrete Columns. Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 11. The Effect of Scale on the Transmission of Heat Through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

Bulletin No. 12. Tests of Reinforced Concrete T-Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 13. An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *None available.*

Bulletin No. 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 15. How to Burn Illinois Coal Without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*

Bulletin No. 16. A Study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*

Bulletin No. 17. The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *None available.*

Bulletin No. 18. The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*

Bulletin No. 19. Comparative Tests of Carbon, Metallized Carbon and Tantalum Filament Lamps, by T. H. Amrine. 1908. *None available.*

Bulletin No. 20. Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 21. Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908. *Fifteen cents.*

Bulletin No. 22. Tests of Cast-Iron and Reinforced Concrete Culvert Pipe, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 23. Voids, Settlement, and Weight of Crushed Stone, by Ira O. Baker. 1908. *Fifteen cents.*

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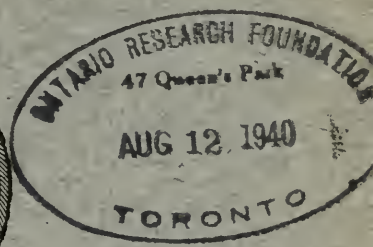
LABORATORY TESTS OF A CONSOLIDATION LOCOMOTIVE

BY

E. C. SCHMIDT, J. M. SNODGRASS

AND

R. B. KELLER



BULLETIN No. 82

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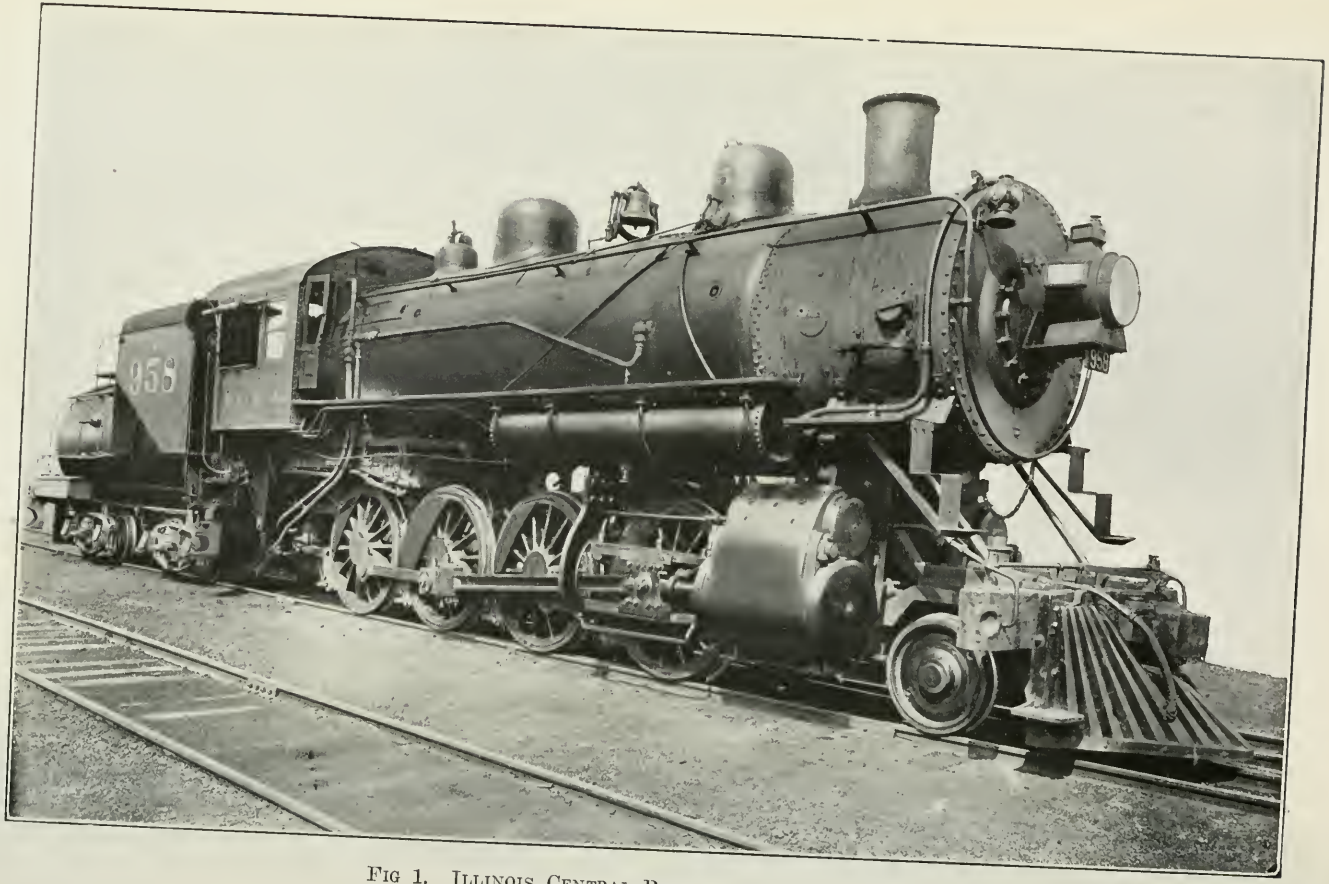

Fig 1. Illinois Central Railroad Locomotive 958.

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 82 SEPTEMBER, 1915

LABORATORY TESTS OF A CONSOLIDATION LOCOMOTIVE

BY

EDWARD C. SCHMIDT,[1] JOHN M. SNODGRASS[2] AND ROBERT B. KELLER.[3]

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LABORATORY TESTS OF A CONSOLIDATION LOCOMOTIVE.

PART I.

I. Introduction.

1. The tests the results of which are here recorded constitute the first work of the recently established locomotive laboratory of the University of Illinois. They relate to a typical consolidation locomotive which was loaned to the University by the Illinois Central Rrailroad.

In Part I of this report the aim has been to present as brief a statement of the conditions and results as is compatible with a clear understanding of the tests. Part II, on the other hand, consists of appendixes in which supplementary detail is fully recorded. In the presentation and discussion of the results in Part I, only the more important facts concerning boiler and engine performance have been included. There remain in the complete record of results given in Appendix 4 many facts which may be of use to those interested in the details of boiler and engine tests. In this, the first publication relating to the work of the laboratory, laboratory equipment and methods are described in detail in order to complete the record and to provide a basis for reference in future publications from which such detail will be omitted.

2. *Purpose of the Tests.*—The locomotive was first tested in the condition in which it was received from service. It was then subjected to certain repairs some of which affected its performance, and was again fully tested. The main purpose of the tests was to determine the general performance of the locomotive and the performance of its boiler and engines after the repairs were made and when the locomotive was in excellent condition.

3. *Acknowledgments.*—The locomotive was loaned for the tests through the interest and courtesy of Mr. W. L. Park, Vice President, and Mr. Morgan K. Barnum, General Superintendent of Motive Power, of the Illinois Central Railroad. During the progress of the tests Mr. R. W. Bell, then General Superintendent of Motive Power, and various members of his staff frequently gave assistance and advice to those in charge of the laboratory. It is a pleasure to record here our appreciation of these services.

Mr. Franklin W. Marquis, formerly Associate in the Department of Railway Engineering, was in immediate charge of the laboratory

from its establishment until the completion of the first ten tests included in this report. He also had a large share in working out the details of the laboratory design. To him is due the successful solution of many of the problems which arose in putting the equipment in operation and in establishing the test procedure.

We would acknowledge also the assistance received from the numerous members of the laboratory staff and especially from Mr. H. H. Dunn.

II. The Locomotive.

The locomotive tested is of the consolidation (2-8-0) type, built by the Baldwin Locomotive Works in 1909. It weighs 223 000 pounds, and has 22 in. x 30 in. simple cylinders using saturated steam. Its principal dimensions are given below, and a detailed description appears in Appendix 1.

Total weight, in working order, lb	223 000
Weight on drivers, lb	200 900
Cylinders (simple), diameter and stroke, in	22 x 30
Diameter of drivers, in	63
Fire-box width, in	66
Grate area, sq. ft	49.55
Heating surface, tubes (fire side), sq. ft	3094
Heating surface, total, sq. ft	3283
Boiler pressure, lb. per sq. in	200

When it was received at the laboratory, the locomotive had been in service three and one-third years and had run 107 800 miles. Immediately preceding the tests the locomotive had been in service only five weeks after receiving general repairs, and was in good condition when it arrived at the laboratory. It was completely tested in this condition and the results of these tests are designated as Series I. The results of this series disclosed a performance not quite so good as had been anticipated and, in the endeavor to do whatever was possible to improve the performance, valves were reset and eccentric straps shimmed; cylinders and valve chambers were re-bored; new pistons and piston rings, new valve bull-rings and packing rings were applied; rod packing renewed; the exhaust nozzle-tip changed from 5¼ in. to 5⅞ in.; and a small leak in one of the steam pipe joints was stopped. Certain incidental repairs having no effect on performance were made at the same time. Following this work the locomotive was run the equivalent of about 1200 miles in wearing down the cylinders and packing before making the tests of Series 2. It should be

emphasized that all of these repairs were resorted to only that nothing which would probably improve the performance be left undone, and that under ordinary service requirements they would have been regarded as quite unnecessary. After their completion the locomotive, then in excellent condition, was subjected to the tests which are designated as Series 2.

Locomotive 958 is a characteristic freight locomotive of whose type there are about twenty thousand on American railways, or one third of the total in service. Its weight and heating surface exceed the average values of these quantities for all consolidation locomotives by about twenty-five per cent. It is in most respects thoroughly representative of its type. Complete laboratory tests of simple consolidation locomotives are not common and include tests of only three different classes, all of which are somewhat smaller than the one here under consideration.*

III. Summary of the Results.

While it is not possible to summarize all the results of the tests further than is done in the curves included beyond, it is feasible briefly to state at this point the main facts defining the range through which the locomotive was worked and to indicate the minimum or maximum values of a few of the more important quantities. The statements apply to the tests of Series 1 and 2 combined.

4. *The Boiler.*—The maximum amount of dry coal fired per hour during any of the tests was 11 127 lb. or 224.5 lb. per square foot of grate per hour, an amount much in excess of what is usual or desirable on hand-fired locomotives in service. The maximum quantity of cinders ejected into the front end and from the stack amounted to 27.4 per cent of the dry coal fired. This cinder loss also is quite unusual and it occurred under conditions which rarely prevail in service, the draft during this test being equivalent to 12.8 inches of water in front of the diaphragm.

During the test in which the heating surface was forced to its greatest activity, the total equivalent evaporation per hour was 57 954 lb., or 17.65 lb. per square foot of heating surface per hour. This rate of evaporation is altogether unusual in service and has been exceeded only rarely under test conditions. The best economic performance of

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the boiler was obtained in test No. 2024 during which the equivalent evaporation per pound of dry coal was 10.07 lb. There is some doubt however about the validity of this result which exceeds the next highest evaporation per pound of coal (8.96 lb.) by 12.4 per cent.

These results were all obtained when using run-of-mine coal from Mission Field Mine, Vermilion County, Illinois, which varied in heating value from 11 835 B.t.u. to 12 848 B.t.u. per pound of dry coal.

5. *The Engines and the Locomotive.*—The maximum indicated horse power developed during the tests was 1654 which occurred in test No. 2093 with a cut-off of 48.6 per cent and a speed of 30.4 miles per hour. This is the greatest power which has been developed during laboratory tests with a locomotive of this type. The maximum drawbar horse power was 1431. The maximum tractive effort developed, 29 240 lb., is only 75 per cent of the rated maximum and is not significant because of the fact that, as in all laboratory tests, it was not feasible to work the locomotive at the lowest speeds and the greatest cut-offs.

The lowest water rate attained was 27.17 lb. of dry steam per indicated horse power per hour. This steam consumption is not so low as has been previously obtained in tests of locomotives of this type under similar conditions, being almost 17 per cent in excess of the lowest figure previously recorded. The minimum heat content of the dry coal fired per indicated horse power per hour was 50 872 B.t.u. and the minimum dry coal fired per hour per indicated horse power was 4.00 lb. The minimum dry coal fired per hour per drawbar horse power was 4.62 lb.

IV. The Tests and the Test Program.

The locomotive was worked during the tests throughout a range of speed corresponding to that which would ordinarily prevail in service. At each of the various speeds the endeavor was made to vary the cut-off throughout as wide a range as the capacity of the boiler or of the grate would permit. The adhesion between the drivers and the supporting wheels in the laboratory is less however than the adhesion between the drivers and the rail on the road, and consequently it was impossible at low speeds to run at maximum cut-offs. The designations for speed and cut-off used in this section are approximate only, and represent the conditions predetermined for each test. The actual

average values attained during the tests appear in Appendix 4. All tests were run with the throttle wide open.

TABLE 1.

TEST PROGRAM—SERIES 2.

SHOWING TESTS RUN AT VARIOUS SPEEDS AND CUT-OFFS.

Approximate Speed		Approximate Cut-off—Per cent of Stroke					
Rev. per Minute	Miles per Hour	16	24	32	40	48	56
55	10		2081 2086	2075 2097	2085 2096	2095 2098	
110	20	2080 2087	2077	2073	2072	2084	2094
165	30	2083	2078	2074 2092	2082	2093	
220	40	2088	2079	2076	2089		

TABLE 2.

TEST PROGRAM—SERIES 1.

SHOWING TESTS RUN AT VARIOUS SPEEDS AND CUT-OFFS.

Approximate Speed		Approximate Cut-off—Per cent of Stroke					
Rev. per Minute	Miles per Hour	16	20	24	32	40	48
55	10			2024	2028		
83	15	2017 2021		2018 2020	2019 2022	2031	
110	20	2026		2027	2029	2035	2033
138	25	2009		2012	2013	2023	
165	30			2030	2032	2037	
193	35	2016	2010	2015	2014	2034	

6. *Series 2.*—Series 2 comprises 25 tests and includes tests 2072 to 2098 (excepting only tests 2090 and 2091 which are referred to beyond). In this series the speed varied from 10 to 40 miles per hour or from 55 to 220 revolutions per minute, while the cut-off ranged from 16 per cent to 56 per cent of the stroke. The distribution of these tests at the different speeds and cut-offs is shown in Table 1.

As elsewhere explained, (see section II and Appendix 1) the locomotive during this group of tests was in excellent condition, valves having been reset, valve chambers and cylinders rebored, the packing for pistons and valves and rods renewed, a leak in one of the steam pipe joints stopped, and the exhaust nozzle tip changed from 5¼ in. to 5⅞ in.

7. *Series 1.*—Series 1 comprises 26 tests and includes tests 2009 to 2037 (excepting No. 2011, 2025, and 2036). Test 2025 is omitted from the record because of errors in water measurement, and tests 2011 and 2036 were discontinued before their completion—one on account of an injector failure, the other on account of a faulty valve in the line supplying oil to the absorption brakes.

In this group of tests the speed varied from 10 to 35 miles per hour or from 55 to 193 revolutions per minute, while the cut-off ranged from 16 per cent to 48 per cent of the stroke. The distribution of these tests at the different speeds and cut-offs is shown in Table 2. During Series 1 the locomotive was in the condition in which it was received at the laboratory, which is distinguished from the condition prevailing during Series 2 by the repairs above cited.

8. *Intermediate Tests.*—Immediately after the completion of the tests of Series 1, the valves of the locomotive were reset, the eccentric straps shimmed, rod packing replaced, and the valve rings and piston rings were renewed and refitted. After these changes eight tests—No. 2038 to 2045—were run.

These changes, intended to improve cylinder performance, did not materially affect it. Because of them, however, these tests are excluded from Series 1 and their results appear only in Appendix 4. They are not included in any of the figures presented in the report. Since during these eight tests the condition of the boiler was exactly the same as during Series 1, their results relating to boiler performance are comparable with those of that series.

During the progress of Series 2, two tests—No. 2090 and 2091—were run with the nozzle tip changed from 5⅞ in. to 5¼ in. With this exception all conditions prevailing in these two tests were the same as in Series 2. These tests are referred to beyond, and their results are separately presented in Appendix 4. They are excluded from Series 2.

In addition to the tests above mentioned, 26 runs (No. 2046-2071) were made for such purposes as to wear down the cylinder and valve chambers after re-boring, to make final choice of exhaust nozzle tip, etc. While these runs were given test numbers, they were incomplete and were not intended to be included in the report. Of the 64 tests made with the expectation that they would be embodied in the report, only the three referred to in paragraph 7 have been excluded from the record.

V. Test Methods and Test Conditions.

9. *Methods and Equipment.*—The methods employed in conducting the tests and in deriving the results are explained in detail in Appendixes 3 and 5. They conform in general to those prescribed by the American Railway Master Mechanics' Association code for conducting laboratory tests of locomotives, published in the Proceedings of the Association for 1914. Whatever deviations from this code have been found desirable are indicated in the appendixes.

The laboratory equipment is described in Appendix 2. While this equipment differs in several details from that of other laboratories, the only difference which has materially affected test methods lies in the presence of a cinder separator, through which all the exhaust gases pass and in which the entire body of cinders is collected. Except during one group of tests conducted at the Pennsylvania Railroad testing plant, when temporary provision was made to collect all the cinders issuing from the stack, the cinder discharge has been determined in other laboratory tests merely by sampling the exhaust gas stream.

The design of this cinder separator is illustrated in Appendix 2. Its operation has been entirely successful. Repeated examinations of the exhaust gases as they issued from the separator, and unsuccessful attempts to collect solid matter in the neighborhood of the laboratory stack have made it clear that the separator collects and retains even the finest cinders under all test conditions.*

10. *Conditions.*—As previously stated the coal used during all the tests came from Mission Field Mine, Vermilion County, Illinois. For all tests to and including No. 2091 run-of-mine coal was used. During tests 2092, 2093, 2094, and 2095 a mixture of run-of-mine and screened lump was used, which in appearance, analysis, and performance was not materially different from the run-of-mine alone. During the last three tests, (2096-2098) on account of a shortage in the supply of run-of-mine coal, 1½-in. screenings were used. Because of this difference in conditions, all data and all results involving coal are excluded from the record of these three tests.

The locomotive during all tests was fired by C. Welker, a skilled fireman, detailed for this purpose by the Illinois Central Railroad from their regular force. Previous to his engagement at the laboratory, he had had four and one-half years' experience as fireman on this road and upon the completion of the tests returned to their service. Dur-

*The term *cinders* is here used to mean particles of appreciable size as distinguished from impalpable dust. Samples of the stack cinders representing the entire range in rate of combustion contained from 10 to 18 per cent of material which passed a 200 mesh screen.

ing some of the tests he was assisted by one of three other firemen who were also detailed at various times from the local Illinois Central force. None of these men had had less than one year's experience. Mr. Welker in these tests, as in those in which he acted alone, remained in charge and responsible for the character of the work.

The condition of the locomotive has been briefly stated in Section II and is more fully explained in Appendix 1. The test program and the conditions of speed and cut-off have been presented in section IV.

VI. The Results of the Tests of Series 2.

All the data and the results of the tests of Series 2 are presented in detail in the tables of Appendix 4. There are included in this section only the more important data and results relating to the performance of the boiler, the engines, and the locomotive. These facts are here presented in both tabular and graphical form. In establishing the relations between results chief reliance is placed upon the figures; and the tabular matter, which is a repetition of parts of Appendix 4, is included for convenience of reference only. Except where otherwise specifically stated, the curves in the figures have been produced by averaging the coordinates of various groups of points, plotting these average values, and passing as nearly as possible through the points thus determined a smooth curve. The test designations which appear in the tables indicate first the approximate speed in revolutions per minute, next the nominal cut-off in per cent, and finally the amount of throttle opening. Thus in test 2072, designated as 110-40-F, the speed was about 110 revolutions per minute, the cut-off approximately 40 per cent, and the throttle—as in all the tests—was "full" or wide open.

A. Boiler Performance.

The more significant data and results pertaining to the performance of the boiler in Series 2 are collected from Appendix 4 and presented here in Tables 3 and 4, which include nearly all the facts used in producing the figures relating to boiler performance. In both of these tables the tests are arranged in the order of the increasing amounts of dry coal fired per hour per square foot of grate (code No. 627). If this arrangement is borne in mind, some of the relations may be more definitely and quite as conveniently studied in the tables as in the curves.

In attempting to draw from these results inferences concerning the performance of locomotives in service, it should be remembered that

Table 3.

Boiler Performance—Series 2.

Test No.	Laboratory Designation	Average Boiler Pressure, lb. per sq. in.	Duration of Test, Minutes	Dry Coal Fired per Hour, lb.		Draft, in. of Water				Temperature, Deg. F.		Quality of the Steam in the Dome
				Total	Per sq. ft. of Grate	In the Ash-pan	In the Fire-box	Back of the Diaphragm	In Front of the Diaphragm	In the Fire-box	In the Front-end	
	Code Item	380		626	627	397	396	395	394	374	367	407
2081	55–24–F	198.2	150	1975	39.9	0.2	0.7	1.5	2.2	1407	507	0.9963
2086	55–24–F	199.1	170	2068	41.7	0.3	0.7	1.4	2.2		506	0.9963
2075	55–32–F	198.1	140	2327	47.0	0.3	0.9	1.8	2.8	1661	543	0.9952
2080	110–16–F	198.8	130	2422	48.9	0.2	1.0	2.0	2.9	1418	534	0.9956
2087	110–16–F	199.2	150	2474	49.9	0.2	0.9	1.9	2.9		524	0.9956
2085	55–40–F	197.9	120	3058	61.7	0.3	1.1	2.5	4.0		545	0.9962
2077	110–24–F	196.0	110	3281	66.2	0.4	1.3	2.5	4.1	1570	565	0.9947
2095	55–48–F	198.1	60	3334	67.3	0.4	1.5	3.0	5.3		567	0.9943
2083	165–16–F	198.7	100	3338	67.4	0.4	1.6	2.7	4.3	1267	563	0.9934
2088	220–16–F	197.8	100	3353	67.7	0.3	1.2	2.9	4.5		563	0.9919
2073	110–32–F	197.6	80	4359	87.9	0.5	1.6	3.5	5.7	1662	595	0.9919
2078	165–24–F	196.4	70	4707	95.0	0.5	1.9	3.9	6.0	1597	595	0.9915
2092	165–32–F	198.4	50	5640	113.8	0.6	2.2	5.1	8.2		643	0.9894
2079	220–24–F	197.4	60	5783	116.7	0.4	2.3	4.8	7.0	1688	614	0.9889
2072	110–40–F	196.7	60	5927	119.6	0.7	2.0	4.9	8.0	1643	620	0.9895
2074	165–32–F	197.1	60	6015	121.3	0.5	2.4	5.3	8.5	1662	637	0.9861
2076	220–32–F	196.0	35	7831	158.0	0.5	2.8	5.7	9.2	1785	675	0.9844
2084	110–48–F	194.0	50	7914	159.7	0.8	2.9	6.3	10.0		653	0.9896
2094	110–56–F	196.3	25	8434	170.2	0.8	3.0	7.3	12.1		679	0.9887
2082	165–40–F	195.2	50	8994	181.5	0.6	3.4	7.3	11.2	1458	673	0.9886
2093	165–48–F	191.5	30	10216	206.2	0.9	3.4	7.8	12.8		702	0.9879
2089	220–40–F	194.9	35	11127	224.5	0.7	3.5	7.1	11.9		703	0.9884

Table 4.

Boiler Performance—Series 2.

Test No.	Dry Coal Fired, per Hour per sq. ft. of Grate, lb.	Equivalent Evaporation, lb.			Calorific Value per lb. of Dry Coal, B. t. u.	Cinders					Efficiency of the Boiler Including the Grate, per cent
		Per Hour	Per Hour per sq. ft. of Heating Surface	Per lb. of Dry Coal		Accumulated in the Front-end per Hour, lb.	Discharged from the Stack per Hour, lb.	Total per Hour, lb.	Total in Per cent of the Dry Coal Fired	Calorific Value in Per cent of the B.t.u. Contained in the Dry Coal	
	627	645	648	658	458	422 & 345	423 & 345	424 & 345	426	460 & 458	666
2081	39.9	17 277	5.26	8.75	12 700	7.2	62	69	3.5	44.93	66.89
2086	41.7	17 308	5.27	8.37	12 586	6.0	110	116	5.6	46.69	64.49
2075	47.0	19 954	6.08	8.57	12 718	5.6	165	171	7.3	55.17	65.46
2080	48.9	20 923	6.37	8.64	12 848	6.9	97	104	4.3	44.99	65.28
2087	49.9	20 846	6.35	8.43	12 272	7.2	118	126	5.1	55.34	66.63
2085	61.7	25 270	7.70	8.26	12 622	8.5	217	226	7.4	77.50	63.54
2077	66.2	26 417	8.05	8.05	12 633	7.7	245	253	7.7	66.50	61.82
2095	67.3	28 614	8.72	8.58	12 315	10.0	250	260	7.8	70.78	67.61
2083	67.4	26 448	8.06	7.92	12 660	12.6	259	271	8.2	73.70	60.72
2088	67.7	26 995	8.22	8.05	12 095	10.2	273	283	8.5	71.29	64.58
2073	87.9	33 719	10.27	7.74	12 751	9.0	553	562	12.9	75.69	58.92
2078	95.0	34 431	10.49	7.31	12 517	13.7	648	662	14.1	76.16	56.66
2092	113.8	41 770	12.72	7.41	12 620	14.5	841	855	15.1	81.42	56.95
2079	116.7	38 155	11.62	6.60	12 767	14.0	873	887	15.3	78.13	50.18
2072	119.6	40 590	12.36	6.85	12 460		1140				53.36
2074	121.3	41 701	12.70	6.93	12 575	3.0	949	952	15.8	79.55	53.50
2076	158.0	46 838	14.27	5.98	12 519	24.1	1822	1847	23.4	88.86	46.41
2084	159.7	46 913	14.29	5.93	12 305	24.1	1499	1523	19.2	88.90	46.76
2094	170.2	54 336	16.55	6.44	12 426	31.0	1931	1962	23.5	85.46	50.33
2082	181.5	49 007	14.93	5.45	12 626	22.9	1976	1999	22.1	87.08	41.87
2093	206.2	57 954	17.65	5.67	12 551	20.0	2782	2802	27.4	83.51	43.82
2089	224.5	54 989	16.75	4.94	12 356	22.4	2979	3002	26.8	91.95	38.77

during the tests the boiler was forced somewhat beyond the limits which would ordinarily be maintained in service; so that the maximum test values of such measures of boiler activity as draft, rate of combustion, and rate of evaporation are somewhat greater than the values which would be maintained on the road for any except very short periods.

11. *General Conditions.*—The average boiler pressure varied during the tests of this series from 191.5 to 199.2 pounds, and the feed temperature ranged between 44.7 and 63.6 degrees. As is common under the uniform conditions of load which are maintained in laboratory tests, the quality of the steam was high and nearly uniform throughout the series, the lowest quality being 0.984 and the highest 0.9963.

The calorific value of the fuel varied between the limits of 10 487 and 11 660 B.t.u. per pound of coal as fired, and from 12 095 to 12 848 B.t.u. per pound of dry coal. The ash in the coal as fired varied from 9.64 to 13.96 per cent.

Of the 25 tests of Series 2, seventeen were of more than one hour's duration. In the remaining eight tests the test period was less than one hour, being in one test only 25 minutes. Even in this test, however, the coal burned amounted to 4095 pounds.

12. *Draft.*—The relation between the draft values and the rate of combustion is indicated in Fig. 2, and their relation to rate of evaporation in Fig. 3. Inspection of the curve of firebox draft in these figures reveals close agreement between the values represented by the individual points and the average value represented by the curve. This fact may be accepted as an indication of the uniformity with which the fire was managed during the tests.

In test 2093 the drafts in front of the diaphragm, back of the diaphragm, in the firebox, and in the ashpan were 12.8, 7.8, 3.4, and 0.9 inches of water respectively. The rate of combustion in this test was 118.66 pounds of coal per hour. The drafts cited are the maxima attained during this series except in the case of firebox draft which was exceeded by 0.1 of an inch in one other test.

13. *Firebox and Front-end Temperatures.*—The temperature of the gases in the firebox varied between 1267 and 1785 degrees during the first twelve tests of this series. This temperature was not recorded during the remaining tests because of a break-down in the pyrometer equipment. The relation of this temperature to both rate of combustion and rate of evaporation is exhibited by the upper curves of Fig. 4 and 5 respectively.

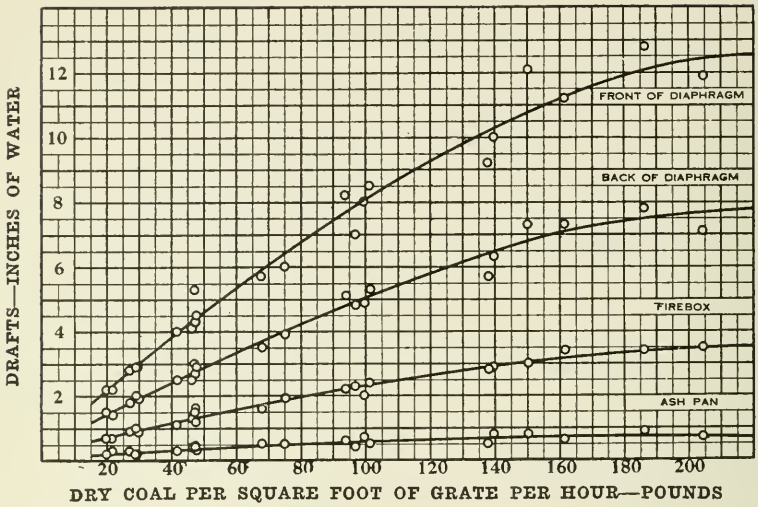


FIG. 2. THE RELATIONS BETWEEN DRAFT AND RATE OF COMBUSTION.

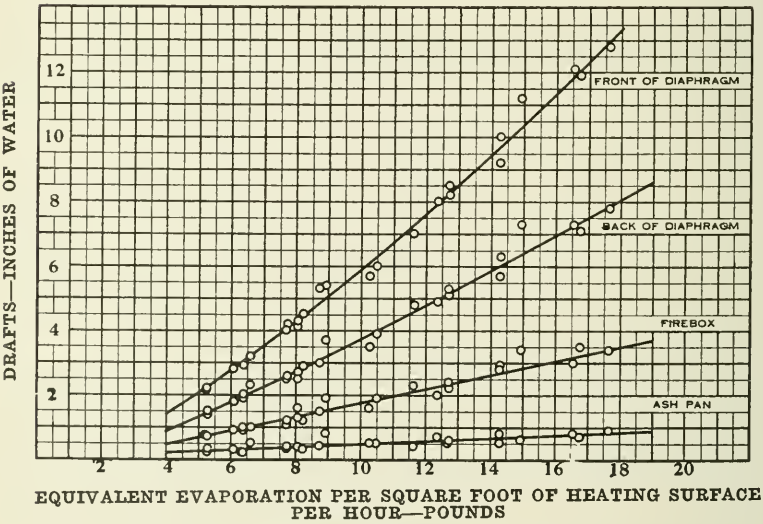


FIG. 3. THE RELATIONS BETWEEN DRAFT AND RATE OF EVAPORATION.

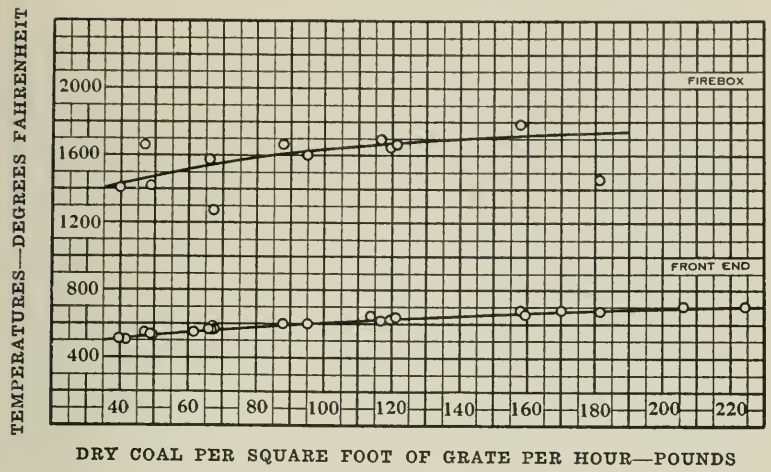


FIG. 4. THE RELATIONS BETWEEN FIREBOX AND FRONT-END TEMPERATURES AND RATE OF COMBUSTION.

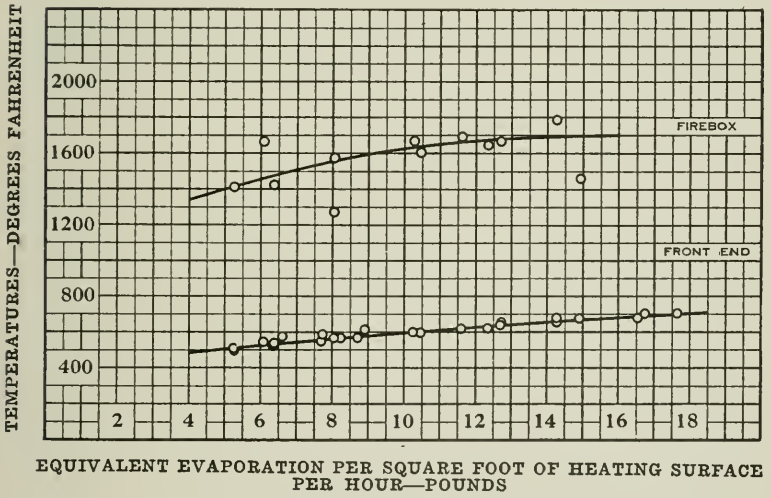


FIG. 5. THE RELATIONS BETWEEN FIREBOX AND FRONT-END TEMPERATURES AND RATE OF EVAPORATION.

The temperature of the gases in the front-end ranged between 506 and 702 degrees and increased very regularly as the activity of the grate and of the heating surface was increased. The relation of front-end temperature to rate of combustion appears in the lower curve of Fig. 4, and its relation to rate of evaporation in Fig. 5.

14. *Coal Consumption.*—The smallest amount of fuel fired during any of the tests was 3799 pounds of moist coal or 3334 pounds of dry coal. The greatest amount per test was 8506 pounds of moist coal or 7495 pounds of dry coal. The rate of firing ranged from 1975 pounds of dry coal per hour in test 2081 to 11 127 pounds of dry coal per hour in test 2089. The rate of combustion varied between 39.9 and 224.5 pounds of dry coal per square foot of grate per hour.

15. *Evaporation.*—The equivalent evaporation per hour varied between the limits of 17 277 and 57 954 pounds. The rate of increase in equivalent evaporation per hour with respect to the hourly consumption of dry coal is exhibited in Fig. 6. In this figure four of the highest values of evaporation are somewhat more divergent from the average represented by the curve than are the values for other tests. These four are all tests of short duration in which the measurement of the coal may be on this account slightly less accurate than in the other tests. Two of them, however, are tests in which the coal used was the mixture of lump and run-of-mine referred to in section V, and this fact may perhaps partially account for their divergence.

The equivalent evaporation per square foot of heating surface per hour varied in this series from 5.26 pounds to 17.65 pounds. Fig. 7 shows the relation of the rate of evaporation to the amount of dry coal fired per square foot of grate per hour.

16. *Boiler Horse Power.*—Under the usual convention of 34.5 pounds of equivalent evaporation per hour per horse power, the boiler of this locomotive developed a maximum horse power of 1680. This maximum is equivalent to one horse power for each 1.95 square feet of heating surface, or for each 0.295 of a square foot of grate area.

17. *Economic Performance.*—The equivalent evaporation per pound of dry coal ranged from a minimum of 4.94 to a maximum of 8.75 pounds. This range represents as good a performance as would be expected from the grade of coal used. The lower evaporations per pound of coal were of course obtained with the higher rates of combustion and evaporation. The rate of this decrease in evaporation per pound of dry coal is shown in Fig. 8 and 9, the former showing the decrease with respect to increase in the rate of combustion and the latter with respect to increase in the rate of evaporation. Either of these figures may serve as an index of the general performance of the boiler.

18. *Boiler Efficiency.*—By efficiency is meant, in this connection, the ratio of the heat absorbed by the boiler to the heat contained in

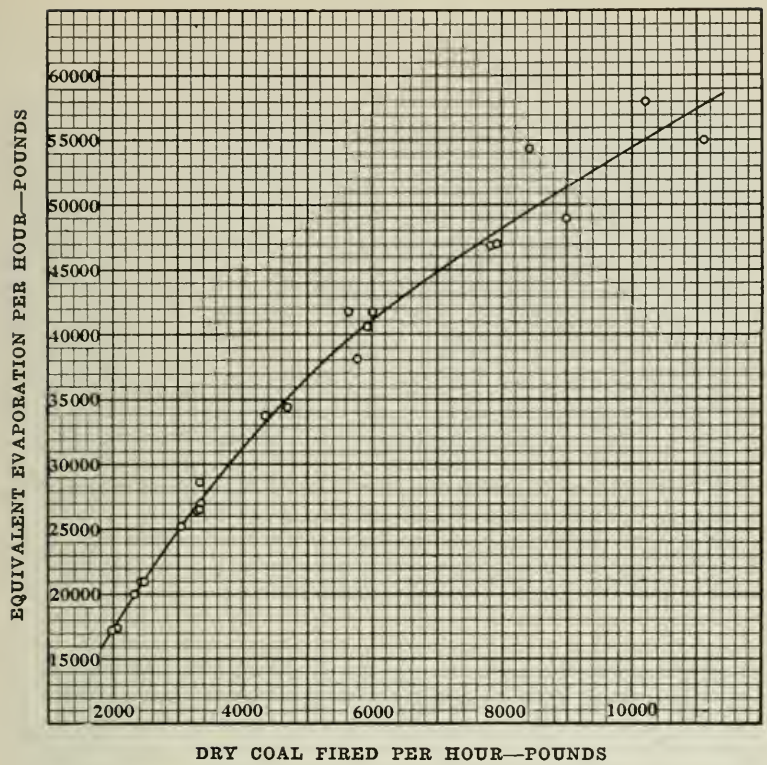


FIG. 6. THE RELATION BETWEEN HOURLY EVAPORATION AND HOURLY COAL CONSUMPTION.

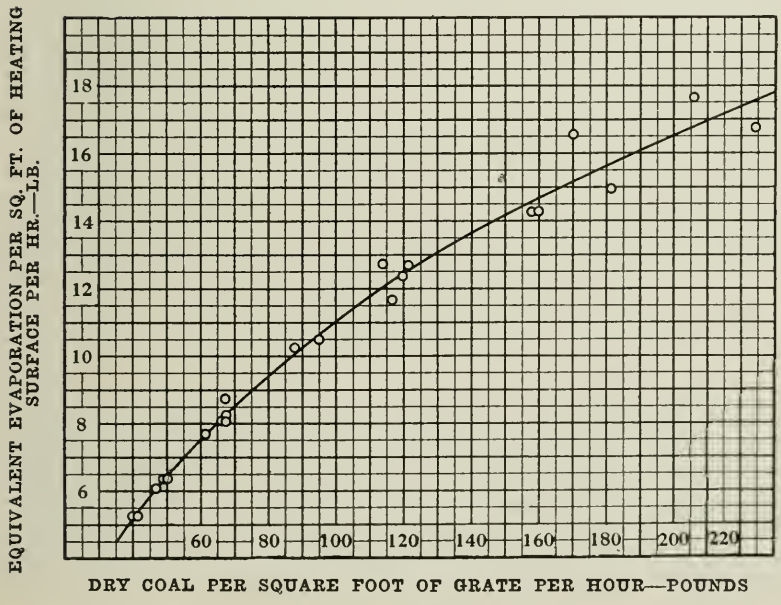


FIG. 7. THE RELATION BETWEEN RATE OF EVAPORATION AND RATE OF COMBUSTION.

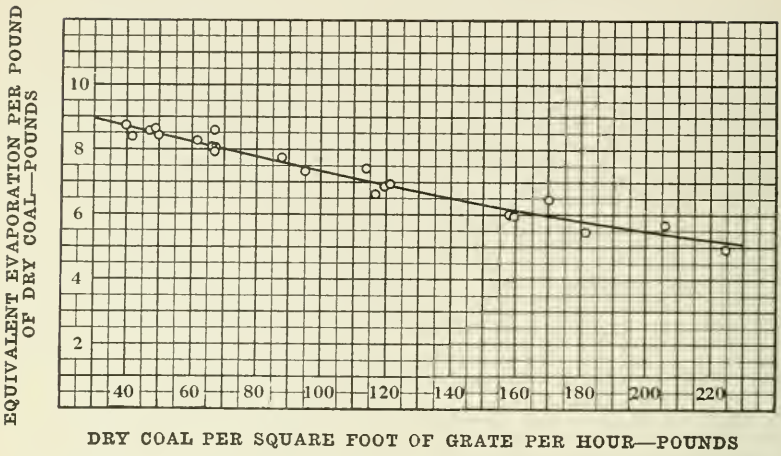


FIG. 8. THE RELATION BETWEEN EVAPORATION PER POUND OF COAL AND RATE OF COMBUSTION.

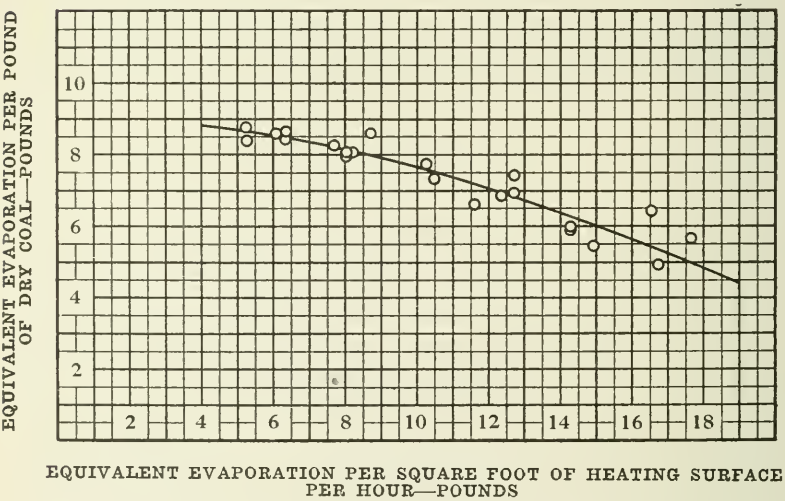


FIG. 9. THE RELATION BETWEEN EVAPORATION PER POUND OF COAL AND RATE OF EVAPORTAION.

the coal in the condition in which it was supplied to the fire, and it represents therefore the combined efficiencies of the furnace and of the boiler proper, in producing and utilizing the heat. The maximum efficiency, 67.61 per cent, was obtained in test 2095 with a rate of combustion of 67.3 pounds of dry coal per square foot of grate per hour, which is not quite the lowest rate of combustion occurring during this

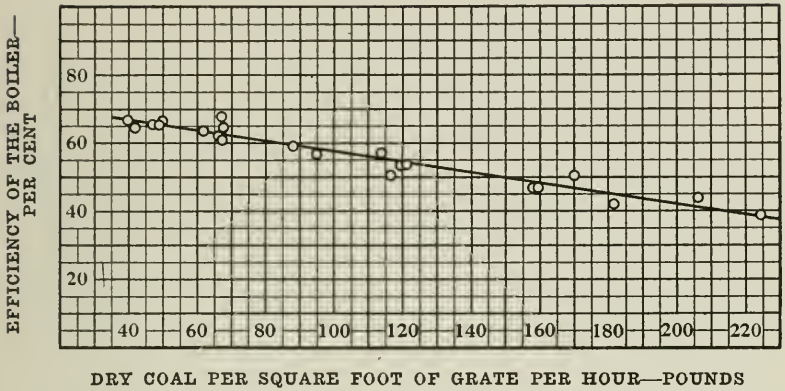


FIG. 10. THE RELATION BETWEEN BOILER EFFICIENCY AND RATE OF COMBUSTION.

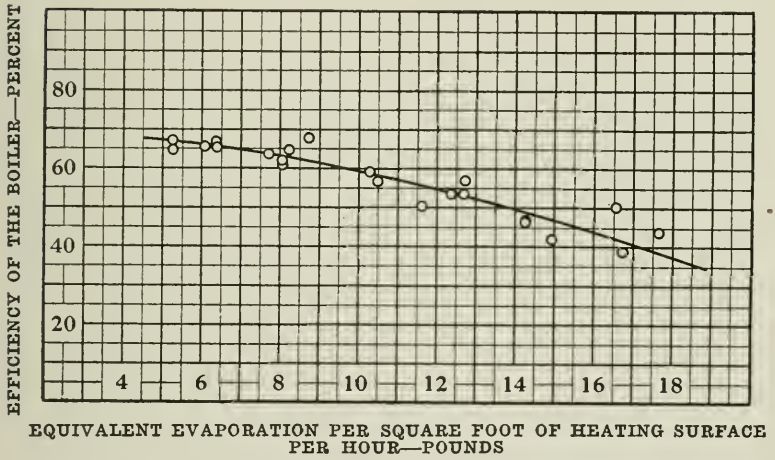


FIG. 11. THE RELATION BETWEEN BOILER EFFICIENCY AND RATE OF EVAPORATION.

series. The minimum efficiency, 38.77 per cent, was obtained in test 2089 in which the highest rate of combustion prevailed, namely 224.5 pounds of dry coal per square foot of grate per hour. The relations of efficiency to rate of combustion and to rate of evaporation are shown in Fig. 10 and 11, respectively.

19. *Cinder Losses.*—The data relating to the cinder losses which occurred during these tests have an especial significance in view of the methods by which they were obtained. The locomotive was equipped with a self-cleaning front-end and the maximum amount of cinders there collected during any test was only 21 pounds. In no test did the weight of front-end cinders amount to more than 0.4 of one per cent of the dry coal fired. For this reason no attempt is made to distinguish between cinders accumulated in the front end and those discharged from the stack, in the discussion here presented, although they are so distinguished in Table 4. In the discussion only the total amounts of cinders formed are referred to. These are substantially the same as the amounts discharged from the stack.

The minimum cinder loss occurred in test 2081, during which cinders were formed at the rate of 69 pounds per hour. The draft in this test was equivalent to 2.2 inches of water in front of the diaphragm, and the rate of combustion was 39.9 pounds of dry coal per square foot of grate per hour. The greatest cinder loss amounted to 2984 pounds per hour and occurred in test 2089, in which the corresponding draft was 11.9 inches of water and the rate of combustion 224.5 pounds of dry coal per square foot of grate per hour. Fig. 12 shows the increase in cinders formed per hour as the amount of coal burned per hour increases.

These cinder losses are more conveniently expressed as percentages of the weight of dry coal fired, and they are so presented in Table 4. Inspection of this table shows the minimum loss to have amounted to 3.5 per cent of the dry coal fired. This loss occurred in test 2081 in which, as above stated, the draft in front of the diaphragm was 2.2 inches of water and 39.9 pounds or dry coal were burned per square foot of grate per hour. The maximum cinder loss amounted to 27.4 per cent of the dry coal fired and occurred in test 2093, when the draft was 12.8 inches and the rate of combustion 206.2 pounds of dry coal per square foot of grate per hour. With few exceptions the cinders increase in amount with every increase in the rate of combustion. Fig. 13 shows the relation between cinder loss in per cent of the dry coal fired and the rate of combustion. The cinder losses have been expressed as percentages of the dry coal rather than of the coal as fired, because of the accidental variations in the moisture content of the latter. For this reason the amounts of dry coal fired have seemed here as elsewhere to offer the more logical basis for computation. If the cinder losses had been based upon moist instead of dry coal they would have been de-

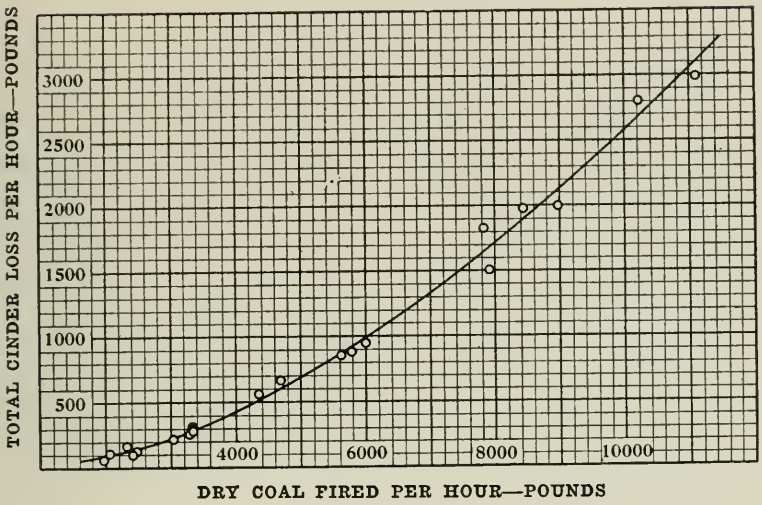


FIG. 12. THE RELATION BETWEEN HOURLY CINDER DISCHARGE AND HOURLY COAL CONSUMPTION.

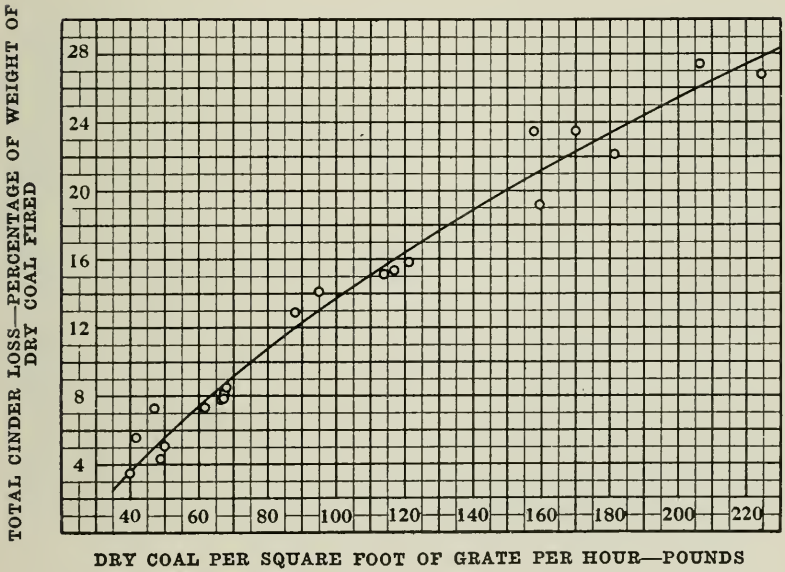


FIG. 13. THE RELATION BETWEEN CINDER DISCHARGE AND RATE OF COMBUSTION.

fined, of course, by smaller percentages. So based, the cinder losses in this series varied between 3.1 and 23.6 per cent.

In service the dry coal fired per square foot of grate per hour probably would rarely exceed 120 pounds. At this rate of combustion Fig. 13 indicates a cinder loss of about 16 per cent. On the road, therefore, except during rare and short intervals, the cinder discharge for this locomotive would probably range between 3 and 16 per cent of the weight of the dry coal fired, when using coal similar to that used during these tests.

The immediate cause of the cinder discharge is of course the intense draft which is essential in locomotive boiler operation. The relation of the cinder loss to draft is shown in Fig. 14 in which the ordinates represent cinder discharge in percentages of the dry coal, and the abscissae the draft in the front-end in front of the diaphragm. Fig. 14 shows the cinder discharge to have been almost directly proportional to draft.

The heating value of the cinders varied greatly, depending on the speed of their transit through the furnace and boiler. This speed is obviously influenced largely by the draft, which in turn determines also the rate of combustion. The increase in the heating value of the cinders as the rate of combustion increases is shown in Fig. 15, in which the ordinates represent the heating value of the cinders expressed as percentages of the B.t.u. contained in the dry coal, and the abscissae represent the rates of combustion. In test 2081 with the lowest rate of combustion and the smallest cinder loss, the calorific value of the cinders was only 44.9 per cent of the calorific value of the dry coal. In test 2089 with the highest rate of combustion and next to the greatest cinder discharge, the calorific value of the cinders was 92 per cent of that of the dry coal, the cinders in this case having passed through the boiler practically unburnt.

20. *Heat Balances.*—The heat balances for the tests of Series 2 are presented in Table 5, in which the various items of the balance are expressed in percentages of the heating value of the coal in the condition in which it was fired. The tests are arranged in the table in the order of the increasing amounts of equivalent evaporation per square foot of heating surface per hour.

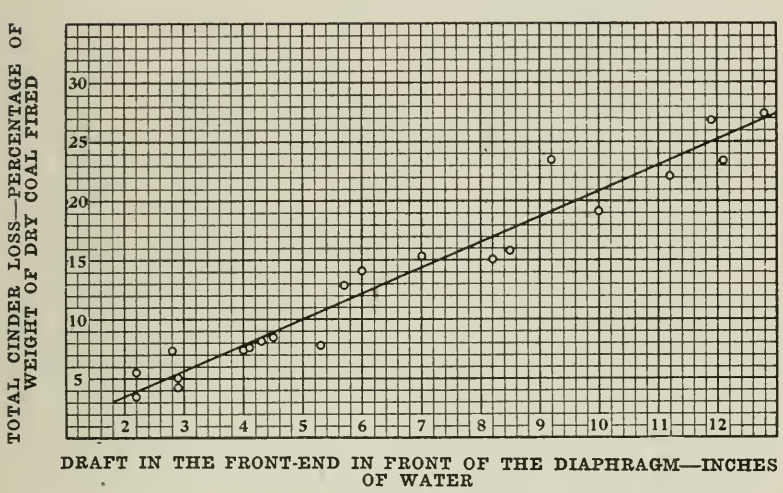


FIG. 14. THE RELATION BETWEEN CINDER DISCHARGE AND DRAFT.

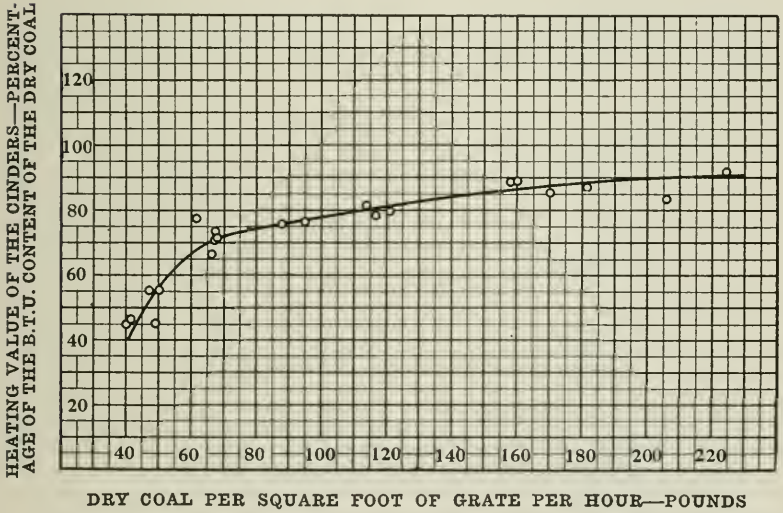


FIG. 15. THE RELATION BETWEEN THE HEATING VALUE OF THE CINDERS AND RATE OF COMBUSTION.

Table 5.
Heat Balance for Series 2.

Test No.	Equivalent Evaporation per sq. ft. of Heating Surface per Hour, lb.	Percentages of the Heating Value of the Coal as Fired									
		Absorbed by the Boiler	To Moisture in the Coal	To Moisture in the Air	To Hydrogen in the Coal	To Escaping Gases	To Incomplete Combustion	To Combustible in the Front-end Cinders	To Combustible in the Stack Cinders	To Combustible in the Ash	To Radiation and Not-accounted for
	648	881	882	883	884	885	886	887	888	890	899
2081	5.26	66.9	1.3	0.3	4.6	18.9	0.0	0.2	1.5	3.2	3.2
2086	5.27	64.5	1.4	0.2	4.5	15.8	0.6	0.1	2.4	3.3	7.2
2075	6.08	65.5	1.5	0.3	4.6	17.9	0.7	0.1	3.9	1.4	4.3
2087	6.35	66.6	1.4	0.3	4.6	15.9	0.0	0.1	2.6	3.5	5.0
2080	6.37	65.3	1.5	0.3	4.6	17.7	1.3	0.1	1.7	2.8	4.7
2085	7.70	63.5	1.5	0.2	4.6	15.6	0.0	0.2	5.3	3.3	5.7
2077	8.05	61.8	1.5	0.2	4.6	16.0	0.3	0.1	4.9	2.0	8.5
2083	8.06	60.7	1.6	0.2	4.6	15.3	0.0	0.2	5.6	4.1	7.6
2088	8.22	64.6	1.6	0.2	4.5	14.5	0.0	0.1	5.7	3.2	5.6
2095	8.72	67.6	1.5	0.2	4.7	15.3	0.0	0.2	5.2	2.9	2.4
2073	10.27	58.9	1.6	0.2	4.7	14.8	0.5	0.1	9.4	2.4	7.4
2078	10.49	56.7	1.6	0.2	4.7	16.0	0.0	0.1	10.4	1.7	8.6
2079	11.62	50.2	1.0	0.2	4.7	15.0	0.0	0.1	11.7	4.3	13.5
2072	12.36										
2074	12.70	53.5	1.3	0.2	4.7	14.0	2.1	0.0	12.3	2.4	9.5
2092	12.72	57.0	1.8	0.2	4.8	13.7	0.2	0.1	12.0	3.0	7.3
2076	14.27	46.4	1.9	0.2	4.9	12.8	0.7	0.1	20.4	2.7	10.0
2084	14.29	46.8	1.6	0.3	4.8	14.6	0.0	0.1	16.3	2.8	12.7
2082	14.93	41.9	1.4	0.2	4.9	13.2	0.0	0.1	18.3	2.9	17.1
2094	16.55	50.3	1.8	0.2	4.9	12.0	0.3	0.2	19.4	3.0	7.9
2089	16.75	38.8	1.5	0.1	4.9	11.1	1.2	0.1	23.5	2.0	16.8
2093	17.65	43.8	1.7	0.1	4.9	11.0	0.6	0.1	22.3	5.0	10.4

Table 6.

Engine and General Performance—Series 2.

Test No.	Laboratory Designation	Duration of Test, Minutes	Revolutions, Average per Minute	Speed in Miles per Hour	Piston Speed in Feet per Minute	Position of Throttle	Average Boiler Pressure, lb. per sq. in.	Drawbar Pull, lb.	Average Cut-off, Per cent of Stroke	Average Least Back Pressure, lb. per sq. in.	Average Mean Effective Pressure, lb. per sq. in.
	Code Item☞		352	353	354	363	380	487	499	615	678
2081	55–24–F	150	50.6	9.2	252.5	Full	198.2		24.1	0.7	77.1
2086	55–24–F	170	51.3	9.3	256.1	Full	199.1	15 532	23.4	0.6	77.1
2075	•55–32–F	140	50.6	9.2	252.3	Full	198.1	20 483	32.1	1.7	98.9
2097	55–32–F	110	52.1	9.5	260.0	Full	198.8	20 820	32.3	2.1	101.5
2085	55–40–F	120	51.1	9.3	255.1	Full	197.9	24 833	41.3	1.8	117.8
2096	55–40–F	90	51.5	9.4	257.0	Full	196.1	24 980	40.4	1.7	120.2
2095	55–48–F	60	51.3	9.3	255.9	Full	198.1	28 922	49.2	1.8	135.4
2098	55–48–F	50	51.7	9.4	258.2	Full	198.2	29 240	49.1	2.0	137.5
2080	110–16–F	130	110.4	20.0	550.7	Full	198.8		16.9	2.0	43.9
2087	110–16–F	150	111.1	20.2	554.4	Full	199.2	8 135	16.6	2.3	43.7
2077	110–24–F	110	110.7	20.1	552.2	Full	196.0	12 512	24.0	3.2	62.4
2073	110–32–F	80	109.4	19.9	545.9	Full	197.6	16 961	29.6	6.5	80.8
2072	110–40–F	60	109.6	19.9	546.8	Full	196.7	20 877	41.5	9.3	97.6
2084	110–48–F	50	110.4	20.0	550.8	Full	194.0	22 403	48.4	12.2	105.7
2094	110–56–F	25	110.9	20.1	553.3	Full	196.3	25 225	57.0	16.3	119.1
2083	165–16–F	100	170.3	30.9	849.8	Full	198.7	7 078	18.4	3.9	37.2
2078	165–24–F	70	169.0	30.7	843.3	Full	196.4	10 188	24.0	7.4	52.0
2074	165–32–F	60	169.6	30.8	846.3	Full	197.1	13 486	28.8	12.5	64.7
2092	165–32–F	50	168.5	30.6	840.8	Full	198.4	13 701	30.4	11.6	65.3
2082	165–40–F	50	169.7	30.8	846.7	Full	195.2	14 783	41.4	18.3	74.5
2093	165–48–F	30	167.4	30.4	835.5	Full	191.5	17 660	48.6	22.1	84.2
2088	220–16–F	100	234.2	42.5	1168.6	Full	197.8	5 568	15.9	5.1	28.1
2079	220–24–F	60	231.9	42.1	1157.2	Full	197.4	8 270	23.4	9.8	41.5
2076	220–32–F	35	229.9	41.7	1147.2	Full	196.0	10 396	32.2	15.8	51.5
2089	220–40–F	35	230.7	41.9	1151.2	Full	194.9	11 831	43.5	22.0	58.7

Table 7.

Engine and General Performance—Series 2.

Test No.	Laboratory Designation	Indicated Horse Power, Total	Dry Coal Consumed per Indicated Horse Power per Hour, lb.	Dry Steam Consumed per Indicated Horse Power per Hour, lb.	Drawbar Horse Power	Dry Coal Consumed per Drawbar Horse Power per Hour, lb.	Dry Steam Consumed per Drawbar Horse Power per Hour, lb.	Machine Friction of Locomotive in Terms of Horse Power	Machine Efficiency of Locomotive, per cent.	Efficiency of Locomotive, per cent.
	Code Item☞	711	734	736	743	744	745	770	778	779
2081	55–24–F	450.5	4.36	31.53						
2086	55–24–F	456.0	4.51	31.18	386.0	5.33	36.83	70.0	84.7	3.80
2075	55–32–F	578.6	4.00	28.40	501.4	4.62	32.76	77.2	86.7	4.33
2097	55–32–F	610.6			525.2		33.91	85.4	86.0	
2085	55–40–F	694.3	4.39	29.69	614.5	4.96	33.55	79.8	88.5	4.07
2096	55–40–F	713.6			622.8		33.46	90.8	87.3	
2095	55–48–F	804.9	4.15	29.37	718.0	4.66	32.92	86.9	89.2	4.44
2098	55–48–F	822.2			732.2		32.56	90.0	89.1	
2080	110–16–F	558.5	4.31	30.87						
2087	110–16–F	560.3	4.39	30.65	437.6	5.63	39.24	122.7	78.1	3.69
2077	110–24–F	795.7	4.10	27.36	670.2	4.87	32.48	125.5	84.2	4.14
2073	110–32–F	1019.3	4.26	27.20	898.3	4.83	30.87	121.0	88.1	4.13
2072	110–40–F	1233.8	4.79	27.19	1107.8	5.33	30.28	126.0	89.8	3.83
2084	110–48–F	1347.5	5.88	28.69	1197.2	6.62	32.30	150.3	88.9	3.13
2094	110–56–F	1521.4	5.58	29.39	1354.1	6.27	33.02	167.3	89.0	3.27
2083	165–16–F	730.4	4.52	29.65	583.6	5.65	37.10	146.8	79.9	3.56
2078	165–24–F	1011.2	4.63	28.09	833.8	5.62	34.06	177.4	82.5	3.62
2074	165–32–F	1264.3	4.73	27.17	1107.3	5.40	31.03	157.0	87.6	3.75
2092	165–32–F	1267.3	4.46	27.34	1117.6	5.06	31.01	149.7	88.2	3.99
2082	165–40–F	1457.3	6.15	27.88	1214.6	7.38	33.45	242.7	83.4	2.73
2093	165–48–F	1633.5	6.31	29.62	1431.6	7.19	33.80	201.9	87.6	2.82
2088	220–16–F	756.1	4.42	29.40	631.3	5.29	35.21	124.8	83.5	3.98
2079	220–24–F	1109.9	5.17	28.25	928.5	6.18	33.84	181.4	83.7	3.23
2076	220–32–F	1364.3	5.71	28.30	1157.1	6.73	33.36	207.2	84.8	3.02
2089	220–40–F	1559.9	7.10	29.18	1321.6	8.38	34.42	238.3	84.7	2.46

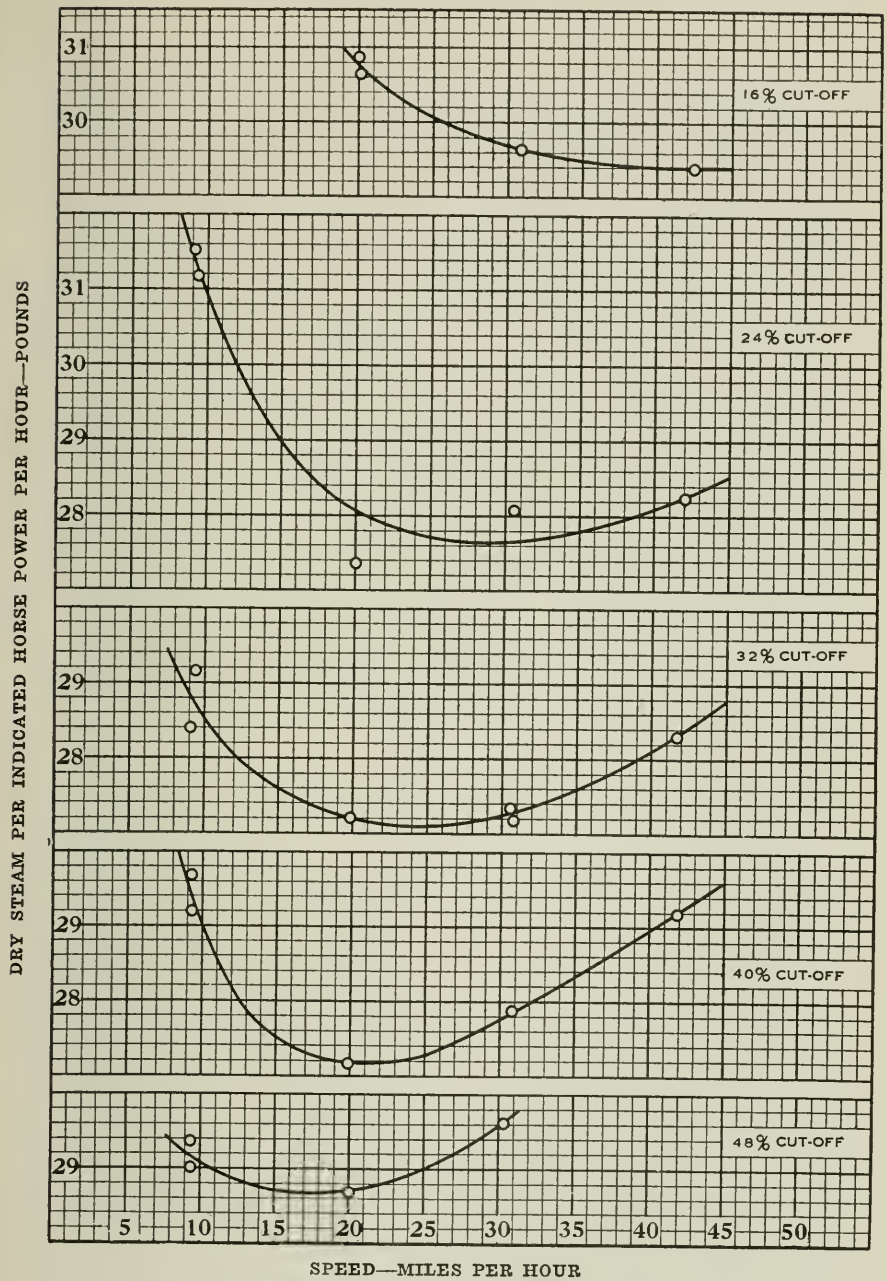


FIG. 16. THE RELATION BETWEEN STEAM CONSUMPTION AND SPEED, AT VARIOUS CUT-OFFS.

B. Engine Performance.

Tables 6 and 7 present information relating to the general conditions and the more important results concerning engine and general performance for all tests of Series 2. These data are arranged in groups with reference to the speed of the locomotive and, within each group, are arranged with reference to the indicated horse power developed, the first test in each group giving the lowest horse power developed at the group speed. Appendix 4 contains data and results for all tests including information concerning cylinder performance as shown by average values taken from indicator diagrams. Fig. 56 and 57, there included, show representative indicator diagrams.

The nominal speeds at which the locomotive was operated were 10, 20, 30, and 40 miles per hour, and the data indicate that the actual speeds obtained closely approximated these figures. The nominal cut-offs at which the locomotive was operated were 16, 24, 32, 40, 48, and 56 per cent of the stroke. The actual cut-offs, as determined from the indicator cards, do not vary greatly from the nominal cut-offs. All tests at a given nominal cut-off were made with the reverse lever in the same notch of the reverse-lever quadrant. In the discussion which follows relative to engine and general performance, speed and cut-off are referred to in terms of the nominal values. All points plotted upon the figures are, however, located with regard to the actual speed and cut-off as determined from test data.

The data in general indicate uniform conditions in those particulars in which uniformity was sought. Test conditions as to nominal speed and nominal cut-off were duplicated in six cases. Such duplicated tests show, in general, satisfactory agreement as regards both test conditions and derived results.

21. *Dry Steam per Indicated Horse Power Hour and per Drawbar Horse Power Hour.*—Fig. 16 and 17 present the relation between dry steam per indicated horse power hour and speed. In Fig. 16 this relation has been shown with a separate water-rate scale for each group of tests at a given cut-off. In Fig. 17 the same curves have been referred to a single water-rate scale in order that the curves may be compared more readily. No curve is drawn for 56 per cent cut-off, since only one test was made at that cut-off.

The minimum water-rate was 27.17 pounds of dry steam, and occurred in test 2074 at a speed of 30 miles per hour and 32 per cent cut-off. The maximum water rate was 31.53 pounds of dry steam, and occurred in test 2081 at a speed of 10 miles per hour and 24 per cent cut-off. The difference between the minimum and maximum water-

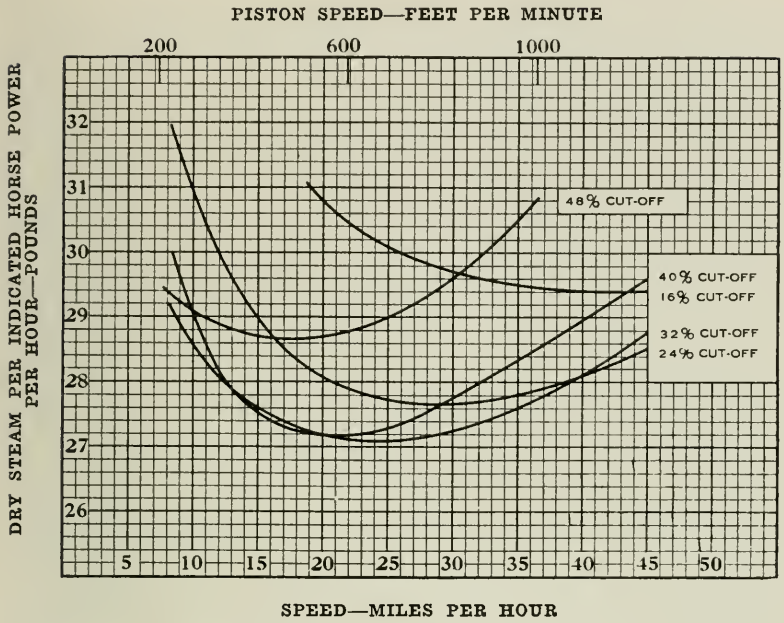


FIG. 17. THE RELATION BETWEEN STEAM CONSUMPTION AND SPEED, AT VARIOUS CUT-OFFS.

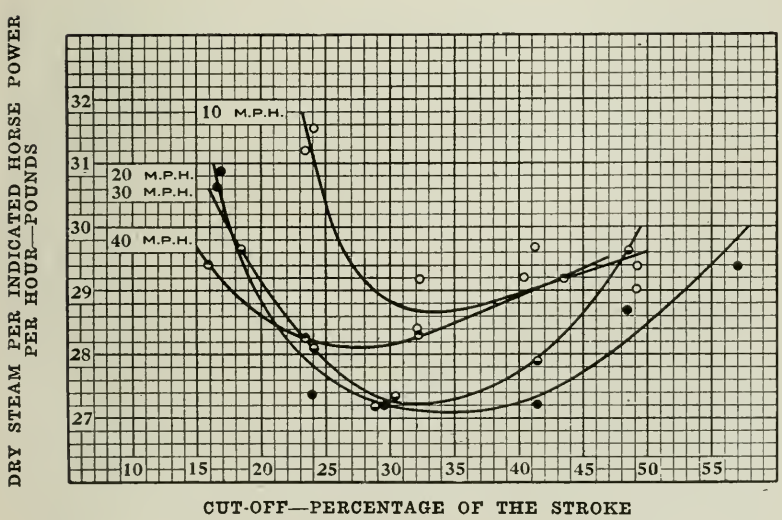


FIG. 18. THE RELATION BETWEEN STEAM CONSUMPTION AND CUT-OFF, AT VARIOUS SPEEDS.

rates for all tests was only 4.36 pounds of dry steam. The corresponding differences between minimum and maximum rates at a given cut-off are in general much smaller.

A decrease in steam consumption per indicated horse power per hour as speed increases is shown until the speed has become from 20 to 30 miles per hour. Further increase of speed is then accompanied by increased steam consumption, as shown by all curves with the exception of that for tests at 16 per cent cut-off. The tests at both short and long cut-off show comparatively high water-rates. The best performance is shown by the curve for tests at 32 per cent cut-off. The tests at 40 per cent cut-off show rather better performance for freight service conditions than those made at 24 per cent cut-off.

Fig. 18 presents curves showing the dry steam consumed per indicated horse power per hour in its relation to cut-off. A curve is shown for each of the four nominal speeds at which tests were made—10, 20, 30, and 40 miles per hour. The tests made at 10 and at 40 miles per hour show much higher water-rates than do the tests made at 20 and 30 miles per hour. This is particularly true for cut-offs between 20 and 50 per cent. At short cut-off the 40 miles per hour curve shows a lower water-rate than the 20 and 30 miles per hour curves. At long cut-off the 10 miles per hour curve appears likewise to show a lower water-rate than the 20 and 30 miles per hour curves.

Except during short periods at starting and on heavy grades, the speed of this locomotive in service would probably vary between about 15 and 35 miles per hour, and the cut-off would range from say 50 to 20 per cent. Under these conditions of speed and cut-off the steam consumption varies between approximately 27 and 29 pounds of steam per indicated horse power per hour. It is probable that this range fairly represents the general average water rate for a very considerable number of freight locomotives in service.

22. *Indicated Horse Power and Drawbar Horse Power.*—Fig. 19 presents indicated horse power in its relation to speed, each curve of the figure representing all of the tests made at a particular nominal cut-off. In addition to the relationship just mentioned this figure shows clearly the range of the tests as to speed, cut-off, and load. It further shows the range covered within each group of tests when the tests are grouped either according to constant speed or to constant cut-off. The six different conditions of speed and cut-off at which duplicate tests were made are evident, and the proximity of the two points representing each pair of such tests indicates the uniformity obtained as to speed and indicated horse power developed.

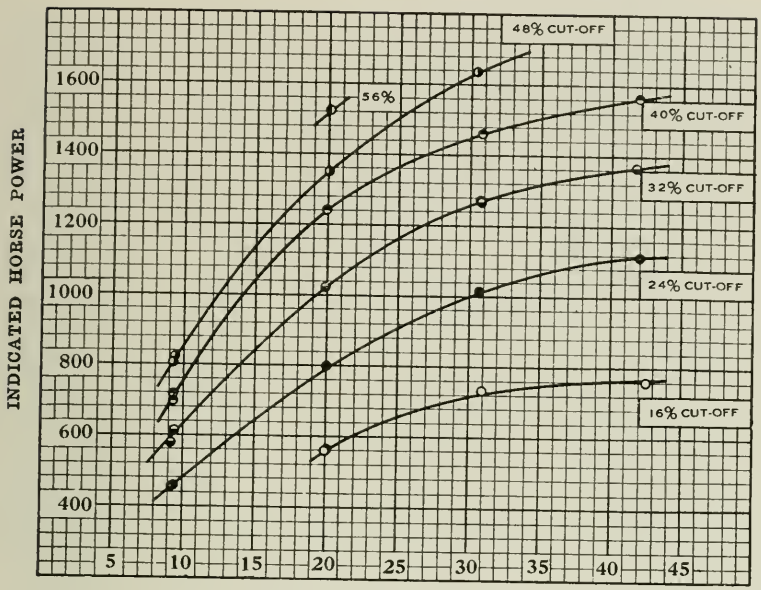


FIG. 19. THE RELATION BETWEEN INDICATED HORSE POWER AND SPEED, AT VARIOUS CUT-OFFS.

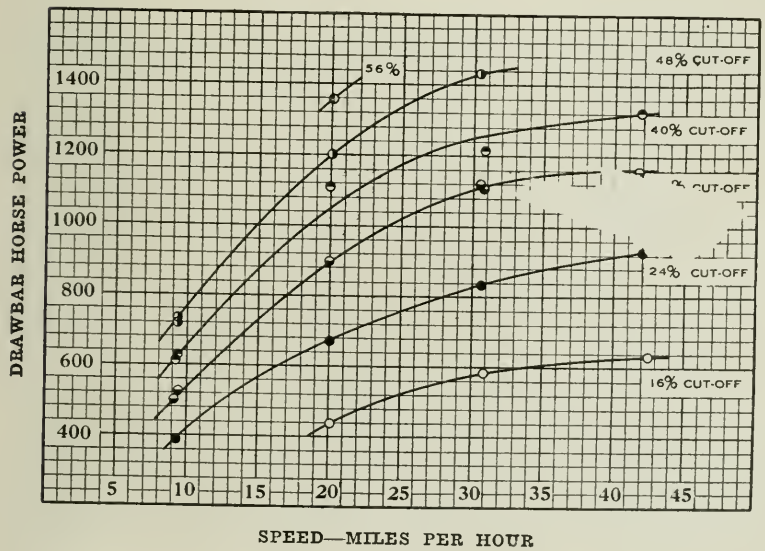


FIG. 20. THE RELATION BETWEEN DRAWBAR HORSE POWER AND SPEED, AT VARIOUS CUT-OFFS.

The maximum indicated horse power was developed in test 2093 at a speed of 30 miles per hour and at 48 per cent cut-off. The average rate of working for this test was 1633.5 indicated horse power. The lowest rate of working was for test 2081, being 450.5 indicated horse power while running at 10 miles per hour with 24 per cent cut-off. The dry steam supplied to the engines per hour when developing 1633.5 indicated horse power was 48 387 pounds. The moist steam delivered to the engines for the same test was 48 812 pounds per hour.

The relations between the horse power developed at the locomotive drawbar and the speed are shown in Fig. 20, in which each of the curves presents this relation for a particular cut-off. The maximum rate of 1431.6 drawbar horse power and the minimum rate of 386.0 drawbar horse power were developed during tests 2093 and 2086 respectively, the former test being at 30 miles per hour and 48 per cent cut-off, and the latter at 10 miles per hour and 24 per cent cut-off. Owing to incomplete dynamometer records for tests 2081 no record is available for the drawbar horse power developed for this test. Tests 2081 and 2086 were made under similar conditions of speed and cut-off.

The plotted points and curves of Fig. 19 and 20 show the engines to have been tested throughout a range of speed, cut-off, and load which would cover all ordinary service conditions above a speed of 10 miles per hour.

23. *Machine Friction.*—The diagrams in Fig. 21 present information concerning machine friction and its relation to indicated horse power for speeds of 10, 20, 30, and 40 miles per hour. Upon each diagram is shown the relation between the indicated horse power developed and machine friction expressed in horse power and also the relation between the indicated horse power developed and machine friction expressed in per cent of indicated horse power. Obviously the ordinates of each pair of curves in this figure bear to each other a definite numerical relation, and the curves have been so drawn that they satisfy this relation and also fairly represent the plotted values for the individual tests.

The range in machine friction is, for the entire series, from 70 to 242.7 horse power. These values were obtained in tests 2086 and 2082 during which 456.0 and 1457.3 indicated horse power respectively were developed. Test 2086 was at 10 miles per hour and 24 per cent cut-off, and test 2082 was at 30 miles per hour and 40 per cent cut-off. Expressed as percent of the indicated horse power developed, the minimum machine friction was 10.2 per cent and occurred in test 2072;

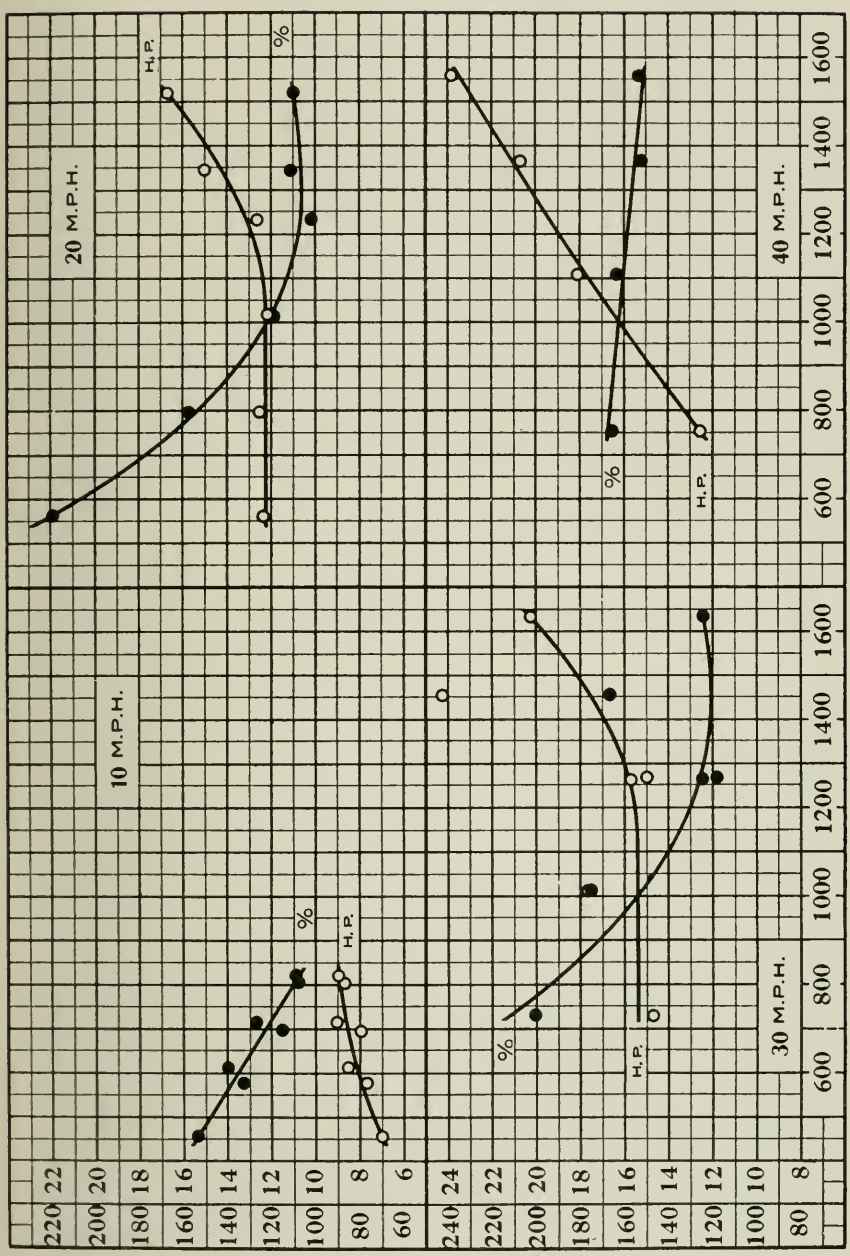


FIG. 21. THE RELATION BETWEEN MACHINE FRICTION AND INDICATED HORSE POWER, AT VARIOUS SPEEDS.

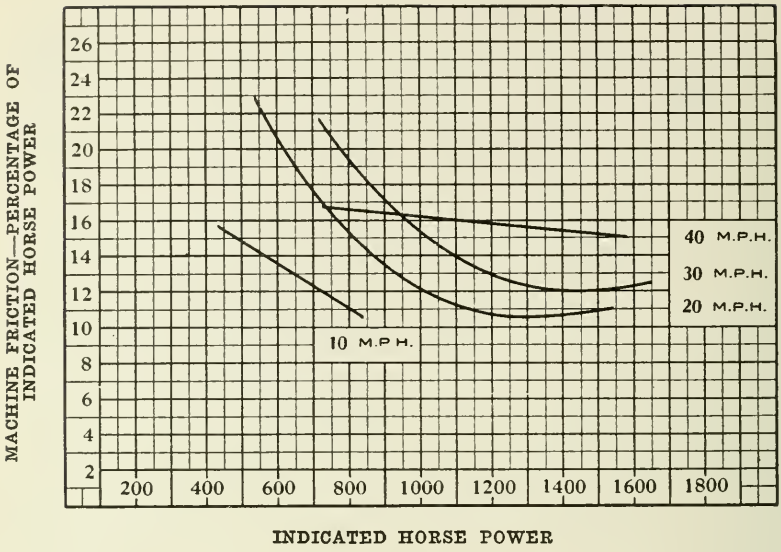


FIG. 22. THE RELATION BETWEEN MACHINE FRICTION AND INDICATED HORSE POWER, AT VARIOUS SPEEDS.

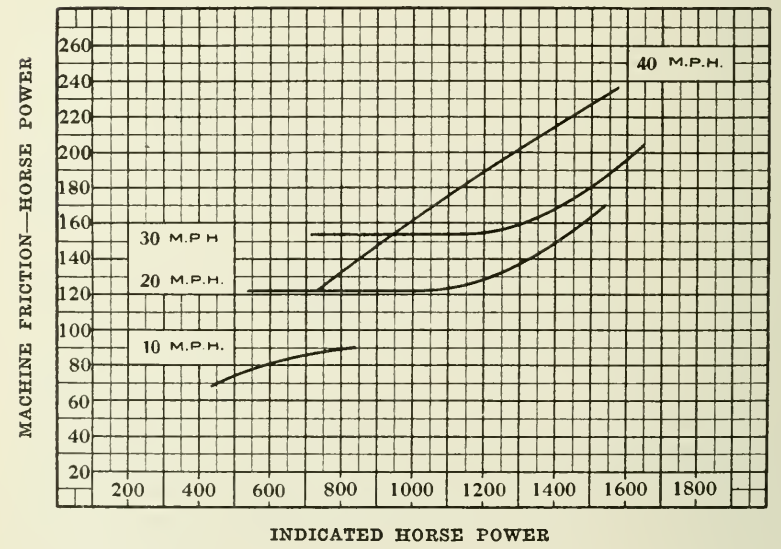


FIG. 23. THE RELATION BETWEEN MACHINE FRICTION POWER AND INDICATED HORSE POWER, AT VARIOUS SPEEDS.

the maximum was 21.9 per cent and occurred in test 2087. The former test was at 20 miles per hour and 40 per cent cut-off, developing 1233.8 indicated horse power, and the latter test at 20 miles per hour and 16 per cent cut-off, developing 560.3 indicated horse power.

While the curves differ more or less for different speeds, they all show the machine friction horse power to increase with increasing indicated horse power. In general, the ratio of machine friction horse power to indicated horse power decreases with increasing load. The rate of decrease of this ratio appears to be quite rapid for loads under 1000 horse power, but at greater loads the ratio becomes fairly constant for a given speed and ranges from 10 per cent to 15 per cent for the different speeds.

Fig. 22 presents upon a single diagram the four curves showing the relation between the machine friction in percentage of indicated horse power and the indicated horse power, which are included in Fig. 21. The curves so grouped indicate that as the speed increased, the percentage of power which was absorbed by machine friction also increased.

Fig. 23 likewise presents upon a single diagram the four curves showing the relation between machine friction and indicated horse power, the machine friction being expressed in terms of horse power. A general increase in machine friction both with increase of speed and with increase of indicated horse power is shown. The curves of Fig. 23, taken as a whole, show a tendency for the machine friction horse power to be fairly constant at a given speed or to increase rather slowly as the load increases up to about 1000 horse power. With loads greater than 1000 to 1200 horse power, the increase in machine friction is more rapid.

Fig. 24, 25, and 26 again present machine friction in its relation to indicated horse power. In these figures however each curve represents the data for all tests at a given nominal cut-off, instead of at a given speed as in the three preceding diagrams. The curves have been located and are presented in a manner similar to that used in connection with Fig. 21, 22, and 23.

These curves indicate that for a given cut-off machine friction horse power increased with the load, and at a rate approximately proportional to the increase of the load. The curves further indicate that, within the range of the tests, machine friction horse power increased with decreasing cut-off when the load remained constant. The per cent of indicated horse power absorbed in machine friction increased rap-

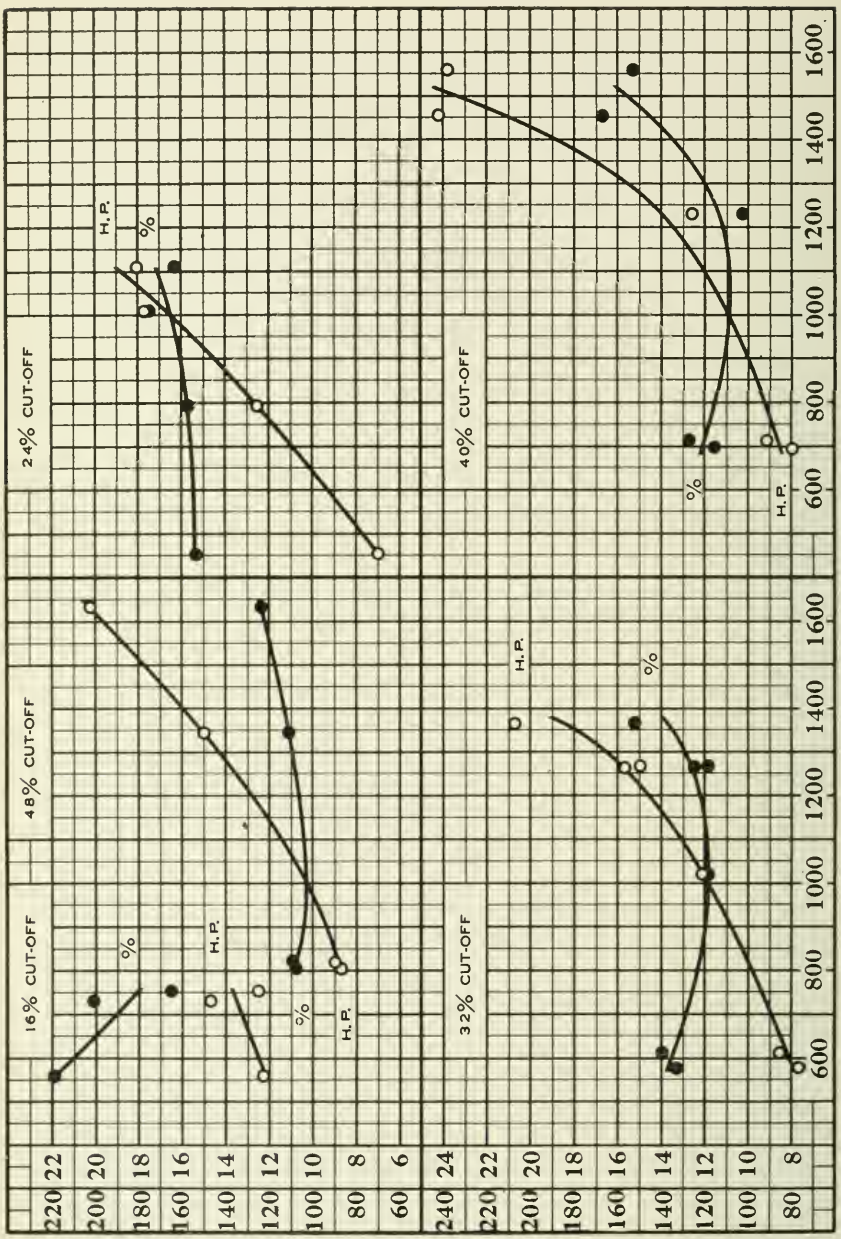


FIG. 24. THE RELATION BETWEEN MACHINE FRICTION AND INDICATED HORSE POWER, AT VARIOUS CUT-OFFS.

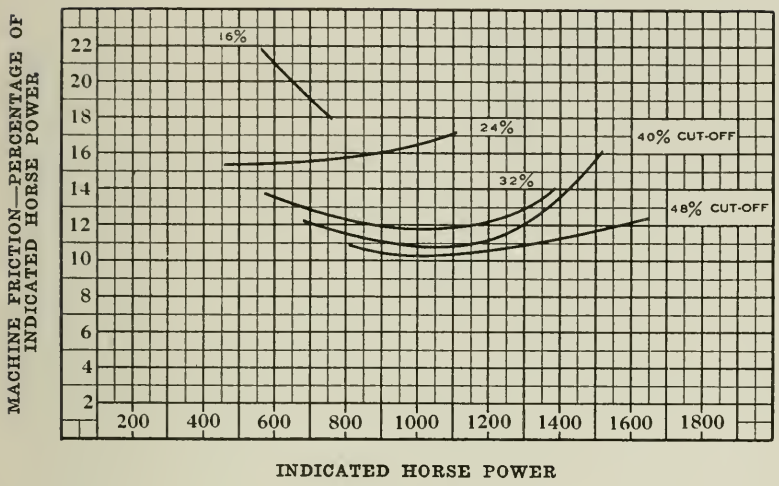


FIG. 25. THE RELATION BETWEEN MACHINE FRICTION AND INDICATED HORSE POWER, AT VARIOUS CUT-OFFS.

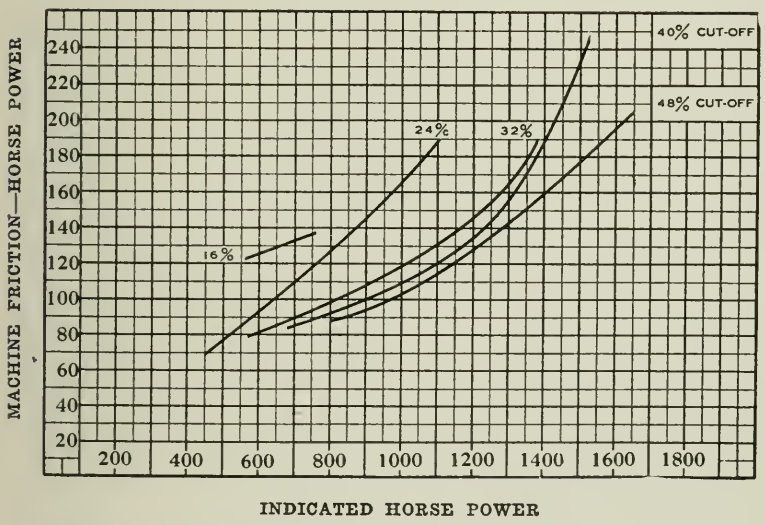


FIG. 26. THE RELATION BETWEEN MACHINE FRICTION POWER AND INDICATED HORSE POWER, AT VARIOUS CUT-OFFS.

idly with decreasing cut-off at constant load, and increased but slightly with increasing load at constant cut-off.

The locomotive tested carried 100.45 tons upon the drivers. The following table presents machine friction in terms of tractive force in pounds per ton upon drivers. The values given in the table were calculated from the curves of Fig. 23, making use of the minimum, maximum, and average values for machine friction horse power as shown by the various curves. In making these calculations nominal speed was employed.

TABLE 8.

MACHINE FRICTION.—SERIES 2.

Speed in Miles per Hour	Machine Friction Expressed as Tractive Force in Pounds per Ton of Weight upon Drivers.		
	Minimum	Average	Maximum
10	25	31	34
20	23	25	31
30	19	20	26
40	12	17	22
Averages	20	23	28

For the locomotive tested the values given in Table 8 show 20 to 23 pounds tractive force per ton of weight on drivers to be values fairly representative of machine friction for practically all speeds when the load does not exceed 1000 to 1200 indicated horse power. For conditions of low speed, however, and for all speeds where the indicated horse power is comparatively high and the maximum tractive effort is approached, the machine friction is materially greater, as shown by the values given in the table for the speed of 10 miles per hour, and by the maximum values, which vary from 22 to 34 lb. per ton of weight upon the drivers.

Fig. 27 and 28 present machine friction in its relation to speed in miles per hour. A curve is drawn for each nominal cut-off. The values presented are the same as have been shown in preceding curves concerning machine friction, and some of the relations already considered in connection with Fig. 21 to 26 may be seen in Fig. 27 and 28. While the curves of Fig. 27 and 28 exhibit considerable lack of uniformity, the former figure shows machine friction horse power to increase more or less uniformly with increasing speed for all cut-offs; and Fig. 28 indicates that at constant speed the ratio of machine friction horse power to indicated horse power decreases as the cut-off increases, and that for any given cut-off this ratio is fairly constant throughout the range of speed shown.

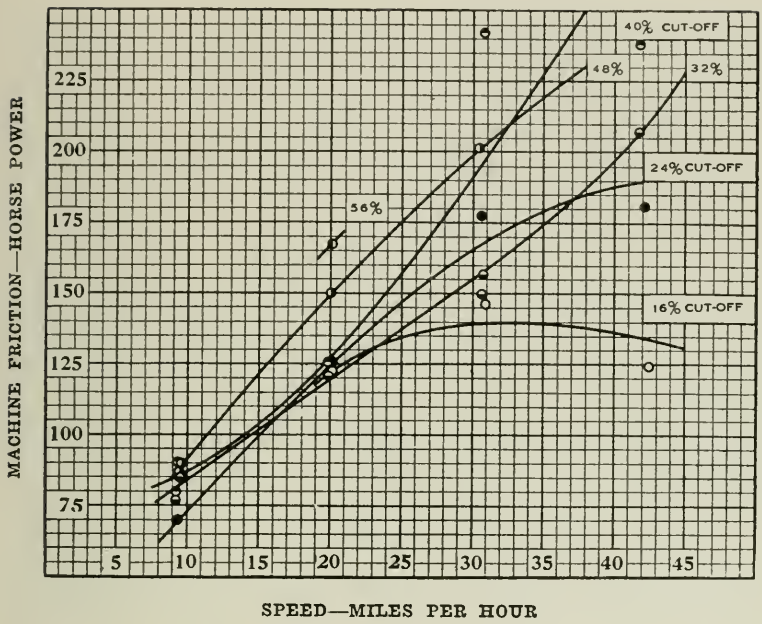


FIG. 27. THE RELATION BETWEEN MACHINE FRICTION POWER AND SPEED, AT VARIOUS CUT-OFFS.

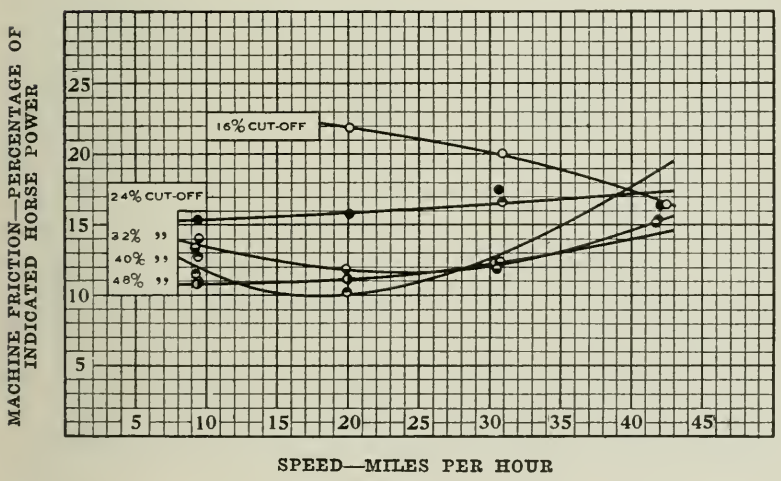


FIG. 28. THE RELATION BETWEEN MACHINE FRICTION AND SPEED, AT VARIOUS CUT-OFFS.

The facts here presented seem to warrant the conclusions that, for this locomotive, the percentage of indicated horse power absorbed by the friction of the machinery varies rather definitely with respect to speed, to cut-off, and to load; and that this machine friction entails a loss in tractive force between the cylinders and the locomotive drawbar which varies from about 15 to about 30 pounds for each ton of weight carried on the driving wheels.

C. General Performance.

24. *Coal Consumption per Indicated Horse Power Hour and per Drawbar Horse Power Hour.*—The curves of Fig. 29 show the relation between speed and the amount of dry coal consumed per indicated horse power per hour. Each of the curves there drawn applies to a particular cut-off. In a similar manner the relation between speed and the dry coal consumed per drawbar horse power per hour is presented in Fig. 30.

The most economical performance was obtained in test 2075, made at a speed of 10 miles per hour and at 32 per cent cut-off. During this test 4.00 pounds and 4.62 pounds of dry coal per indicated horse power hour and per drawbar horse power hour respectively were used. The highest coal rate occurred in test 2089, made at a speed of 40 miles per hour and at 40 per cent cut-off, during which 7.10 pounds and 8.38 pounds of dry coal per indicated horse power hour and per drawbar horse power hour respectively were used. Both figures show a more rapid increase in coal consumption with increase of speed at long cut-off than with increase of speed at short cut-off. This is in conformity with the results presented in Fig. 19 where the relation between indicated horse power and speed is shown. The curves of Fig. 29 and 30 show that the economy was fairly constant, or increased slowly as speed was increased from 10 to 20 miles per hour. Tests at 24 per cent cut-off show an economy apparently better at 15 to 20 miles per hour than at 10 miles per hour. As speed increased above 20 miles per hour the coal consumption increased more rapidly than at lower speeds, with the exception of the tests made at 16 per cent cut-off.

25. *General Efficiency.*—By general efficiency is meant, in this connection, the ratio of the heat equivalent of the work done at the locomotive drawbar to the heat content of the coal. This ratio is a measure of the economic performance of the locomotive as a whole. General efficiency and its relation to speed are shown in Fig. 31, in which a separate curve is presented for each nominal cut-off.

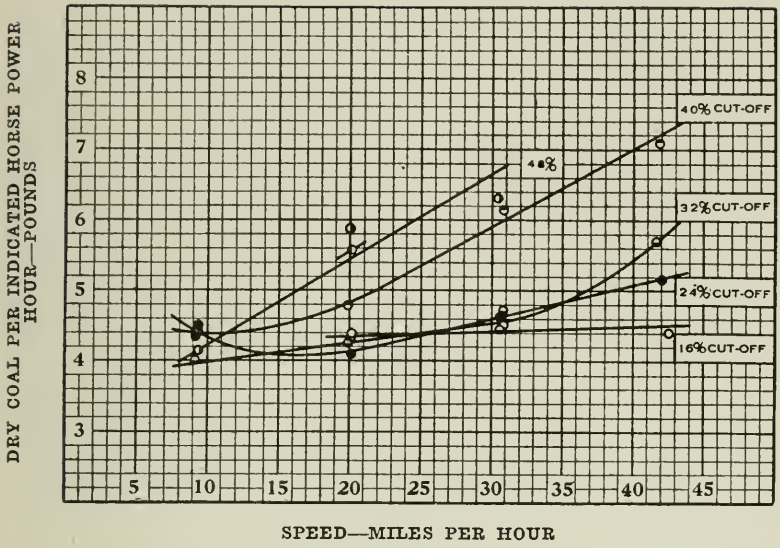


FIG. 29. THE RELATION BETWEEN COAL CONSUMED PER INDICATED HORSE POWER HOUR AND SPEED, AT VARIOUS CUT-OFFS.

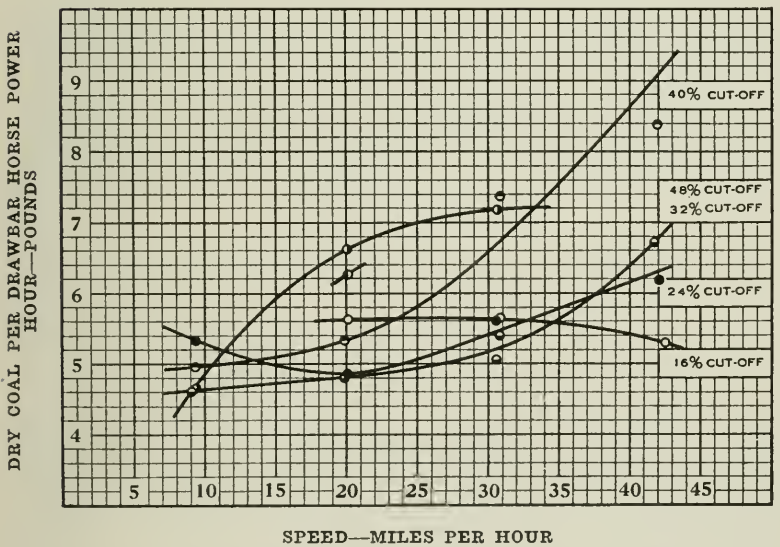


FIG. 30. THE RELATION BETWEEN COAL CONSUMED PER DRAWBAR HORSE POWER HOUR AND SPEED, AT VARIOUS CUT-OFFS.

The maximum efficiency obtained was 4.44 per cent, and occurred in test 2095 which was made at a speed of 10 miles per hour with 48 per cent cut-off, while developing 804.9 indicated horse power. The minimum efficiency obtained was 2.46 per cent, and occurred in test 2089 made at a speed of 40 miles per hour with 40 per cent cut-off, while developing 1559.9 indicated horse power. This group of curves shows substantially the same relations as were shown by Fig. 29 and 30 which presented coal consumption per indicated horse power hour and drawbar horse power hour. Collectively the curves indicate a fairly constant efficiency of about 4 per cent, for speeds from 10 to 20 miles per hour. As the speed increases above 20 miles per hour the efficiency decreases from about 4 per cent to about 3 per cent.

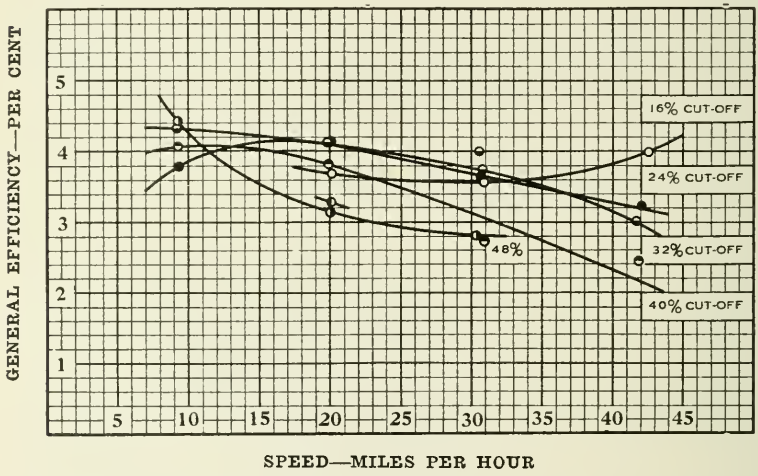


Fig. 31. The Relation Between General Efficiency and Speed, at Various Cut-offs.

VII. The Results of the Tests of Series I.

Series 1 comprises tests 2009 to 2037 inclusive. The conditions which prevailed during the tests of this series have been set forth in sections IV and V, and the differences in the condition of the locomotive during Series 1 as compared with Series 2 are stated in section II and in Appendix 1. It is sufficient here to recall the fact that during Series 1 the locomotive was in the condition in which it was deliv-

ered to the laboratory. The repairs made between Series 1 and 2 did not prove to have affected radically the performance of the locomotive, and most of the relations presented in the preceding section relating to Series 2 remain substantially the same for Series 1. For these reasons it has seemed unnecessary to present the results for Series 1 in very great detail. All the results are given in Appendix 4; but only the more important measures of performance are here included and discussed.

D. Boiler Performance.

26. *The Range of Performance.*—The more significant data and results pertaining to the performance of the boiler during Series 1 are given in Table 9, in which the tests are arranged in the order of the increasing amounts of dry coal fired per hour. In order to exhibit the range of the boiler performance, the minimum and maximum values of the main data and results are assembled in the table immediately following. As in all the tables, the quantities cited are the average values prevailing during the tests.

	Minimum	*Maximum*
Duration of test, minutes	40	180
Boiler pressure, lb. per sq. in.	189.9	198.1
Feed water temperature, deg. F.	57.7	72.2
Quality of the steam in the dome	0.9833	0.9956
Calorific value of coal as fired, B.t.u.	9929	11 376
Calorific value of dry coal, B.t.u.	11 835	12 757
Ash content of coal as fired—per cent	10.68	14.27
Draft in front of diaphragm, inches of water	2.2	10.7
Firebox temperature, deg. F.	1552	2081
Front-end temperature, deg. F.	494	761
Dry coal fired per test, lb	3568	10031
Dry coal fired per hour, lb	1814	7767
Dry coal fired per hour per square foot of grate, lb	36.6	156.8
Equivalent evaporation per hour, lb.	16 934	46 380
Equivalent evaporation per hour per square foot of heating surface, lb.	5.16	14.13
Total cinder loss, per hour, lb	66	1509
Total cinder loss, per cent of dry coal	3.4	20.8

27. *Economic Performance.*—In Series 1 the equivalent evaporation per pound of dry coal ranged from a minimum of 5.97 pounds to a maximum of 10.07 pounds. This latter value, applying to test 2024,

Table 9.

Boiler Performance—Series 1.

Test No.	Laboratory Designation	Average Boiler Pressure, lb. per sq. in.	Dry Coal Fired per Hour, lb.		Duration of Test, Minutes	Quality of the Steam in the Dome	Cinders, lb.		Calorific Value per lb. of Dry Coal, B. t. u.	Equivalent Evaporation, lb.		Efficiency of the Boiler Including the Grate, per cent
			Total	Per sq. ft. of Grate			Total per Hour	Total in Per cent of the Dry Coal Fired		Per Hour per sq. ft. of Heating Surface	Per lb. of Dry Coal	
	Code Item☞	380	626	627		407	424 & 345	426	458	648	658	666
2024	55–24–F	196.3	1814	36.6	120	.9950	68	3.7	12 712	5.56	10.07	76.87
2017	83–16–F	193.4	1957	39.5	180	.9910	66	3.4	12 422	5.33	8.96	69.88
2021	83–16–F	193.7	2211	44.6	120	.9945	133	6.0	12 274	5.16	7.66	60.56
2026	110–16–F	196.9	2293	46.3	130	.9895	140	6.1	12 309	6.00	8.59	67.75
2028	55–32–F	198.1	2406	48.6	140	.9912	166	6.9	12 653	6.14	8.37	64.21
2020	83–24–F	190.7	2472	49.9	120	.9929	151	6.1	12 302	6.42	8.53	67.33
2018	83–24–F	194.2	2537	51.2	160	.9930	153	6.1	12 265	6.50	8.41	66.49
2009	138–16–F	190.5	2647	53.4	150	.9900	176	6.6	12 553	6.60	8.19	63.30
2019	83–32–F	193.8	3215	64.9	130	.9910	277	8.6	12 523	8.17	8.35	64.68
2016	193–16–F	193.9	3255	65.7	140	.9890	297	9.1	11 992	7.56	7.62	61.68
2027	110–24–F	196.8	3256	65.7	150	.9914	306	9.4	12 688	7.68	7.74	59.22
2022	83–32–F	189.9	3673	74.1	120	.9956	453	12.3	11 875	8.02	7.17	58.54
2012	138–24–F	191.8	3707	74.8	110	.9890	359	9.7	12 280	8.69	7.69	60.80
2010	193–20–F	192.0	3834	77.4	70	.9870	438	11.4	12 433	8.71	7.45	57.63
2030	165–24–F	196.5	4013	81.0	100	.9894	519	13.0	12 757	9.60	7.85	59.71
2029	110–32–F	197.1	4242	85.6	90	.9902	589	13.9	12 486	9.78	7.57	58.85
2031	83–40–F	196.4	4244	85.6	90	.9896	638	15.0	11 989	10.14	7.84	63.43
2013	138–32–F	190.1	4749	95.8	90	.9850	623	13.1	12 242	10.54	7.29	57.78
2015	193–24–F	192.1	4927	99.5	90	.9870	736	14.9	12 307	10.04	6.69	52.84
2033	110–48–F	196.0	5126	103.5	80	.9867	924	18.0	12 243	11.77	7.54	59.76
2032	165–32–F	196.8	5352	108.0	40	.9862	1003	18.8	12 242	11.26	6.91	54.79
2035	110–40–F	194.3	5565	112.3	70	.9833	803	14.5	12 329	12.06	7.11	55.94
2014	193–32–F	191.5	6199	125.1	70	.9840	901	14.6	12 184	11.83	6.27	49.92
2037	165–40–F	196.1	6554	132.3	60	.9860	1217	18.6	12 441	13.97	7.00	54.54
2023	138–40–F	190.8	6687	135.0	90	.9930	1388	20.8	12 311	12.89	6.33	49.86
2034	193–40–F	192.1	7767	156.8	60	.9857	1509	19.4	11 835	14.13	5.97	48.95

is however so divergent from the other values for similar rates of combustion as to raise doubt of its validity, although errors can not be found in the data. The next highest evaporation per pound of dry coal is 8.96 pounds. The rate of decrease in the evaporation per pound of coal with respect to increase in rate of combustion is shown in Fig. 32. The rate of this decrease with respect to increase in rate of evaporation is shown in Fig. 33. These two figures are comparable with Fig. 8 and 9 of Series 2.

28. *Boiler Efficiency.*—As previously explained, efficiency in this connection means the ratio of the heat absorbed by the steam generated, to the heat contained in the coal as it was fired. If, for the reason above suggested, we exclude test 2024, the highest efficiency obtained during Series 1 was 69.88 per cent, which occurred during test 2017 with a rate of combustion of 39.5 pounds of dry coal per square foot of grate per hour. The lowest efficiency, 48.95 per cent, prevailed during test 2034 in which the rate of combustion was 156.8 pounds of dry coal per square foot of grate per hour. The relation between efficiency and rate of combustion is shown in Fig. 34.

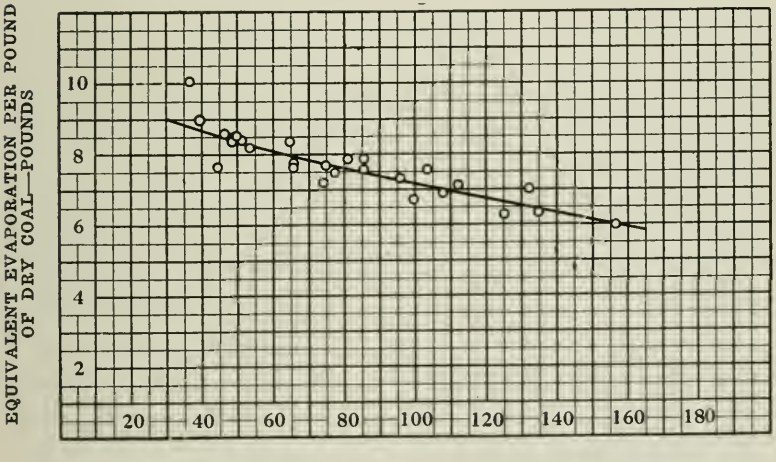


FIG. 32. THE RELATION BETWEEN EVAPORATION PER POUND OF COAL AND RATE OF COMBUSTION.

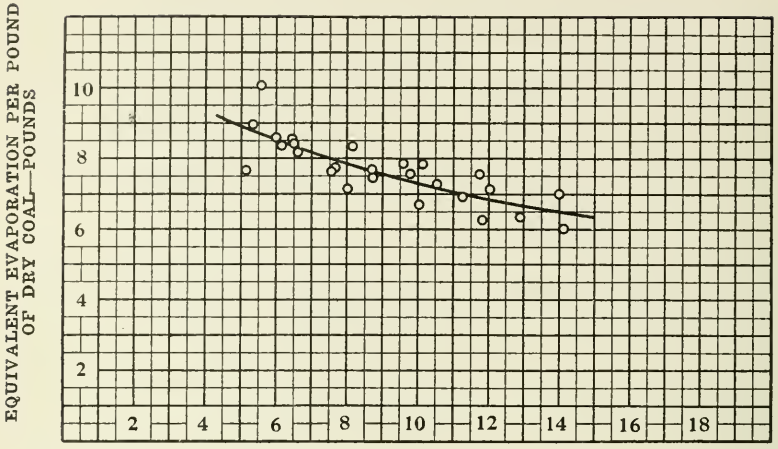


FIG. 33. THE RELATION BETWEEN EVAPORATION PER POUND OF COAL AND RATE OF EVAPORTION.

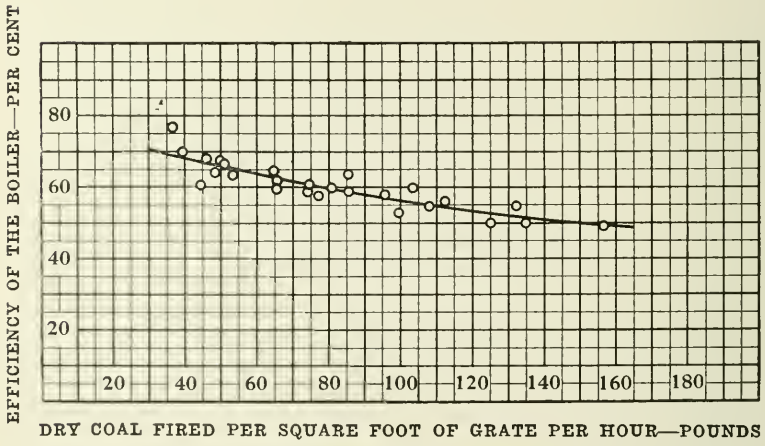


FIG. 34. THE RELATION BETWEEN BOILER EFFICIENCY AND RATE OF COMBUSTION.

E. Engine Performance and General Performance.

The more important data and results pertaining to the performance of the engines and to the general performance of the locomotive during Series 1 are assembled in Table 10. The remaining data and results appear in Appendix 4. In the table all tests run at like speed are grouped and, within these groups, the tests are arranged in the order of the values of cut-off. The tests of Series 1 were made at speeds varying from 10 to 35 miles per hour, and at cut-offs of 16, 20, 24, 32, 40, and 48 per cent. Only one test however was made at 20 per cent cut-off and one at 48 per cent cut-off, and these are omitted from the figures here included.

29. *Dry Steam per Indicated Horse Power Hour.*—In Fig. 35 is shown the relation between steam consumption and speed for each of the four cut-offs. The curves here drawn show the minimum steam consumption to have occurred in each case at a speed of from 20 to 25

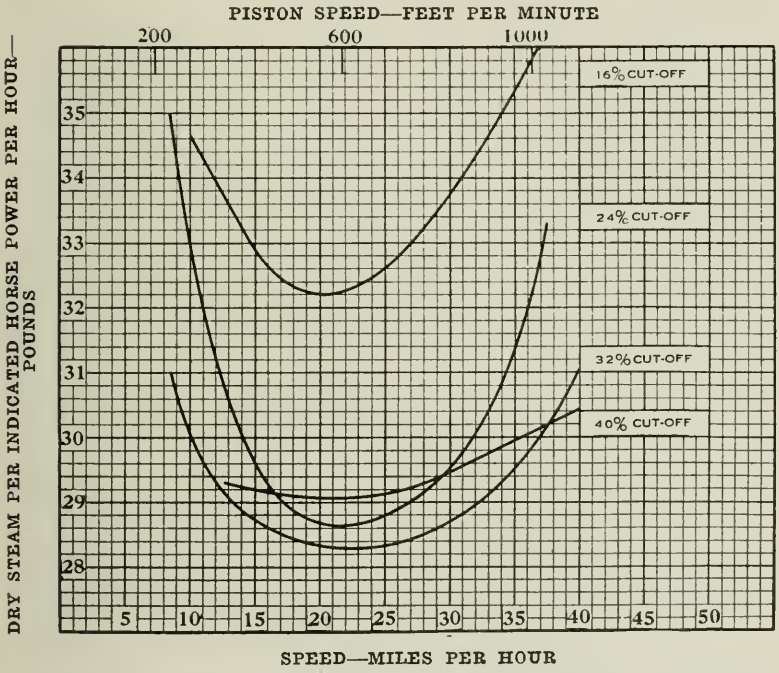


Fig. 35. The Relation Between Steam Consumption and Speed, at Various Cut-offs.

Table 10.

Engine and General Performance—Series 1.

Test No.	Laboratory Designation	Duration of Test, Minutes	Speed in Miles per Hour	Piston Speed in Feet per Minute	Average Cut-off, Per cent of Stroke	Average Least Back Pressure, lb. per sq. in.	Indicated Horse Power, Total	Dry Steam Consumed per I.H.P. per Hour, lb.	Drawbar Horse Power	Machine Efficiency of Locomotive, per cent	Efficiency of Locomotive, per cent
	Code Item☞		353	354	499	615	711	736	743	778	779
2025	55–16–F	120	9.4	257.0			301.6	47.16	231.7	76.8	3.11
2024	55–24–F	120	9.2	253.0			431.0	34.74	355.6	82.5	3.97
2028	55–32–F	140	9.1	251.0	31.7		548.7	30.57	488.1	89.0	4.10
2017	83–16–F	180	14.5	399.2	16.9	2.6	428.1	34.08	357.1	83.4	3.76
2021	83–16–F	120	14.5	399.7	14.9	4.4	428.6	32.10	346.3	80.8	3.31
2020	83–24–F	120	14.6	401.7	22.4	5.1	584.4	29.99	508.9	87.1	4.27
2018	83–24–F	160	14.5	399.2	22.6	4.0	593.8	29.71	511.7	86.2	4.21
2019	83–32–F	130	14.6	402.0	30.6	5.2	765.7	28.96	683.1	89.2	4.35
2022	83–32–F	120	14.6	402.2	30.3	7.2	773.9	28.14	674.6	87.2	3.94
2031	83–40–F	90	15.6	428.1	39.5	10.3	953.9	28.75	869.7	91.2	4.38
2026	110–16–F	130	19.9	545.9	19.1	3.1	515.0	31.67	415.1	80.6	3.75
2027	110–24–F	150	20.0	548.9	24.3	6.1	749.1	27.84	633.6	84.6	3.91
2029	110–32–F	90	19.9	547.9	30.7	10.5	968.6	27.51	820.8	84.7	3.94
2035	110–40–F	70	20.3	557.9	39.9	14.7	1119.1	29.43	942.9	84.3	3.49
2033	110–48–F	80	20.0	548.4	40.7	15.2	1142.3	27.83	1007.9	88.2	4.12
2009	138–16–F	150	25.2	694.6	17.1	5.3	545.5	33.06			
2012	138–24–F	110	25.3	696.3	23.3	7.7	802.8	29.44	684.9	85.3	3.84
2013	138–32–F	90	25.4	697.2	30.8	11.6	986.3	29.14	863.7	87.6	3.78
2023	138–40–F	90	25.2	693.6	39.4	19.5	1188.7	29.10	1070.5	90.1	3.36
2030	165–24–F	100	30.8	845.3	23.4	11.5	899.6	28.99	725.8	80.7	3.62
2032	165–32–F	40	30.5	838.8	30.2		1094.6	27.84	922.8	84.3	3.61
2037	165–40–F	60	30.7	844.8	40.1	18.1	1277.7	29.56	1045.3	81.8	3.29
2016	193–16–F	140	36.3	998.0	16.4	6.4	583.8	35.18	418.2	71.6	2.74
2010	193–20–F	71	35.7	981.8	19.2	8.2	737.0	32.07			
2015	193–24–F	90	36.3	998.0	22.7	12.1	845.6	32.32	626.2	74.1	2.63
2014	193–32–F	70	36.3	996.4	31.4	17.1	1079.0	29.82	853.1	79.1	2.88
2034	193–40–F	60	36.0	990.5	41.4	24.1	1276.7	30.02	961.7	75.3	2.67

miles per hour. The best performance was obtained during the tests made at 32 per cent cut-off. The minimum steam consumption was 27.51 pounds of dry steam per indicated horse power per hour, and the maximum 35.18 pounds. The difference between these quantities, 7.67 pounds, represents a comparatively small variation in water rate, considering the variety and range of the test conditions. Fig. 35 is comparable with Fig. 17, and a comparison of these figures shows the steam consumption during Series 1 to have been considerably greater than in Series 2, at all cut-offs.

30. *Indicated Horse Power.*—The relation of indicated horse power to speed is shown in Fig. 36, in which each of the curves represents all the tests made at a particular cut-off. This figure shows also the range of the test conditions for Series 1. The maximum load, 1278 indicated horse power, was developed during test 2037 when the speed was 30 miles per hour and the cut-off 40 per cent. The minimum average load, 428 indicated horse power, occurred in test 2017 at a speed of 15 miles per hour and 16 per cent cut-off. The work performed at the locomotive drawbar varied from 346 to 1071 horse power.

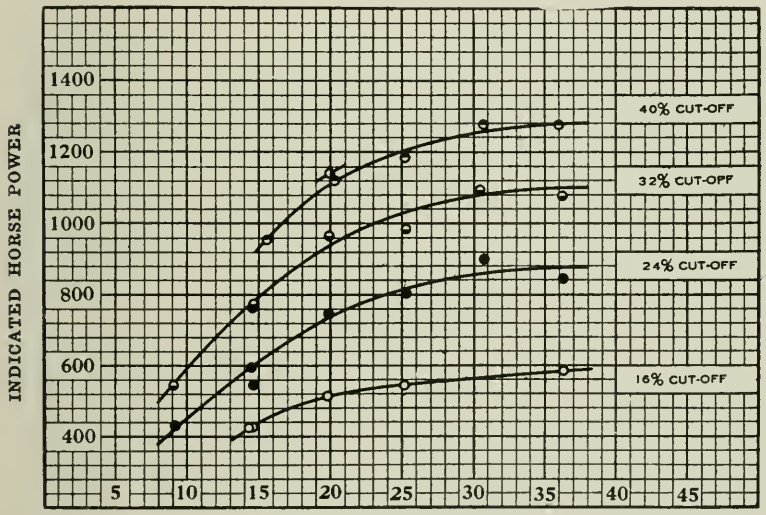


FIG. 36. THE RELATION BETWEEN INDICATED HORSE POWER AND SPEED, AT VARIOUS CUT-OFFS.

31. *Machine Friction.*—Machine friction horse power during Series 1 and its relation to speed are shown in Fig. 37. This figure indicates also the influence of cut-off on machine friction, and is comparable with Fig. 27 which presents the same relations for Series 2.

An analysis of machine friction similar to that developed in section VI for Series 2 has been made for Series 1, although the figures are not here included. This analysis shows that during Series 1 the machine friction was somewhat greater than during Series 2—a result which was expected, in view of the work done upon the machinery between these two groups of tests. This analysis indicates also that for constant cut-off the machine friction increased more rapidly with increasing load during Series 1 than during Series 2. With these two exceptions, the relations shown in the discussion of machine friction for Series 2, and the conclusions there drawn, remain substantially the same for Series 1.

32. *Coal Consumption per Indicated Horse Power.*—The dry coal consumed per indicated horse power per hour, and the relation of this coal consumption to speed are shown in the curves of Fig. 38. The curves show this coal rate to vary in general between 4 and 6 pounds per hour for the range of conditions which prevailed in Series 1. Again excluding test 2024, the lowest coal rate, 4.18 pounds per hour, was obtained during test 2019 at 15 miles per hour and 32 per cent cut-off. The highest coal rate, 6.07 pounds per hour, was obtained during test 2034, at 35 miles per hour and 40 per cent cut-off. Test 2034 is the one during which the lowest boiler efficiency prevailed.

33. *General Efficiency.*—The general efficiency of the locomotive and the relation of this efficiency to speed are shown in Fig. 39, which is comparable with Fig. 31 of Series 2. The greatest efficiency was 4.38 per cent, and the lowest 2.63 per cent. This is practically the same range in efficiency as is represented in Fig. 31.

VIII. Comparison of the Results of Series 1 and 2.

A few of the differences in the performance of the locomotive during the tests of Series 1 and Series 2 have already been referred to incidentally in the preceding discussion. It is the purpose to discuss in this section the effects on the general performance of the boiler and engines caused by the repairs and changes which were made between these two groups of tests. While these changes are elsewhere enumer-

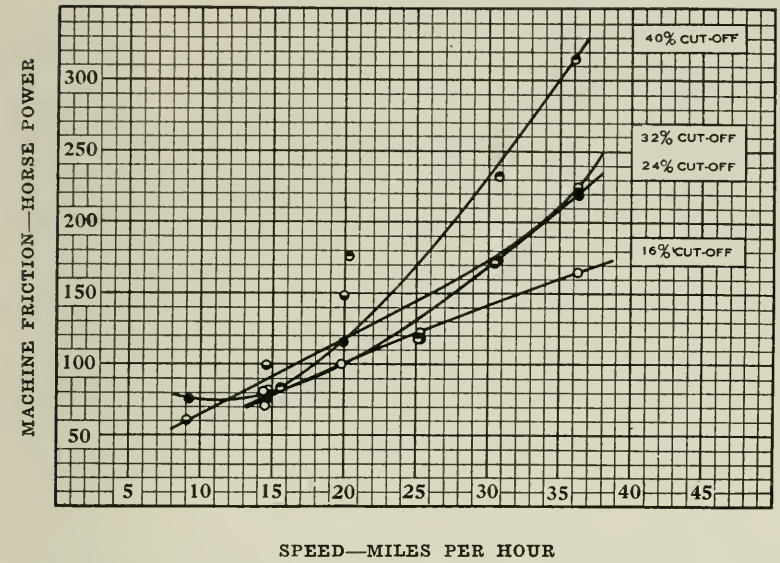


FIG. 37. THE RELATION BETWEEN MACHINE FRICTION POWER AND SPEED, AT VARIOUS CUT-OFFS.

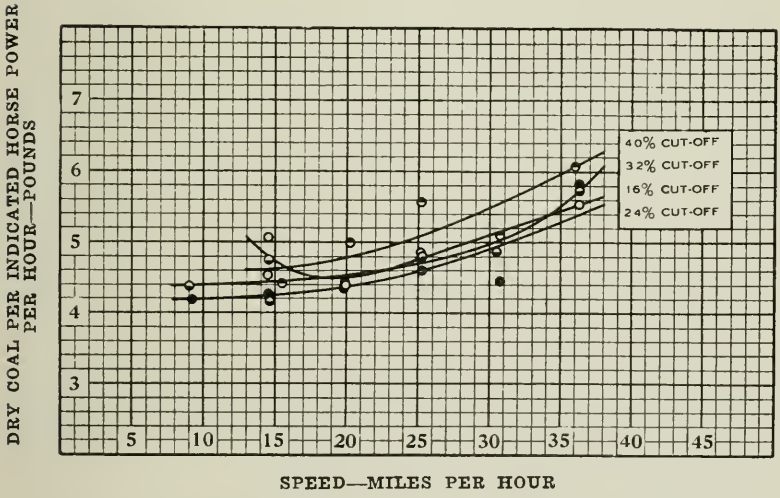


FIG. 38. THE RELATION BETWEEN COAL CONSUMED PER INDICATED HORSE POWER PER HOUR AND SPEED, AT VARIOUS CUT-OFFS.

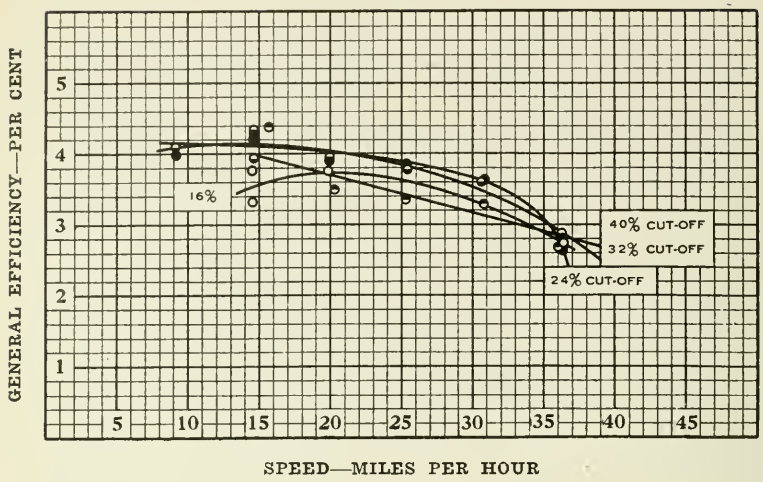


Fig. 39. The Relation Between General Efficiency and Speed, at Various Cut-offs.

ated, it will be convenient to have restated at this point those which might have affected the general performance. They were:

1. The nozzle tip, 5¼ in. in diameter, used during Series 1 was replaced by a 5⅞-in. tip.
2. A small leak in one of the branch-pipe joints was stopped.
3. The cylinders and valve chambers were re-bored.
4. New pistons and new piston rings were applied.
5. New valve bull-rings and valve packing rings were applied.
6. Lost motion in the eccentric straps was taken up.
7. The valves were reset.
8. The piston rods were trued and the rod packing renewed on both sides.
9. The side-rod bushings were renewed.
10. Three new driving-wheel tires were applied.

34. *Comparison of the Boiler Performance.*—Perhaps the best basis for a comparison of the boiler performance during the two series is to be found in the curves of Fig. 8 and 32, which present the average values of the equivalent evaporation per pound of dry coal per hour for Series 2 and Series 1 respectively. These two curves, when plotted on the same diagram, almost coincide. The curve for Series 1 lies below that for Series 2 by an amount which at no rate of combus-

tion exceeds one-fifth of a pound of equivalent evaporation per pound of coal. At no point throughout the range of the rate of combustion is the performance of the boiler in Series 2 better than in Series 1 by more than 3 per cent. A comparison of boiler efficiency based on Fig. 10 and 34 shows an even closer agreement in the boiler performance of the two series. Of the various changes cited above only item 1, the change in nozzle tip, could have affected boiler performance, and the facts just stated seem to warrant the conclusion that this had no substantial effect upon the general performance of the boiler and furnace.

35. *Comparison of the Cylinder Performance.*—Any of the first seven items in the list of changes given might conceivably have affected the cylinder performance. The steam leak referred to in item 2, however, was proved at the time to have been inconsiderable in amount. The combined effect of these seven items should be disclosed by a comparison of the steam consumption per indicated horse power hour for all tests of both series which are comparable as regards speed and cut-off. The values of steam consumption for such tests are brought together in the following table. In four instances in this table a pair of tests from Series 2 is compared with a single test from Series 1, and in these cases the water rate presented for Series 2 is the average rate for the pair.

TABLE 11.

STEAM CONSUMPTION FOR COMPARABLE TESTS OF SERIES 1 AND 2.

Speed, m. p. h.	Cut-off, per cent	Test Numbers		Steam Consumption, lb. per i. h. p. hour		Difference in Steam Consumption, Percentage of Consumption for Series 2.
		Series 1	Series 2	Series 1	Series 2	
10	24	2024	2081 2086 2075	34.74	31.35	10.8
"	32	2028	2097 2080	30.57	28.40	7.6
20	16	2026	2087	31.67	30.76	2.9
"	24	2027	2077	27.84	27.36	1.8
"	32	2029	2073	27.51	27.20	1.1
"	40	2035	2072	29.43	27.19	8.2
"	48	2033	2084	27.83	28.69	−3.0
30	24	2030	2078 2074	28.99	28.09	3.2
"	32	2032	2092	27.84	27.25	2.2
"	40	2037	2082	29.56	27.88	6.0
					Average	4.1

Except for the two tests run at 20 miles per hour and 48 per cent cut-off, all the tests of Series 2 show a lower steam consumption than the corresponding tests of Series 1. The differences in steam consumption are shown in the last column of the table, where they are expressed in percentages of the water rate for Series 2. With the one exception cited, the improvement in water rate ranged from 1.1 to 10.8 per cent and the average improvement for all tests amounted to 4.1 per cent. Neither this average nor the range is very great, and these facts emphasize the statement previously made that all the repairs and changes were such as would have been regarded as unnecessary under ordinary conditions, and that they were resorted to only that nothing which would probably improve performance should be left undone.

In the data for tests 2090 and 2091 means are at hand for roughly differentiating the effect of the changes in nozzle tip from the effect of the other changes. It will be recalled that during these two tests the conditions were identical with those prevailing during the tests of Series 2, except that the 5⅞-in. nozzle tip used in Series 2 was replaced by the 5¼-in. tip which had been used in Series 1. A comparison of the water rates for these two tests with the water rates for the comparable tests of Series 2 is exhibited below.

TABLE 12.

COMPARISON OF WATER RATE WITH 5⅞-IN. NOZZLE TIP AND 5¼-IN. NOZZLE TIP.

Speed, m. p. h.	Cut-off, per cent	Test Numbers		Steam Consumption, lb. per i. h. p. hour		Difference in Steam Consumption, Percentage of Consumption for Series 2
		Tests with 5¼-in. Tip	Tests with 5⅞-in. Tip, Series 2	Tests with 5¼-in. Tip	Tests with 5⅞-in. Tip, Series 2	
20	24	2090	2077 2074	28.99	27.36	6.0
30	32	2091	2092	29.10	27.25	6.8

It is apparent that in both tests 2090 and 2091, with the smaller nozzle tip, the water rate was higher than in the corresponding tests of Series 2 in which the larger tip was used. The average difference based on the tests of Series 2 is 6.4 per cent. In view of the range in the differences in water rate for all tests of the two series, which is exhibited in the first table in this section, it is doubtless unsafe to draw too sweeping a conclusion from a showing which rests on a comparison of two pairs of tests only. Since, however, the average difference in

steam consumption for all tests of both series was only 4.1 per cent, and since such information as is available concerning the effect of the change in nozzle tip shows that it made an average change in steam consumption of 6.4 per cent; the inference is perhaps warranted, that practically all the improvement effected by the changes and repairs was accomplished by the increase in the size of the exhaust nozzle tip, through its influence on back pressure.

PART II.

APPENDIX I.

The Locomotive.

Illinois Central Railroad locomotive 958 is of the consolidation type. It was built by the Baldwin Locomotive Works in December, 1909, and in the classification of the Associated Lines is designated as C — 63 — $\frac{22}{30}$ — 39.2. The locomotive uses saturated steam in simple cylinders twenty-two inches in diameter by thirty inches stroke, weighs 223 000 pounds, and has a rated tractive effort of 39 180 pounds. This tractive effort assumes a driving wheel diameter of 63 inches. The drivers however had been turned to 61 inches for which the rated tractive effort would be 40 470 pounds. The general design of the locomotive is shown in Fig. 40 and 42, and its appearance in service is shown in Fig. 1. The period of its service and its mileage have been stated in section II.

Repairs and Changes.—The repairs and changes which have been referred to in Part I were as follows.

After test No. 2037. The valves were reset; the valve and cylinder packing-rings were renewed; lost motion in the eccentric straps was taken up; the piston-rod packing was renewed; cylinder cocks were repaired or renewed; side-rod bushings were renewed; a new injector was applied; the boiler seams were caulked; three new tires were applied; and certain minor adjustments were made in order to take up lost motion.

After test No. 2045. The cylinders and the valve chambers were rebored; the pistons and piston packing-rings were renewed; the valve bull-rings and packing-rings were renewed; the piston rods were trued; a small leak in one of the branch pipe joints was stopped; and the 5¼-in. nozzle tip previously in use was replaced by a 5⅞-in. tip. The 5¼-in. tip was again used during tests 2090 and 2091.

The Boiler.—The boiler, which carried a working pressure of 200 pounds, was of the crown-bar type, with straight top and wide firebox. Its general design appears in Fig. 41, 43, and 44. The principal dimensions of the boiler are given in the following list.

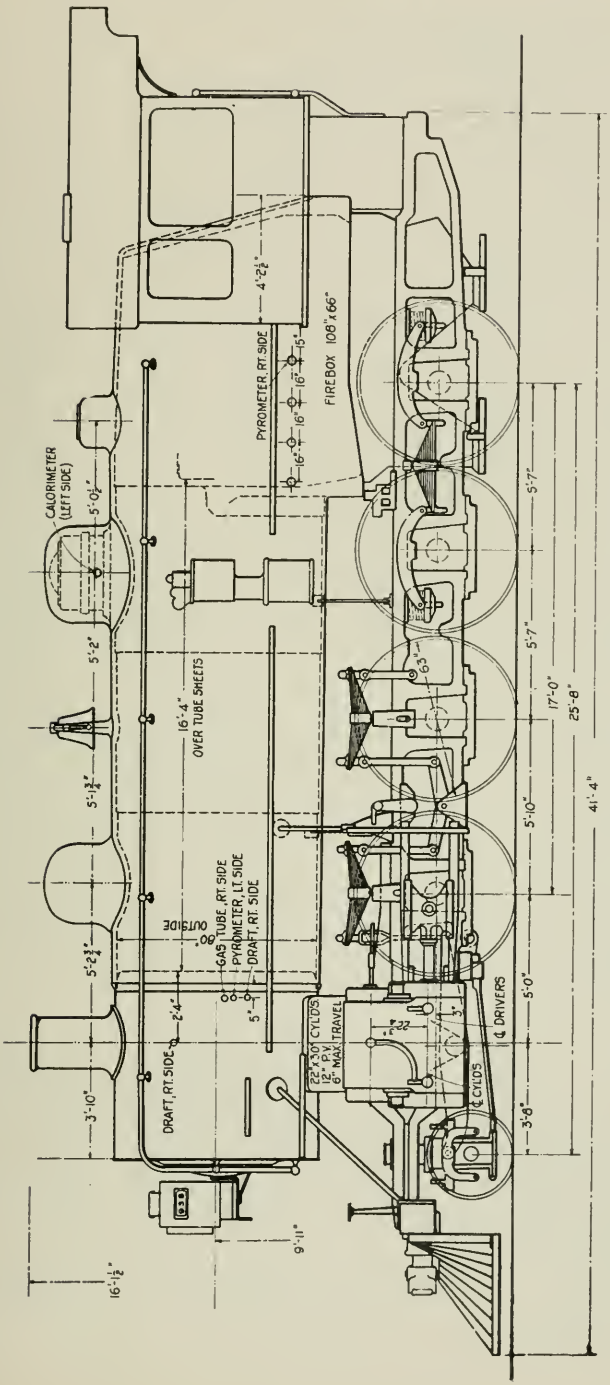


FIG. 40. SIDE ELEVATION OF ILLINOIS CENTRAL RAILROAD LOCOMOTIVE 958.

Outside diameter of first ring *(205)**	80	inches
Thickness of sheets:		
Cylindrical courses	¾	"
Wrapper sheet	⅝	"
Flue sheets	½	"
Firebox sheets	⅜	"
Number of tubes *(211)*	413	
Outside diameter of tubes *(212)*	2	inches
Thickness of tubes *(213)*	0.133	"
Length between tube sheets *(214)*	195.2	"
Length of firebox, inside *(234)*	108.25	"
Width of firebox, inside *(235)*	66.13	"
Volume of firebox *(238)*	244	cu. ft.
Heating surface of the tubes, fire side *(272)*	3094	sq. ft.
Heating surface of front tube sheet *(277)*	21	" "
Heating surface of the firebox, fire side *(273)*	168	" "
Total heating surface, fire side *(275)*	3283	" "
Water space in the boiler *(282)*	429	cu. ft.
Steam space in the boiler *(283)*	107	" "
Grates, of the interlocking finger type, area *(252)*	49.55	sq. ft.

The Cylinders and Valves.—The cylinders and valve chambers were of cast-iron with cast iron bushings. The pistons were of the box type, cast in one piece. The steam distribution was controlled by piston valves with inside admission. The nominal piston travel was 6 inches, and the actual travel 5-55/64 inches on the right side, and 5-27/32 on the left side. The valves were set with 1/32 inch lead in full gear and were actuated by an indirect shifting link motion.

The principal cylinder and valve dimensions during the various tests included in this report were as follows:

	During Tests 2009-2045	During Tests 2072-2098
Cylinder Diameter		
Right side *(68)*	22.071 inches	22.107 inches
Left side *(69)*	22.282 "	22.314 "
Valve Chamber Diameter		
Right side	12.031 "	12.102 "
Left side	12.020 "	12.078 "

*Code item number.

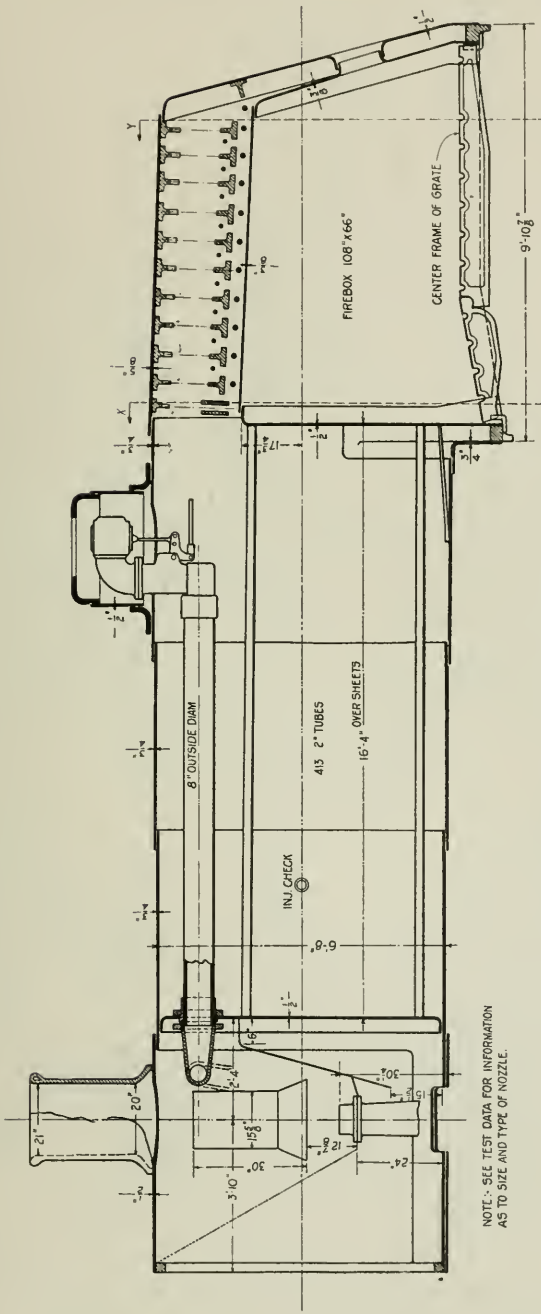


FIG. 41. LONGITUDINAL SECTION THROUGH THE BOILER.

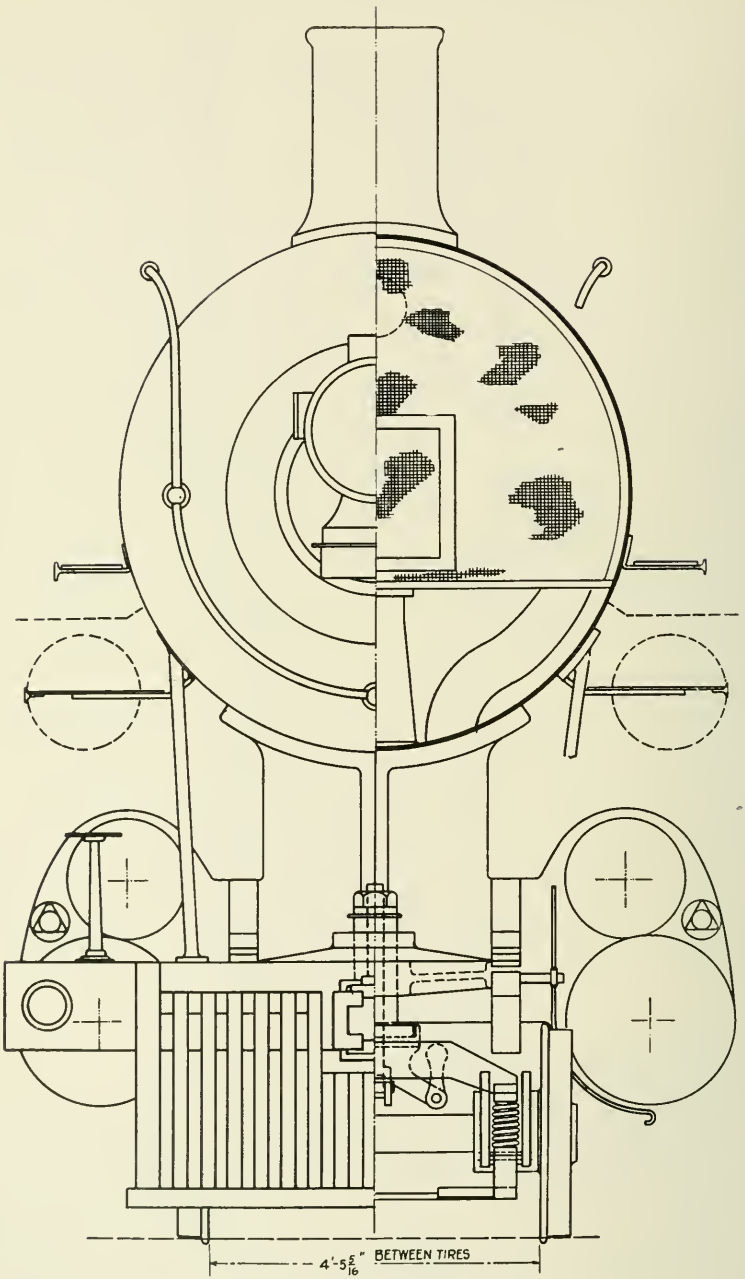


FIG. 42. FRONT ELEVATION OF THE LOCOMOTIVE.

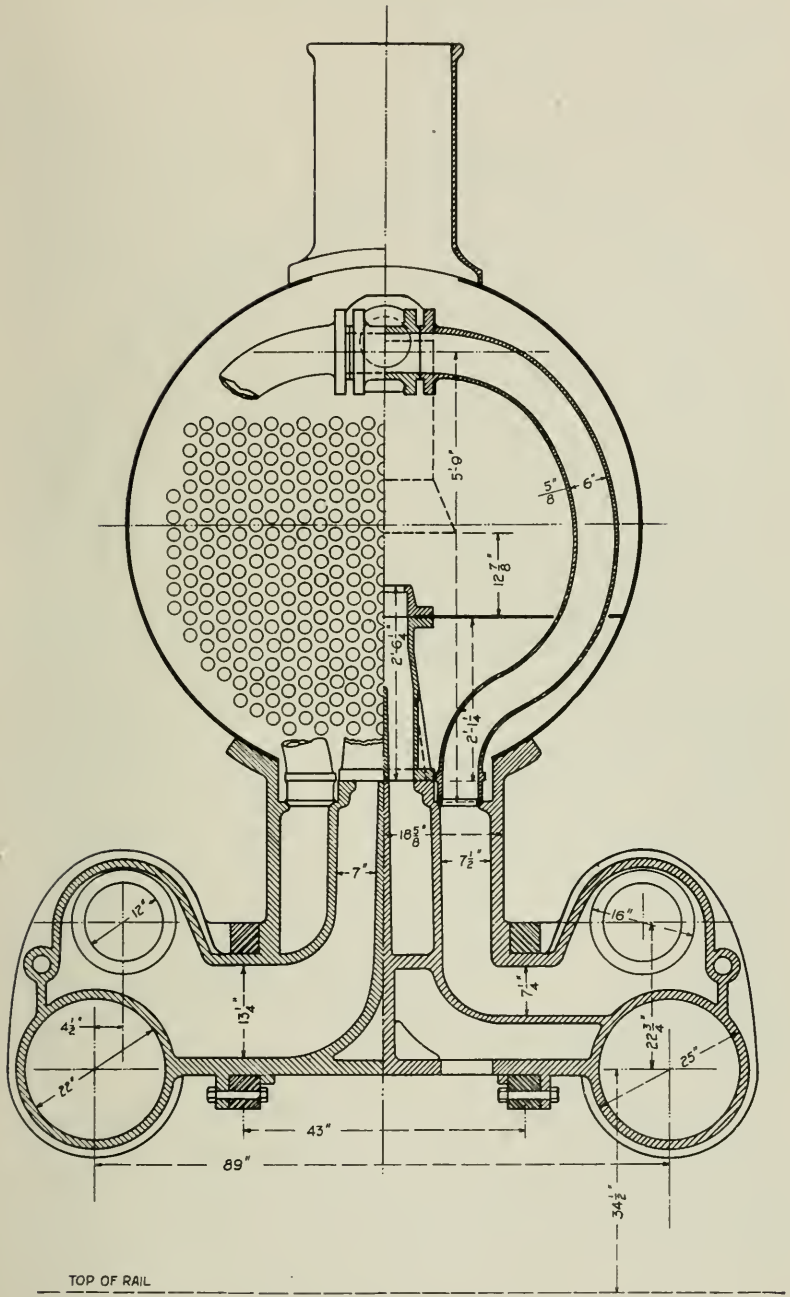


FIG. 43. CROSS SECTION THROUGH THE FRONT-END AND CYLINDERS.

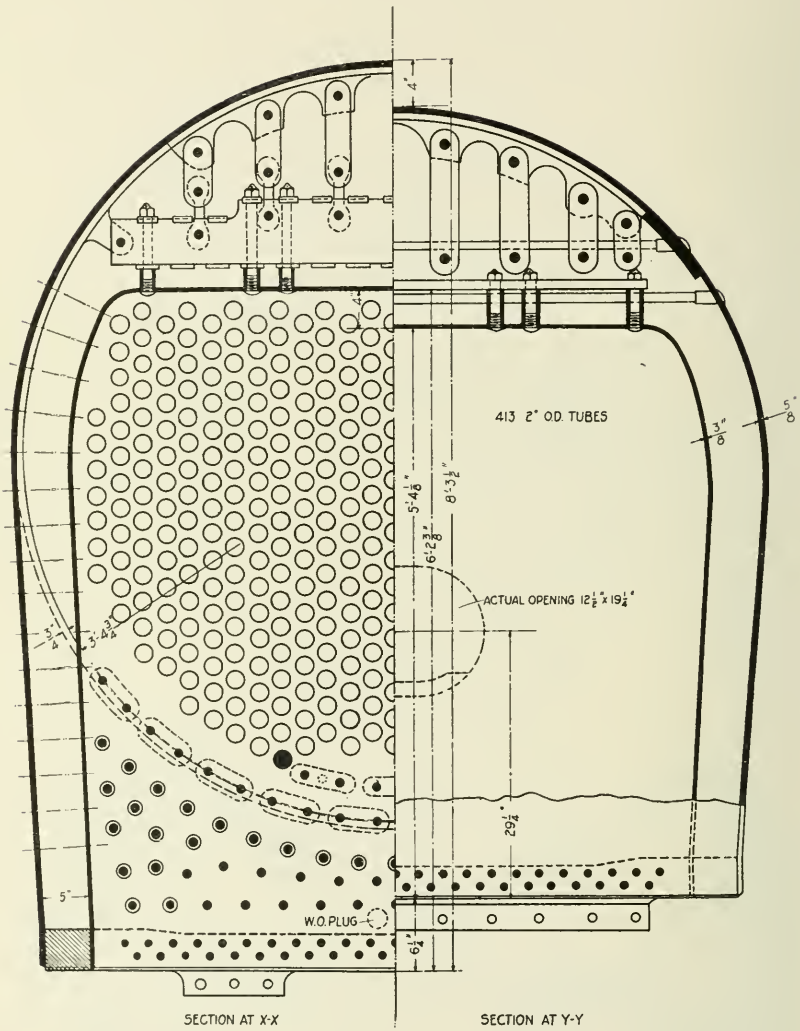


FIG. 44. CROSS SECTIONS THROUGH THE FIREBOX

	During Tests 2009-2045	During Tests 2072-2098
Piston Stroke		
Right side *(77)**	29.92 inches	29.94 inches
Left side *(78)*	29.94 "	29.94 "
Piston Rod Diameter		
Right side *(135)*	3.917 "	3.907 "
Left side *(136)*	3.928 "	3.907 "
Cylinder Volume (both sides)	13.199 cu. ft.	13.199 cu. ft.

	During Tests 2009-2037	During Tests 2038-2098
Clearance Volume		
Ride side, head end *(86)*	9.89 per cent	11.42 per cent
Right side, crank end *(87)*	9.74 "	11.01 "
Left side, head end *(88)*	9.73 "	10.60 "
Left side, crank end *(89)*	10.06 "	11.01 "

General Dimensions.—The principal general dimensions not already cited are shown in the following list.

Total weight in working order *(63)*	223 000 lb.
Weight on drivers *(64)*	200 900 "
Weight on leading truck *(48)*	22 100 "
Weight of tender, loaded	135 000 "
Weight of locomotive and tender, in working order	358 000 "
Driving wheel base *(39)*	17 ft. – 0 in.
Total wheel base *(41)*	25 ft. – 8 in.
Driving wheel diameter, over tires (nominal) *(2)*	63 in.
Truck wheel diameter *(27)*	33½ in.
Driving journal, main	10 in. x 12 in.
Driving journals, other	9 in. x 12 in.
Truck journals	6 in. x 10 in.

The actual average driving wheel diameter was 61.01 inches during tests 2009–2037, and 61.03 inches during tests 2038–2098. The corresponding actual average circumferences *(code No. 19)* were 15.972 and 15.978 feet respectively. The principal ratios are given below. Where two values of the ratio appear, the first is based on nominal dimensions, the second on actual dimensions.

*Code item number.

$$\frac{\text{Weight on drivers}}{\text{Tractive effort}} = \frac{200\,900}{39\,180} = 5.12$$

$$\frac{\text{Weight on drivers}}{\text{Tractive effort}} = \frac{200\,900}{40\,470} = 4.96$$

$$\frac{\text{Total weight}}{\text{Tractive effort}} = \frac{223\,000}{39\,180} = 5.69$$

$$\frac{\text{Total weight}}{\text{Tractive effort}} = \frac{223\,000}{40\,470} = 5.51$$

$$\frac{\text{Tractive effort} \times \text{diameter of drivers}}{\text{Total heating surface}} = \frac{39\,180 \times 63}{3283} = 751.8$$

$$\frac{\text{Tractive effort} \times \text{diameter of drivers}}{\text{Total heating surface}} = \frac{40\,470 \times 61}{3283} = 751.8$$

$$\frac{\text{Firebox heating surface}}{\text{Total heating surface}} = \frac{168}{3283} = .0513$$

$$\frac{\text{Weight on drivers}}{\text{Total heating surface}} = \frac{200\,900}{3283} = 61.19$$

$$\frac{\text{Total weight}}{\text{Total heating surface}} = \frac{223\,000}{3283} = 67.92$$

$$\frac{\text{Heating surface}}{\text{Grate area}} = \frac{3283}{49.55} = 66.26$$

$$\frac{\text{Tube surface}}{\text{Firebox heating surface}} = \frac{3094}{168} = 18.41$$

$$\frac{\text{Total heating surface}}{\text{Cylinder volume}} = \frac{3283}{13.199} = 248.8$$

Horse Power Constants.—The constants used in computing the test results are as follows:

For dynamometer horse power (power developed when the speed is one revolution per minute and the pull is one pound) the constants are

For tests 2009 to 2037 *(318)**	.0004840
For tests 2038 to 2098 *(318)*	.0004842

*Code item number.

For indicated horse power (power developed at one revolution per minute and one pound mean effective pressure) the constants are

	Tests 2009-2045	2072-2098
For right cylinder, head end *(319)**	.02893	.02902
For right cylinder, crank end *(320)*	.02802	.02811
For left cylinder, head end *(321)*	.02948	.02957
For left cylinder, crank end *(322)*	.02857	.02866

*Code item number.

APPENDIX 2.

The Laboratory.

The general purpose underlying the design of this and of all other locomotive laboratories is to provide means whereby the locomotive machinery may be run and the locomotive worked throughout its range of capacity, while the locomotive as a whole remains stationary; thus permitting all test measurements to be made with the degree of accuracy possible in a stationary power plant test.

The laboratory equipment consists of, first, a means for so supporting the locomotive that its driving wheels may be rotated and that the power developed may be absorbed and dissipated; second, a means for anchoring the locomotive when so mounted and for measuring the tractive effort developed; third, means for supplying and measuring coal and water; and finally, means for disposing of the waste gases and exhaust steam.

The Building.—The building in which the plant is housed is shown in Fig. 45. It is 40 feet wide and 115 feet long, with a height of 22 feet under the roof trusses. At the rear end of the building is a coal room, above which are a platform for the exhaust fan and a wash-room. A basement extends under all of the main floor, except under the space occupied by the coal room. The walls are laid up both inside and out with faced brick, the floors are of reinforced concrete, and the roof is of the same material covered with slate. With the exception of the coal room all portions of the building are served by a ten-ton traveling crane.

Supporting Wheels and Axles.—The supporting mechanism consists primarily of pairs of wheels, whose location may be adjusted to suit the wheel base of any locomotive. Fig. 51 shows the general design of wheels, axles, bearings, and bed-plates. The supporting element for each pair of locomotive drivers consists of an axle, two wheels, and two bearings. The supporting wheels are 52 inches in diameter, have plain 5-inch tires, and are pressed on 11½-inch axles.

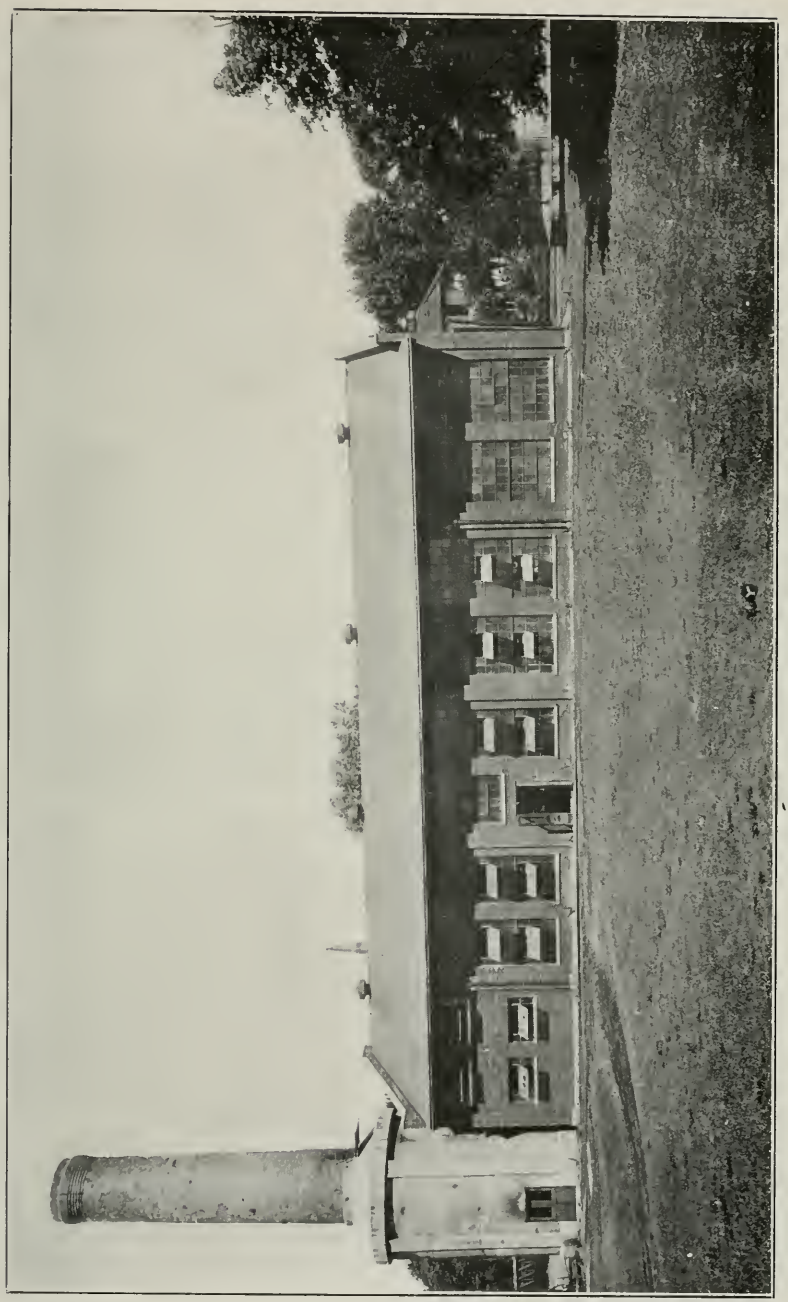


FIG. 45. THE LOCOMOTIVE LABORATORY.

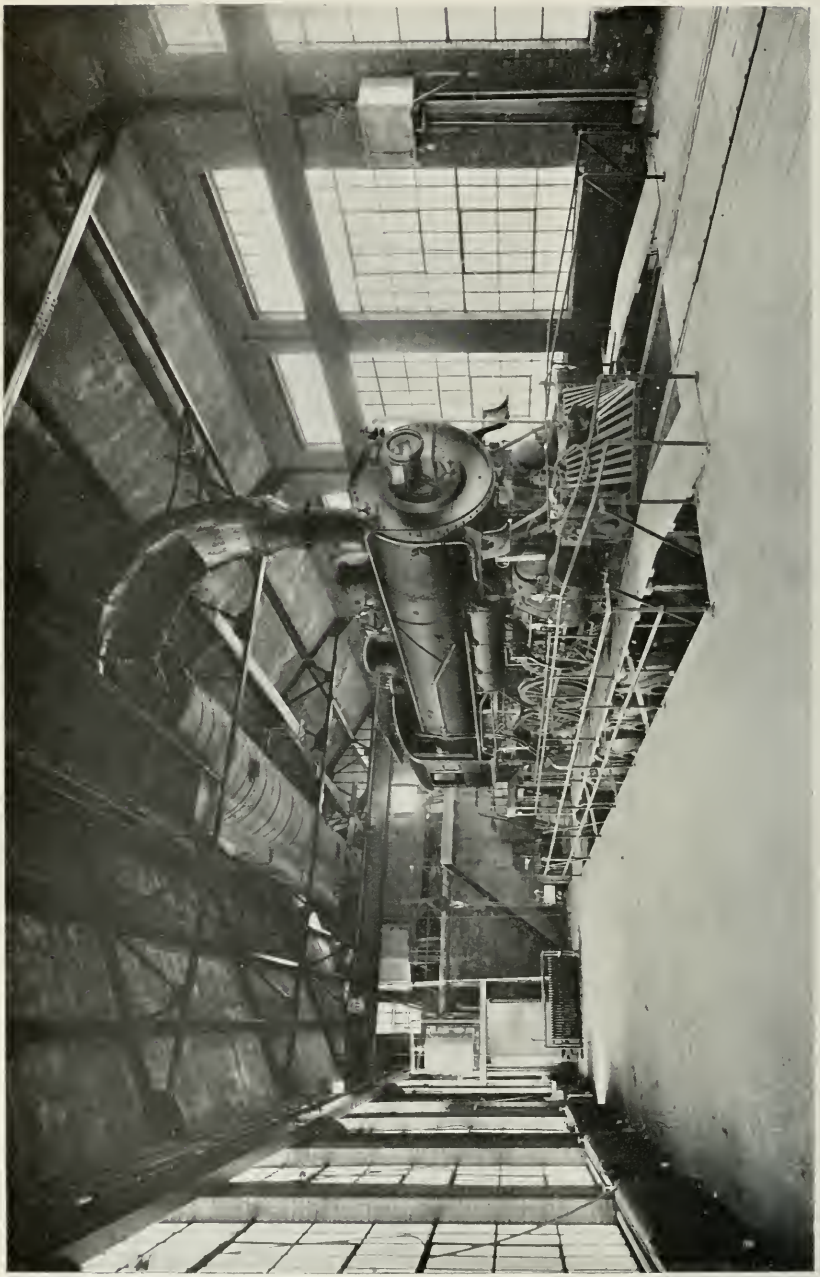


FIG. 46. AN INTERIOR VIEW OF THE LABORATORY WITH LOCOMOTIVE 958 IN POSITION.

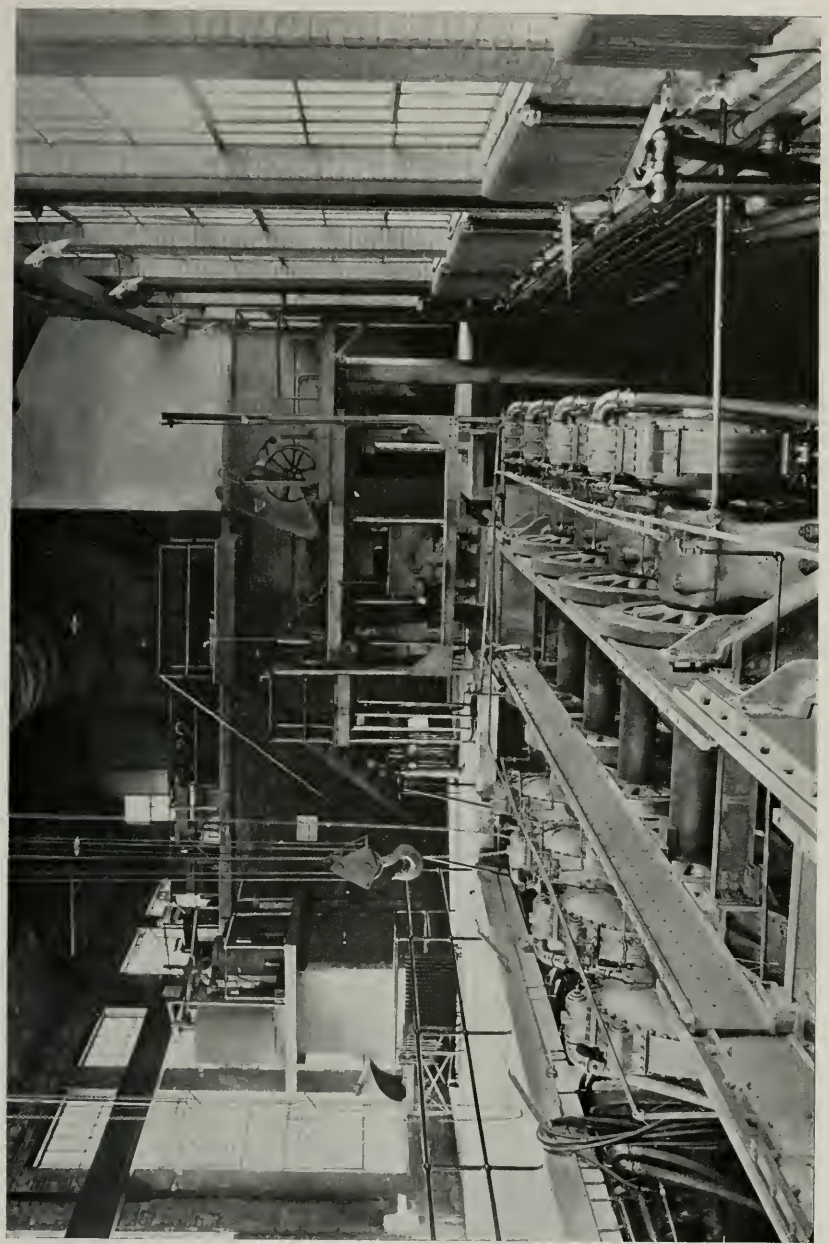


FIG. 47. THE REAR END OF THE TESTING PIT, SHOWING THE REMOVABLE TRACK, THE SUPPORTING WHEELS AND THEIR BEARINGS, AND THE BRAKES.

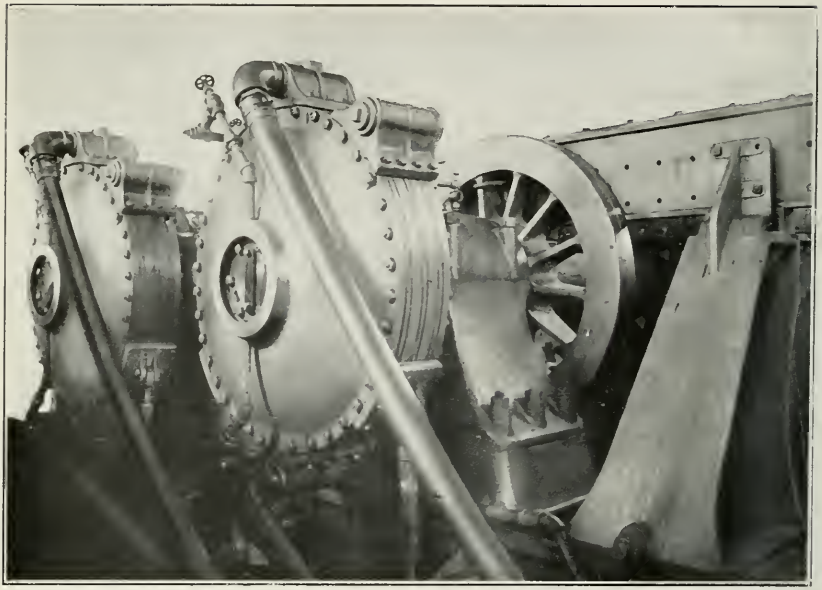


FIG. 48. AN EXTERIOR VIEW OF THE BRAKES.

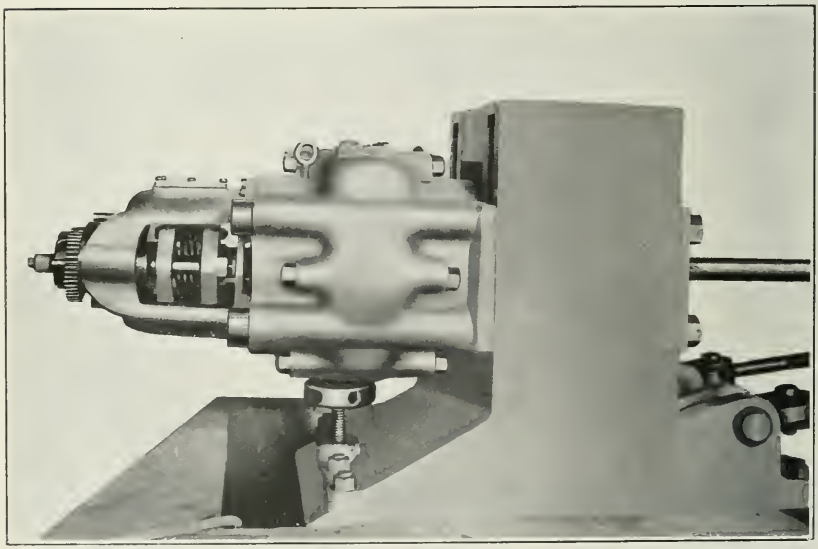


FIG. 49. THE WEIGHING HEAD AND PEDESTAL OF THE DYNAMOMETER.

FIG. 50. THE BRAKE CONTROL VALVES, THE DYNAMOMETER SCALE, THE DRAWBAR AND SAFETY LINKS, AND THE FIRING PLATFORM.

The axles and tires are of the highest grade of heat-treated carbon steel and were donated by the Midvale Steel Company. Provision has been made for replacing the 52-inch wheels by 72-inch wheels for testing high speed locomotives, where the use of the smaller wheels would involve rotating speeds as high as 530 revolutions per minute.

Bearings.—The bearings for the supporting-wheel axles are self-aligning, their shells being carried in spherical sockets which form the upper part of the pedestal. The journals are 9½ inches by 20 inches, and the axles bear on the underside only. Oil for lubrication enters the bearing cap at two points and is supplied under head from an elevated tank. The pedestal is made in two parts, so that by removing the lower section, its height may be adjusted to provide for the 72-inch supporting wheels. This arrangement will continue to bring the top edge of the larger supporting wheels level with the outside track. The base of the pedestal is secured to a massive cast-iron bed-plate by T-bolts held in slots running the entire length of the bed. Each bed-plate consists of three sections placed end to end, 18 inches in height and 36 inches wide over all. The length of the present bed-plate is 42 feet, which provides for a maximum driving-wheel base of 36 feet, and the foundation is built to receive two more 14-foot sections of bed-plate. The supporting machinery rests on a concrete foundation 12 feet wide and 93 feet long, which varies in thickness from 3½ feet at the front to 5 feet at the rear end. The rear end is surmounted by a pyramidal base of reinforced concrete, to which the dynamometer is bolted.

Hydraulic Brakes.—Supported in this way the driving wheels are free to turn and the power produced at the driving wheel rim is absorbed by means of the brakes shown in Fig. 47, 48, and 52. One of these brakes is mounted on each end of each supporting-wheel axle. Each brake consists essentially of three cast iron discs (C, Fig. 52) which, bolted to the cast iron hub (F), are keyed to the supporting axle and form with it an integral revolving element. These three discs rotate between 1/16-inch copper diaphragms (D), bolted to the rim of a stationary housing (H), and flanged over the edges of the floating rings (E) and of the housing, to which they are secured by means of the expanding rings (G). The housing is prevented from turning by means of the links (L) attached to the bed-plate. The rubbing surfaces of the discs and diaphragms are lubricated with a medium grade of cylinder oil which enters the brake under pressure through the oil-

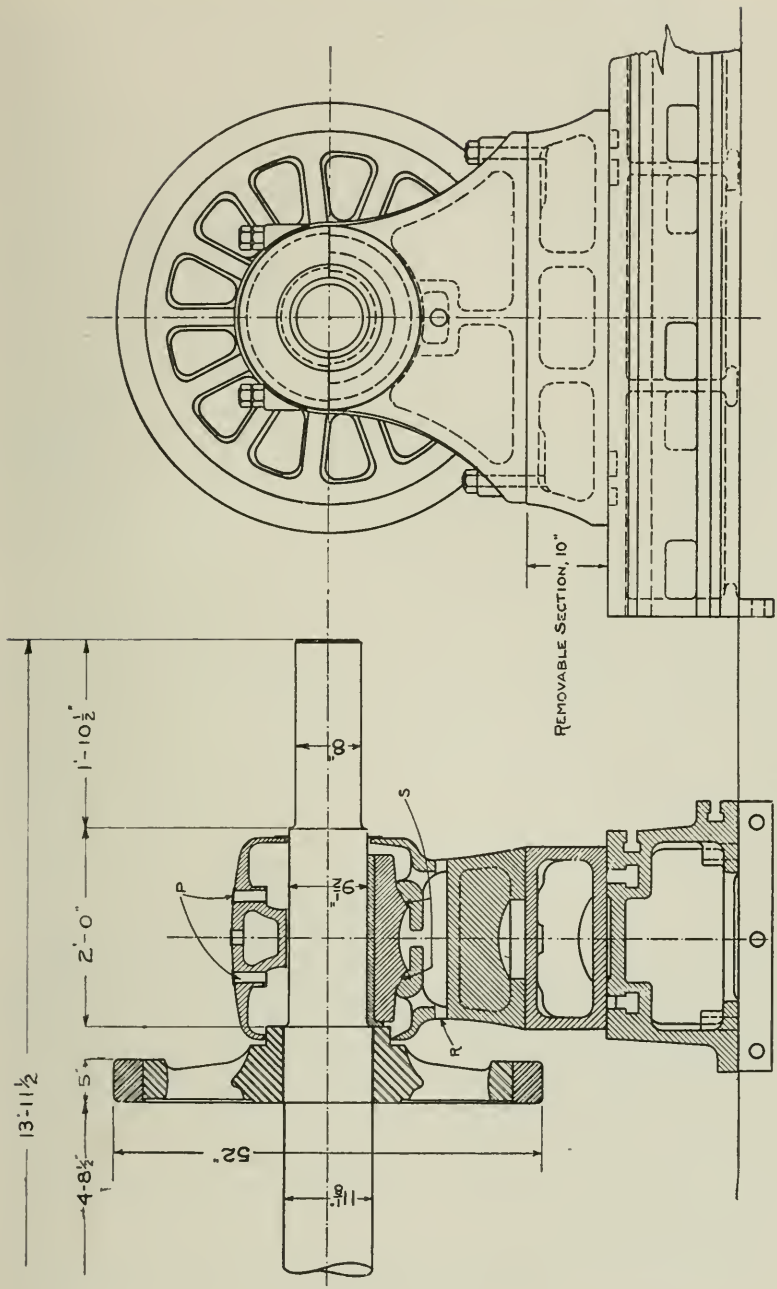


FIG. 51. ONE OF THE SUPPORTING WHEELS, WITH ITS AXLE, BEARING, PEDESTAL, AND BED PLATE.

header (N) at the periphery of the discs, and through the oil-duct (K). The oil is taken off at M as it oozes out around the disc hub. The diaphragms form also within the casing four water compartments which have no communication whatever with the compartments in which the discs rotate. Water at about 60 deg. F. is forced through 3-inch hose connections into the brake at the lower header (B) and leaves through the upper header (A). The amount of water passing through any individual brake and the water pressure within the brake may be regulated at will by means of suitable valves in the piping. The brake load is controlled by thus modifying the water pressure. This is accomplished simultaneously for all of the brakes by means of the large control-valve in the brake supply main, shown in Fig. 50. The auxiliary brake-valves and gages shown at the left in this same figure permit the separate adjustment of load on each brake. Each of these brakes has the capacity of absorbing 450 horse power, having been designed to develop a resisting torque of 18 000 pounds-feet at speeds up to 130 revolutions per minute. This capacity allows for a considerable increase in wheel loads above that which could be imposed by the most heavily loaded driver of the present day.

Placing the Locomotive.—Fig. 47 shows the mounting machinery arranged to receive an eight-driver locomotive. The top of the supporting-wheels is level with the main floor of the building and one-fourth inch higher than the outside track. Before the locomotive to be tested is placed upon the plant, its wheel-spacing is determined and the supporting-wheel centers spaced accordingly. The tender having been removed, the locomotive is backed into the laboratory and onto the temporary track shown in place between the supporting-wheels. The drivers run on their flanges over the temporary track, which leaves their treads free to engage the supporting-wheels, so that when the locomotive is properly placed the supporting-wheels carry all of the weight except, of course, that borne by leading or trailing trucks. The temporary track being relieved, may be removed. Mounted in this way, the locomotive is held in place and prevented from moving forward or backward by means of the dynamometer draw-bar, which is supplemented by two safety-bars that come into play in case of failure of the draw-bar. These three bars are shown in Fig. 50. Forward and trailing-truck wheels are carried on track sections which are level with the supporting wheels.

The Dynamometer.—The dynamometer, the chief function of which is to permit the tractive force of the locomotive to be measured, is

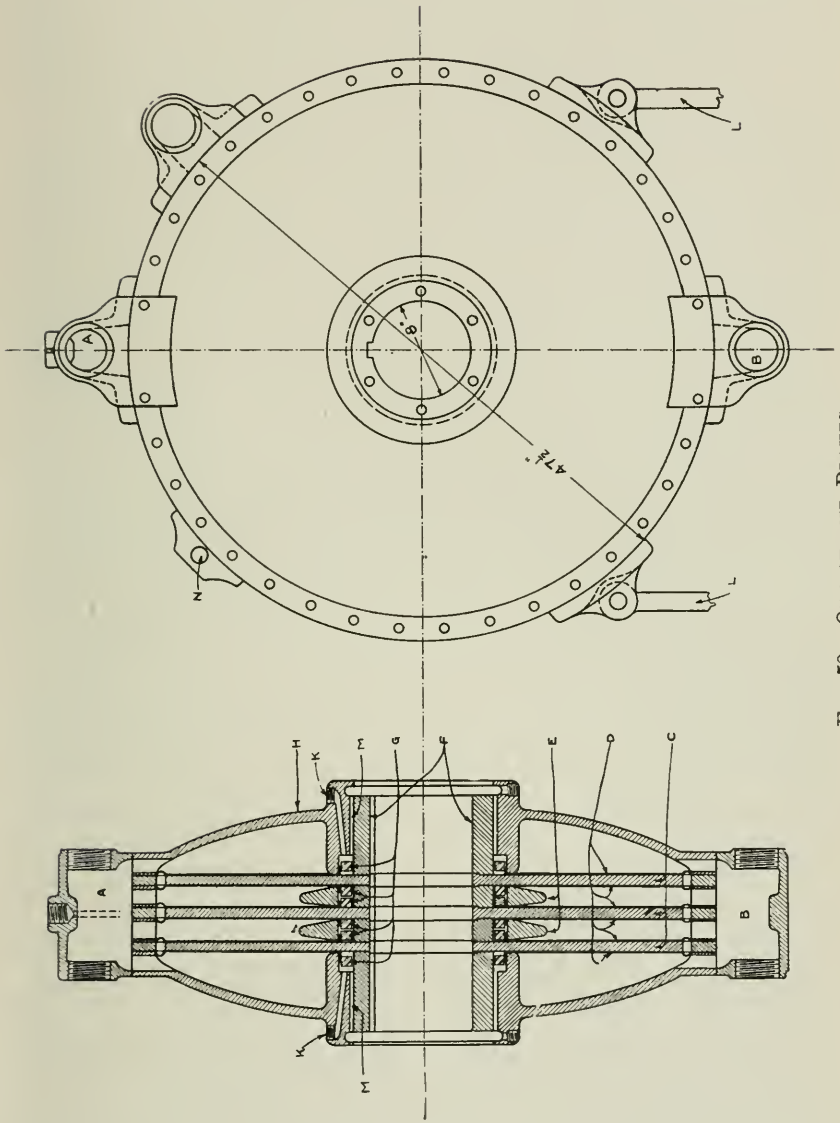


FIG. 52. ONE OF THE BRAKES.

shown in Fig. 47, 49, and 50. It is of the Emery type and consists essentially of two parts: the weighing head, carried on a pedestal and shown in Fig. 49, and the measuring and recording scale shown in Fig. 50. The weighing head may be raised or lowered to suit the height of the drawbar of any locomotive. Within the weighing head is an enclosed oil-chamber with a flexible wall or diaphragm, which receives and balances any force transmitted through the drawbar of the locomotive. The pressure within this oil-chamber varies with the load, and is transmitted through a copper tube of small bore to a smaller oil-chamber known as the reducing chamber, located in the case with the measuring apparatus. The pressure thus produced in the reducing chamber moves the beam of a substantial but sensitive scale which measures the tractive force of the locomotive.

In order to prevent undue shocks from taking place within the weighing head of the dynamometer on account of variations in the force in the drawbar, an initial load of 50 000 pounds is imposed upon the oil behind the diaphragm by means of the capstan and springs located at the rear of the weighing head and shown at the left in Fig. 49. The weighing head of the dynamometer is so designed that by an adjustment of the capstan the tractive effort may be measured whether the locomotive drivers are turning forward or backward. For the sake of accuracy in determining the drawbar pull it is essential that the locomotive drivers be placed and maintained with their centers precisely above the centers of the supporting-wheels. To satisfy this requirement the longitudinal travel of the dynamometer drawbar from no load to full load must be reduced to a minimum. In this instrument the range of movement is only three one-thousandths of an inch. The scale beam reads directly to 20 000 pounds in 100-pound divisions, and a vernier gives readings to ten pounds. For drawbar pulls of more than 20 000 pounds, weights may be added as required. The dynamometer will measure drawbar pulls as high as 125 000 pounds.

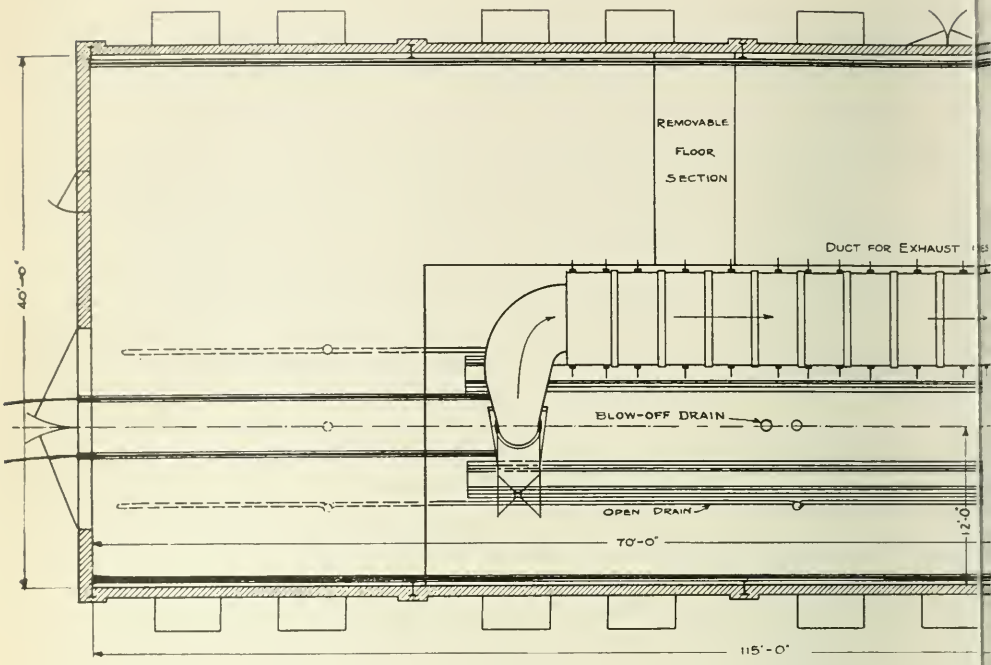
A feature of interest in the design of the scale lies in the fact that the adjustment of the poise weight on the scale beam may be made automatically. This is accomplished by means of a small motor which is mounted on the scale beam and geared to a screw which passes through the poise weight. Attached to the scale beam is a contact arm, and any movement of the beam in either direction causes a series of mercury-cup contacts; the number of contacts depending on the amount of deflection of the beam, which in turn is caused by a change

in the load. When contact is made, an electrical circuit is closed and the motor runs in the direction required to bring the poise weight to a position of equilibrium. As soon as the load is balanced, the circuit is broken and the motor stops. This operation is repeated as often as the load changes, and is practically continuous. The action of the poise weight may also be controlled by a hand switch.

Water and Coal Supply.—The general water supply of the University is from driven wells, the demand upon which at times approaches their full capacity. In order therefore that the water which passes through the brakes shall not be wasted, provision has been made for collecting, cooling, and recirculating it. For this purpose a 100 000-gallon concrete storage reservoir (see Fig. 45) has been built in the ground outside of the building. A supply pump for the brakes draws water from this reservoir through a 6-inch main and pumps it through the main control valve to a header, whence it is distributed through auxiliary supply control valves to the several brakes; after which it flows back through another set of auxiliary back-pressure control valves to a sump located in the basement of the laboratory. (See Fig. 53). The water is drawn from the sump by another pump and forced through five 2-inch whirling-spray nozzles above the surface of the water in the reservoir. Water direct from the University mains may also be used in the brakes when desirable.

Water for the locomotive boiler may be drawn from the reservoir or direct from the University mains, and forced by a separate pump to two elevated tanks which are shown in Fig. 47 and 54. Each of these tanks has a capacity of 2000 pounds and rests permanently on a platform scale. At a supply pressure of 45 pounds, each tank can be filled, weighed, and emptied in two and one-half minutes. From the weighing tanks, the water falls into the 18 000-pound capacity feed tank below, and thence passes through two 4-inch supply pipes to the locomotive injectors. Water for the hydraulic elevator used in raising coal from the main floor to the firing platform may be taken from the University main or from the storage tank. In either case the pressure is maintained at 60 pounds by a throttle-control valve on the supply pump. By these provisions in the piping, reservoir water alone may be used for feed-water, brakes, and elevator.

The coal-room shown in Fig. 53 occupies the rear end of the building. It is 22 feet wide and 40 feet long, and has a storage capacity of 300 tons. Coal for the tests is loaded into 1000-pound capacity wagons, run out onto the scales, raised by the elevator to the firing plat-



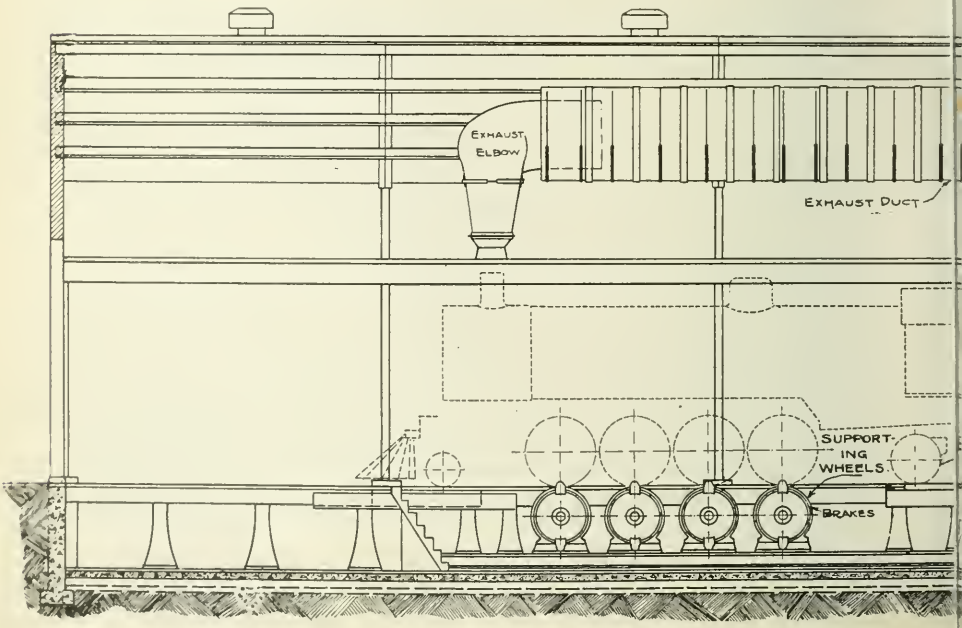
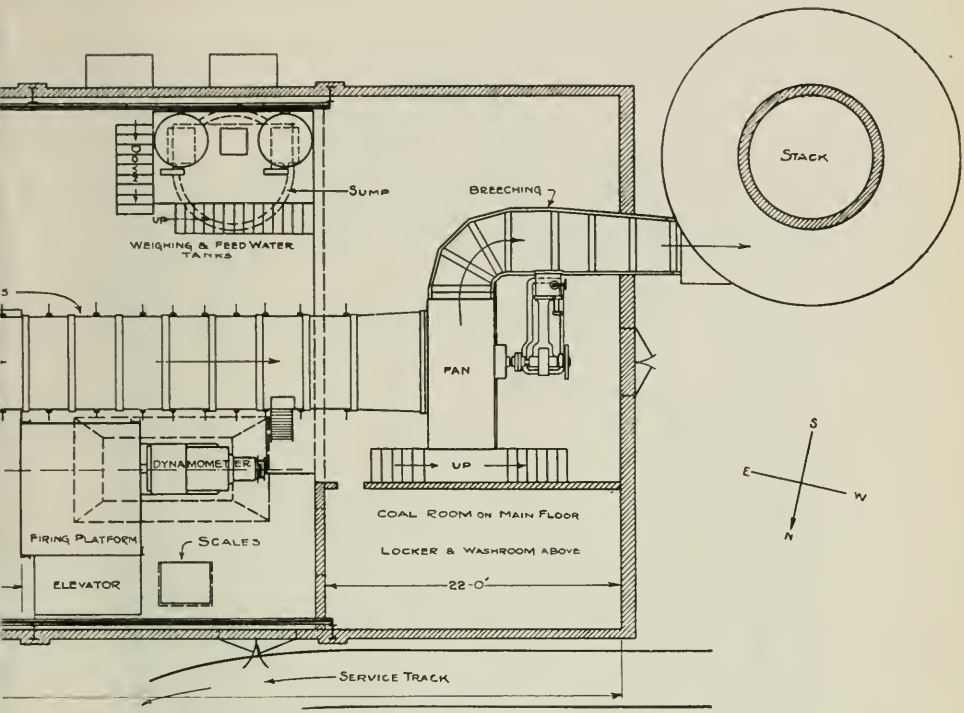
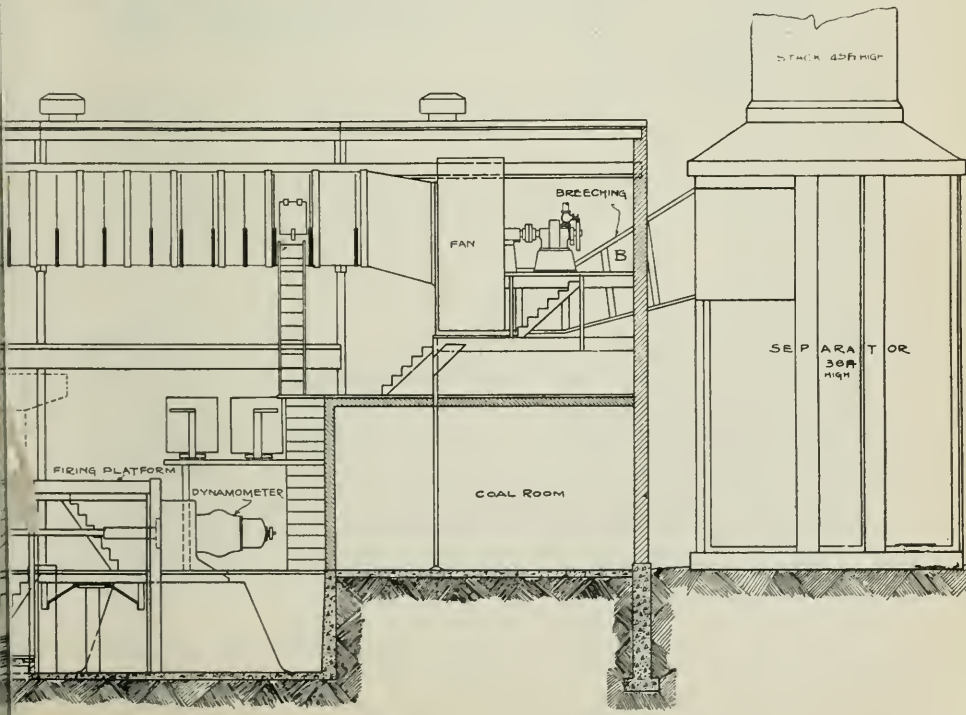


FIG. 53. SECTIONAL PLAN AND


STACK
SUMP
BREECHING
UP
WEIGHING & FEED WATER TANKS
FAN
DYNAMOMETER
UP
COAL ROOM ON MAIN FLOOR
LOCKER & WASHROOM ABOVE
S
E
W
N
FIRING PLATFORM
SCALES
ELEVATOR
22-0"
SERVICE TRACK


STACK 45 FT HIGH
BREECHING
FAN
B
SEPARATOR 30 FT HIGH
FIRING PLATFORM
DYNAMOMETER
COAL ROOM

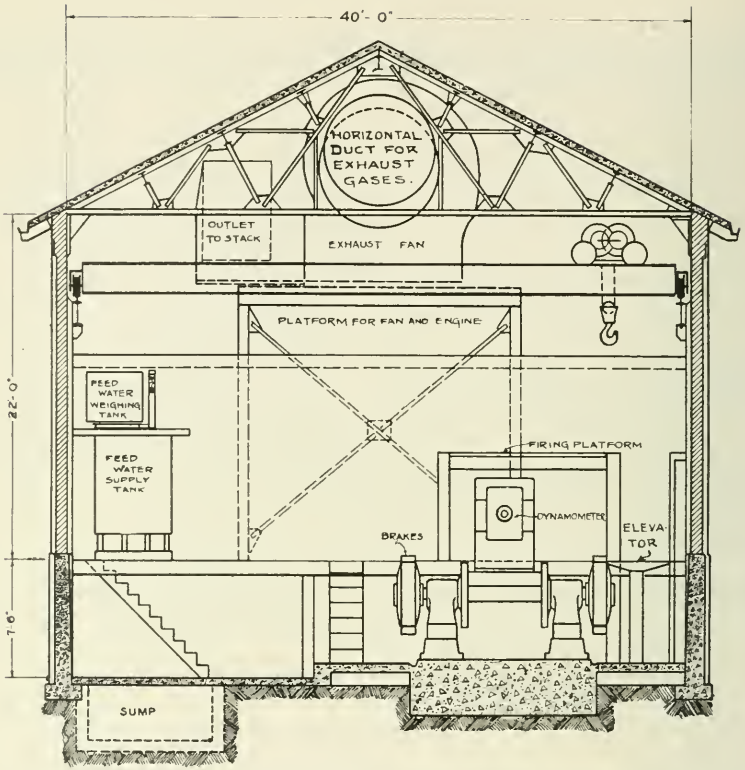


FIG. 54. A CROSS SECTION THROUGH THE MIDDLE OF THE LABORATORY.

form, and there dumped. The firing platform is adjustable in height so as to suit the deck of any locomotive cab. The elevator has a capacity of 2000 pounds. It is also used to raise ashes from the level of the basement.

The Exhaust System.—Recognizing the value of accurate determinations of the total amount of cinders lost through the stack of the locomotive, it was early decided that if possible some means should be incorporated in this plant to collect all of the solid matter which passes through the locomotive front end. Preliminary designs of a cinder catcher which should have sufficient capacity to pass the total volume of waste gas, exhaust steam, and entrained air, and at the same time collect all the cinders from the largest modern locomotive working at high power, made it clear that such a collector would be too

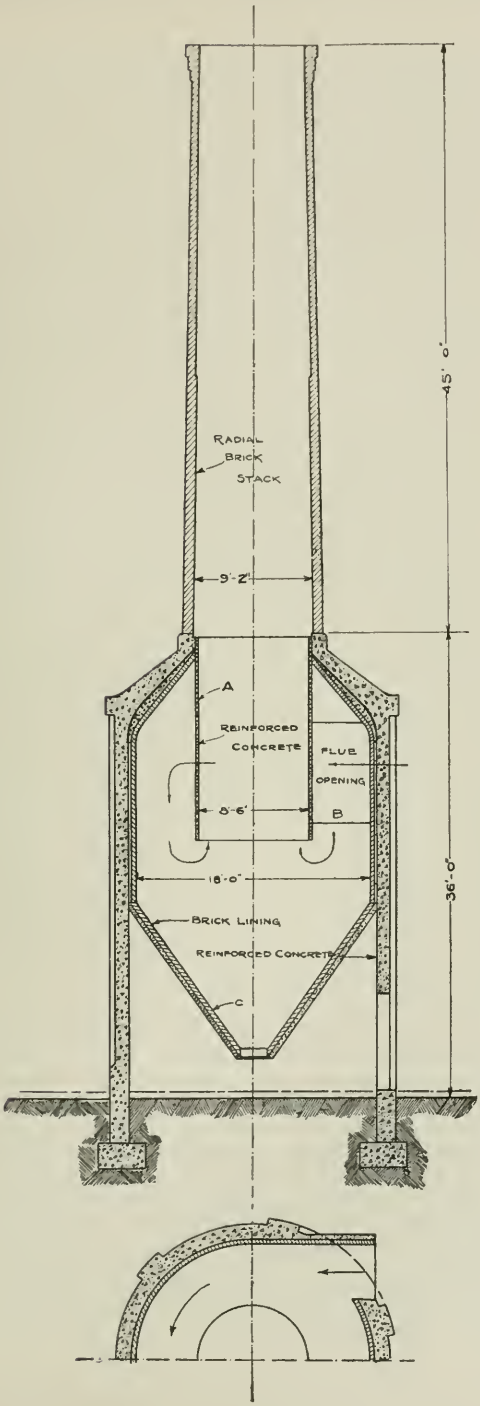


FIG. 55. CROSS SECTION THROUGH CINDER COLLECTOR AND STACK.

large to be located conveniently within the building. Another point considered in the design of the exhaust system was the necessity of a stack of sufficient height to insure that the exhaust gases would be discharged far enough above ground to prove inoffensive to occupants of neighboring residences and University buildings. For this purpose it was decided that a stack 8 feet in diameter and 80 feet high would be required. Further study made it apparent that these two decisions could be embodied in one structure combining the cinder separator and the stack. This has been accomplished in the construction represented in cross-section by Fig. 55, and which is located outside and at the rear of the laboratory.

The system will be most easily understood by following the course of the exhaust gases after they leave the locomotive stack. (Fig. 53 and 54.) The gases and exhaust steam are discharged across an open space above the locomotive stack into a steel exhaust elbow, which carries them up and over to a horizontal duct running through the center of the roof-trusses of the building. The gases, exhaust steam, and solid matter are drawn through this elbow and duct by the exhaust fan, located near the roof at the rear end of the building. They are then passed through a breeching or flue to the separator previously referred to, the action within which may be best explained by reference to Fig. 55. The cinder-laden gases enter the separator at B and in order to leave, they must pass downward and around the sleeve A. This operation gives them a whirling motion, which causes the cinders by centrifugal force to move toward the outside wall, along which they drop to the hopper below, while the gases pass downward and out through the mouth of the sleeve. The cinders collecting at the bottom of the hopper are drawn off, weighed, and analysed between tests. The separator is surmounted by a 45-foot radial brick stack, through which the gases and steam are finally discharged.

On account of the corrosive nature of the mixture of exhaust gases and steam, it was necessary to avoid the use of metal throughout the exhaust system, as far as it was possible to do so. The exhaust elbow which receives the gases from the locomotive stack is necessarily made of steel and needs occasionally to be renewed. The horizontal duct, running through the center of the roof-trusses, is made of a hard and tough asbestos board known as "Transite," which is proof against corrosion. This duct is seven feet in diameter, and is built up in sections so that its length may be varied to suit the position of the locomotive stack. The final adjustment of the elbow above the stack of the

locomotive is obtained through the medium of a telescopic connection between the elbow and the duct. The fan has a runner six feet in diameter, and at a maximum speed of 300 revolutions per minute, will pass 140 000 cubic feet of gas per minute. The breeching between the fan and separator is also built of transite, and has a minimum cross-sectional area of about 24 square feet. The outer shell of the separator is built of reinforced concrete, and it is lined throughout with a course of hard red brick as protection from the corroding action of the gases. Between the lining and the shell is a 2-inch air space which acts as an insulator to protect the shell from overheating. Any leakage of gas through the lining into the air space is vented to the outside air through openings which are provided in the shell, and which serve also to circulate cool air through the air-space. Both the inside sleeve and the hopper are built of reinforced concrete. The stack itself is unlined, but is laid up with acid-proof cement. Provision was made in the design for traps in the bottom of the horizontal duct, whereby any solid matter that should accumulate within the duct could be removed and weighed. Experience has proved this to be unnecessary, as all portions of the duct and breeching have been self-cleaning under all test conditions thus far encountered.

APPENDIX 3.

Test Methods.

The test methods employed were, in general, those outlined in the "Method of Conducting Locomotive and Road Tests" published in the Proceedings of the American Railway Master Mechanics' Association, volume 47, page 538.

Each test was made under predetermined conditions of speed and cut-off. Throughout each test all conditions subject to control were maintained as nearly constant as possible. Variations between different tests, or groups of tests, relative to engine conditions and fuel used have been recorded and discussed.

The test methods employed were, with minor exceptions, the same throughout all tests. All instruments were known to be correct within reasonable limits or were calibrated at intervals and suitable corrections applied to the observed data. Observations were, in general, taken every ten minutes. Indicator diagrams, particularly on comparatively short tests, were often taken at five minute intervals. The locations of the more important instruments and apparatus are indicated in Fig. 40 in Appendix 1, and in the figures of Appendix 2. The methods of applying the load to the engine, of measuring the drawbar-pull, and of collecting the stack cinders are made clear by the description of the Locomotive Laboratory in Appendix 2.

Duration of Tests.—The tests reported in Appendix 4 varied in length from 30 minutes to 3 hours. Tests were in general of such duration that from 120 to 180 pounds of coal would be burned per square foot of grate during the test. This is equivalent to a coal consumption of approximately 6000 to 9000 pounds per test. An examination of the data shows that for 39 tests the coal consumption was within this range; for 15 tests it was less than 6000 pounds; and for 4 tests it was more than 9000 pounds per test.

Starting and Closing a Test.—In general, fires were built upon a clean grate for each test. With sufficient steam pressure the locomotive was started and gradually brought to the required conditions of

speed and cut-off. The locomotive was operated for a short time under the required conditions and until a satisfactory fire and satisfactory boiler pressure were being maintained. On signal, the ash pan and cinder separator were closed, observations of water levels and steam pressure were made, and the test thereby started.

In closing a test, simultaneous observations were made upon water levels, steam pressure, and condition of fire. The locomotive was then stopped as quickly as conditions permitted. As soon as possible after stopping the locomotive, ashes were removed from the ash pan and cinders were removed from the front-end and from the cinder separator. In closing tests, it was sometimes advisable to remove some of the ash from the fire previous to the close of the test in order to bring the fire to the desired condition, and it was also occasionally advisable to remove some ash immediately after the close of the test. In all cases it was endeavored to have the same amount of combustible matter upon the grate at the close of the test as at the start. The removal of ash from the fire in connection with the closing of the test was primarily for the purpose of judging the amount of combustible upon the grate, not for the purpose of collecting the ash. The endeavor was made to have the boiler pressure and water level in the boiler substantially the same at the close as at the beginning of the test. Corrections were made for such irregularities as occurred.

Temperatures, Pressures, Gas Samples, Etc.—Temperatures in the fire-box were observed by means of a radiation pyrometer and temperatures in the front-end by means of a thermo-couple. Mercury thermometers were used at other points where temperature observations were made. Boiler pressure observations were taken from a gage located in the engine cab. Draft pressures were measured by means of U-tubes with water. Quality of steam was determined by means of a throttling calorimeter fitted with a suitable sampling tube. Front-end gas samples were collected through a sampling pipe provided with numerous small holes along the pipe, through which the gas was drawn. The time during which a single sample was collected varied from 20 to 60 minutes, depending mainly upon the total length of the test. The taking of samples usually covered the entire test period. Gas samples were analysed immediately after collection by means of the Orsat apparatus. Speed was measured by means of a stroke counter operated through the indicator reducing-motion.

Indicator Diagrams.—Four indicators were used, one at each end of each cylinder. During a majority of the tests, indicators were used

which, through the operation of electrical attachments, took the four diagrams simultaneously. On account of minor accidents, it was sometimes necessary to use indicators where the pencil applications on some or all of the indicators were made by hand. In all cases, however, the applications were practically simultaneous.

Coal and Water.—Coal was delivered to the firing platform in lots of approximately 1000 pounds each. The time of firing the last shovelful of each lot was recorded. Water was weighed by means of two tanks upon platform scales. Each tank holds approximately 2000 pounds of water. The weighing tanks emptied into the feed-water supply-tank which has a capacity of 18 000 pounds. The height of water in the feed-water tank was maintained at an approximately constant level throughout a test. Observations were so taken that the amount of water furnished to the boiler could be calculated for intervals determined by the emptying of each weighing tank. Fig. 47 and 53 in Appendix 2 show the arrangement of the coal and water weighing apparatus.

Firing.—The locomotive was hand fired during all tests. The method of level firing was used, single shovelfuls of coal at a fairly constant rate being distributed uniformly over the grate. All large pieces of coal were broken, before firing, to lumps whose greatest dimension was from 3 to 4 inches. Two experienced firemen were employed. One of these men, however, was held entirely responsible and did practically all of the firing.

Samples of Coal, Ash, and Cinders.—Following the close of a test the ashes collected in the ash pan, the cinders collected in the front-end, and the cinders collected in the cinder separator were weighed and sampled.

A coal sample weighing from 200 to 500 lb. was collected during each test. This sample was collected while loading the cars taking coal to the firing platform, by placing about 50 lb. in the sampling can for each 1000-lb. car loaded. Care was exercised to make the sample representative.

The front-end cinders after being weighed were thoroughly mixed and about two pounds of cinders set aside as a sample. A sample of the stack cinders, weighing from 25 to 50 lb., was collected as the cinders were being weighed, a small amount being taken from each barrow load after passing over the scales. A sample of ash, weighing from 50 to 100 lb. was collected as the ash was being weighed, representative portions of the ash being selected.

The large samples of coal, ash, and stack cinders were ground or crushed as necessary to reduce them to a maximum size of ¼ inch, then thoroughly mixed and reduced by "quartering" to samples weighing about two pounds. The two pound samples of coal, ash, front-end and stack cinders were submitted to the Chemical Laboratory for analysis.

Chemical Analysis.—The chemical analyses and heat determinations were made by the Department of Chemistry of the University of Illinois. The methods employed were substantially those which have been recommended in the preliminary reports of the Joint Committee on Coal Analysis, of the American Society for Testing Materials and the American Chemical Society.

Proximate analyses and B.t.u. determinations were made for the coal sample for each test. Four ultimate analyses of coal were made —one for tests 2009 to 2019 inclusive, one for tests 2020 to 2045, one for tests 2046 to 2071, and one for tests 2072 to 2095. The ultimate analyses were made from composite samples. Each composite sample was made by combining from each air-dried sample of the tests to be represented, an amount proportional to the coal burned during the test. The ultimate analyses for the individual tests which appear in the tabulated results are based upon the percentages of moisture, ash, and sulphur as determined by the proximate analysis and upon the assumption that the carbon, hydrogen, oxygen, and nitrogen are proportional to the percentages as determined for the composite sample by ultimate analysis.

Each ash and cinder sample was subjected to analysis to determine carbon, earthy matter, and moisture. B.t.u. determinations for ash, front-end and stack cinders were estimated in the following manner. B.t.u. determinations were made upon ten ash samples of Series 1, upon three samples of Series 2, and upon one composite sample representing all tests 2072 to 2095 inclusive. Upon the assumptions that the heat content of the ash was entirely due to its carbon content, and that the heat content of the carbon was uniform in all of the ash, an average value for the heat content of one pound of carbon in the ash was computed. This value was determined as 14 672 B.t.u. per pound of carbon contained in the ash. Using this value as a basis the heat content per pound of ash was calculated for each test.

B.t.u. determinations were made for 10 front-end cinder samples of Series 1 and for one composite sample representing tests 2072 to 2095 inclusive. In a manner similar to that outlined in the case of

the ash, it was computed that the average value for the heat content of one pound of carbon contained in the front-end cinders was 14 336 B.t.u. and with this value as a basis the heat content per pound of front-end cinders was calculated for each test.

B.t.u. determinations were made for 10 stack cinder samples of Series 1, for a composite sample representing 10 tests during which the draft ranged from 2.2 to 4.5 inches of water, for a composite sample representing 9 tests during which the draft ranged from 5.3 to 9.2 inches of water, for a composite sample representing 5 tests during which the draft ranged from 10 to 12.8 inches of water, and for 3 individual tests during which the drafts were respectively 2.9, 7.0, and 11.9 inches of water. The 24 tests represented by the composite samples were tests 2072 to 2095 inclusive. The 3 individual tests mentioned were 2087, 2079, and 2089. With these heat determinations and in a manner involving the same assumptions as were made in the case of the ash and front-end cinders, it was computed that the heat content of one pound of carbon contained in the stack cinders was equivalent to $(14\,932 - 44D)$ B.t.u. In this expression D is the draft in front of the diaphragm, expressed as inches of water. With the values determined by means of this formula the heat content per pound of stack cinders was calculated for each test.

APPENDIX 4.

Detailed Data and Results.

The purpose of this appendix is to present in detail the data and results of all the tests. It consists of 24 tables and 3 figures. Tables 13 to 35 inclusive contain the data and results for 61 tests, arranged in four groups. Table 36 contains information relating to the representative indicator diagrams which are shown in Fig. 56, 57, and 58 at the end of the appendix.

The first of the four groups of tests comprises tests 2009 to 2037 and has been designated as Series 1. The third group, designated as Series 2, comprises tests 2072 to 2098 (excepting No. 2090 and 2091). The results of Series 1 and 2 have been presented and discussed in sections VI to VIII in Part I. The data and results of the two remaining groups appear only in this appendix, and are elsewhere merely referred to incidentally.

Section IV of Part I defines these four groups of tests, and states the difference between them as regards the condition of the locomotive. The locomotive's condition has also been explained in Section II and in Appendix 1. The differences in condition as regards fuel are stated in Section V of Part I. The evaporative efficiencies recorded for tests 2024 and 2038 are enough higher than the corresponding results recorded for other tests to raise some question as to the correctness of their results. The conditions, however, under which they were made were such that high boiler efficiency was to be expected. The data and results for these tests have been included in the tables, and have been used throughout the discussion except as may have been specifically indicated.

The data and results are presented under 182 headings. The column-heading numbers are included between the numbers 344 and 900 and are arranged consecutively in the tables. Tables throughout the bulletin carry corresponding column-headings and numbers wherever the same data are presented. In general the column-headings and column-heading numbers are the same as used in the Code for Testing Locomotives published in the Proceedings of the American Railway Master Mechanics' Association, volume 47, page 538. The methods of calculation, unless entirely obvious, are given in detail in Appendix 5.

Table 13.
General Conditions

Test No.	Laboratory Designation	Duration of Test, Hours	Speed				Position of Levers	
			Revolutions		Equivalent		Reverse Lever, Notches from Center	Throttle
			Total	Average per Minute	Speed in Miles per Hour	Piston Speed in Feet per Minute		
	Code Item☞	345	351	352	353	354	360	363
2009	138–16–F	2.50	20 878	139.2	25.2	694.6	4	Full
2010	193–20–F	1.18	13 969	196.8	35.7	981.8	5	Full
2012	138–24–F	1.83	15 349	139.5	25.3	696.3	6	Full
2013	138–32–F	1.50	12 575	139.7	25.4	697.2	8	Full
2014	193–32–F	1.17	13 981	199.7	36.3	996.4	8	Full
2015	193–24–F	1.50	18 000	200.0	36.3	998.0	6	Full
2016	193–16–F	2.33	27 998	200.0	36.3	998.0	4	Full
2017	83–16–F	3.00	14 398	80.0	14.5	399.2	4	Full
2018	83–24–F	2.67	12 802	80.0	14.5	399.2	6	Full
2019	83–32–F	2.17	10 474	80.6	14.6	402.0	8	Full
2020	83–24–F	2.00	9 656	80.5	14.6	401.7	6	Full
2021	83–16–F	2.00	9 615	80.1	14.5	399.7	4	Full
2022	83–32–F	2.00	9 670	80.6	14.6	402.2	8	Full
2023	138–40–F	1.50	12 515	139.0	25.2	693.6	10	Full
2024	55–24–F	2.00	6 080	50.7	9.2	253.0	6	Full
2026	110–16–F	2.17	14 220	109.4	19.9	545.9	4	Full
2027	110–24–F	2.50	16 500	110.0	20.0	548.9	6	Full
2028	55–32–F	2.33	7 050	50.3	9.1	251.0	8	Full
2029	110–32–F	1.50	9 880	109.8	19.9	547.9	8	Full
2030	165–24–F	1.67	16 935	169.4	30.8	845.3	6	Full
2031	83–40–F	1.50	7 723	85.8	15.6	428.1	10	Full
2032	165–32–F	0.67	6 726	168.1	30.5	838.8	8	Full
2033	110–48–F	1.33	8 793	109.9	20.0	548.4	12	Full
2034	193–40–F	1.00	11 910	198.5	36.0	990.5	10	Full
2035	110–40–F	1.17	7 830	111.8	20.3	557.9	10	Full
2037	165–40–F	1.00	10 160	169.3	30.7	844.8	10	Full
2038	55–24–F	1.83	5 586	50.8	9.2	253.4	6	Full
2039	110–32–F	1.50	9 896	110.0	20.0	548.7	8	Full
2040	165–40–F	1.00	10 127	168.8	30.7	842.2	10	Full
2041	110–40–F	1.17	7 709	110.1	20.0	549.5	10	Full
2042	110–24–F	2.00	13 190	109.9	20.0	548.5	6	Full
2043	110–48–F	1.17	7 681	109.7	19.9	547.6	12	Full
2044	110–56–F	1.00	6 585	109.8	19.9	547.7	14	Full
2045	110–16–F	2.17	14 277	109.8	19.9	548.0	4	Full
2072	110–40–F	1.00	6 575	109.6	19.9	546.8	10	Full
2073	110–32–F	1.33	8 751	109.4	19.9	545.9	8	Full
2074	165–32–F	1.00	10 178	169.6	30.8	846.3	8	Full
2075	55–32–F	2.33	7 081	50.6	9.2	252.3	8	Full
2076	220–32–F	0.58	8 047	229.9	41.7	1147.2	8	Full
2077	110–24–F	1.83	12 174	110.7	20.1	552.2	6	Full
2078	165–24–F	1.17	11 830	169.0	30.7	843.3	6	Full
2079	220–24–F	1.00	13 914	231.9	42.1	1157.2	6	Full
2080	110–16–F	2.17	14 346	110.4	20.0	550.7	4	Full
2081	55–24–F	2.50	7 593	50.6	9.2	252.5	6	Full
2082	165–40–F	0.83	8 484	169.7	30.8	846.7	10	Full
2083	165–16–F	1.67	17 031	170.3	30.9	849.8	4	Full
2084	110–48–F	0.83	5 519	110.4	20.0	550.8	12	Full
2085	55–40–F	2.00	6 136	51.1	9.3	255.1	10	Full
2086	55–24–F	2.83	8 725	51.3	9.3	256.1	6	Full
2087	110–16–F	2.50	16 660	111.1	20.2	554.4	4	Full
2088	220–16–F	1.67	23 418	234.2	42.5	1168.6	4	Full
2089	220–40–F	0.58	8 075	230.7	41.9	1151.2	10	Full
2092	165–32–F	0.83	8 425	168.5	30.6	840.8	8	Full
2093	165–48–F	0.50	5 023	167.4	30.4	835.5	12	Full
2094	110–56–F	0.42	2 772	110.9	20.1	553.3	14	Full
2095	55–48–F	1.00	3 077	51.3	9.3	255.9	12	Full
2096	55–40–F	1.50	4 636	51.5	9.4	257.0	10	Full
2097	55–32–F	1.83	5 732	52.1	9.5	260.0	8	Full
2098	55–48–F	0.83	2 587	51.7	9.4	258.2	12	Full
2090	110–24–F	1.00	6 640	110.7	20.1	552.2	6	Full
2091	165–32–F	0.50	5 079	169.3	30.7	844.8	8	Full

Table 14.

Temperature and Pressure

Test No.	Laboratory Designation	Temperature, Deg. Fahr.					Pressure, lb. per sq. in.	
		Front-end	Laboratory		Feed Water	Fire Box	Boiler, Average	Laboratory Barometric
			Dry Bulb	Wet Bulb				
	Code Item	367	368	369	373	374	380	388
2009	138–16–F		93	75	64.1	1950	190.5	14.3
2010	193–20–F	761	62	57	59.3	2081	192.0	14.4
2012	138–24–F	712	86	69	61.7	1959	191.8	14.4
2013	138–32–F	754	87	70	62.2	1957	190.1	14.4
2014	193–32–F	751	94	75	62.9		191.5	14.4
2015	193–24–F	702	97	75	61.7		192.1	14.4
2016	193–16–F	671	94	75	63.4		193.9	14.4
2017	83–16–F	619	93	76	72.2		193.4	14.4
2018	83–24–F	649	92	71	64.0		194.2	14.3
2019	83–32–F	684	86	73	62.1		193.8	14.3
2020	83–24–F	499	70	61	64.9		190.7	14.4
2021	83–16–F	494	63	53	57.8	1552	193.7	14.5
2022	83–32–F	554	66	56	58.3	1808	189.9	14.5
2023	138–40–F	639	64	55	57.7	1898	190.8	14.4
2024	55–24–F	517	66	51	59.3	1829	196.3	14.5
2026	110–16–F	531	58	52	61.0	1748	196.9	14.5
2027	110–24–F	552	64	56	60.2	1677	196.8	14.4
2028	55–32–F	515	66	58	70.4	1700	198.1	14.4
2029	110–32–F	560	73	65	60.9	1690	197.1	14.4
2030	165–24–F	565	76	68	60.2	1636	196.5	14.3
2031	83–40–F	570	72	66	60.6	1811	196.4	14.3
2032	165–32–F	613	71	66	59.6	1630	196.8	14.3
2033	110–48–F	603	73	67	60.6	1806	196.0	14.3
2034	193–40–F	632	76	69	59.8	1879	192.1	14.3
2035	110–40–F	589	73	65	61.7	1663	194.3	14.3
2037	165–40–F	651	75	69	60.8	1800	196.1	14.4
2038	55–24–F	510	64	52	58.2	1544	198.3	14.5
2039	110–32–F	578	62	55	59.1	1815	197.1	14.5
2040	165–40–F	640	64	55	57.9	1828	190.3	14.5
2041	110–40–F	621	58	52	55.0	1800	192.3	14.5
2042	110–24–F	557	59	53	57.1	1758	196.8	14.5
2043	110–48–F	646	55	50	58.4	1856	194.4	14.5
2044	110–56–F	686	54	48	56.8	1871	190.1	14.4
2045	110–16–F	551	61	54	59.6	1775	197.1	14.4
2072	110–40–F	620	59	54	59.5	1643	196.7	14.4
2073	110–32–F	595	53	48	56.9	1662	197.6	14.5
2074	165–32–F	637	59	52	59.7	1662	197.1	14.4
2075	55–32–F	543	58	52	58.1	1661	198.1	14.4
2076	220–32–F	675	63	54	60.1	1785	196.0	14.4
2077	110–24–F	565	58	51	58.7	1570	196.0	14.3
2078	165–24–F	595	62	53	58.4	1597	196.4	14.5
2079	220–24–F	614	64	54	58.8	1688	197.4	14.5
2080	110–16–F	534	60	53	59.7	1418	198.8	14.6
2081	55–24–F	507	63	54	57.9	1407	198.2	14.6
2082	165–40–F	673	55	51	63.6	1458	195.2	14.3
2083	165–16–F	563	62	54	60.5	1267	198.7	14.4
2084	110–48–F	653	60	56	52.6		194.0	14.5
2085	55–40–F	545	62	55	44.7		197.9	14.4
2086	55–24–F	506	67	57	56.2		199.1	14.4
2087	110–16–F	524	69	59	59.5		199.2	14.3
2088	220–16–F	563	73	61	58.1		197.8	14.2
2089	220–40–F	703	59	50	58.4		194.9	14.4
2092	165–32–F	643	52	49	59.4		198.4	14.4
2093	165–48–F	702	60	52	59.3		191.5	14.3
2094	110–56–F	679	65	58	59.0		196.3	14.2
2095	55–48–F	567	60	53	60.0		198.1	14.2
2096	55–40–F	584			61.4		196.1	14.4
2097	55–32–F	573			58.6		198.8	14.4
2098	55–48–F	611			59.2		198.2	14.4
2090	110–24–F	552	67	58	60.1		199.0	14.4
2091	165–32–F	626	64	57	59.5		198.6	14.4

TABLE 15.

DRAFT, INJECTORS, QUALITY OF STEAM.

Test No.	Laboratory Designation	Draft, in. of Water				Injectors in Action				Quality of Steam in Dome	Factor of Correction for Quality of Steam
		Front-end		Fire Box	Ash Pan	Right, Total Hours	Left, Total Hours	Right, No. of Times	Left, No. of Times		
		Front of Diaphragm	Back of Diaphragm								
	Code Item☞	394	395	396	397	403	404	405	406	407	412
2009	138–16–F	3.7	2.6	1.6						0.990	0.9919
2010	193–20–F	5.5	3.9	2.1						0.987	0.9908
2012	138–24–F	5.8	4.0	2.1						0.989	0.9918
2013	138–32–F	7.6	5.1	2.7						0.985	0.9888
2014	193–32–F	9.1	6.0	3.2						0.984	0.9883
2015	193–24–F	7.3	4.9	2.8						0.987	0.9908
2016	193–16–F	5.0	3.5	2.0						0.989	0.9919
2017	83–16–F	2.7	1.9	0.9						0.991	0.9935
2018	83–24–F	3.9	2.7	1.5						0.993	0.9949
2019	83–32–F	5.4	3.7	2.0						0.991	0.9935
2020	83–24–F	3.4	2.3	1.2		1.6	0.0	19	0	0.9929	0.9949
2021	83–16–F	2.2	1.4	0.7		1.3	0.0	29	0	0.9945	0.9961
2022	83–32–F	4.8	3.0	1.6		2.0	0.0	3	0	0.9956	0.9968
2023	138–40–F	9.0	5.6	2.1		0.5	1.5	26	1	0.9930	0.9950
2024	55–24–F	2.6	1.6	1.0		1.2	0.0	37	0	0.9950	0.9964
2026	110–16–F	2.9	1.9	1.0		1.4	0.0	50	0	0.9895	0.9925
2027	110–24–F	4.7	2.7	1.5		2.1	0.0	33	0	0.9914	0.9938
2028	55–32–F	3.2	2.2	1.1		2.1	0.0	32	0	0.9912	0.9936
2029	110–32–F	6.1	3.9	1.8		1.5	0.0	3	0	0.9902	0.9930
2030	165–24–F	6.2	3.9	2.0		1.7	0.0	1	0	0.9894	0.9924
2031	83–40–F	6.8	4.3	2.2		1.5	0.1	1	4	0.9896	0.9926
2032	165–32–F	8.4	4.9	2.3		0.7	0.1	1	5	0.9862	0.9901
2033	110–48–F	8.2	5.1	2.2		0.3	1.3	16	1	0.9867	0.9904
2034	193–40–F	10.7	6.6	2.3		0.6	1.0	17	1	0.9857	0.9898
2035	110–40–F	8.4	5.2	2.7		0.5	1.2	13	1	0.9833	0.9879
2037	165–40–F	10.2	6.2	3.2		0.5	0.0	24	0	0.9860	0.9900
2038	55–24–F	2.6	1.7	1.1	0.3	1.8	0.0	1	0	0.9934	0.9953
2039	110–32–F	6.9	4.5	2.4	0.6	1.5	0.0	1	0	0.9917	0.9941
2040	165–40–F	9.9	6.8	3.2	0.6	0.6	1.0	16	1	0.9917	0.9940
2041	110–40–F	8.7	5.5	2.2	0.5	0.3	1.2	20	1	0.9900	0.9852
2042	110–24–F	4.9	3.2	1.7	0.4	2.0	0.0	1	0	0.9932	0.9952
2043	110–48–F	9.7	5.9	2.4	0.7	0.6	1.2	24	1	0.9866	0.9904
2044	110–56–F	11.8	7.5	3.2	0.7	0.8	1.0	5	1	0.9900	0.9928
2045	110–16–F	3.2	2.2	1.1	0.3	2.2	0.0	1	0	0.9934	0.9953
2072	110–40–F	8.0	4.9	2.0	0.7	0.3	1.0	20	1	0.9895	0.9925
2073	110–32–F	5.7	3.5	1.6	0.5	1.3	0.0	1	0	0.9919	0.9942
2074	165–32–F	8.5	5.3	2.4	0.5	0.5	0.9	21	1	0.9861	0.9901
2075	55–32–F	2.8	1.8	0.9	0.3	2.3	0.0	1	0	0.9952	0.9970
2076	220–32–F	9.2	5.7	2.8	0.5	3.8	0.6	10	1	0.9844	0.9888
2077	110–24–F	4.1	2.5	1.3	0.4	1.8	0.0	3	0	0.9947	0.9963
2078	165–24–F	6.0	3.9	1.9	0.5	1.2	0.1	3	3	0.9915	0.9939
2079	220–24–F	7.0	4.8	2.3	0.4	0.1	1.0	13	1	0.9889	0.9921
2080	110–16–F	2.9	2.0	1.0	0.2	2.2	0.0	1	0	0.9956	0.9968
2081	55–24–F	2.2	1.5	0.7	0.2	2.5	0.0	4	0	0.9963	0.9974
2082	165–40–F	11.2	7.3	3.4	0.6	0.4	0.8	18	1	0.9886	0.9917
2083	165–16–F	4.3	2.7	1.6	0.4	1.7	0.0	1	0	0.9934	0.9953
2084	110–48–F	10.0	6.3	2.9	0.8	0.8	0.4	21	1	0.9896	0.9926
2085	55–40–F	4.0	2.5	1.1	0.3	2.0	0.0	1	0	0.9962	0.9973
2086	55–24–F	2.2	1.4	0.7	0.3	2.8	0.0	1	0	0.9963	0.9974
2087	110–16–F	2.9	1.9	0.9	0.2	2.5	0.0	1	0	0.9956	0.9968
2088	220–16–F	4.5	2.9	1.2	0.3	1.7	0.0	1	0	0.9919	0.9942
2089	220–40–F	11.9	7.1	3.5	0.7	0.6	0.6	3	1	0.9884	0.9917
2092	165–32–F	8.2	5.1	2.2	0.6	0.3	0.8	16	1	0.9894	0.9924
2093	165–48–F	12.8	7.8	3.4	0.9	0.3	0.5	3	1	0.9879	0.9913
2094	110–56–F	12.1	7.3	3.0	0.8	0.4	0.4	2	1	0.9887	0.9919
2095	55–48–F	5.3	3.0	1.5	0.4	1.0	0.0	1	0	0.9943	0.9959
2096	55–40–F	4.2	2.6	1.2	0.4	1.5	0.1	1	1	0.9891	0.9922
2097	55–32–F	3.2	2.3	1.0	0.5	1.8	0.0	1	0	0.9853	0.9895
2098	55–48–F	5.4	3.7	1.9	0.8	0.8	0.0	1	0	0.9840	0.9885
2090	110–24–F	4.2	2.9	1.1	0.3	1.0	0.0	1	0	0.9952	0.9966
2091	165–32–F	8.2	4.9	2.1	0.6	0.2	0.5	8	1	0.9911	0.9936

TABLE 16.

COAL, CINDERS, ASH, SMOKE, AND HUMIDITY.

Test No.	Laboratory Designation	Coal Fired Total, lb.	Dry Coal Fired Total, lb.	Combustible by Analysis Total, lb.	Ash by Analysis Total, lb.	Front-end Cinders Total, lb.	Stack Cinders Total, lb.	Front End and Stack Cinders Total, lb.
	Code Item☞	418	419	420	421	422	423	424
2009	138–16–F	7497	6618	5786	832	26	413	439
2010	193–20–F	5147	4537	3943	593	10	507	517
2012	138–24–F	7657	6797	5835	962	9	648	657
2013	138–32–F	7832	7124	6119	1005	10	925	935
2014	193–32–F	7978	7230	6153	1077	9	1045	1054
2015	193–24–F	8298	7391	6365	1026	12	1092	1104
2016	193–16–F	8603	7595	6368	1228	11	681	692
2017	83–16–F	6535	5872	5079	793	12	187	199
2018	83–24–F	7589	6765	5795	970	29	380	409
2019	83–32–F	7793	6965	6096	865	14	586	600
2020	83–24–F	5416	4943	4318	625	16	286	302
2021	83–16–F	5040	4422	3837	585	22	243	265
2022	83–32–F	8198	7346	6222	1124	8	898	906
2023	138–40–F	11556	10031	8637	1394	7	2075	2082
2024	55–24–F	4104	3628	3171	456	20	115	135
2026	110–16–F	5693	4969	4265	704	25	279	304
2027	110–24–F	9322	8139	7075	1064	20	745	765
2028	55–32–F	6414	5615	4887	729	14	372	386
2029	110–32–F	7257	6363	5445	918	15	869	884
2030	165–24–F	7501	6689	5888	801	11	856	867
2031	83–40–F	7686	6366	5414	952	13	944	957
2032	165–32–F	4104	3568	3037	532	10	662	672
2033	110–48–F	7940	6835	5839	996	10	1219	1229
2034	193–40–F	8916	7767	6501	1266	10	1499	1509
2035	110–40–F	7590	6493	5575	918	6	934	940
2037	165–40–F	7625	6554	5631	923	11	1206	1217
2038	55–24–F	4202	3688	2985	703	19	126	145
2039	110–32–F	7678	6775	5639	1136	17	808	825
2040	165–40–F	8515	7482	6325	1157	10	1617	1627
2041	110–40–F	7957	6838	5851	987	6	1218	1224
2042	110–24–F	7771	6713	5720	992	6	663	669
2043	110–48–F	9979	8637	7431	1205	17	1829	1846
2044	110–56–F	9623	8361	7080	1282	10	1771	1781
2045	110–16–F	6061	5258	4369	889	23	453	476
2072	110–40–F	6802	5927	5103	824		1140	
2073	110–32–F	6703	5812	5116	694	12	736	748
2074	165–32–F	6766	6015	5206	805	3	949	952
2075	55–32–F	6236	5430	4791	639	13	385	398
2076	220–32–F	5394	4568	3983	585	14	1057	1071
2077	110–24–F	6896	6015	5240	775	14	449	463
2078	165–24–F	6332	5492	4786	706	16	758	774
2079	220–24–F	6332	5783	5121	662	14	873	887
2080	110–16–F	6031	5248	4666	581	15	210	225
2081	55–24–F	5591	4937	4355	582	18	155	173
2082	165–40–F	8506	7495	6624	871	19	1640	1659
2083	165–16–F	6453	5564	4847	717	21	432	453
2084	110–48–F	7592	6595	5633	963	20	1244	1264
2085	55–40–F	6991	6116	5353	763	17	434	451
2086	55–24–F	6660	5860	5082	778	17	310	327
2087	110–16–F	7004	6185	5337	847	18	296	314
2088	220–16–F	6445	5588	4688	900	17	456	473
2089	220–40–F	7401	6491	5558	933	13	1728	1741
2092	165–32–F	5491	4700	4128	572	12	698	710
2093	165–48–F	5933	5108	4444	664	10	1391	1401
2094	110–56–F	4095	3514	3055	459	13	811	824
2095	55–48–F	3799	3334	2854	480	10	250	260
2096	55–40–F							
2097	55–32–F							
2098	55–48–F							
2090	110–24–F	3605	3176	2772	404	14	312	326
2091	165–32–F	3028	2626	2302	324	12	431	443

Table 17.

Coal, Cinders, Ash, Smoke, and Humidity.

Test No.	Laboratory Designation	Cinder Loss, Per cent of Total Dry Coal Fired	Stack Cinder Loss, Per cent of Total Dry Coal Fired	Ash from Ash Pan			Smoke	Humidity
				Total, lb.	Per cent of Total Dry Coal Fired	Per cent of Ash by Analysis	Per cent of Blackness by Ringlemann Chart	Moisture per lb. of Dry Air, lb.
	Code Item	426	427	428	429	430	431	435
2009	138–16–F	6.6	6.2	172	2.6	20.7		.014
2010	193–20–F	11.4	11.2	69	1.5	11.6		.008
2012	138–24–F	9.7	9.5	318	4.7	33.0		.011
2013	138–32–F	13.1	13.0	301	4.2	30.0		.011
2014	193–32–F	14.6	14.5	159	2.2	14.7		.014
2015	193–24–F	14.9	14.8	557	7.5	54.2		.014
2016	193–16–F	9.1	9.0	445	5.9	36.2		.014
2017	83–16–F	3.4	3.2	295	5.0	37.2		.015
2018	83–24–F	6.1	5.6	309	4.6	31.9		.011
2019	83–32–F	8.6	8.4	387	5.6	44.5		.014
2020	83–24–F	6.1	5.8	151	3.1	24.2		.010
2021	83–16–F	6.0	5.5	455	10.3	77.8		.006
2022	83–32–F	12.3	12.2	568	7.7	50.5		.010
2023	138–40–F	20.8	20.7	631	6.3	45.3		.007
2024	55–24–F	3.7	3.2	202	5.6	44.3		.005
2026	110–16–F	6.1	5.6	390	7.9	55.4	45	.007
2027	110–24–F	9.4	9.2	513	6.3	48.2	43	.008
2028	55–32–F	6.9	6.6	429	7.6	58.9	20	.009
2029	110–32–F	13.9	13.7	503	7.9	54.8	44	.011
2030	165–24–F	13.0	12.8	374	5.6	46.7	55	.013
2031	83–40–F	15.0	14.8	434	6.8	45.6		.012
2032	165–32–F	18.8	18.6	222	6.2	41.7	51	.013
2033	110–48–F	18.0	17.8	409	6.0	41.1		.013
2034	193–40–F	19.4	19.3	645	8.3	51.0		.014
2035	110–40–F	14.5	14.4	518	8.0	56.4		.011
2037	165–40–F	18.6	18.4	445	6.8	48.2		.014
2038	55–24–F	3.9	3.4	280	7.6	39.8		.006
2039	110–32–F	12.2	11.9	476	7.0	41.9		.008
2040	165–40–F	21.8	21.6	393	5.3	34.0		.007
2041	110–40–F	17.9	17.8	400	5.9	40.5		.007
2042	110–24–F	10.0	9.9	410	6.1	41.3		.007
2043	110–48–F	21.4	21.2	463	5.4	38.4		.006
2044	110–56–F	21.3	21.2	410	4.9	32.0		.006
2045	110–16–F	9.0	8.6	400	7.6	45.0	32	.007
2072	110–40–F		19.2	273	4.6	33.1	42	.008
2073	110–32–F	12.9	12.7	336	5.8	48.4	35	.006
2074	165–32–F	15.8	15.8	416	6.9	51.7	45	.007
2075	55–32–F	7.3	7.1	216	4.0	33.8	20	.007
2076	220–32–F	23.4	23.1	313	6.9	53.5		.007
2077	110–24–F	7.7	7.5	200	3.3	25.8		.006
2078	165–24–F	14.1	13.8	282	5.1	39.9		.007
2079	220–24–F	15.3	15.1	569	9.8	86.0		.007
2080	110–16–F	4.3	4.0	428	8.2	73.7		.007
2081	55–24–F	3.5	3.1	427	8.7	73.4		.007
2082	165–40–F	22.1	21.9	576	7.7	66.1		.007
2083	165–16–F	8.2	7.8	564	10.1	78.7		.007
2084	110–48–F	19.2	18.9	501	7.6	52.0		.008
2085	55–40–F	7.4	7.1	530	8.7	69.5		.008
2086	55–24–F	5.6	5.3	555	9.5	71.3		.008
2087	110–16–F	5.1	4.8	604	9.8	71.3		.008
2088	220–16–F	8.5	8.2	523	9.4	58.1		.008
2089	220–40–F	26.8	26.6	429	6.6	46.0		.006
2092	165–32–F	15.1	14.9	357	7.6	62.4		.006
2093	165–48–F	27.4	27.2	495	9.7	74.5		.006
2094	110–56–F	23.5	23.1	234	6.7	51.0		.008
2095	55–48–F	7.8	7.5	256	7.7	53.3		.007
2096	55–40–F							
2097	55–32–F							
2098	55–48–F							
2090	110–24–F	10.3	9.8	315	9.9	78.0		.008
2091	165–32–F	16.9	16.4	260	9.9	80.3		.008

TABLE 18.

COAL ANALYSIS.

Test No.	Laboratory Designation	Proximate Analysis Coal as Fired					Calorific Value per lb. of Coal as Fired, B.t.u.	Ultimate Analysis Coal as Fired			
		Fixed Carbon, per cent	Volatile Matter, per cent	Moisture, per cent	Ash, per cent	Sulphur Separately Determined, per cent		Carbon, per cent	Hydrogen, per cent	Nitrogen, per cent	Oxygen, per cent
	Code Item	437	438	440	441	442	443	449	450	451	452
2009	138–16–F	38.21	38.97	11.72	11.10	2.33	11 083	61.68	4.48	0.85	7.84
2010	193–20–F	37.71	38.90	11.86	11.53	3.54	10 959	60.22	4.37	0.83	7.65
2012	138–24–F	38.08	38.12	11.23	12.57	3.43	10 901	59.97	4.36	0.83	7.62
2013	138–32–F	38.92	39.21	9.04	12.83	3.56	11 135	61.45	4.46	0.85	7.81
2014	193–32–F	38.14	38.99	9.37	13.50	3.43	11 042	60.74	4.41	0.84	7.72
2015	193–24–F	38.16	38.55	10.92	12.37	3.51	10 963	60.32	4.38	0.83	7.66
2016	193–16–F	36.86	37.16	11.71	14.27	4.16	10 588	57.57	4.18	0.79	7.31
2017	83–16–F	38.80	38.92	10.15	12.13	3.50	11 179	61.17	4.44	0.84	7.72
2018	83–24–F	38.25	38.11	10.86	12.78	3.12	10 932	60.36	4.38	0.83	7.67
2019	83–32–F	37.68	40.55	10.62	11.15	3.13	11 192	61.89	4.43	0.85	7.86
2020	83–24–F	39.40	40.33	8.73	11.54	3.86	11 228	62.42	3.80	1.61	8.03
2021	83–16–F	37.59	38.54	12.27	11.60	3.69	10 768	59.60	3.63	1.54	7.67
2022	83–32–F	37.57	38.33	10.39	13.71	4.36	10 642	58.86	3.59	1.52	7.57
2023	138–40–F	36.41	38.33	13.20	12.06	4.38	10 686	57.89	3.53	1.50	7.45
2024	55–24–F	37.02	40.25	11.61	11.12	3.41	11 236	60.77	3.70	1.57	7.82
2026	110–16–F	35.94	38.98	12.72	12.36	3.68	10 743	58.61	3.57	1.51	7.54
2027	110–24–F	37.38	38.52	12.69	11.41	3.42	11 078	59.64	3.63	1.54	7.67
2028	55–32–F	37.11	39.08	12.45	11.36	3.21	11 077	60.05	3.66	1.55	7.72
2029	110–32–F	35.06	39.97	12.32	12.65	3.67	10 948	58.71	3.58	1.52	7.55
2030	165–24–F	38.03	40.46	10.83	10.68	3.61	11 376	61.61	3.75	1.59	7.92
2031	83–40–F	34.11	36.33	17.18	12.38	3.15	9 929	55.36	3.37	1.43	7.12
2032	165–32–F	35.55	38.44	13.05	12.96	4.36	10 644	57.29	3.49	1.48	7.37
2033	110–48–F	35.99	37.55	13.92	12.54	4.16	10 539	57.08	3.48	1.48	7.34
2034	193–40–F	34.86	38.05	12.89	14.20	3.99	10 309	56.71	3.45	1.47	7.29
2035	110–40–F	34.41	39.04	14.46	12.09	4.09	10 547	57.07	3.48	1.48	7.34
2037	165–40–F	35.69	38.10	14.05	12.10	4.31	10 693	57.22	3.49	1.48	7.36
2038	55–24–F	34.50	36.54	12.24	16.72	3.63	10 041	55.46	3.38	1.43	7.13
2039	110–32–F	35.80	37.65	11.76	14.79	4.36	10 355	56.85	3.46	1.47	7.31
2040	165–40–F	35.58	38.70	12.13	13.59	4.19	10 688	57.67	3.51	1.49	7.42
2041	110–40–F	35.64	37.89	14.06	12.41	3.55	10 550	57.58	3.51	1.49	7.41
2042	110–24–F	36.24	37.37	13.62	12.77	3.73	10 602	57.50	3.50	1.49	7.39
2043	110–48–F	36.48	37.99	13.45	12.08	3.50	10 841	58.39	3.56	1.51	7.51
2044	110–56–F	36.25	37.32	13.11	13.32	3.56	10 594	57.60	3.51	1.49	7.41
2045	110–16–F	35.43	36.66	13.25	14.66	4.55	10 310	55.57	3.39	1.44	7.15
2072	110–40–F	37.21	37.81	12.87	12.11	4.28	10 857	57.75	4.27	2.10	6.62
2073	110–32–F	38.60	37.72	13.33	10.35	3.78	11 051	59.21	4.38	2.15	6.79
2074	165–32–F	38.79	38.16	11.15	11.90	3.36	11 173	60.07	4.44	2.19	6.89
2075	55–32–F	38.85	37.98	12.93	10.24	3.59	11 074	59.79	4.42	2.18	6.86
2076	220–32–F	35.81	38.03	15.31	10.85	2.79	10 602	58.00	4.29	2.11	6.65
2077	110–24–F	37.94	38.04	12.78	11.24	3.64	11 019	59.05	4.37	2.15	6.77
2078	165–24–F	37.88	37.71	13.26	11.15	3.66	10 858	58.72	4.34	2.14	6.73
2079	220–24–F	40.44	40.44	8.67	10.45	3.56	11 660	63.12	4.67	2.30	7.24
2080	110–16–F	36.71	40.66	12.99	9.64	3.32	11 178	60.45	4.47	2.20	6.93
2081	55–24–F	38.17	39.72	11.70	10.41	3.29	11 214	60.90	4.51	2.22	6.98
2082	165–40–F	37.92	39.95	11.89	10.24	3.27	11 125	60.90	4.51	2.22	6.98
2083	165–16–F	36.71	38.41	13.77	11.11	3.16	10 916	58.74	4.35	2.14	6.74
2084	110–48–F	35.11	39.08	13.13	12.68	3.51	10 689	57.70	4.27	2.10	6.62
2085	55–40–F	37.53	39.04	12.52	10.91	3.13	11 042	59.95	4.44	2.18	6.87
2086	55–24–F	36.86	39.45	12.01	11.68	3.58	11 075	59.37	4.39	2.16	6.81
2087	110–16–F	36.77	39.43	11.70	12.10	3.68	10 836	59.20	4.38	2.15	6.79
2088	220–16–F	35.86	36.88	13.30	13.96	4.04	10 487	56.08	4.15	2.04	6.43
2089	220–40–F	36.54	38.56	12.30	12.60	3.33	10 837	58.59	4.33	2.13	6.72
2092	165–32–F	37.35	37.83	14.40	10.42	3.02	10 802	58.90	4.36	2.14	6.75
2093	165–48–F	37.16	37.74	13.90	11.20	2.69	10 807	58.95	4.36	2.14	6.76
2094	110–56–F	36.92	37.68	14.20	11.20	3.41	10 662	58.11	4.30	2.11	6.66
2095	55–48–F	37.26	37.87	12.24	12.63	2.89	10 808	58.97	4.36	2.15	6.76
2096	55–40–F										
2097	55–32–F										
2098	55–48–F										
2090	110–24–F	36.99	39.90	11.91	11.20	3.51	11 094	59.90	4.43	2.18	6.87
2091	165–32–F	37.15	38.87	13.28	10.70	3.99	10 965	58.80	4.35	2.14	6.74

Table 19.

Calorific Value of Coal and Cinders, Analysis of Front-end Gases.

Test No.	Laboratory Designation	Calorific Value, B.t.u. per lb.					Analysis of Front-end Gases			
		Dry Coal	Combustible	Front-end Cinders	Stack Cinders	Ash	Oxygen O_2	Carbon Monoxide CO	Carbon Dioxide CO_2	Nitrogen N_2
	Code Item	458	459	461	462	463	466	467	468	469
2009	138–16–F	12 553	14 360	7796	6685	5069	10.9	0.0	8.1	81.0
2010	193–20–F	12 433	14 305	5544	8947	4587	13.7	0.0	6.0	80.3
2012	138–24–F	12 280	14 306	6242	8042	4515	13.7	0.0	5.7	80.7
2013	138–32–F	12 242	14 252	5312	8779	4785	13.5	0.1	6.0	80.6
2014	193–32–F	12 184	14 316	5611	9704	4046	11.4	0.0	7.4	81.1
2015	193–24–F	12 307	14 291	2586	9438	5297	11.9	0.0	7.3	80.8
2016	193–16–F	11 992	14 304	6104	8359	4844	12.1	0.0	7.1	80.7
2017	83–16–F	12 422	14 384	4821	6112	4624	11.3	0.1	8.0	80.5
2018	83–24–F	12 265	14 316	6155	7515	4134	11.0	0.2	8.3	80.6
2019	83–32–F	12 523	14 307	6523	8218	3272	10.3	0.2	8.9	80.6
2020	83–24–F	12 302	14 083	5342	6850	6528	10.7	0.0	8.7	80.6
2021	83–16–F	12 274	14 144	6573	7659	5267	11.8	0.0	7.7	80.5
2022	83–32–F	11 875	14 021	3907	9492	3850	11.8	0.0	8.0	80.2
2023	138–40–F	12 311	14 298	7357	10341	4842	6.6	0.0	11.2	82.2
2024	55–24–F	12 712	14 541	5657	6459	4261	10.9	0.0	8.0	81.1
2026	110–16–F	12 309	14 339	7003	8311	6064	11.6	0.0	7.7	80.7
2027	110–24–F	12 688	14 596	6996	8274	4853	10.8	0.0	8.1	81.1
2028	55–32–F	12 653	14 539	7109	7864	4032	10.4	0.0	8.1	81.5
2029	110–32–F	12 486	14 591	4841	8914	5429	9.5	0.0	9.4	81.1
2030	165–24–F	12 757	14 494	7007	9867	5618	8.9	0.0	9.7	81.4
2031	83–40–F	11 989	14 096	2985	9677	4262	8.2	0.2	9.8	81.8
2032	165–32–F	12 242	14 386	7539	4922	6021				
2033	110–48–F	12 243	14 331	2798	9888	4685	7.6	0.0	10.2	82.2
2034	193–40–F	11 835	14 139	6172	10324	5327	7.0	0.0	10.4	82.6
2035	110–40–F	12 329	14 359	5839	9698	5547	8.5	0.0	8.3	83.2
2037	165–40–F	12 441	14 479	6543	10098	5942	6.0	0.0	10.8	83.2
2038	55–24–F	11 442	14 134	6650	5772	6168	10.0	0.0	8.4	81.6
2039	110–32–F	11 734	14 098	6127	8557	4341	8.1	0.0	10.2	81.7
2040	165–40–F	12 164	14 389	4986	10227	5659	6.7	0.1	11.5	81.7
2041	110–40–F	12 276	14 348	6656	9634	5122	8.1	0.1	10.5	81.3
2042	110–24–F	12 273	14 403	6850	8425	5361	9.3	0.0	9.7	81.0
2043	110–48–F	12 523	14 558	1512	10046	4840	7.0	0.4	11.7	80.9
2044	110–56–F	12 192	14 400	3755	10654	4400	5.5	0.4	11.1	83.0
2045	110–16–F	11 885	14 302	7518	6890	4862	11.5	0.0	7.3	81.2
2072	110–40–F	12 460	14 472	4659	9926	3920	7.4	0.3	10.4	81.9
2073	110–32–F	12 751	14 480	4934	9485	5216	7.7	0.1	9.8	82.4
2074	165–32–F	12 575	14 520	5873	9780	4331	7.4	0.5	10.3	81.8
2075	55–32–F	12 718	14 414	6273	6914	4450	10.7	0.1	8.0	81.2
2076	220–32–F	12 519	14 358	6331	11014	4871	6.8	0.2	11.0	82.0
2077	110–24–F	12 633	14 502	8064	8289	7618	9.1	0.1	9.2	81.7
2078	165–24–F	12 517	14 364	6129	9454	4168	9.2	0.0	9.1	81.7
2079	220–24–F	12 767	14 416	6024	9867	5587	8.1	0.0	9.9	82.0
2080	110–16–F	12 848	14 448	6337	5522	4451	10.7	0.2	7.9	81.2
2081	55–24–F	12 700	14 398	5995	6097	4697	11.5	0.0	7.1	81.3
2082	165–40–F	12 626	14 287	6573	10548	4792	6.3	0.0	11.2	82.5
2083	165–16–F	12 660	14 531	7740	9157	5126	9.5	0.0	9.2	81.3
2084	110–48–F	12 305	14 408	3484	10655	4604	7.0	0.0	9.8	83.1
2085	55–40–F	12 622	14 421	7364	9496	4865	9.4	0.0	8 9	81.7
2086	55–24–F	12 586	14 513	3244	5777	4358	11.0	0.1	8.1	80.8
2087	110–16–F	12 272	14 220	5656	6711	4393	10.1	0.0	8.6	81.3
2088	220–16–F	12 095	14 418	2770	8456	4132	8.7	0.0	9.6	81.7
2089	220–40–F	12 356	14 431	5914	10926	3691	4.3	0.4	12.2	83.1
2092	165–32–F	12 620	14 368	5266	10165	5022	6.0	0.1	11.5	82.5
2093	165–48–F	12 551	14 429	5928	10295	6473	4.7	0.2	12.4	82.7
2094	110–56–F	12 426	14 292	6159	10447	5525	6.0	0.1	11.8	82.1
2095	55–48–F	12 315	14 385	8983	8508	4670	9.4	0.0	9.7	80.9
2096	55–40–F									
2097	55–32–F									
2098	55–48–F									
2090	110–24–F	12 594	14 429	7406	8440	5118	8.9	0.1	9.8	81.3
2091	165–32–F	12 626	14 423	7885	9831	5370	7.2	0.0	10.5	82.3

TABLE 20.
WATER AND DRAWBAR PULL.

Test No.	Laboratory Designation	Water						Drawbar Pull, lb.
		Delivered to Boiler by Injectors, lb.	Weight of Water in Boiler at Start of Test Minus Weight in Boiler at Close of Test, lb.	Correction for Change of Water Level and Steam Pressure in Boiler, Start to Close, lb.	Loss From Boiler, lb.	Loss From Boiler Corrected, lb.	Presumably Evaporated, lb.	
	Code Item	476	477	478	479	480	481	487
2009	138–16–F	45 314	+294	+122			45 436	
2010	193–20–F	28 122	+142	+147			28 269	
2012	138–24–F	43 727	+ 47	+ 34			43 761	10 140
2013	138–32–F	43 335	+317	+277			43 612	12 772
2014	193–32–F	38 210	−185	−114			38 096	8 823
2015	193–24–F	41 286	+140	+136			41 422	6 469
2016	193–16–F	48 590	− 94	− 76			48 514	4 320
2017	83–16–F	44 488	−182	−149			44 339	9 222
2018	83–24–F	47 725	−279	−184			47 542	13 215
2019	83–32–F	48 834	−313	−224			48 610	17 522
2020	83–24–F	35 196	+ 94	+104			35 300	13 072
2021	83–16–F	28 176	− 50	− 44			28 132	8 931
2022	83–32–F	43 759	0	− 18			43 741	17 292
2023	138–40–F	52 634	+141	+165			52 799	15 911
2024	55–24–F	30 466	−144	−111			30 355	14 528
2026	110–16–F	35 634	+ 49	+ 43			35 677	7 839
2027	110–24–F	52 496	+ 47	+ 60			52 556	11 903
2028	55–32–F	39 512	+ 51	+ 37			39 549	20 048
2029	110–32–F	40 044	+250	+179			40 223	15 444
2030	165–24–F	43 818	+ 47	+ 59			43 876	8 852
2031	83–40–F	41 688	+ 49	+ 17			41 704	20 947
2032	165–32–F	20 484	+144	+138			20 622	11 343
2033	110–48–F	42 984	+203	+137			43 121	18 946
2034	193–40–F	38 656	+193	+186			38 842	10 009
2035	110–40–F	38 650	+288	+152			38 802	17 426
2037	165–40–F	38 911	−659	−471			38 440	12 756
2038	55–24–F	27 068	−203	−129			26 939	14 998
2039	110–32–F	41 779	+ 51	+ 87			41 886	15 477
2040	165–40–F	37 933	+152	+197			38 130	13 869
2041	110–40–F	39 277	−152	− 5			39 272	18 895
2042	110–24–F	45 085	+ 50	− 7			45 078	12 680
2043	110–48–F	44 973	+355	+341			45 314	21 800
2044	110–56–F	43 838	+101	+260			44 098	23 666
2045	110–16–F	37 376	− 51	− 37			37 339	8 212
2072	110–40–F	33 886	0	+ 24			33 910	20 877
2073	110–32–F	37 169	+260	+203			37 372	16 961
2074	165–32–F	34 914	0	− 18			34 896	13 486
2075	55–32–F	38 781	−156	−111			38 670	20 483
2076	220–32–F	23 011	−153	−109			22 902	10 396
2077	110–24–F	40 308	−106	− 45			40 263	12 512
2078	165–24–F	33 546	−102	− 98			33 448	10 188
2079	220–24–F	32 074	−253	−225			31 849	8 270
2080	110–16–F	37 696	0	− 10			37 686	
2081	55–24–F	35 815	0	0			35 815	
2082	165–40–F	34 198	0	+ 34			34 232	14 783
2083	165–16–F	36 739	− 50	− 36			36 703	7 078
2084	110–48–F	32 266	+260	+177			32 443	22 403
2085	55–40–F	41 387	+102	+ 72			41 459	24 833
2086	55–24–F	40 553	+ 51	+ 46			40 599	15 532
2087	110–16–F	43 365	−102	− 79			43 286	8 135
2088	220–16–F	37 413	0	+ 18			37 431	5 568
2089	220–40–F	26 703	+102	+ 73			26 776	11 831
2092	165–32–F	28 808	+360	+247			29 055	13 701
2093	165–48–F	23 767	+675	+441			24 208	17 660
2094	110–56–F	18 810	+ 51	+ 88			18 898	25 225
2095	55–48–F	23 813	0	− 8			23 805	28 922
2096	55–40–F	31 887	−102	− 55			31 832	24 980
2097	55–32–F	33 084	+197	+140			33 224	20 820
2098	55–48–F	20 353	+ 60	+ 62			20 415	29 240
2090	110–24–F	21 688	0	0			21 688	11 477
2091	165–32–F	16 786	− 51	− 19			16 767	12 024

Table 21.

Events of Stroke from Indicator Cards—Cut-off and Release.

Test No.	Laboratory Designation	Cut Off, Per cent of Stroke					Release, Per cent of Stroke				
		Right Side		Left Side		Average	Right Side		Left Side		Average
		Head End	Crank End	Head End	Crank End		Head End	Crank End	Head End	Crank End	
	Code Item☞	495	496	497	498	499	510	511	512	513	514
2009	138–16–F	14.0	19.0	17.4	17.8	17.1	51.1	57.4	54.0	59.7	55.6
2010	193–20–F	20.1	20.6	18.0	18.0	19.2	55.4	62.3	61.0	61.9	60.2
2012	138–24–F	24.4	22.7	22.4	23.8	23.3	63.9	66.9	63.7	65.9	65.1
2013	138–32–F	29.6	33.5	30.5	29.7	30.8	67.1	69.6	67.8	70.8	68.8
2014	193–32–F	33.4	33.1	29.0	30.1	31.4	67.0	71.2	67.3	68.0	68.4
2015	193–24–F	22.2	24.0	21.8	22.9	22.7	61.1	64.9	59.1	65.8	62.7
2016	193–16–F	16.1	17.6	14.2	17.8	16.4	53.6	60.8	55.9	60.5	57.7
2017	83–16–F	15.0	17.1	16.3	19.2	16.9	49.9	56.4	52.4	57.5	54.1
2018	83–24–F	20.7	22.9	22.7	23.9	22.6	56.4	63.2	59.3	60.3	59.8
2019	83–32–F	28.3	32.3	28.7	33.2	30.6	67.6	66.7	65.2	65.2	66.2
2020	83–24–F	18.3	25.5	21.4	24.4	22.4	57.1	62.6	60.4	61.8	60.5
2021	83–16–F	12.3	15.2	14.3	17.6	14.9	48.9	54.3	50.9	54.6	52.2
2022	83–32–F	27.0	31.7	28.0	34.5	30.3	62.9	66.8	66.0	70.4	66.5
2023	138–40–F	34.0	43.1	39.3	41.2	39.4	72.1	74.5	73.2	74.9	73.7
2024	55–24–F										
2026	110–16–F	15.2	24.3	18.1	18.7	19.1	52.6	55.5	56.1	59.3	55.9
2027	110–24–F	22.0	25.4	22.6	27.0	24.3	62.3	64.5	63.8	66.9	64.4
2028	55–32–F	28.6	32.1	31.4	34.8	31.7	65.6	66.7	68.9	69.9	67.8
2029	110–32–F	28.3	30.9	30.0	33.7	30.7	69.4	70.1	68.3	72.8	70.2
2030	165–24–F	20.8	22.9	23.3	26.4	23.4	57.8	65.0	68.0	64.8	63.9
2031	83–40–F	35.9	40.4	39.1	42.4	39.5	71.6	73.9	73.9	73.9	73.3
2032	165–32–F	28.5	31.4	27.9	33.0	30.2	65.1	71.1	70.8	72.9	70.0
2033	110–48–F	37.7	40.4	40.8	43.8	40.7	73.9	76.4	75.3	74.9	75.1
2034	193–40–F	38.3	44.9	39.1	43.4	41.4	77.0	78.2	78.3	79.3	78.2
2035	110–40–F	36.3	39.6	41.8	41.9	39.9	73.7	75.0	76.4	75.6	75.2
2037	165–40–F	38.9	41.6	36.7	43.0	40.1	74.5	75.3	75.7	77.2	75.7
2038	55–24–F										
2039	110–32–F	29.9	34.9	31.8	33.4	32.5	65.9	70.7	68.8	70.3	68.9
2040	165–40–F	41.1	41.4	41.9	41.4	41.5	75.2	76.7	76.1	76.8	76.2
2041	110–40–F	39.6	42.2	41.9	40.8	41.1	73.6	76.9	75.8	75.5	75.5
2042	110–24–F	23.6	23.5	27.9	24.3	24.8	59.5	62.8	63.5	63.8	62.4
2043	110–48–F	47.6	49.6	48.0	48.7	48.5	79.5	79.3	83.0	79.5	80.3
2044	110–56–F	56.2	56.9	60.5	56.5	57.5	81.6	82.2	86.3	81.2	71.8
2045	110–16–F	18.4	16.8	23.5	18.2	19.2	52.0	55.5	56.5	55.6	54.9
2072	110–40–F	41.5	41.5	41.8	41.0	41.5	75.1	74.8	75.0	75.6	75.1
2073	110–32–F	25.6	29.8	31.6	31.5	29.6	73.4	68.5	70.4	69.8	70.5
2074	165–32–F	20.3	32.7	32.5	29.7	28.8	67.7	70.8	72.3	69.5	70.1
2075	55–32–F	29.9	33.9	33.7	31.0	32.1	66.7	68.5	70.5	69.2	68.7
2076	220–32–F	29.3	33.3	31.4	34.9	32.2	68.3	68.1	63.2	67.3	66.7
2077	110–24–F	21.5	24.4	25.5	24.4	24.0	56.1	61.7	63.0	63.1	61.0
2078	165–24–F	22.0	24.7	26.6	22.8	24.0	58.9	64.6	64.5	65.4	63.4
2079	220–24–F	24.1	23.6	22.7	23.1	23.4	69.7	67.3	66.8	63.5	66.8
2080	110–16–F	15.3	16.6	18.9	16.8	16.9	50.6	53.0	61.0	59.3	56.0
2081	55–24–F	23.1	24.8	26.2	22.4	24.1	57.8	60.1	62.2	61.8	60.5
2082	165–40–F	36.0	45.3	41.9	42.3	41.4	74.2	72.9	76.9	75.7	74.9
2083	165–16–F	16.2	18.2	21.8	17.4	18.4	51.4	57.5	60.8	58.7	57.1
2084	110–48–F	47.9	49.9	48.9	47.0	48.4	79.2	79.8	80.9	78.4	79.6
2085	55–40–F	39.7	42.0	43.3	40.2	41.3	70.3	74.4	77.6	74.6	74.2
2086	55–24–F	22.0	23.8	25.7	22.0	23.4	58.5	61.3	63.2	62.3	61.3
2087	110–16–F	15.9	15.7	17.4	17.4	16.6	49.3	54.0	56.5	54.2	53.5
2088	220–16–F	16.4	16.9	14.9	15.3	15.9	59.3	58.7	57.4	56.3	57.9
2089	220–40–F	41.2	45.4	42.0	45.4	43.5	74.5	77.8	78.4	77.4	77.0
2092	165–32–F	30.1	31.1	30.8	29.5	30.4	70.0	70.2	70.8	70.3	70.3
2093	165–48–F	45.9	50.5	48.4	49.7	48.6	78.5	79.6	79.7	77.1	78.7
2094	110–56–F	55.7	56.3	61.1	54.7	57.0	83.1	84.5	86.9	82.5	**84.3**
2095	55–48–F	47.8	50.1	51.5	47.5	49.2	80.0	79.8	81.8	79.6	80.3
2096	55–40–F	38.1	41.3	43.2	39.1	40.4	73.2	74.0	77.6	74.7	74.9
2097	55–32–F	30.6	33.2	35.0	30.5	32.3	66.4	69.4	71.0	69.0	69.0
2098	55–48–F	47.7	50.1	51.5	47.2	49.1	79.5	80.2	82.7	80.2	80.7
2090	110–24–F	20.9	22.3	23.4	24.6	22.8	56.5	66.1	64.2	64.2	62.8
2091	165–32–F	27.5	34.0	27.6	26.0	28.8	68.5	70.5	71.8	71.3	70.5

TABLE 22.

EVENTS OF STROKE AND PRESSURE FROM INDICATOR CARDS—BEGINNING OF COMPRESSION AND INITIAL PRESSURE.

Test No.	Laboratory Designation	Beginning of Compression, Per cent of Stroke					Initial Pressure, lb. per sq. in.				
		Right Side		Left Side		Average	Right Side		Left Side		Average
		Head End	Crank End	Head End	Crank End		Head End	Crank End	Head End	Crank End	
	Code Item ☞	525	526	527	528	529	540	541	542	543	544
2009	138–16–F	56.0	58.3	56.7	56.2	56.8	170.9	188.6	173.0	186.2	179.7
2010	193–20–F	57.5	66.8	71.1	59.7	63.8	150.5	159.6	144.5	162.6	154.3
2012	138–24–F	50.2	59.5	48.2	55.8	53.4	166.4	182.3	170.8	180.8	175.1
2013	138–32–F	47.4	58.7	50.4	49.1	51.4	165.0	177.9	167.7	173.1	170.9
2014	193–32–F	77.5	76.0	80.8	59.1	73.4	156.4	154.3	166.8	173.0	162.5
2015	193–24–F	79.7	73.9	78.2	76.9	77.2	159.7	155.7	162.9	160.9	159.8
2016	193–16–F	64.0	71.2	68.5	67.2	67.7	156.1	163.6	150.1	165.1	158.7
2017	83–16–F	50.4	58.1	49.4	56.5	53.6	177.3	187.4	177.1	181.8	180.9
2018	83–24–F	43.9	50.8	43.3	53.1	47.8	178.8	179.4	176.4	190.2	181.2
2019	83–32–F	38.2	42.4	35.8	37.6	38.5	178.3	178.6	176.5	178.7	178.0
2020	83–24–F	39.2	43.6	39.2	42.8	41.2	175.1	177.0	177.6	180.0	177.4
2021	83–16–F	46.2	51.5	46.1	51.6	48.9	182.9	182.9	181.8	185.5	183.3
2022	83–32–F	31.2	42.6	29.0	35.5	34.6	175.5	156.6	178.7	182.1	173.2
2023	138–40–F	33.7	35.8	30.8	34.9	33.8	168.7	178.6	169.9	178.8	174.0
2024	55–24–F										
2026	110–16–F	51.5	54.6	51.6	55.8	53.4	191.1	181.1	187.5	175.9	183.9
2027	110–24–F	44.9	47.6	44.6	44.8	45.5	185.4	174.5	189.2	176.2	181.3
2028	55–32–F	32.1	35.0	35.6	36.6	34.8	186.1	184.7	192.2	191.7	188.7
2029	110–32–F	38.3	45.1	35.6	41.0	40.0	181.8	180.4	186.8	192.2	185.3
2030	165–24–F	43.6	63.4	48.3	53.6	52.2	161.3	171.9	167.4	179.7	170.1
2031	83–40–F	28.7	32.3	30.0	31.9	30.7	183.7	177.9	187.7	186.9	184.1
2032	165–32–F	42.1	48.4	77.5	68.3	59.1	185.8	166.2	162.6	166.0	170.2
2033	110–48–F	29.7	33.7	30.5	34.1	32.0	185.6	178.4	184.2	189.3	184.4
2034	193–40–F	62.0	70.0	71.8	71.6	68.9	168.7	172.9	179.7	170.8	173.0
2035	110–40–F	33.5	36.7	30.0	33.1	33.3	178.7	172.9	183.3	187.7	180.7
2037	165–40–F	61.6	68.5	67.4	66.5	66.0	176.7	161.4	159.3	179.8	169.3
2038	55–24–F										
2039	110–32–F	41.2	46.3	40.0	39.8	41.8	175.0	194.2	181.8	189.6	185.2
2040	165–40–F	73.9	75.5	74.5	74.6	74.6	184.5	195.7	192.1	184.6	189.2
2041	110–40–F	34.5	37.3	36.2	32.8	35.2	175.5	186.1	179.6	183.1	181.1
2042	110–24–F	46.3	51.8	47.2	46.9	48.1	180.9	176.1	188.0	173.9	179.7
2043	110–48–F	28.2	30.2	30.4	25.7	28.6	191.7	188.2	181.1	194.7	188.9
2044	110–56–F	23.8	23.3	25.7	22.4	23.8	180.1	190.8	191.4	187.4	187.4
2045	110–16–F	54.9	56.7	52.8	53.4	54.5	185.9	181.1	193.4	195.9	189.1
2072	110–40–F	30.5	30.7	31.3	29.3	30.5	183.8	187.4	185.5	186.7	185.9
2073	110–32–F	36.8	41.4	37.7	35.5	37.9	184.8	190.3	189.7	190.2	188.8
2074	165–32–F	43.3	43.0	41.2	44.2	42.9	178.9	173.7	178.1	173.9	176.2
2075	55–32–F	31.5	36.6	37.8	33.2	34.8	192.6	195.7	193.8	197.1	194.8
2076	220–32–F	51.4	50.5	47.4	48.9	49.6	182.7	184.5	193.1	186.5	186.7
2077	110–24–F	48.6	47.2	48.5	47.1	47.9	185.5	193.9	190.8	191.1	190.3
2078	165–24–F	49.5	51.3	48.6	50.4	50.0	179.9	177.5	168.7	173.6	175.0
2079	220–24–F	52.2	54.7	52.3	53.5	53.2	185.1	166.4	179.6	186.1	179.3
2080	110–16–F	54.6	54.6	53.1	52.6	53.7	190.3	178.5	176.4	179.2	181.1
2081	55–24–F	41.3	45.5	44.8	41.8	43.4	193.0	192.8	190.3	197.3	193.4
2082	165–40–F	33.2	36.4	34.0	33.0	34.2	191.9	170.6	163.6	171.6	174.4
2083	165–16–F	51.6	59.7	48.5	54.3	53.5	169.8	182.1	173.9	176.9	175.7
2084	110–48–F	25.7	24.8	16.4	22.8	24.9	179.4	181.8	179.8	187.1	182.0
2085	55–40–F	27.3	28.1	29.6	23.7	27.2	189.4	193.6	192.9	194.2	192.5
2086	55–24–F	39.2	41.7	39.8	37.4	39.2	192.7	192.1	189.4	193.4	191.9
2087	110–16–F	53.1	51.1	50.7	49.1	51.0	176.3	175.3	174.2	177.8	175.9
2088	220–16–F	54.9	52.5	52.1	52.4	53.0	136.4	134.0	160.9	159.9	147.8
2089	220–40–F	44.0	43.7	42.9	41.2	43.0	163.2	162.0	160.2	170.8	164.1
2092	165–32–F	39.1	41.5	39.9	41.3	40.5	163.3	172.5	162.7	171.8	167.6
2093	165–48–F	27.6	26.2	25.5	24.8	26.0	165.4	165.4	157.2	166.7	163.7
2094	110–56–F	19.5	17.7	20.7	15.5	18.4	172.4	171.3	170.6	178.9	173.3
2095	55–48–F	20.4	22.0	22.5	19.2	21.0	187.3	189.9	187.9	190.7	189.0
2096	55–40–F	25.1	27.1	27.8	23.6	25.9	187.8	187.9	189.1	190.2	188.8
2097	55–32–F	30.4	33.6	34.3	29.9	32.1	192.9	191.9	192.4	192.4	192.4
2098	55–48–F	21.6	22.0	23.3	19.5	21.6	188.7	191.4	191.8	192.9	191.2
2090	110–24–F	44.4	46.1	41.4	42.6	43.6	184.7	172.4	171.2	173.4	175.4
2091	165–32–F	39.4	43.0	36.1	39.6	39.5	164.4	174.1	165.4	175.5	169.9

Table 23.

Pressure from Indicator Cards—Cut-off and Release.

Test No.	Laboratory Designation	Pressure at Cut-Off, lb. per sq. in.					Pressure at Release, lb. per sq. in.				
		Right Side		Left Side		Average	Right Side		Left Side		Average
		Head End	Crank End	Head End	Crank End		Head End	Crank End	Head End	Crank End	
	Code Item ☞	566	567	568	569	570	581	582	583	584	585
2009	138–16–F	146.6	129.7	130.6	132.0	134.7	44.0	45.7	46.4	42.8	44.7
2010	193–20–F	108.5	129.0	119.5	131.0	122.0	40.5	45.5	39.5	43.5	42.3
2012	138–24–F	130.7	138.0	131.3	136.8	134.2	45.6	54.0	48.4	53.4	50.4
2013	138–32–F	123.8	134.6	122.0	139.4	130.0	52.9	64.9	55.3	63.3	59.1
2014	193–32–F	101.3	123.5	114.0	125.0	116.0	49.3	57.8	48.2	57.0	53.1
2015	193–24–F	114.8	123.6	114.0	131.8	121.1	42.8	49.4	47.2	49.1	47.1
2016	193–16–F	113.9	133.0	114.6	132.0	123.4	38.8	41.3	34.8	42.2	39.3
2017	83–16–F	152.0	162.7	146.0	155.3	154.0	51.7	55.5	52.6	54.5	53.6
2018	83–24–F	148.0	157.7	142.0	156.7	151.1	57.9	60.5	59.1	66.4	61.0
2019	83–32–F	150.3	158.0	145.9	155.1	152.3	58.9	75.0	66.6	79.6	70.0
2020	83–24–F	156.2	147.3	154.1	155.6	153.3	58.0	61.6	60.8	66.4	61.7
2021	83–16–F	162.0	168.9	164.5	167.0	165.6	51.1	56.3	52.6	61.7	56.7
2022	83–32–F	149.1	129.2	151.4	154.7	146.1	65.6	66.0	70.2	77.7	69.9
2023	138–40–F	136.9	133.6	121.1	146.2	134.5	65.7	76.8	66.3	81.3	72.5
2024	55–24–F										
2026	110–16–F	145.6	123.6	133.2	151.6	138.4	45.1	54.7	45.1	51.6	49.1
2027	110–24–F	139.7	146.8	143.9	144.3	143.7	49.0	58.1	51.1	57.3	53.9
2028	55–32–F	154.9	165.3	161.7	167.3	162.3	67.4	80.1	72.2	81.7	75.4
2029	110–32–F	135.3	150.7	143.7	154.5	141.1	52.0	67.9	62.7	74.0	64.2
2030	165–24–F	122.0	139.7	129.1	136.4	131.8	48.4	56.1	43.4	61.2	52.3
2031	83–40–F	149.0	150.5	149.5	154.6	150.9	72.5	80.5	77.0	85.8	79.0
2032	165–32–F	126.2	131.8	133.0	134.7	131.4	56.4	62.5	52.6	61.8	58.3
2033	110–48–F	139.7	146.0	141.0	145.6	143.1	70.0	75.3	75.3	82.3	75.7
2034	193–40–F	111.5	112.2	111.1	123.4	114.5	51.8	62.1	53.1	64.1	57.8
2035	110–40–F	139.5	141.4	133.9	148.2	140.8	66.1	74.7	71.0	80.3	73.0
2037	165–40–F	117.9	124.2	125.3	132.2	124.9	58.2	69.5	60.5	72.1	65.1
2038	55–24–F										
2039	110–32–F	135.4	152.6	139.7	150.3	146.8	60.8	73.2	62.4	70.6	66.8
2040	165–40–F	119.7	134.9	120.3	127.4	125.6	63.7	71.2	63.6	66.8	66.3
2041	110–40–F	136.3	150.8	141.0	149.7	144.5	71.6	80.3	75.3	78.0	76.3
2042	110–24–F	140.7	158.6	135.7	149.6	146.2	55.5	62.6	56.7	56.5	57.8
2043	110–48–F	146.5	156.1	149.8	153.0	151.4	85.1	94.5	83.8	91.9	88.8
2044	110–56–F	144.8	154.4	139.5	152.8	147.9	96.8	103.9	92.8	105.9	99.9
2045	110–16–F	136.5	164.1	121.3	146.7	142.2	47.4	54.4	47.4	51.6	50.2
2072	110–40–F	139.2	149.1	145.2	149.4	145.7	73.1	80.9	79.4	79.7	78.3
2073	110–32–F	155.4	157.7	149.4	148.1	152.7	51.6	70.9	68.1	67.1	64.2
2074	165–32–F	153.4	135.2	129.1	139.7	139.4	56.8	63.5	56.2	62.4	59.7
2075	55–32–F	166.3	167.0	166.1	163.8	165.8	77.1	79.9	79.8	75.4	78.1
2076	220–32–F	116.7	117.4	113.2	116.2	115.9	51.1	58.8	55.5	57.5	55.7
2077	110–24–F	142.3	148.9	142.3	145.7	144.8	57.4	59.5	57.1	58.4	58.1
2078	165–24–F	135.2	139.0	125.8	135.9	134.0	51.7	56.0	50.5	51.1	52.3
2079	220–24–F	112.9	121.6	124.3	119.9	119.7	36.0	47.3	44.5	43.7	42.9
2080	110–16–F	147.9	161.2	146.3	148.8	151.1	48.9	54.1	45.2	43.3	47.9
2081	55–24–F	163.2	169.9	164.6	170.6	167.1	68.8	70.5	72.0	65.1	69.1
2082	165–40–F	134.3	124.3	123.6	124.8	126.8	64.1	76.3	64.3	68.6	68.3
2083	165–16–F	139.3	142.1	129.1	132.7	135.8	46.2	47.0	43.2	41.5	44.5
2084	110–48–F	139.1	147.6	145.8	149.1	145.4	83.8	88.6	85.6	87.1	86.3
2085	55–40–F	162.1	169.1	164.4	165.0	165.2	91.4	90.5	90.6	85.3	89.5
2086	55–24–F	167.3	173.7	167.2	175.4	170.9	67.4	68.2	72.0	65.3	68.2
2087	110–16–F	145.9	164.8	152.6	147.8	152.8	50.9	53.4	49.6	51.6	51.4
2088	220–16–F	99.6	108.0	126.6	130.4	116.2	25.5	32.6	34.3	35.2	31.9
2089	220–40–F	107.3	112.0	107.8	108.2	108.8	57.7	63.1	54.5	60.6	59.0
2092	165–32–F	128.3	138.9	132.7	139.8	134.9	52.4	63.6	57.7	59.3	58.3
2093	165–48–F	128.5	131.7	127.2	129.8	129.3	73.9	82.5	75.4	81.7	78.4
2094	110–56–F	148.6	154.6	144.9	156.4	151.1	98.9	99.9	98.1	102.8	99.9
2095	55–48–F	170.2	170.6	169.3	171.3	170.4	101.0	102.7	105.2	101.2	102.5
2096	55–40–F	169.9	171.0	168.2	172.6	170.4	90.8	93.4	92.7	89.9	91.7
2097	55–32–F	165.4	171.4	165.9	172.9	168.9	81.2	81.0	83.2	78.2	80.9
2098	55–48–F	168.9	176.5	171.9	175.5	173.2	100.5	105.2	106.1	100.6	103.1
2090	110–24–F	145.1	155.8	151.2	143.4	148.9	55.8	53.4	54.4	55.0	54.7
2091	165–32–F	133.8	129.8	143.4	147.9	138.7	54.2	63.9	56.7	58.8	58.4

Table 24.

Pressure from Indicator Cards—Beginning of Compression and Least Back Pressure.

Test No.	Laboratory Designation	Pressure at Beginning of Compression, lb. per sq. in.					Least Back Pressure, lb. per sq. in.				
		Right Side		Left Side		Average	Right Side		Left Side		Average
		Head End	Crank End	Head End	Crank End		Head End	Crank End	Head End	Crank End	
	Code Item #	596	597	598	599	600	611	612	613	614	615
2009	138–16–F	7.2	9.3	10.9	9.1	9.1	3.9	3.8	7.0	6.4	5.3
2010	193–20–F	10.4	11.5	9.0	13.3	11.1	7.3	7.6	9.0	9.0	8.2
2012	138–24–F	11.5	12.2	14.0	10.4	12.0	7.0	7.8	7.8	8.2	7.7
2013	138–32–F	15.0	16.8	16.4	16.0	16.1	9.8	12.7	10.4	13.4	11.6
2014	193–32–F	15.8	17.5	16.8	17.3	16.9	15.4	17.9	16.6	18.3	17.1
2015	193–24–F	11.6	13.0	13.8	13.3	12.9	10.2	13.5	11.3	13.4	12.1
2016	193–16–F	9.2	8.8	10.5	10.8	9.8	6.3	6.7	5.8	6.9	6.4
2017	83–16–F	2.8	4.6	4.1	3.1	3.7	2.7	2.1	2.4	3.2	2.6
2018	83–24–F	4.5	6.6	5.9	4.8	5.5	4.6	3.4	3.1	4.7	4.0
2019	83–32–F	7.1	10.6	9.1	8.0	8.7	4.5	5.6	5.1	5.4	5.2
2020	83–24–F	5.3	8.0	9.3	8.4	7.8	3.6	2.4	6.6	7.8	5.1
2021	83–16–F	4.0	5.9	3.8	5.1	6.0		2.2	6.1	5.3	4.4
2022	83–32–F	6.7	9.4	13.3	11.2	10.2	4.1	7.2	8.2	9.4	7.2
2023	138–40–F	22.4	27.3	28.2	28.4	26.6	17.1	18.5	20.0	22.3	19.5
2024	55–24–F										
2026	110–16–F	5.8	8.1	9.6	5.7	7.3	2.3	2.8	4.1	3.2	3.1
2027	110–24–F	8.2	11.3	11.0	9.4	10.0	4.7	5.9	6.3	7.4	6.1
2028	55–32–F	2.2	5.5	5.9	1.5	3.8					
2029	110–32–F	8.2	12.7	16.5	16.0	13.4	8.1	10.4	10.0	13.6	10.5
2030	165–24–F	13.4	13.7	16.6	20.6	16.1	10.5	12.1	9.0	14.2	11.5
2031	83–40–F	10.5	11.7	14.4	13.4	12.5	9.6	10.0	10.6	10.8	10.3
2032	165–32–F	18.8	20.0	16.2	18.7	18.4					
2033	110–48–F	15.4	19.9	21.4	18.8	18.9	13.8	15.3	16.9	14.6	15.2
2034	193–40–F	21.6	23.8	24.7	26.7	24.2	22.8	24.0	24.1	25.6	24.1
2035	110–40–F	14.9	17.2	22.1	18.5	18.2	11.9	15.7	16.0	15.0	14.7
2037	165–40–F	21.1	22.8	22.1	25.1	22.8	21.8	23.4	22.5	24.7	18.1
2038	55–24–F										
2039	110–32–F	12.6	16.8	15.8	14.4	14.9	8.5	11.5	8.4	11.1	9.9
2040	165–40–F	24.4	24.8	23.8	23.5	24.1	22.5	24.8	22.9	24.7	23.7
2041	110–40–F	18.5	21.1	21.4	18.0	19.8	12.4	16.0	14.0	15.3	14.4
2042	110–24–F	10.4	10.7	11.2	9.6	10.5	5.0	7.0	5.4	6.4	6.0
2043	110–48–F	23.0	26.8	26.8	25.1	25.4	18.5	22.0	20.0	21.4	20.5
2044	110–56–F	27.9	30.6	29.9	28.7	29.3	22.5	24.9	22.9	22.4	23.2
2045	110–16–F	5.9	7.2	7.6	7.1	7.0	3.0	3.8	3.1	3.3	3.3
2072	110–40–F	12.8	15.1	14.9	13.7	14.1	8.3	10.1	8.9	10.0	9.3
2073	110–32–F	9.1	10.9	11.6	10.0	10.4	5.9	7.1	5.6	7.2	6.5
2074	165–32–F	15.5	19.5	20.5	18.5	18.5	11.4	13.9	11.3	13.5	12.5
2075	55–32–F	4.1	3.1	2.1	2.4	2.9	2.2	1.9	1.2	1.6	1.7
2076	220–32–F	18.3	21.9	21.5	22.5	21.1	14.5	17.9	14.0	16.9	15.8
2077	110–24–F	5.6	8.5	6.6	6.5	6.8	3.0	3.4	2.8	3.5	3.2
2078	165–24–F	10.3	14.2	12.5	12.0	12.3	5.1	8.8	7.1	8.4	7.4
2079	220–24–F	12.3	15.9	15.7	15.3	14.8	8.3	12.0	8.2	10.7	9.8
2080	110–16–F	4.3	5.6	5.6	4.5	5.0	2.0	2.1	2.0	2.0	2.0
2081	55–24–F	2.5	2.4	1.9	2.1	2.2	2.0	0.3	0.3	0.3	0.7
2082	165–40–F	21.6	24.4	26.6	25.1	24.4	17.2	20.0	17.7	18.2	18.3
2083	165–16–F	7.7	8.9	11.5	9.1	9.3	3.3	4.9	3.3	4.2	3.9
2084	110–48–F	15.4	17.9	18.4	17.0	17.2	12.6	13.3	10.0	13.0	12.2
2085	55–40–F	3.5	4.1	4.2	3.8	3.9	1.5	1.2	2.6	1.9	1.8
2086	55–24–F	2.7	2.3	2.1	2.8	2.5	1.6	0.4	0.0	0.5	0.6
2087	110–16–F	4.1	6.2	6.0	4.9	5.3	2.2	2.2	2.2	2.7	2.3
2088	220–16–F	7.5	9.4	10.4	11.2	9.6	4.8	7.5	3.4	4.7	5.1
2089	220–40–F	25.2	30.3	30.2	29.7	28.9	19.8	24.5	20.8	23.0	22.0
2092	165–32–F	15.5	18.0	18.3	17.2	17.3	9.6	14.0	10.8	12.0	11.6
2093	165–48–F	28.7	29.9	31.7	29.5	30.0	20.4	24.0	21.1	23.0	22.1
2094	110–56–F	21.9	22.1	22.9	22.3	22.3	16.0	17.1	16.3	15.8	16.3
2095	55–48–F	5.2	4.1	4.2	4.2	4.4	2.3	1.7	1.1	2.0	1.8
2096	55–40–F	6.2		4.2	3.8		2.8	1.0	1.4	1.5	1.7
2097	55–32–F	5.4	3.0	3.8	3.6	4.0	3.1	1.9	2.1	1.2	2.1
2098	55–48–F	4.6	4.8	5.1	4.0	4.6	1.7	2.0	2.3	1.9	2.0
2090	110–24–F	8.7	11.5	11.3	10.0	10.4	5.4	6.0	5.0	5.4	5.5
2091	165–32–F	21.7	24.4	26.5	23.4	24.0	14.2	17.1	14.7	16.4	15.6

Table 25.

Boiler Performance—Coal and Evaporation.

Test No.	Laboratory Designation	Dry Coal Fired, lb.		Evaporation					Steam Used at Calorimeter, Safety Valve, Leaks etc., lb.	Dry Steam to Engine per Hour, lb.	Factor of Evaporation
					Dry Steam, lb.						
		Per Hour	Per Hour per sq. ft. of Grate Surface	Moist Steam per Hour, lb.	Per Hour	Per Hour per sq. ft. of Heating Surface	Per lb. of Dry Coal	Per lb. of Coal as Fired			
	Code Items	626	627	633	634	635	636	637	638	639	641
2009	138–16–F	2647	53.4	18 174	18 027	5.49	6.80	6.01	183	18 023	1.192
2010	193–20–F	3834	77.4	23 887	23 668	7.21	6.17	5.44	39	23 631	1.196
2012	138–24–F	3707	74.8	23 869	23 674	7.21	6.39	5.67	89	23 632	1.194
2013	138–32–F	4749	95.8	29 076	28 751	8.76	6.05	5.51	50	28 742	1.190
2014	193–32–F	6199	125.1	32 648	32 272	9.83	5.21	4.72	39	32 173	1.189
2015	193–24–F	4927	99.5	27 617	27 363	8.33	5.55	4.95	50	27 330	1.193
2016	193–16–F	3255	65.7	20 792	20 623	6.28	6.34	5.59	183	20 538	1.193
2017	83–16–F	1957	39.5	14 778	14 683	4.47	7.50	6.74	246	14 590	1.186
2018	83–24–F	2537	51.2	17 828	17 737	5.40	6.99	6.23	157	17 643	1.195
2019	83–32–F	3215	64.9	22 429	22 288	6.79	6.93	6.20	150	22 173	1.196
2020	83–24–F	2472	49.9	17 650	17 560	5.35	7.10	6.48	64	17 523	1.195
2021	83–16–F	2211	44.6	14 066	14 011	4.27	6.34	5.56	502	13 758	1.204
2022	83–32–F	3673	74.1	21 871	21 801	6.64	5.94	5.32	64	21 778	1.204
2023	138–40–F	6687	135.0	35 199	35 025	10.67	5.24	4.54	623	34 594	1.202
2024	55–24–F	1814	36.6	15 178	15 123	4.61	8.34	7.37	265	14 974	1.203
2026	110–16–F	2293	46.3	16 461	16 341	4.98	7.13	6.22	71	16 313	1.196
2027	110–24–F	3256	65.7	21 022	20 892	6.36	6.42	5.60	92	20 850	1.198
2028	55–32–F	2406	48.6	16 950	16 841	5.13	7.00	6.13	169	16 775	1.188
2029	110–32–F	4242	85.6	26 815	26 629	8.11	6.28	5.50	52	26 640	1.197
2030	165–24–F	4013	81.0	26 326	26 126	7.96	6.51	5.80	63	26 081	1.197
2031	83–40–F	4244	85.6	27 804	27 598	8.41	6.50	5.39	303	27 419	1.197
2032	165–32–F	5352	108.0	30 933	30 627	9.33	5.72	4.98	105	30 480	1.195
2033	110–48–F	5126	103.5	32 341	32 030	9.76	6.25	5.38	389	31 791	1.194
2034	193–40–F	7767	156.8	38 841	38 445	11.71	4.95	4.31	414	38 330	1.194
2035	110–40–F	5565	112.3	33 253	32 856	10.01	5.90	5.05	38	32 940	1.190
2037	165–40–F	6554	132.3	38 440	38 056	11.59	5.81	4.99	102	37 769	1.193
2038	55–24–F	2012	40.6	14 967	14 625	4.45	7.27	6.38	709	14 199	1.203
2039	110–32–F	4517	91.2	27 927	27 762	8.46	6.15	5.42	109	27 663	1.201
2040	165–40–F	7482	151.0	38 130	37 901	11.54	5.07	4.45	70	37 787	1.201
2041	110–40–F	5861	118.3	33 656	33 163	10.10	5.66	4.86	290	32 794	1.193
2042	110–24–F	3356	67.7	22 539	22 431	6.83	6.68	5.77	425	22 247	1.204
2043	110–48–F	7403	149.4	38 840	38 468	11.72	5.20	4.50	314	38 213	1.196
2044	110–56–F	8361	168.7	44 098	43 780	13.34	5.23	4.55	242	43 382	1.201
2045	110–16–F	2427	49.0	17 228	17 151	5.22	7.07	6.13	388	16 968	1.201
2072	110–40–F	5927	119.6	33 910	33 656	10.25	5.68	4.95	78	33 554	1.197
2073	110–32–F	4359	87.9	28 029	27 866	8.49	6.39	5.54	238	27 731	1.203
2074	165–32–F	6015	121.3	34 896	34 551	10.52	5.74	5.11	216	34 354	1.195
2075	55–32–F	2327	47.0	16 574	16 523	5.03	7.10	6.18	171	16 431	1.204
2076	220–32–F	7831	158.0	39 261	38 820	11.82	4.96	4.20	82	38 608	1.193
2077	110–24–F	3281	66.2	21 959	21 878	6.64	6.67	5.82	141	21 770	1.203
2078	165–24–F	4707	95.0	28 668	28 493	8.68	6.05	5.25	101	28 404	1.201
2079	220–24–F	5783	116.7	31 849	31 597	9.62	5.46	4.99	224	31 347	1.198
2080	110–16–F	2422	48.9	17 392	17 336	5.28	7.16	6.23	214	17 244	1.203
2081	55–24–F	1975	39.9	14 326	14 289	4.35	7.24	6.39	211	14 205	1.206
2082	165–40–F	8994	181.5	41 078	40 738	12.41	4.53	3.99	60	40 625	1.193
2083	165–16–F	3338	67.4	22 022	21 918	6.68	6.57	5.66	425	21 656	1.201
2084	110–48–F	7914	159.7	38 932	38 646	11.77	4.88	4.24	60	38 671	1.205
2085	55–40–F	3058	61.7	20 730	20 674	6.30	6.76	5.91	146	20 616	1.219
2086	55–24–F	2068	41.7	14 329	14 290	4.35	6.91	6.08	213	14 219	1.208
2087	110–16–F	2474	49.9	17 314	17 259	5.26	6.98	6.16	189	17 175	1.204
2088	220–16–F	3353	67.7	22 459	22 328	6.80	6.66	5.77	142	22 233	1.202
2089	220–40–F	11127	224.5	45 902	45 521	13.87	4.09	3.59	42	45 498	1.198
2092	165–32–F	5640	113.8	34 866	34 601	10.54	6.13	5.25	64	34 660	1.198
2093	165–48–F	10216	206.2	48 416	47 994	14.62	4.70	4.04	36	48 387	1.197
2094	110–56–F	8434	170.2	45 355	44 988	13.71	5.33	4.58	80	44 709	1.198
2095	55–48–F	3334	67.3	23 805	23 707	7.22	7.11	6.24	73	23 643	1.202
2096	55–40–F			21 222	21 057	6.41			285	20 837	1.196
2097	55–32–F			18 124	17 933	5.46			291	17 807	1.196
2098	55–48–F			24 498	24 216	7.38			303	23 843	1.194
2090	110–24–F	3176	64.1	21 688	21 614	6.58	6.80	6.00	73	21 542	1.203
2091	165–32–F	5252	106.0	33 534	33 320	10.15	6.34	5.50	39	33 178	1.200

TABLE 26.

BOILER PERFORMANCE—EQUIVALENT EVAPORATION, HORSE POWER, AND EFFICIENCY.

Test No.	Laboratory Designation	Dry Steam Loss per Hour Due to Calorimeter, Leaks, Corrections etc., lb.	Dry Coal Loss per Hour Equivalent to Steam Loss, lb.	Equivalent Evaporation From and at 212°F., lb.						Boiler Horse Power	Efficiency of Boiler, per cent
				Per Hour	Per Hour per sq. ft. of Total Heating Surface	Per Hour per sq. ft. of Grate Area	Per lb. of Coal as Fired	Per lb. of Dry Coal	Per lb. of Combustible		
	Code Item	642	643	645	648	656	657	658	659	660	666
2009	138–16–F	4	1	21 669	6.60	437.3	7.23	8.19	9.36	628.1	63.30
2010	193–20–F	37	6	28 564	8.71	576.5	6.57	7.45	8.57	827.9	57.63
2012	138–24–F	42	6	28 514	8.69	575.4	6.83	7.69	8.96	826.5	60.80
2013	138–32–F	9	1	34 612	10.54	698.5	6.63	7.29	8.48	1003.3	57.78
2014	193–32–F	99	18	38 820	11.83	783.5	5.68	6.27	7.36	1125.2	49.92
2015	193–24–F	33	6	32 958	10.04	665.2	5.97	6.69	7.77	955.3	52.84
2016	193–16–F	85	14	24 804	7.56	500.7	6.73	7.62	9.09	719.1	61.68
2017	83–16–F	93	12	17 513	5.33	353.4	8.05	8.96	10.35	507.6	69.88
2018	83–24–F	94	13	21 326	6.50	430.4	7.49	8.41	9.81	618.1	66.49
2019	83–32–F	115	16	26 834	8.17	541.6	7.46	8.35	9.54	777.8	64.68
2020	83–24–F	37	5	21 092	6.42	425.7	7.79	8.53	9.77	611.4	67.33
2021	83–16–F	253	40	16 934	5.16	341.8	6.72	7.66	8.83	490.8	60.56
2022	83–32–F	123	4	26 332	8.02	531.4	6.42	7.17	8.46	763.3	58.54
2023	138–40–F	431	82	42 329	12.89	854.3	5.49	6.33	7.35	1226.9	49.86
2024	55–24–F	149	18	18 258	5.56	368.5	8.90	10.07	11.52	529.2	76.87
2026	110–16–F	28	4	19 700	6.00	397.6	7.50	8.59	10.01	571.0	67.75
2027	110–24–F	42	7	25 206	7.68	508.7	6.76	7.74	8.91	730.6	59.22
2028	55–32–F	66	9	20 143	6.14	406.5	7.33	8.37	9.62	583.9	64.21
2029	110–32–F	−11	− 2	32 108	9.78	648.0	6.64	7.57	8.84	930.7	58.85
2030	165–24–F	45	7	31 517	9.60	636.1	7.00	7.85	8.92	913.5	59.71
2031	83–40–F	179	28	33 281	10.14	671.7	6.49	7.84	9.22	964.7	63.43
2032	165–32–F	147	26	36 967	11.26	746.1	6.01	6.91	8.11	1071.5	54.79
2033	110–48–F	239	38	38 624	11.77	779.5	6.49	7.54	8.82	1119.5	59.76
2034	193–40–F	115	23	46 380	14.13	936.0	5.20	5.97	7.13	1344.4	48.95
2035	110–40–F	−84	−14	39 578	12.06	798.8	6.08	7.11	8.28	1147.2	55.94
2037	165–40–F	287	49	45 859	13.97	925.5	6.01	7.00	8.14	1329.3	54.54
2038	55–24–F	426	59	17 676	5.38	356.7	7.71	8.79	10.86	512.4	74.51
2039	110–32–F	99	16	33 529	10.21	676.7	6.55	7.42	8.90	971.9	61.38
2040	165–40–F	114	23	45 802	13.95	824.4	5.38	6.12	7.24	1327.6	48.84
2041	110–40–F	369	65	40 168	12.24	810.7	5.89	6.85	8.01	1164.3	54.18
2042	110–24–F	184	27	27 132	8.26	547.6	6.98	8.08	9.49	787.0	63.89
2043	110–48–F	255	49	46 472	14.16	937.9	5.43	6.28	7.30	1347.0	48.60
2044	110–56–F	398	76	52 948	16.13	1068.6	5.50	6.33	7.48	1534.7	50.38
2045	110–16–F	183	26	20 704	6.31	417.9	7.40	8.53	10.27	600.1	69.65
2072	110–40–F	102	18	40 590	12.36	819.2	5.97	6.85	7.95	1176.5	53.36
2073	110–32–F	135	21	33 719	10.27	680.5	6.71	7.74	8.79	977.4	58.92
2074	165–32–F	197	34	41 701	12.70	841.6	6.16	6.93	8.01	1208.7	53.50
2075	55–32–F	92	13	19 954	6.08	402.7	7.47	8.57	9.72	578.4	65.46
2076	220–32–F	212	43	46 838	14.27	945.3	5.07	5.98	6.86	1357.6	46.41
2077	110–24–F	108	16	26 417	8.05	533.1	7.02	8.05	9.24	765.7	61.82
2078	165–24–F	89	15	34 431	10.49	694.9	6.34	7.31	8.39	998.0	56.66
2079	220–24–F	250	46	38 155	11.62	770.0	6.03	6.60	7.45	1105.9	50.18
2080	110–16–F	92	13	20 923	6.37	422.3	7.52	8.64	9.72	606.5	65.28
2081	55–24–F	84	12	17 277	5.26	348.7	7.73	8.75	9.92	500.8	66.89
2082	165–40–F	113	25	49 007	14.93	989.0	4.80	5.45	6.15	1420.5	41.87
2083	165–16–F	262	40	26 448	8.06	533.8	6.83	7.92	9.09	766.6	60.72
2084	110–48–F	−25	− 5	46 913	14.29	946.7	5.15	5.93	6.94	1359.8	46.76
2085	55–40–F	58	9	25 270	7.70	509.9	7.23	8.26	9.44	732.5	63.54
2086	55–24–F	71	10	17 308	5.27	349.3	7.36	8.37	9.65	501.7	64.49
2087	110–16–F	84	12	20 846	6.35	420.7	7.44	8.43	9.77	604.2	66.63
2088	220–16–F	95	14	26 995	8.22	544.8	6.99	8.05	9.60	782.5	64.58
2089	220–40–F	23	6	54 989	16.75	1109.7	4.33	4.94	5.77	1593.9	38.77
2092	165–32–F	−59	−10	41 770	12.72	842.9	6.34	7.41	8.43	1210.7	56.95
2093	165–48–F	−393	−84	57 954	17.65	1169.5	4.88	5.67	6.52	1678.8	43.82
2094	110–56–F	279	52	54 336	16.55	1096.5	5.53	6.44	7.41	1575.0	50.33
2095	55–48–F	64	9	28 614	8.72	577.4	7.53	8.58	10.03	829.4	67.61
2096	55–40–F	220		25 382	7.73	512.2				735.7	
2097	55–32–F	126		21 676	6.60	437.4				628.3	
2098	55–48–F	373		29 251	8.91	590.3				**847.9**	
2090	110–24–F	72	11	26 091	7.95	526.5	7.24	8.22	9.41	756.3	63.33
2091	165–32–F	142	23	40 240	12.26	812.0	6.64	7.66	8.74	1166.4	58.76

Table 27.

Engine Performance—Mean Effective Pressure and Number of Expansions.

Test No.	Laboratory Designation	Mean Effective Pressure, lb. per sq. in.					Number of Expansions			
		Right Side		Left Side		Average	Right Side		Left Side	
		Head End	Crank End	Head End	Crank End		Head End	Crank End	Head End	Crank End
	Code Item☞	674	675	676	677	678	697	698	699	700
2009	138–16–F	32.8	32.9	32.9	37.7	34.1	2.51	2.34	2.35	2.50
2010	193–20–F	28.3	36.9	29.0	36.4	32.6	2.18	2.38	2.55	2.56
2012	138–24–F	45.5	53.1	48.0	53.7	50.1	2.15	2.36	2.29	2.24
2013	138–32–F	56.8	65.5	57.4	65.9	61.4	1.95	1.84	1.93	2.03
2014	193–32–F	44.7	52.2	41.6	49.7	47.1	1.78	1.89	1.99	1.94
2015	193–24–F	33.8	40.3	32.8	40.3	36.8	2.21	2.21	2.18	2.30
2016	193–16–F	23.0	28.4	21.1	29.3	25.5	2.44	2.58	2.74	2.53
2017	83–16–F	42.9	46.8	45.8	50.8	46.6	2.40	2.47	2.39	2.31
2018	83–24–F	59.5	65.4	63.7	69.7	64.6	2.17	2.24	2.13	2.07
2019	83–32–F	76.2	85.1	79.9	89.6	82.7	2.03	1.82	1.95	1.74
2020	83–24–F	58.2	63.5	62.4	68.6	63.2	2.38	2.05	2.25	2.08
2021	83–16–F	42.2	43.8	47.3	52.7	46.5	2.65	2.57	2.53	2.34
2022	83–32–F	77.2	86.7	80.6	89.5	83.5	1.97	1.85	2.01	1.80
2023	138–40–F	70.2	76.4	69.8	81.3	74.4	1.87	1.59	1.69	1.66
2024	55–24–F	69.8	74.8	72.5	78.8	74.0				
2026	110–16–F	36.5	40.6	38.1	48.7	41.0	2.49	1.92	2.37	2.41
2027	110–24–F	55.3	59.3	57.3	65.2	59.3	2.26	2.11	2.28	2.08
2028	55–32–F	88.2	93.2	94.2	103.9	94.9	1.96	1.83	1.91	1.78
2029	110–32–F	71.8	76.6	73.8	84.7	76.7	2.08	1.97	1.96	1.89
2030	165–24–F	42.4	47.5	45.5	49.4	46.2	2.21	2.29	2.35	2.05
2031	83–40–F	91.9	95.8	95.3	104.6	96.9	1.78	1.67	1.71	1.60
2032	165–32–F	53.8	56.8	53.6	62.5	56.7	1.95	1.97	2.14	1.93
2033	110–48–F	88.9	91.6	88.1	93.1	90.4	1.75	1.72	1.68	1.58
2034	193–40–F	54.7	56.4	52.2	60.6	56.0	1.80	1.61	1.80	1.67
2035	110–40–F	83.3	85.0	86.8	93.2	87.1	1.81	1.72	1.67	1.65
2037	165–40–F	63.1	65.4	62.9	71.2	65.7	1.73	1.66	1.84	1.64
2038	55–24–F	66.6	73.6	73.1	80.3	73.4				
2039	110–32–F	69.3	77.0	71.9	79.4	74.4	1.87	1.78	1.87	1.83
2040	165–40–F	61.8	66.2	62.1	68.1	64.5	1.65	1.67	1.65	1.68
2041	110–40–F	83.3	90.4	85.8	91.1	87.7	1.67	1.65	1.65	1.67
2042	110–24–F	54.3	60.7	60.8	63.8	59.9	2.03	2.14	1.92	2.12
2043	110–48–F	97.4	101.3	98.3	102.5	99.9	1.54	1.49	1.60	1.52
2044	110–56–F	103.0	107.0	104.8	108.7	105.9	1.38	1.37	1.36	1.37
2045	110–16–F	37.2	42.0	40.3	45.0	41.1	2.13	2.39	1.97	2.28
2072	110–40–F	94.1	99.3	96.9	100.2	97.6	1.64	1.63	1.63	1.67
2073	110–32–F	77.4	81.7	80.4	83.6	80.8	2.29	1.95	1.92	1.90
2074	165–32–F	61.7	67.8	61.8	67.5	64.7	2.50	1.87	1.92	1.98
2075	55–32–F	93.8	99.9	101.7	100.3	98.9	1.89	1.77	1.83	1.91
2076	220–32–F	53.0	53.2	47.1	52.6	51.5	1.96	1.79	1.76	1.71
2077	110–24–F	56.0	63.9	61.9	67.6	62.4	2.05	2.05	2.04	2.09
2078	165–24–F	48.2	55.6	50.0	54.0	52.0	2.10	2.12	2.02	2.26
2079	220–24–F	40.9	43.2	40.2	41.8	41.5	2.28	2.26	2.32	2.18
2080	110–16–F	38.8	45.5	43.5	47.7	43.9	2.32	2.32	2.43	2.53
2081	55–24–F	72.1	78.1	80.8	77.6	77.1	2.01	1.99	1.98	2.18
2082	165–40–F	71.7	78.3	71.9	75.9	74.5	1.81	1.49	1.67	1.63
2083	165–16–F	33.2	39.3	39.0	37.3	37.2	2.28	2.35	2.20	2.45
2084	110–48–F	103.3	108.1	103.5	107.8	105.7	1.53	1.49	1.54	1.54
2085	55–40–F	112.7	120.0	120.3	118.1	117.8	1.60	1.61	1.64	1.67
2086	55–24–F	71.9	77.2	81.1	78.0	77.1	2.09	2.08	2.03	2.22
2087	110–16–F	39.2	44.5	43.7	47.6	43.7	2.22	2.43	2.40	2.30
2088	220–16–F	28.0	27.1	29.7	27.5	28.1	2.54	2.50	2.67	2.56
2089	220–40–F	57.8	60.8	56.2	60.1	58.7	1.63	1.57	1.69	1.57
2092	165–32–F	61.9	68.0	63.9	67.3	65.3	1.96	1.93	1.97	2.01
2093	165–48–F	81.1	86.9	82.0	86.6	84.2	1.57	1.47	1.53	1.45
2094	110–56–F	115.8	120.8	118.5	121.3	119.1	1.41	1.42	1.36	1.42
2095	55–48–F	131.8	136.7	136.7	136.2	135.4	1.54	1.49	1.49	1.55
2096	55–40–F	115.1	121.2	123.2	121.3	120.2	1.72	1.63	1.64	1.71
2097	55–32–F	97.0	103.7	103.2	102.2	101.5	1.85	1.82	1.79	1.93
2098	55–48–F	133.0	138.1	141.3	137.7	137.5	1.54	1.49	1.50	1.57
2090	110–24–F	52.7	59.1	59.2	61.4	58.1	2.10	2.32	2.20	2.11
2091	165–32–F	54.2	60.2	58.1	60.9	58.3	2.05	1.81	2.16	2.22

Table 28.

Engine Performance—Indicated Horse Power.

Test No.	Laboratory Designation	Indicated Horse Power					
		Right Side		Left Side		Total	Maximum
		Head End	Crank End	Head End	Crank End		
	Code Item☞	707	708	709	710	711	721
2009	138–16–F	131.9	128.8	135.1	149.7	545.5	687.5
2010	193–20–F	161.2	203.4	167.9	204.5	737.0	902.5
2012	138–24–F	183.9	207.4	197.3	214.2	802.8	1023.7
2013	138–32–F	229.6	256.5	237.0	263.2	986.3	1224.6
2014	193–32–F	258.4	292.0	245.0	283.6	1079.0	1374.9
2015	193–24–F	195.8	225.9	193.4	230.5	845.6	1051.3
2016	193–16–F	133.1	158.9	124.6	167.2	583.8	774.8
2017	83–16–F	99.3	104.8	107.9	116.1	428.1	624.5
2018	83–24–F	137.6	146.6	150.3	159.3	593.8	742.0
2019	83–32–F	177.6	192.1	189.7	206.3	765.7	948.8
2020	83–24–F	135.4	143.2	148.1	157.7	584.4	596.3
2021	83–16–F	97.8	98.3	111.7	120.8	428.6	441.7
2022	83–32–F	180.0	195.9	191.8	206.2	773.9	797.9
2023	138–40–F	282.4	297.5	286.0	322.8	1188.7	1217.6
2024	55–24–F	102.3	106.2	108.3	114.2	431.0	450.9
2026	110–16–F	115.5	124.5	122.7	152.3	515.0	527.6
2027	110–24–F	175.9	182.7	185.7	204.8	749.1	
2028	55–32–F	128.4	131.3	139.7	149.3	548.7	557.6
2029	110–32–F	228.1	235.8	238.9	265.8	968.6	974.7
2030	165–24–F	208.0	225.2	227.1	239.3	899.6	924.4
2031	83–40–F	226.9	229.5	241.1	256.4	953.9	992.4
2032	165–32–F	261.4	267.3	265.7	300.2	1094.6	1125.8
2033	110–48–F	282.6	282.0	285.4	292.3	1142.3	1155.3
2034	193–40–F	314.0	313.6	305.3	343.8	1276.7	1299.0
2035	110–40–F	269.4	266.1	285.9	297.7	1119.1	1148.6
2037	165–40–F	309.1	310.1	313.9	344.6	1277.7	1307.5
2038	55–24–F	97.8	104.7	109.4	116.5	428.4	442.6
2039	110–32–F	220.5	237.1	233.0	249.4	940.0	967.5
2040	165–40–F	301.4	312.8	308.7	328.1	1251.0	1294.3
2041	110–40–F	265.4	279.3	278.7	287.0	1110.4	1133.7
2042	110–24–F	172.8	186.5	197.0	197.4	753.9	799.9
2043	110–48–F	309.0	311.3	318.1	321.2	1259.6	1284.3
2044	110–56–F	326.7	328.7	338.7	340.6	1334.7	1391.1
2045	110–16–F	118.2	129.2	130.5	141.2	519.1	534.7
2072	110–40–F	299.3	305.9	313.8	314.8	1233.8	1253.2
2073	110–32–F	245.8	251.3	260.1	262.1	1019.3	1047.8
2074	165–32–F	303.7	322.9	309.7	328.0	1264.3	1282.6
2075	55–32–F	137.9	142.3	152.7	145.7	578.6	597.2
2076	220–32–F	353.5	344.0	320.0	346.8	1364.3	1390.0
2077	110–24–F	179.9	198.8	202.7	214.3	795.7	813.9
2078	165–24–F	236.4	263.8	249.5	261.5	1011.2	1023.2
2079	220–24–F	275.4	281.1	275.3	278.1	1109.9	1152.3
2080	110–16–F	124.3	141.3	142.0	150.9	558.5	576.0
2081	55–24–F	105.9	111.2	120.9	112.5	450.5	467.6
2082	165–40–F	353.1	373.7	361.1	369.4	1457.3	1484.1
2083	165–16–F	163.9	188.0	196.4	182.1	730.4	756.1
2084	110–48–F	331.6	336.0	338.5	341.4	1347.5	1366.2
2085	55–40–F	167.2	172.4	181.7	173.0	694.3	705.6
2086	55–24–F	106.5	111.4	123.2	114.9	456.0	466.4
2087	110–16–F	126.5	138.9	143.5	151.4	560.3	574.9
2088	220–16–F	189.5	177.7	204.9	184.0	756.1	791.4
2089	220–40–F	386.6	393.6	382.6	397.1	1559.9	1588.6
2092	165–32–F	302.2	321.4	319.6	324.1	1267.3	1427.0
2093	165–48–F	396.0	411.5	408.3	417.7	1633.5	1654.2
2094	110–56–F	372.2	375.9	388.3	385.0	1521.4	1570.1
2095	55–48–F	196.2	197.1	211.4	200.2	804.9	822.7
2096	55–40–F	172.0	175.3	187.4	178.9	713.6	733.1
2097	55–32–F	146.7	152.0	159.0	152.8	610.6	621.0
2098	55–48–F	200.0	201.1	216.6	204.5	822.2	837.5
2090	110–24–F	169.7	184.2	194.2	195.0	743.1	757.3
2091	165–32–F	266.6	286.6	291.0	295.8	1140.0	1170.8

Table 29.

Engine Performance—Coal, Steam, and B.t.u. per Indicated Horse Power Hour.

Test No.	Laboratory Designation	Consumed per Indicated Horse Power per Hour			
		Dry Coal, lb.	B.t.u. in Coal	Dry Steam, lb.	B.t.u. in Steam Above 32°F.
	Code Item☞	734	735	736	737
2009	138–16–F	4.85	60 882	33.06	
2010	193–20–F	5.19	64 527	32.07	
2012	138–24–F	4.61	56 611	29.44	
2013	138–32–F	4.81	58 884	29.14	
2014	193–32–F	5.72	69 692	29.82	
2015	193–24–F	5.82	71 627	32.32	
2016	193–16–F	5.55	66 556	35.18	
2017	83–16–F	4.54	56 396	34.08	
2018	83–24–F	4.25	52 126	29.71	
2019	83–32–F	4.18	52 346	28.96	
2020	83–24–F	4.22	51 914	29.99	35 943
2021	83–16–F	5.07	62 229	32.10	38 478
2022	83–32–F	4.75	56 406	28.14	33 723
2023	138–40–F	5.56	68 449	29.10	34 876
2024	55–24–F	4.17	53 009	34.74	41 650
2026	110–16–F	4.44	54 652	31.67	37 969
2027	110–24–F	4.34	55 066	27.84	33 377
2028	55–32–F	4.37	55 294	30.57	36 653
2029	110–32–F	4.38	55 188	27.51	32 984
2030	165–24–F	4.45	56 769	28.99	34 756
2031	83–40–F	4.42	52 991	28.75	34 468
2032	165–32–F	4.87	59 619	27.84	33 365
2033	110–48–F	4.45	54 481	27.83	33 363
2034	193–40–F	6.07	71 838	30.02	35 982
2035	110–40–F	4.99	61 522	29.43	35 278
2037	165–40–F	5.09	63 325	29.56	35 439
2038	55–24–F	4.56	52 176	33.14	39 745
2039	110–32–F	4.79	56 206	29.43	35 287
2040	165–40–F	5.96	72 497	30.21	36 206
2041	110–40–F	5.22	64 081	29.54	35 407
2042	110–24–F	4.42	54 247	29.52	35 391
2043	110–48–F	5.84	73 134	30.34	36 372
2044	110–56–F	6.21	75 712	32.50	38 948
2045	110–16–F	4.63	55 028	32.69	39 195
2072	110–40–F	4.79	59 683	27.19	32 601
2073	110–32–F	4.26	54 319	27.20	32 586
2074	165–32–F	4.73	59 480	27.17	32 574
2075	55–32–F	4.00	50 872	28.40	34 052
2076	220–32–F	5.71	71 483	28.30	33 929
2077	110–24–F	4.10	51 795	27.36	32 788
2078	165–24–F	4.63	57 954	28.09	33 677
2079	220–24–F	5.17	66 005	28.25	33 869
2080	110–16–F	4.31	55 375	30.87	37 016
2081	55–24–F	4.36	55 372	31.53	37 808
2082	165–40–F	6.15	77 650	27.88	33 423
2083	165–16–F	4.52	57 223	29.65	35 553
2084	110–48–F	5.88	72 353	28.69	34 391
2085	55–40–F	4.39	55 411	29.69	35 598
2086	55–24–F	4.51	56 763	31.18	37 388
2087	110–16–F	4.39	53 383	30.65	36 752
2088	220–16–F	4.42	53 460	29.40	35 251
2089	220–40–F	7.10	87 728	29.18	34 978
2092	165–32–F	4.46	56 285	27.34	32 783
2093	165–48–F	6.31	79 197	29.62	35 500
2094	110–56–F	5.58	69 337	29.39	35 233
2095	55–48–F	4.15	51 107	29.37	35 215
2096	55–40–F				35 005
2097	55–32–F				34 978
2098	55–48–F				34 783
2090	110–24–F	4.26	53 650	28.99	34 762
2091	165–32–F	4.58	58 080	29.10	34 894

Table 30.

General Performance—Drawbar Horse Power and Millions of Foot Pounds at Drawbar.

Test No.	Laboratory Designation	Drawbar Horse Power	Consumed per D.H.P. Hour			Millions of Foot Pounds at Drawbar per Hour	Per Million Ft. lb. at Drawbar		
			Dry Coal, lb.	Dry Steam, lb.	B.t.u.		Dry Coal, lb.	Dry Steam, lb.	B.t.u.
	Code Item	743	744	745	746	750	752	753	754
2009	138–16–F								
2010	193–20–F								
2012	138–24–F	684.9	5.40	34.50	66 312	1357	2.73	17.4	33 524
2013	138–32–F	863.7	5.50	33.28	67 331	1711	2.77	16.8	33 910
2014	193–32–F	853.1	7.25	37.71	88 334	1689	3.66	19.0	44 593
2015	193–24–F	626.2	7.86	43.64	96 733	1240	3.97	22.0	48 859
2016	193–16–F	418.2	7.75	49.11	92 938	828	3.92	24.8	47 009
2017	83–16–F	357.1	5.45	40.85	67 700	707	2.75	20.6	34 161
2018	83–24–F	511.7	4.93	34.48	60 466	1014	2.49	17.4	30 540
2019	83–32–F	683.1	4.68	32.46	58 608	1353	2.36	16.4	29 554
2020	83–24–F	508.9	4.85	34.46	59 665	1009	2.44	17.4	30 017
2021	83–16–F	346.3	6.27	39.72	76 958	686	3.16	20.0	38 786
2022	83–32–F	674.6	5.44	32.29	64 600	1336	2.75	16.3	32 656
2023	138–40–F	1070.5	6.16	32.31	75 836	2120	3.11	16.3	38 287
2024	55–24–F	355.6	5.05	42.34	64 196	706	2.54	21.2	32 288
2026	110–16–F	415.1	5.51	39.31	67 823	822	2.78	19.8	34 219
2027	110–24–F	633.6	5.13	32.92	65 089	1255	2.59	16.6	32 862
2028	55–32–F	488.1	4.91	34.37	62 126	967	2.48	17.3	31 379
2029	110–32–F	820.8	5.17	32.50	64 553	1626	2.61	16.4	32 588
2030	165–24–F	725.8	5.52	35.95	70 419	1438	2.79	18.1	35 592
2031	83–40–F	869.7	4.85	31.52	58 147	1723	2.45	15.9	29 373
2032	165–32–F	922.8	5.77	33.22	70 636	1828	2.91	16.7	35 624
2033	110–48–F	1007.9	5.05	31.54	61 827	1996	2.55	15.9	31 220
2034	193–40–F	961.7	8.05	39.86	95 272	1905	4.07	20.1	48 168
2035	110–40–F	942.9	5.92	34.99	72 988	1868	2.99	17.6	36 864
2037	165–40–F	1045.3	6.22	36.09	77 383	2070	3.14	18.2	39 065
2038	55–24–F	368.4	5.30	38.54	60 643	730	2.68	19.4	30 665
2039	110–32–F	824.2	5.46	33.56	64 068	1631	2.76	17.0	32 386
2040	165–40–F	1133.6	6.58	33.33	80 039	2244	3.32	16.8	40 384
2041	110–40–F	1007.7	5.75	32.54	70 587	1995	2.91	16.4	35 723
2042	110–24–F	674.8	4.93	32.97	60 506	1336	2.49	16.7	30 560
2043	110–48–F	1158.5	6.35	32.98	79 521	2293	3.21	16.7	40 199
2044	110–56–F	1257.7	6.59	34.49	80 345	2490	3.33	17.4	40 599
2045	110–16–F	436.6	5.50	38.86	65 368	865	2.78	19.6	33 040
2072	110–40–F	1107.8	5.33	30.28	66 412	2193	2.69	15.3	33 517
2073	110–32–F	898.3	4.83	30.87	61 587	1779	2.44	15.6	31 112
2074	165–32–F	1107.3	5.40	31.03	67 905	2193	2.73	15.7	34 330
2075	55–32–F	501.4	4.62	32.76	58 757	993	2.33	16.5	29 633
2076	220–32–F	1157.1	6.73	33.36	84 253	2291	3.40	16.9	42 565
2077	110–24–F	670.2	4.87	32.48	61 523	1328	2.46	16.4	31 077
2078	165–24–F	833.8	5.62	34.06	70 346	1651	2.84	17.1	35 548
2079	220–24–F	928.5	6.18	33.84	78 900	1839	3.12	17.0	39 833
2080	110–16–F								
2081	55–24–F								
2082	165–40–F	1214.6	7.38	33.45	93 180	2405	3.73	16.9	47 095
2083	165–16–F	583.6	5.65	37.10	71 529	1156	2.85	18.7	36 081
2084	110–48–F	1197.2	6.62	32.30	81 459	2371	3.34	16.3	41 099
2085	55–40–F	614.5	4.96	33.55	62 605	1217	2.51	16.9	31 681
2086	55–24–F	386.0	5.33	36.83	67 083	764	2.69	18.6	33 856
2087	110–16–F	437.6	5.63	39.24	69 091	867	2.84	19.8	34 852
2088	220–16–F	631.3	5.29	35.21	63 983	1250	2.67	17.8	32 294
2089	220–40–F	1321.6	8.38	34.42	103 543	2617	4.25	17.4	52 513
2092	165–32–F	1117.6	5.06	31.01	63 857	2213	2.55	15.7	32 181
2093	165–48–F	1431.6	7.19	33.80	90 242	2835	3.63	17.1	45 560
2094	110–56–F	1354.1	6.27	33.02	77 911	2681	3.17	16.7	39 390
2095	55–48–F	718.0	4.66	32.92	57 388	1422	2.35	16.6	28 940
2096	55–40–F	622.8		33.46		1234		16.9	
2097	55–32–F	525.2		33.91		1040		17.1	
2098	55–48–F	732.2		32.56		1450		16.4	
2090	110–24–F	614.9	5.15	35.03	64 859	1218	2.60	17.7	32 744
2091	165–32–F	985.6	5.31	33.66	67 044	1952	2.68	17.0	33 838

Table 31.

General Performance—Indicated Horse Power, Drawbar Horse Power, and Tractive Force.

Test No.	Laboratory Designation	Indicated Horse Power		Drawbar Horse Power		Tractive Force Based on M.E.P., lb.
		Per sq. ft. of Heating Surface	Per sq. ft. of Grate Surface	Per sq. ft. of Heating Surface	Per sq. ft. of Grate Surface	
	Code Item☞	755	756	757	758	764
2009	138–16–F	0.17	11.01			8 096
2010	193–20–F	0.22	14.89			7 745
2012	138–24–F	0.24	16.23	0.21	13.84	11 876
2013	138–32–F	0.30	19.91	0.26	17.43	14 581
2014	193–32–F	0.33	21.84	0.26	17.27	11 153
2015	193–24–F	0.26	17.09	0.19	12.65	8 736
2016	193–16–F	0.18	11.83	0.13	8.48	6 031
2017	83–16–F	0.13	8.69	0.11	7.25	11 050
2018	83–24–F	0.18	12.04	0.16	10.39	15 325
2019	83–32–F	0.23	15.53	0.21	13.86	19 621
2020	83–24–F	0.18	11.81	0.16	10.29	14 994
2021	83–16–F	0.13	8.81	0.11	7.12	11 050
2022	83–32–F	0.24	15.64	0.21	13.62	19 827
2023	138–40–F	0.37	24.29	0.33	21.87	17 659
2024	55–24–F	0.13	8.79	0.11	7.25	17 555
2026	110–16–F	0.16	10.41	0.13	8.39	9 728
2027	110–24–F	0.23	15.15	0.19	12.82	14 065
2028	55–32–F	0.17	11.11	0.15	9.89	22 533
2029	110–32–F	0.29	19.54	0.25	16.56	18 216
2030	165–24–F	0.27	18.19	0.22	14.67	10 967
2031	83–40–F	0.29	19.38	0.27	17.66	22 967
2032	165–32–F	0.34	22.20	0.28	18.71	13 445
2033	110–48–F	0.35	23.22	0.31	20.49	21 459
2034	193–40–F	0.39	25.85	0.29	19.47	13 280
2035	110–40–F	0.34	22.53	0.29	18.98	20 674
2037	165–40–F	0.39	25.99	0.32	21.26	15 593
2038	55–24–F	0.13	8.91	0.12	7.66	17 431
2039	110–32–F	0.29	19.04	0.25	16.69	17 659
2040	165–40–F	0.38	25.33	0.35	22.95	15 304
2041	110–40–F	0.34	22.65	0.31	20.57	20 819
2042	110–24–F	0.23	15.33	0.21	13.73	14 148
2043	110–48–F	0.39	25.59	0.36	23.54	23 710
2044	110–56–F	0.41	27.22	0.39	25.63	25 115
2045	110–16–F	0.16	10.59	0.13	8.91	9 769
2072	110–40–F	0.38	24.97	0.34	22.43	23 256
2073	110–32–F	0.31	20.67	0.27	18.22	19 249
2074	165–32–F	0.39	25.67	0.34	22.48	15 387
2075	55–32–F	0.18	11.75	0.15	10.18	23 627
2076	220–32–F	0.42	27.68	0.35	23.48	12 247
2077	110–24–F	0.24	16.14	0.21	13.60	14 850
2078	165–24–F	0.31	20.47	0.25	16.88	12 351
2079	220–24–F	0.34	22.58	0.29	18.89	9 893
2080	110–16–F	0.17	11.33			10 451
2081	55–24–F	0.14	9.14			18 382
2082	165–40–F	0.45	29.49	0.37	24.58	17 741
2083	165–16–F	0.23	14.92	0.18	11.92	8 860
2084	110–48–F	0.41	27.19	0.36	24.16	25 218
2085	55–40–F	0.21	14.05	0.19	12.43	28 047
2086	55–24–F	0.14	9.25	0.12	7.83	18 361
2087	110–16–F	0.17	11.37	0.13	8.87	10 409
2088	220–16–F	0.23	15.33	0.19	12.79	6 671
2089	220–40–F	0.48	31.50	0.40	26.68	13 962
2092	165–32–F	0.39	25.53	0.34	22.51	15 531
2093	165–48–F	0.49	32.69	0.43	28.66	20 158
2094	110–56–F	0.47	30.89	0.42	27.50	28 336
2095	55–48–F	0.25	16.28	0.22	14.53	32 426
2096	55–40–F	0.22	14.55	0.19	12.70	28 605
2097	55–32–F	0.19	12.41	0.16	10.68	24 205
2098	55–48–F	0.25	16.85	0.23	15.01	32 818
2090	110–24–F	0.23	15.05	0.19	12.45	13 879
2091	165–32–F	0.35	23.10	0.30	19.98	13 900

Table 32.

General Performance—Machine Friction, Efficiencies, and Ratios.

Test No.	Laboratory Designation	Machine Friction of Locomotive in Terms of				Machine Efficiency of Locomotive, per cent	Efficiency of Locomotive per cent	Ratios	
		Horse Power	Mean Effective Pressure, lb. per sq. in.	Drawbar Pull, lb.	Per cent of Indicated Horse Power			Total Weight of Locomotive to Maximum I.H.P.	Total Heating Surface to Maximum I.H.P.
	Code Item	770	771	772	773	778	779	785	786
2009	138–16–F							324.4	4.8
2010	193–20–F							247.1	3.6
2012	138–24–F	117.9	7.35	1746	14.7	85.3	3.84	217.8	3.2
2013	138–32–F	122.6	7.63	1812	12.4	87.6	3.78	182.1	2.7
2014	193–32–F	225.9	9.86	2337	20.9	79.1	2.88	162.2	2.4
2015	193–24–F	219.4	9.56	2267	26.0	74.1	2.63	212.1	3.1
2016	193–16–F	165.6	7.22	1711	28.4	71.6	2.74	287.8	4.2
2017	83–16–F	71.0	7.72	1837	16.6	83.4	3.76	357.1	5.3
2018	83–24–F	82.1	8.93	2120	13.8	86.2	4.21	300.5	4.4
2019	83–32–F	82.6	8.92	2118	10.8	89.2	4.35	235.0	3.5
2020	83–24–F	75.5	8.16	1938	12.9	87.1	4.27	374.0	5.5
2021	83–16–F	82.3	8.93	2123	19.2	80.8	3.31	504.9	7.4
2022	83–32–F	99.3	10.71	2545	12.8	87.2	3.94	279.5	4.1
2023	138–40–F	118.2	7.40	1757	9.9	90.1	3.36	183.2	2.7
2024	55–24–F	75.4	12.94	3073	17.5	82.5	3.97	494.6	7.3
2026	110–16–F	99.9	7.95	1887	19.4	80.6	3.75	422.7	6.2
2027	110–24–F	115.5	9.14	2169	15.4	84.6	3.91		
2028	55–32–F	60.6	10.47	2489	11.0	89.0	4.10	399.9	5.9
2029	110–32–F	147.8	11.71	2781	15.3	84.7	3.94	235.3	3.4
2030	165–24–F	173.8	8.93	2118	19.3	80.7	3.62	241.2	3.6
2031	83–40–F	84.2	8.56	2023	8.8	91.2	4.38	224.7	3.3
2032	165–32–F	171.8	8.90	2112	15.7	84.3	3.61	198.1	2.9
2033	110–48–F	134.4	10.64	2527	11.8	88.2	4.12	193.0	2.8
2034	193–40–F	315.0	13.81	3279	24.7	75.3	2.67	171.7	2.5
2035	110–40–F	176.2	13.70	3256	15.7	84.3	3.49	194.2	2.9
2037	165–40–F	232.4	11.94	2836	18.2	81.8	3.29	170.6	2.5
2038	55–24–F	60.0	10.28	2440	14.0	86.0	4.20	503.8	7.4
2039	110–32–F	115.8	9.17	2175	12.3	87.7	3.97	230.5	3.4
2040	165–40–F	117.4	6.05	1437	9.4	90.6	3.18	172.3	2.5
2041	110–40–F	102.7	8.11	1923	9.3	90.8	3.61	196.8	2.9
2042	110–24–F	78.9	6.27	1483	10.5	89.5	4.21	278.9	4.1
2043	110–48–F	101.1	8.02	1903	8.0	92.0	3.20	173.7	2.6
2044	110–56–F	77.0	6.10	1449	5.8	94.2	3.17	160.3	2.4
2045	110–16–F	82.5	6.53	1551	15.9	84.1	3.90	417.1	6.1
2072	110–40–F	126.0	9.97	2375	10.2	89.8	3.83	177.9	2.6
2073	110–32–F	121.0	9.59	2284	11.9	88.1	4.13	212.8	3.1
2074	165–32–F	157.0	8.04	1913	12.4	87.6	3.75	173.9	2.6
2075	55–32–F	77.2	13.20	3154	13.3	86.7	4.33	373.4	5.5
2076	220–32–F	207.2	7.81	1861	15.2	84.8	3.02	160.4	2.4
2077	110–24–F	125.5	9.83	2342	15.8	84.2	4.14	274.0	4.0
2078	165–24–F	177.4	9.10	2169	17.5	82.5	3.62	217.9	3.2
2079	220–24–F	181.4	6.78	1615	16.3	83.7	3.23	193.5	2.9
2080	110–16–F							387 2	5.7
2081	55–24–F							476.9	7.0
2082	165–40–F	242.7	12.40	2954	16.7	83.4	2.73	150.3	2.2
2083	165–16–F	146.8	7.46	1780	20.1	79.9	3.56	294.9	4.3
2084	110–48–F	150.3	11.79	2813	11.2	88.9	3.13	163.2	2.4
2085	55–40–F	79.8	13.54	3224	11.5	88.5	4.07	316.0	4.7
2086	55–24–F	70.0	11.83	2817	15.4	84.7	3.80	478.1	7.0
2087	110–16–F	122.7	9.57	2280	21.9	78.1	3.69	387.9	5.7
2088	220–16–F	124.8	4.64	1101	16.5	83.5	3.98	281.8	4.2
2089	220–40–F	238.3	8.97	2134	15.3	84.7	2.46	140.4	2.1
2092	165–32–F	149.7	7.71	1834	11.8	88.2	3.99	156.3	2.3
2093	165–48–F	201.9	10.41	2491	12.4	87.6	2.82	134.8	2.0
2094	110–56–F	167.3	13.10	3117	11.0	89.0	3.27	142.0	2.1
2095	55–48–F	86.9	14.62	3501	10.8	89.2	4.44	271.1	4.0
2096	55–40–F	90.8	15.28	3641	12.7	87.3		304.2	4.5
2097	55–32–F	85.4	14.20	3385	14.0	86.0		359.1	5.3
2098	55–48–F	90.0	15.06	3592	11.0	89.1		266.3	3.9
2090	110–24–F	128.2	10.02	2394	17.3	82.8	3.93	294.5	4.3
2091	165–32–F	154.4	7.89	1884	13.5	86.5	3.80	190.5	2.8

Table 33.

Analysis of Ash, Front-end Cinders, and Stack Cinders.

Test No.	Laboratory Designation	Analysis of Ash			Analysis of Front-end Cinders			Analysis of Stack Cinders		
		Carbon, per cent	Earthy Matter, per cent	Moisture, per cent	Carbon, per cent	Earthy Matter, per cent	Moisture, per cent	Carbon, per cent	Earthy Matter, per cent	Moisture, per cent
	Code Item	831	832	833	841	842	843	846	847	848
2009	138–16–F	34.90	51.97	10.92	57.47	35.56	4.50	44.35	51.75	0.84
2010	193–20–F	30.26	60.22	7.33	42.75	54.85	0.58	61.97	34.90	0.54
2012	138–24–F	29.34	63.97	4.21	42.52	54.33	0.73	52.71	43.88	0.84
2013	138–32–F	31.53	62.82	2.86	37.46	59.41	0.66	60.26	37.00	0.54
2014	193–32–F	25.41	66.84	4.66	38.60	57.42	1.50	66.76	28.48	2.23
2015	193–24–F	33.03	58.52	4.48	18.19	79.68	0.62	63.90	32.16	1.26
2016	193–16–F	30.75	59.56	6.43	39.95	55.40	0.74	56.47	40.76	0.81
2017	83–16–F	30.12	63.55	3.78	29.99	65.99	1.34	40.17	50.54	7.02
2018	83–24–F	27.49	67.01	3.28	44.08	52.97	0.88	50.62	44.44	2.64
2019	83–32–F	24.59	71.63	1.58	45.95	48.93	2.71	55.30	40.83	1.27
2020	83–24–F	44.49	50.94	4.57	37.26	44.31	18.43	46.34	52.40	1.26
2021	83–16–F	35.90	62.95	1.15	45.85	53.00	1.15	51.63	46.90	1.47
2022	83–32–F	26.24	72.63	1.13	27.25	72.05	0.70	64.48	33.95	1.57
2023	138–40–F	33.00	64.76	2.24	51.32	47.83	0.85	71.14	28.56	0.30
2024	55–24–F	29.04	68.25	2.70	39.46	38.60	21.94	43.59	55.04	1.37
2026	110–16–F	41.33	51.24	7.43	48.85	50.37	0.78	56.14	42.77	1.09
2027	110–24–F	33.08	63.71	3.21	48.80	50.38	0.82	56.19	43.24	0.57
2028	55–32–F	27.48	70.63	1.89	49.59	49.88	0.53	53.17	44.05	2.78
2029	110–32–F	37.00	60.61	2.39	33.77	65.56	0.67	60.79	38.27	0.94
2030	165–24–F	38.29	56.74	4.97	48.88	51.11	0.01	67.31	32.34	0.35
2031	83–40–F	29.05	69.49	1.46	20.82	78.66	0.52	66.13	32.96	0.91
2032	165–32–F	41.04	55.71	3.25	52.59	46.76	0.65	33.80	65.28	0.92
2033	110–48–F	31.93	66.04	2.03	19.52	79.89	0.59	67.86	31.48	0.66
2034	193–40–F	36.31	61.59	2.10	43.05	56.35	0.60	71.39	28.04	0.57
2035	110–40–F	37.81	60.00	2.19	40.73	58.72	0.55	66.60	32.69	0.71
2037	165–40–F	40.50	56.91	2.59	45.64	53.40	0.96	69.72	29.61	0.67
2038	55–24–F	42.04	57.58	0.38	46.39	52.05	1.56	38.95	60.16	0.89
2039	110–32–F	29.59	69.28	1.13	42.74	55.44	1.82	58.50	40.89	0.61
2040	165–40–F	38.57	59.98	1.45	34.78	64.71	0.51	70.55	28.96	0.49
2041	110–40–F	34.91	63.69	1.40	46.43	52.35	1.22	66.22	33.30	0.48
2042	110–24–F	36.54	62.11	1.35	47.78	51.63	0.59	57.25	42.08	0.67
2043	110–48–F	32.99	64.35	2.66	10.55	89.37	0.08	69.26	30.14	0.60
2044	110–56–F	29.99	67.45	2.56	26.19	73.50	0.31	73.92	25.64	0.44
2045	110–16–F	33.14	65.06	1.80	52.44	47.41	0.15	46.58	53.03	0.39
2072	110–40–F	26.72	70.98	2.30	32.50	67.18	0.32	68.08	31.37	0.55
2073	110–32–F	35.55	60.82	3.63	34.42	65.08	0.50	64.61	34.96	0.43
2074	165–32–F	29.52	68.03	2.45	40.97	58.68	0.35	67.18	32.41	0.41
2075	55–32–F	30.33	69.36	0.31	43.76	56.18	0.06	46.69	52.84	0.47
2076	220–32–F	33.20	64.08	2.72	44.16	55.50	0.34	75.82	23.79	0.39
2077	110–24–F	51.92	40.16	7.92	56.25	43.34	0.41	56.19	43.00	0.81
2078	165–24–F	28.41	69.20	2.39	42.75	56.88	0.37	64.45	34.98	0.57
2079	220–24–F	38.08	60.98	0.94	42.02	57.69	0.29	67.47	32.03	0.50
2080	110–16–F	30.34	67.79	1.87	44.20	53.12	2.68	37.30	61.96	0.74
2081	55–24–F	32.01	66.82	1.17	41.82	57.98	0.20	41.10	58.37	0.53
2082	165–40–F	32.66	64.62	2.72	45.85	53.65	0.50	73.05	26.41	0.54
2083	165–16–F	34.94	61.87	3.19	53.99	45.66	0.35	62.11	37.19	0.70
2084	110–48–F	31.38	66.85	1.97	24.30	53.52	22.18	73.52	25.92	0.56
2085	55–40–F	33.16	66.19	0.65	51.37	48.42	0.21	64.35	35.23	0.42
2086	55–24–F	29.70	69.64	0.66	22.63	54.43	22.94	38.94	53.81	7.25
2087	110–16–F	29.94	69.47	0.59	39.45	60.35	0.20	45.33	53.70	0.97
2088	220–16–F	28.16	71.15	0.69	19.32	74.04	6.64	57.39	41.93	0.68
2089	220–40–F	25.16	72.85	1.99	41.25	58.48	0.27	75.83	23.84	0.33
2092	165–32–F	34.23	63.49	2.28	36.73	62.95	0.32	69.76	29.65	0.59
2093	165–48–F	44.12	54.79	1.09	41.35	58.42	0.23	71.65	28.01	0.34
2094	110–56–F	37.66	61.46	0.88	42.96	55.06	1.98	72.55	26.94	0.51
2095	55–48–F	31.83	67.47	0.70	62.66	37.00	0.34	57.88	41.66	0.46
2096	55–40–F									
2097	55–32–F									
2098	55–48–F									
2090	110–24–F	34.88	64.52	0.60	51.66	48.03	0.31	57.23	41.93	0.84
2091	165–32–F	36.60	61.98	1.42	55.00	44.32	0.68	67.47	31.79	0.74

Table 34.

Heat Balance—British Thermal Units.

Test No.	Laboratory Designation	B.t.u. Absorbed by Boiler per lb. of Coal as Fired	B.t.u. Loss Per Pound of Coal as Fired								
			Due to Moisture in Coal	Due to Moisture in Air	Due to Hydrogen in Coal	Due to Escaping Gases	Due to Incomplete Combustion	Due to Combustible in Front-end Cinders	Due to Combustible in Stack Cinders	Due to Combustible in Ash	Due to Radiation, and Unaccounted-for
	Code Item ☞	851	852	853	854	855	856	857	858	860	869
2009	138–16–F	7007	143	46	491	1691	0	27	368	116	1194
2010	193–20–F	6376	163	57	542	3688	0	11	882	61	−821
2012	138–24–F	6618	150	77	522	3525	30	7	681	187	−896
2013	138–32–F	6435	122	78	542	3506	47	7	1036	184	−821
2014	193–32–F	5512	126	75	532	2698	22	7	1272	81	713
2015	193–24–F	5638	144	64	518	2171	0	4	1243	355	826
2016	193–16–F	6531	153	68	490	2323	15	8	662	250	87
2017	83–16–F	7812	130	64	512	2213	106	9	175	209	− 50
2018	83–24–F	7269	141	49	510	2220	106	24	377	168	268
2019	83–32–F	7239	140	62	532	2211	124	11	618	162	93
2020	83–24–F	7560					0	16	362	182	
2021	83–16–F	6521	154	22	409	1786	0	28	370	476	1003
2022	83–32–F	6231	133	34	412	1811	0	4	1039	267	711
2023	138–40–F	5317	174	19	419	1336	0	4	1856	264	1297
2024	55–24–F	8637	146	18	419	1942	0	28	181	210	−345
2026	110–16–F	7278	162	26	409	1928	0	31	408	415	85
2027	110–24–F	6550	162	29	418	1908	0	15	661	267	1069
2028	55–32–F	7113	157	30	414	1812	0	16	457	270	808
2029	110–32–F	6414	157	33	410	1512	0	10	1068	376	968
2030	165–24–F	6793	138	40	429	1566	0	10	1127	280	993
2031	83–40–F	6299	220	33	388	1360	96	5	1188	240	91
2032	165–32–F	5822	171		411	1431	0	19	794	326	
2033	110–48–F	6289	180	35	405	1400	0	3	1519	241	467
2034	193–40–F	5045	168	36	405	1347	0	7	1736	386	1180
2035	110–40–F	5900	186	39	403	1707	0	4	1191	379	738
2037	165–40–F	5813	185	39	413	1400	0	9	1599	347	889
2038	55–24–F	7482	154	18	383	1622	0	29	173	411	−231
2039	110–32–F	6356	152	22	402	1478	0	13	900	269	762
2040	165–40–F	5180	160	18	417	1271	38	6	1941	261	1396
2041	110–40–F	5716	185	20	416	1455	45	5	1477	258	974
2042	110–24–F	6774	175	22	405	1557	0	5	719	283	662
2043	110–48–F	5231	179	16	426	1299	154	2	1838	225	1471
2044	110–56–F	5306	177	17	427	1408	156	4	1961	187	551
2045	110–16–F	7180	170	21	391	1474	0	3	515	321	236
2072	110–40–F	5793	169		505			0	1664	157	
2073	110–32–F	6511	175	19	516	1638	56	9	1042	261	824
2074	165–32–F	5978	148	21	529	1568	229	2	1372	266	1061
2075	55–32–F	7249	166	28	509	1981	69	13	427	154	478
2076	220–32–F	4920	205	18	516	1352	74	16	2159	283	1059
2077	110–24–F	6812	165	22	507	1765	29	16	540	221	941
2078	165–24–F	6152	172	22	508	1738	0	16	1132	185	932
2079	220–24–F	5851	113	22	549	1676	0	13	1361	502	1573
2080	110–16–F	7297	166	28	512	1982	142	16	193	316	527
2081	55–24–F	7501	147	29	511	2121	0	19	169	359	358
2082	165–40–F	4658	160	20	545	1468	0	14	2034	324	1902
2083	165–16–F	6629	177	23	504	1672	0	26	613	448	824
2084	110–48–F	4998	175	26	510	1566	0	9	1746	304	1354
2085	55–40–F	7016	160	26	510	1725	0	18	590	369	628
2086	55–24–F	7142	151	26	495	1750	68	8	269	363	803
2087	110–16–F	7220	147	28	496	1725	0	14	284	379	543
2088	220–16–F	6773	169	25	476	1517	0	7	599	335	586
2089	220–40–F	4202	167	14	528	1207	126	10	2551	214	1819
2092	165–32–F	6152	192	18	522	1475	22	11	1292	326	792
2093	165–48–F	4736	188	15	530	1187	62	10	2414	540	1125
2094	110–56–F	5666	190	21	517	1282	35	20	2069	315	847
2095	55–48–F	7307	158	23	505	1656	0	24	560	315	260
2096	55–40–F										
2097	55–32–F										
2098	55–48–F										
2090	110–24–F	7026	152	25	508	1524	26	29	730	447	627
2091	165–32–F	6443	174	23	514	1470	0	31	1399	461	449

Table 35.

Heat Balance—Percentage.

Test No.	Laboratory Designation	Per cent of Heat of Coal as Fired									
		Absorbed by Boiler	To Moisture in Coal	To Moisture in Air	To Hydrogen in Coal	To Escaping Gases	To Incomplete Combustion	To Combustible in Front-end Cinders	To Combustible in Stack Cinders	To Combustible in Ash	To Radiation, and Unaccounted-for
	Code Item	881	882	883	884	885	886	887	888	890	899
2009	138–16–F	63.2	1.3	0.4	4.4	15.3	0.0	0.2	3.3	1.1	10.8
2010	193–20–F	58.2	1.5	0.5	5.0	33.7	0.0	0.1	8.1	0.6	– 7.5
2012	138–24–F	60.7	1.4	0.7	4.8	32.3	0.3	0.1	6.3	1.7	– 8.2
2013	138–32–F	57.8	1.1	0.7	4.9	31.5	0.4	0.1	9.3	1.7	– 7.4
2014	193–32–F	49.9	1.1	0.7	4.8	24.4	0.2	0.1	11.5	0.7	6.5
2015	193–24–F	51.4	1.3	0.6	4.7	19.8	0.0	0.0	11.3	3.2	7.5
2016	193–16–F	61.7	1.4	0.6	4.6	21.9	0.1	0.1	6.3	2.4	0.8
2017	83–16–F	69.9	1.2	0.6	4.6	19.8	1.0	0.1	1.6	1.9	– 0.4
2018	83–24–F	66.5	1.3	0.5	4.7	20.3	1.0	0.2	3.4	1.5	2.5
2019	83–32–F	64.7	1.3	0.6	4.8	19.8	1.1	0.1	5.5	1.5	0.8
2020	83–24–F	67.3					0.0	0.1	3.2	1.6	
2021	83–16–F	60.6	1.4	0.2	3.8	16.6	0.0	0.3	3.4	4.4	9.3
2022	83–32–F	58.6	1.3	0.3	3.9	17.0	0.0	0.0	9.8	2.5	6.7
2023	138–40–F	49.8	1.6	0.2	3.9	12.5	0.0	0.0	17.4	2.5	12.1
2024	55–24–F	76.9	1.3	0.2	3.7	17.3	0.0	0.3	1.6	1.9	– 3.1
2026	110–16–F	67.8	1.5	0.2	3.8	18.0	0.0	0.3	3.8	3.9	0.8
2027	110–24–F	59.1	1.5	0.3	3.8	17.2	0.0	0.1	6.0	2.4	9.7
2028	55–32–F	64.2	1.4	0.3	3.7	16.4	0.0	0.1	4.1	2.4	7.3
2029	110–32–F	58.6	1.4	0.3	3.8	13.8	0.0	0.1	9.8	3.4	8.8
2030	165–24–F	59.7	1.2	0.4	3.8	13.8	0.0	0.1	9.9	2.5	8.7
2031	83–40–F	63.4	2.2	0.3	3.9	13.7	1.0	0.1	12.0	2.4	0.9
2032	165–32–F	54.7	1.6		3.9		0.0	0.2	7.5	3.1	
2033	110–48–F	59.7	1.7	0.3	3.8	13.3	0.0	0.0	14.0	2.3	4.4
2034	193–40–F	48.9	1.6	0.4	3.9	13.1	0.0	0.1	16.8	3.7	11.5
2035	110–40–F	55.9	1.8	0.4	3.8	16.2	0.0	0.0	11.3	3.6	7.0
2037	165–40–F	54.4	1.7	0.4	3.9	13.1	0.0	0.1	15.0	3.2	8.3
2038	55–24–F	74.5	1.5	0.2	3.8	16.2	0.0	0.3	1.7	4.1	– 2.3
2039	110–32–F	61.4	1.5	0.2	3.9	14.3	0.0	0.1	8.7	2.6	7.4
2040	165–40–F	48.5	1.5	0.2	3.9	11.9	0.4	0.1	18.2	2.5	13.1
2041	110–40–F	54.2	1.8	0.2	3.9	13.8	0.4	0.0	14.0	2.4	9.2
2042	110–24–F	63.9	1.7	0.2	3.8	14.7	0.0	0.1	6.8	2.7	6.3
2043	110–48–F	48.3	1.7	0.2	3.9	12.0	1.4	0.0	17.0	2.1	13.6
2044	110–56–F	50.1	1.7	0.2	4.0	13.3	1.5	0.0	18.5	1.8	5.2
2045	110–16–F	69.6	1.7	0.2	3.8	14.3	0.0	0.0	5.0	3.1	2.3
2072	110–40–F										
2073	110–32–F	58.9	1.6	0.2	4.7	14.8	0.5	0.1	9.4	2.4	7.4
2074	165–32–F	53.5	1.3	0.2	4.7	14.0	2.1	0.0	12.3	2.4	9.5
2075	55–32–F	65.5	1.5	0.3	4.6	17.9	0.7	0.1	3.9	1.4	4.3
2076	220–32–F	46.4	1.9	0.2	4.9	12.8	0.7	0.1	20.4	2.7	10.0
2077	110–24–F	61.8	1.5	0.2	4.6	16.0	0.3	0.1	4.9	2.0	8.5
2078	165–24–F	56.7	1.6	0.2	4.7	16.0	0.0	0.1	10.4	1.7	8.6
2079	220–24–F	50.2	1.0	0.2	4.7	15.0	0.0	0.1	11.7	4.3	13.5
2080	110–16–F	65.3	1.5	0.3	4.6	17.7	1.3	0.1	1.7	2.8	4.7
2081	55–24–F	66.9	1.3	0.3	4.6	18.9	0.0	0.2	1.5	3.2	3.2
2082	165–40–F	41.9	1.4	0.2	4.9	13.2	0.0	0.1	18.3	2.9	17.1
2083	165–16–F	60.7	1.6	0.2	4.6	15.3	0.0	0.2	5.6	4.1	7.6
2084	110–48–F	46.8	1.6	0.3	4.8	14.6	0.0	0.1	16.3	2.8	12.7
2085	55–40–F	63.5	1.5	0.2	4.6	15.6	0.0	0.2	5.3	3.3	5.7
2086	55–24–F	64.5	1.4	0.2	4.5	15.8	0.6	0.1	2.4	3.3	7.2
2087	110–16–F	66.6	1.4	0.3	4.6	15.9	0.0	0.1	2.6	3.5	5.0
2088	220–16–F	64.6	1.6	0.2	4.5	14.5	0.0	0.1	5.7	3.2	5.6
2089	220–40–F	38.8	1.5	0.1	4.9	11.1	1.2	0.1	23.5	2.0	16.8
2092	165–32–F	57.0	1.8	0.2	4.8	13.7	0.2	0.1	12.0	3.0	7.3
2093	165–48–F	43.8	1.7	0.1	4.9	11.0	0.6	0.1	22.3	5.0	10.4
2094	110–56–F	50.3	1.8	0.2	4.9	12.0	0.3	0.2	19.4	3.0	7.9
2095	55–48–F	67.6	1.5	0.2	4.7	15.3	0.0	0.2	5.2	2.9	2.4
2096	55–40–F										
2097	55–32–F										
2098	55–48–F										
2090	110–24–F	63.3	1.4	0.2	4.6	13.7	0.2	0.3	6.6	4.0	5.6
2091	165–32–F	58.8	1.6	0.2	4.7	13.4	0.0	0.3	12.8	4.2	4.1

Table 36.

Information Concerning the Indicator Diagrams Shown in Fig. 56, 57, and 58.

Diagram No.	Right or Left Side	Head or Crank End	Test No.	Laboratory Designation	Nominal Speed, M. P. H.	Speed, R. P. M.
1	R	H	2086	55–24–F	10	51.3
2	R	C	"	"	"	"
3	L	H	"	"	"	"
4	L	C	"	"	"	"
5	R	H	2077	110–24–F	20	110.7
6	R	C	"	"	"	"
7	L	H	"	"	"	"
8	L	C	"	"	"	"
9	R	H	2083	165–16–F	30	170.3
10	R	C	"	"	"	"
11	L	H	"	"	"	"
12	L	C	"	"	"	"
13	R	H	2088	220–16–F	40	234.2
14	R	C	"	"	"	"
15	L	H	"	"	"	"
16	L	C	"	"	"	"
17	R	H	2095	55–48–F	10	51.3
18	R	C	"	"	"	"
19	L	H	"	"	"	"
20	L	C	"	"	"	"
21	R	H	2084	110–48–F	20	110.4
22	R	C	"	"	"	"
23	L	H	"	"	"	"
24	L	C	"	"	"	"
25	R	H	2093	165–48–F	30	167.4
26	R	C	"	"	"	"
27	L	H	"	"	"	"
28	L	C	"	"	"	"
29	R	H	2089	220–40–F	40	230.7
30	R	C	"	"	"	"
31	L	H	"	"	"	"
32	L	C	"	"	"	"
33	R	H	2028	55–32–F	10	50.3
34	R	C	"	"	"	"
35	L	H	"	"	"	"
36	L	C	"	"	"	"
37	R	H	2029	110–32–F	20	109.8
38	R	C	"	"	"	"
39	L	H	"	"	"	"
40	L	C	"	"	"	"
41	R	H	2030	165–24–F	30	169.4
42	R	C	"	"	"	"
43	L	H	"	"	"	"
44	L	C	"	"	"	"
45	R	H	2034	193–40–F	35	198.5
46	R	C	"	"	"	"
47	L	H	"	"	"	"
48	L	C	"	"	"	"

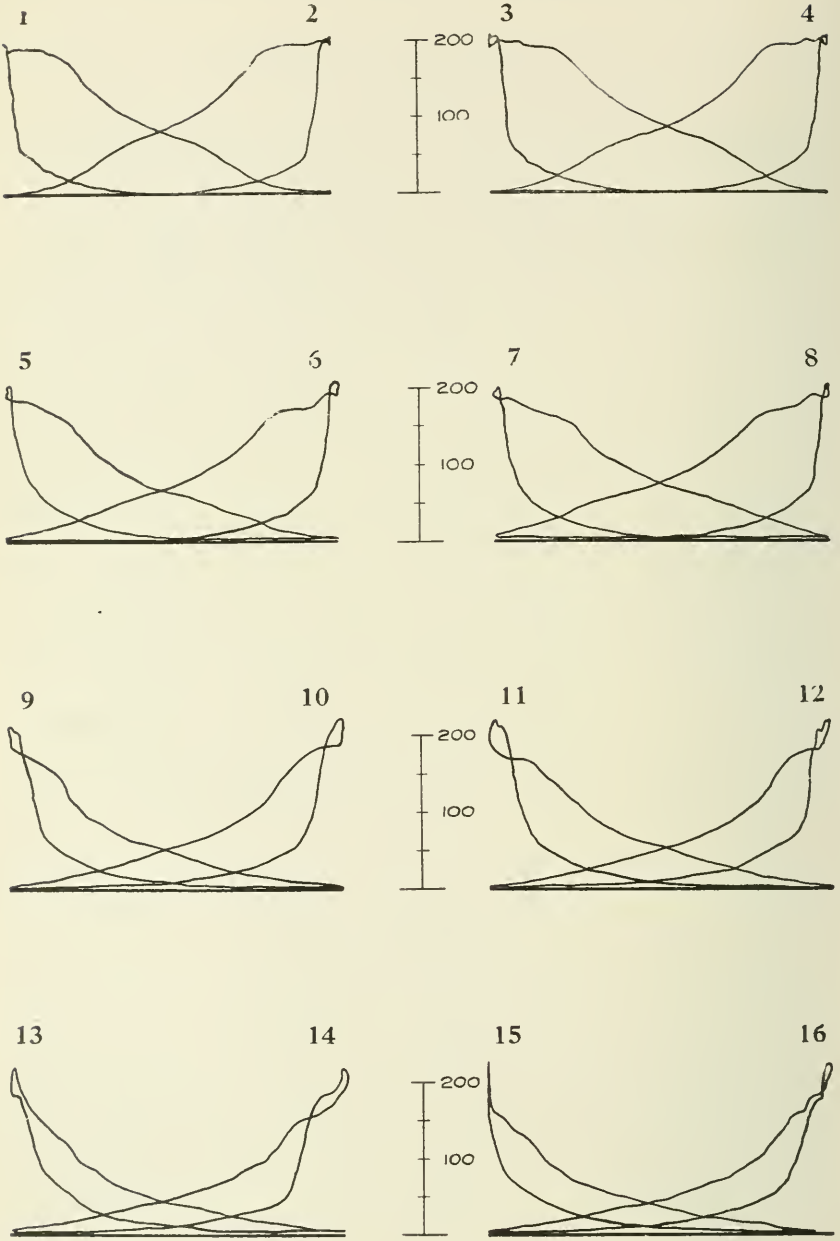


FIG. 56. REPRESENTATIVE INDICATOR DIAGRAMS FOR SERIES 2.
For Data, See Table 36.

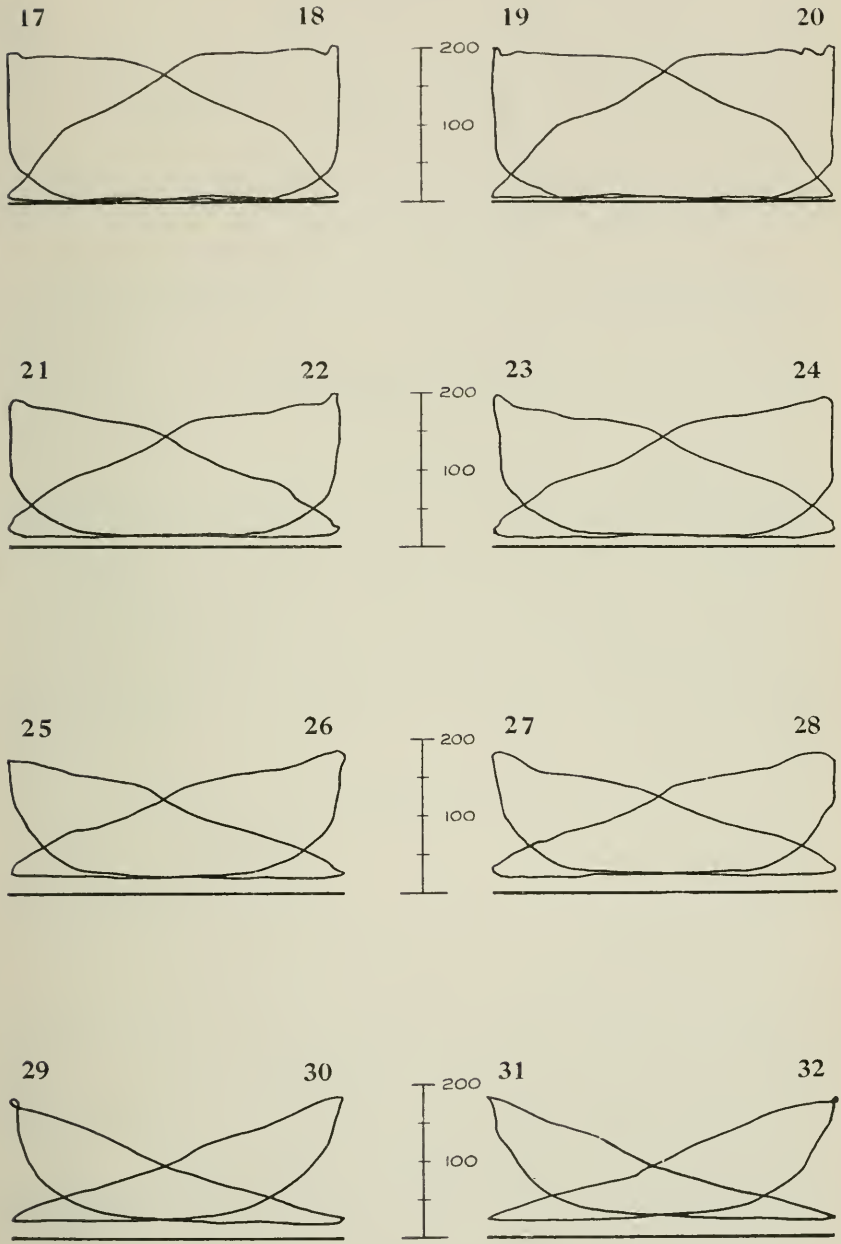


FIG. 57. REPRESENTATIVE INDICATOR DIAGRAMS FOR SERIES 2.
For Data, See Table 36.

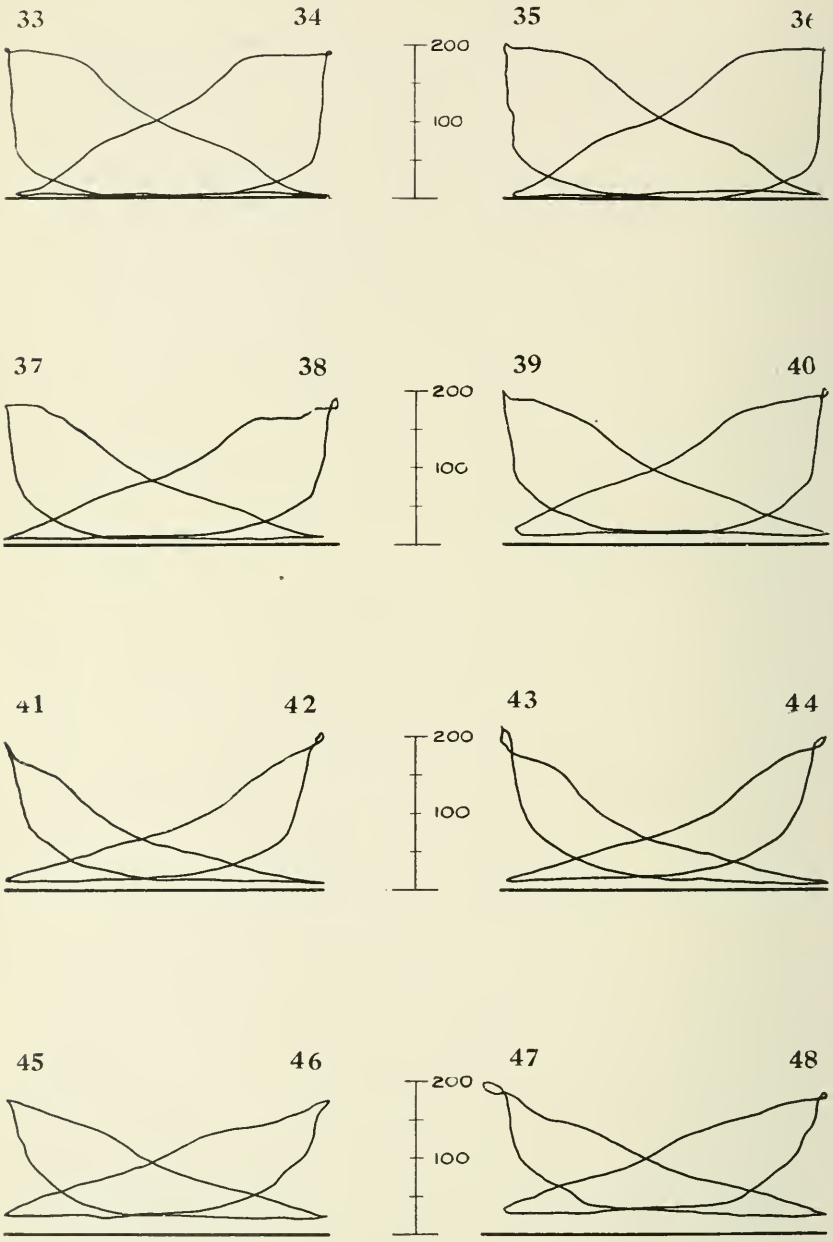


FIG. 58. REPRESENTATIVE INDICATOR DIAGRAMS FOR SERIES 1.
For Data, See Table 36.

APPENDIX 5.

Methods of Calculation.

Appendix 5 presents in detail the methods of calculation used in determining all results except for those items whose determination is self-evident.

The Marks and Davis steam tables for saturated and superheated steam have been used in all calculations pertaining to the properties of steam.

The events of the stroke and the corresponding pressures were determined for each indicator diagram by inspection and measurement. Horse power calculations were made, in like manner, for each indicator diagram. The values tabulated in Appendix 4 are averages of the determinations made from the individual diagrams.

Methods of estimating the ultimate analyses for the individual coal samples and of estimating the calorific values for the samples of ash and cinders have been presented in Appendix 4 in the consideration of Test Methods.

Item 318. Constant for dynamometer horse power.

$$\frac{\text{Item } 19}{33\,000}$$

Item 319. Constant for indicated horse power. Right, head end.

$$.000001983 \times (\text{Item } 68)^2 \times \text{Item } 77$$

Item 320. Constant for indicated horse power. Right, crank end.

$$.000001983 \times [(\text{Item } 68)^2 - (\text{Item } 135)^2] \times \text{Item } 77$$

Item 321. Constant for indicated horse power. Left, head end.

$$.000001983 \times (\text{Item } 69)^2 \times \text{Item } 78$$

Item 322. Constant for indicated horse power. Left, crank end.

$$.000001983 \times [(\text{Item } 69)^2 - (\text{Item } 136)^2 \times \text{Item } 78$$

Item 332. Constant for piston displacement. Right, head end.

$$229.17 \times \text{Item } 319$$

Item 333. Constant for piston displacement. Right, crank end.

$$229.17 \times \text{Item } 320$$

Item 334. Constant for piston displacement. Left, head end.

$$229.17 \times \text{Item } 321$$

Item 335. Constant for piston displacement. Left, crank end.

$$229.17 \times \text{Item } 322$$

Item 352. Average revolutions per minute.

$$\frac{\text{Item } 351}{60 \times \text{Item } 345}$$

Item 353. Equivalent speed, miles per hour.

$$\frac{\text{Item } 352 \times \text{Item } 19}{88}$$

Item 354. Equivalent piston speed in feet per minute.

$$\text{Item } 352 \times \left[\frac{\text{Item } 77 + \text{Item } 78}{12}\right]$$

Item 388. Barometric pressure in laboratory. The observed value has been corrected for the expansion of the mercury and brass by means of the formula:

$H = h_1[1.0026 - 0.000091\ t_1]$.

This method is in accordance with that of the United States Weather Bureau as described in Bulletin No. 472, page 29.

Item 407. Quality of steam, average.

Quality of steam in the dome has been determined by means of a throttling calorimeter and the formula:

$$x_o = \frac{H_o + 0.47 \times [t_s - t_o] - q_o}{r_o}$$

x_o = quality of steam
t_s = observed temperature in calorimeter
t_o = temperature of saturated steam at pressure in calorimeter
q_o = heat of liquid due to boiler pressure
H_o = total heat of dry steam at calorimeter pressure
r_o = latent heat of dry steam due to boiler pressure

Item 412. Factor of correction for quality of steam.

$$\frac{q + xr - h}{q + r - h}$$

q = heat of liquid due to average boiler pressure
h = heat of liquid due to average feed water temperature
x = quality of steam, average
r = latent heat of dry steam due to average boiler pressure.

Item 419. Total pounds of dry coal fired.

$$\text{Item } 418 \times \left[\frac{100 - \text{Item } 440}{100}\right]$$

Item 420. Total pounds of combustible by analysis.

$$\text{Item } 418 \times \left[\frac{100 - (\text{Item } 440 + \text{Item } 441)}{100}\right]$$

Item 421. Total pounds of ash by analysis.

$$\text{Item } 418 \times \left[\frac{\text{Item } 441}{100}\right]$$

Item 424. Total pounds of front end and stack cinders.

Item 422 + Item 423.

Item 435. Pounds of moisture per pound of dry air has been obtained from item 368, item 369, and the psychrometric chart and formula described by W. H. Carrier in the November, 1911, Journal of the American Society of Mechanical Engineers.

Item 458. Calorific value of dry coal in B.t.u. per pound.

$$\left[\frac{\text{Item } 443}{100 - \text{Item } 440}\right] \times 100$$

Item 459. Calorific value of combustible in B.t.u. per pound.

$$\left[\frac{\text{Item } 443}{100 - [\text{Item } 440 + \text{Item } 441]}\right] \times 100$$

Item 478. Correction for change of water level and pressure in the boiler from start to close of test has been calculated by the formula:

$$\frac{W_i\ [q + xr - q_i] - W_f\ [q + xr - q_f]}{q + xr - h}$$

W_i = initial weight of water in the boiler, pounds
W_f = final weight of water in the boiler, pounds
q = heat of liquid due to average boiler pressure
x = quality of steam, average
r = latent heat of dry steam due to average boiler pressure
q_i = heat of liquid at start of test
q_f = heat of liquid at close of test
h = heat of liquid due to average feed water temperature

Item 480. Total hot water losses, corrected, pounds.

$$\text{Item } 479 \times \left[\frac{xr}{q + xr - h}\right]$$

Item 481. Water delivered to boiler and presumably evaporated, pounds.

Item 476 — Item 480 + Item 478

Item 626. Dry coal fired per hour, pounds.

$$\frac{\text{Item } 419}{\text{Item } 345}$$

Item 627. Dry coal fired per hour per square foot of grate surface, pounds.

$$\frac{\text{Item 626}}{\text{Item 252}}$$

Item 633. Moist steam evaporated per hour, pounds.

$$\frac{\text{Item 481}}{\text{Item 345}}$$

Item 634. Dry steam evaporated per hour, pounds.

$$\text{Item 633} \times \text{Item 412}$$

Item 635. Dry steam evaporated per hour per square foot of heating surface, pounds.

$$\frac{\text{Item 634}}{\text{Item 275}}$$

Item 636. Dry steam evaporated per pound of dry coal, pounds.

$$\frac{\text{Item 634}}{\text{Item 626}}$$

Item 637. Dry steam evaporated per pound of coal as fired, pounds.

$$\text{Item 634} \div \left[\frac{\text{Item 418}}{\text{Item 345}}\right]$$

Item 639. Dry steam to engine per hour, pounds.

$$[\text{Item 476} + \text{Item 477} - \text{Item 479} - \text{Item 638}]$$

$$\times \left[\frac{\text{Item 412}}{\text{Item 345}}\right]$$

Item 641. Factor of evaporation.

$$\frac{q + xr - h}{970.4}$$

Item 642. Dry steam loss per hour due to calorimeter, leaks, corrections, etc., pounds.

Item 634 — Item 639

Item 642. Dry coal loss per hour equivalent to steam loss, pounds.

$$\frac{\text{Item } 642}{\text{Item } 636}$$

Item. 645. Equivalent evaporation per hour from and at 212°F., pounds.

Item 633 × Item 641.

Item 648. Equivalent evaporation per hour per square foot of total heating surface, pounds.

$$\frac{\text{Item } 645}{\text{Item } 275}$$

Item 656. Equivalent evaporation per hour per square foot of grate surface, pounds.

$$\frac{\text{Item } 645}{\text{Item } 252}$$

Item 657. Equivalent evaporation per hour per pound of coal as fired, pounds.

$$\text{Item } 645 \div \left[\frac{\text{Item } 418}{\text{Item } 345}\right]$$

Item 658. Equivalent evaporation per hour per pound of dry coal, pounds.

$$\frac{\text{Item } 645}{\text{Item } 626}$$

Item 659. Equivalent evaporation per hour per pound of combustible, pounds.

$$\text{Item } 645 \div \left[\frac{\text{Item } 420}{\text{Item } 345}\right]$$

Item 660. Boiler horse power.

$$\frac{\text{Item } 645}{34.5}$$

Item 666. Efficiency of the boiler, per cent.

$$\frac{\text{Item } 657 \times 970.4 \times 100}{\text{Item } 443.}$$

Item 697. Number of expansions, right, head end.

$$\frac{\text{Item } 510 + \text{Item } 86}{\text{Item } 495 + \text{Item } 86}$$

Item 698. Number of expansions, right, crank end.

$$\frac{\text{Item } 511 + \text{Item } 87}{\text{Item } 496 + \text{Item } 87}$$

Item 699. Number of expansions, left, head end.

$$\frac{\text{Item } 512 + \text{Item } 88}{\text{Item } 497 + \text{Item } 88}$$

Item 700. Number of expansions, left, crank end.

$$\frac{\text{Item } 513 + \text{Item } 89}{\text{Item } 498 + \text{Item } 89}$$

Item 734. Dry coal used by engine per indicated horse power per hour, pounds.

$$\left[\frac{\text{Item } 639}{\text{Item } 636}\right] \div \text{Item } 711$$

Item 735. B.t.u. in dry coal per indicated horse power per hour.

$$\text{Item } 734 \times \text{Item } 458$$

Item 736. Dry steam per indicated horse power per hour, pounds.

$$\frac{\text{Item } 639}{\text{Item } 711}$$

Item 737. B.t.u. in steam above 32°F. per indicated horse power per hour.

$$\text{Item } 736 \times [\, q + r]$$

Item 743. Drawbar horse power.

$$\text{Item } 318 \times \text{Item } 352 \times \text{Item } 487$$

Item 744. Dry coal per drawbar horse power per hour, pounds.

$$\left[\frac{\text{Item } 639}{\text{Item } 636}\right] \div \text{Item } 743$$

Item 745. Dry steam per drawbar horse power per hour, pounds.

$$\frac{\text{Item } 639}{\text{Item } 743}$$

Item 746. B.t.u. per drawbar horse power per hour.

$$\text{Item } 744 \times \text{Item } 458$$

Item 750. Millions of foot pounds at drawbar per hour.

$$\frac{\text{Item } 487 \times \text{Item } 19 \times \text{Item } 351}{\text{Item } 345 \times 1\,000\,000}$$

Item 752. Dry coal per million foot pounds at drawbar, pounds.

$$\left[\frac{\text{Item } 639}{\text{Item } 636}\right] \div \text{Item } 750$$

Item 753. Dry steam per million foot pounds at drawbar, pounds.

$$\frac{\text{Item } 639}{\text{Item } 750}$$

Item 754. B.t.u. per million foot pounds at drawbar.

$$\text{Item } 752 \times \text{Item } 458$$

Item 755. Indicated horse power per square foot of heating surface.

$$\left[\frac{\text{Item } 711}{\text{Item } 275}\right] \times \left[\frac{\text{Item } 634}{\text{Item } 639}\right]$$

Item 756. Indicated horse power per square foot of grate surface.

$$\left[\frac{\text{Item } 711}{\text{Item } 252}\right] \times \left[\frac{\text{Item } 634}{\text{Item } 639}\right]$$

Item 757. Drawbar horse power per square foot of heating surface.

$$\left[\frac{\text{Item } 743}{\text{Item } 275}\right] \times \left[\frac{\text{Item } 634}{\text{Item } 639}\right]$$

Item 758. Drawbar horse power per square foot of grate surface.

$$\left[\frac{\text{Item } 743}{\text{Item } 252}\right] \times \left[\frac{\text{Item } 634}{\text{Item } 639}\right]$$

Item 764. Tractive force based on mean effective pressure, pounds.

$$\left[\frac{33\,000}{\text{Item } 19}\right] \times \left[\frac{\text{Item } 711}{\text{Item } 352}\right]$$

Item 770. Machine friction of the locomotive in terms of horse power.

$$\text{Item } 711 - \text{Item } 743$$

Item 771. Machine friction of the locomotive in terms of mean effective pressure, pounds.

$$\frac{\text{Item } 770}{\text{Item } 352 \times [\text{Item } 319 + \text{Item } 320 + \text{Item } 321 + \text{Item } 322]}$$

Item 772. Machine friction of the locomotive in terms of drawbar pull, pounds.

$$\left[\frac{33\,000}{\text{Item } 19}\right] \times \left[\frac{\text{Item } 770}{\text{Item } 352}\right]$$

Item 773. Machine friction of the locomotive in per cent of indicated horse power.

$$\left[\frac{\text{Item } 770}{\text{Item } 711}\right] \times 100$$

Item 778. Machine efficiency of the locomotive, per cent.

$$\left[\frac{\text{Item } 743}{\text{Item } 711}\right] \times 100$$

Item 779. Efficiency of the locomotive, per cent.

$$\frac{254\,655.8}{\text{Item } 746}$$

$$\text{Constant } 254\,655.8 = \left[\frac{33\,000 \times 60}{777.52}\right] \times 100$$

Item 785. Ratio of total weight of the locomotive to the maximum indicated horse power.

$$\frac{\text{Item } 63}{\text{Item } 721}$$

Item 786. Ratio of total heating surface to maximum indicated horse power.

$$\frac{\text{Item } 275}{\text{Item } 721}$$

Item 851. B.t.u. absorbed by the boiler per pound of coal as fired.

$$\text{Item } 657 \times 970.4$$

Items 852, 853, 854, 855, and 856 see next page.

Item 857. B.t.u. loss due to combustible in front-end cinders.

$$\frac{\text{Item } 422 \times \text{Item } 461}{\text{Item } 418}$$

Item 858. B.t.u. loss due to combustible in stack cinders.

$$\frac{\text{Item } 423 \times \text{Item } 462}{\text{Item } 418}$$

Item 860. B.t.u. loss due to combustible in ash.

$$\frac{\text{Item } 428 \times \text{Item } 463}{\text{Item } 418}$$

Item 869. B.t.u. loss due to radiation and unaccounted-for.

$$\text{Item } 443 - [\text{Item } 851 + \text{Item } 852 + \text{Item } 853 + \text{Item } 854 + \text{Item } 855 + \text{Item } 856 + \text{Item } 857 + \text{Item } 858 + \text{Item } 860]$$

Item 852. B.t.u. loss per pound of coal as fired due to moisture in the coal.

$$\frac{\text{Item } 440}{100} \times \left[(211 - \text{Item } 368) + 970.4 + 0.47 \times (\text{Item } 367 - 211) \right]$$

Item 853. B.t.u. loss per pound of coal as fired due to moisture in the air.

$$\left[\frac{[\text{Item } 418 \times \text{Item } 449] - [\text{Item } 428 \times \text{Item } 831] - [\text{Item } 422 \times \text{Item } 841] - [\text{Item } 423 \times \text{Item } 846]}{\text{Item } 418 \times 100} \right] \times \left[\frac{3.032 \times \text{Item } 469}{\text{Item } 468 + \text{Item } 467} \right] \times \left[0.47 \times (\text{Item } 367 - \text{Item } 368) \right] \times \text{Item } 435$$

Item 854. B.t.u. loss per pound of coal as fired due to hydrogen in the coal.

$$9 \times \left[\frac{\text{Item } 450}{100} \right] \times \left[(211 - \text{Item } 368) + 970.4 + 0.47 \times (\text{Item } 367 - 211) \right]$$

Item 855. B.t.u. loss per pound of coal as fired due to escaping gases.

$$\left[\frac{[\text{Item } 418 \times \text{Item } 449] - [\text{Item } 428 \times \text{Item } 831] - [\text{Item } 422 \times \text{Item } 841] - [\text{Item } 423 \times \text{Item } 846]}{\text{Item } 418 \times 100} \right] \times \left[\frac{[4 \times \text{Item } 468] + \text{Item } 466 + 700}{3 \times [\text{Item } 468 + \text{Item } 467]} \right] \times \left[0.24 \times [\text{Item } 367 - \text{Item } 368] \right]$$

Item 856. B.t.u. loss per pound of coal as fired due to incomplete combustion.

$$\left[\frac{[\text{Item } 418 \times \text{Item } 449] - [\text{Item } 428 \times \text{Item } 831] - [\text{Item } 422 \times \text{Item } 841] - [\text{Item } 423 \times \text{Item } 846]}{\text{Item } 418 \times 100} \right] \times \left[\frac{\text{Item } 467}{\text{Item } 468 + \text{Item } 467} \right] \times 10\,150$$

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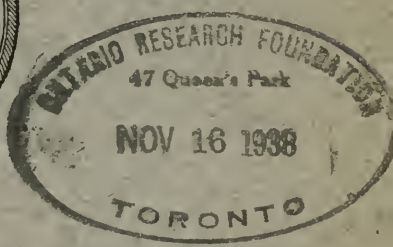
MAGNETIC AND OTHER PROPERTIES OF IRON-SILICON ALLOYS, MELTED IN VACUO

BY

TRYGVE D. YENSEN



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ENGINEERING EXPERIMENT STATION

BULLETIN No. 83 NOVEMBER, 1915

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BY

TRYGVE D. YENSEN

Assistant, Engineering Experiment Station,
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I. Introduction.*

1. *Preliminary.*—Probably no series of iron alloys has received more attention in recent years than has the iron-silicon series. This is largely due to the fact that some of these alloys have proved their superiority over pure iron and other iron alloys for certain very important purposes in the electrical industry. The iron-silicon alloys have been thoroughly investigated by chemists, metallurgists, and physicists, so that their properties are generally very well known. The reason that this series of alloys has again been the object of an investigation is the discovery by the writer about two years ago that the magnetic properties of pure iron that is melted in vacuo are far superior to any other grade of iron known at that time.† It was evident that this superiority was due to the purity of the iron produced by the vacuum process, the iron thus produced containing only 0.01 per cent carbon. A new standard of iron purity being thus produced, it became desirable to repeat previous investigations on the properties of iron alloys with this new standard as a basis, and the present bulletin deals primarily with iron-silicon alloys containing from 0.001 to 8.5 per cent silicon.‡ Besides the magnetic properties the bulletin contains data concerning the electrical and mechanical properties and gives chemical analyses. Photomicrographs are also shown for the various alloys.

The writer wishes to express his appreciation of the sympathy shown and assistance rendered by members of various departments in the University of Illinois. In particular he wishes to mention Mr. W. A. Gatward, Fellow in the Engineering Experiment Station, for the conscientious work done during the year, largely in connection with the magnetic testing, but also in many other ways. The chemical analysis has been done by Mr. J. M. Lindgren and Mr. F. H. Whittum of the Chemistry department who deserve great credit for the thoroughness with which they have accomplished their part of the investigation.

II. Historical Review.

2. *Mechanical Properties.*—The effect of silicon upon the mechanical properties of iron was noticed as early as the beginning of

*A paper containing much of the information given in this bulletin was presented before the American Institute of Electrical Engineers, St. Louis, October 20, 1915. (Trans. Am. Inst. of Elect. Engrs. Vol. 34, No. 10, Oct., 1915).

†Magnetic and other Properties of Electrolytic Iron Melted in Vacuo. Bulletin No. 72, Engineering Experiment Station. Univ. of Illinois, 1914. Trans. Am. Inst. of Elect. Engrs. Vol. 33, Part I, p. 451, 1914. (A. I. E. E. Proceedings, February, 1915, p. 237).

‡Bulletin No. 77 published by the Engineering Experiment Station deals with iron-boron alloys, melted in vacuo, the maximum boron content being 0.50 per cent.

the last century, when Mushet found that quartz sand applied to molten iron made the iron harder and more brittle. These results were later confirmed by other investigators,* and by 1880 it was generally accepted that silicon increases the strength of iron and at the same time it tends to make it hard and brittle if added in quantities above a few per cent. It was not until towards the end of the century, however, that systematic investigations of the iron-silicon alloys were made. In 1887 Tilden, Roberts-Austen, and Turner† carried on extensive investigations on the mechanical properties of low silicon alloys, containing from 0.10 to 0.50 per cent silicon. These investigators found that silicon increases the hardness, raises the elastic limit and ultimate strength, and decreases the elongation. The investigations were continued by Hadfield in 1889,‡ who carried the silicon content up to 8.83 per cent. With a silicon content of 7.23 per cent and 8.83 per cent, however, the ingots could not be forged, so that the highest silicon alloy tested contained 4.90 per cent silicon. The results obtained by Hadfield as well as by some of the later investigators are shown graphically in Figs. 1a, b, and 2a, b. Pages 7, 8 and 9. As the carbon content in the alloys used in the above investigations was comparatively high, varying from 0.14 to 0.26 per cent, Baker,¶ in his research on iron-silicon alloys in 1903, reduced the carbon content to 0.04 per cent. This was accomplished by first melting the iron alone, and then, after removing the slag, adding the ferrosilicon. The charge was kept molten for 15 minutes longer, and was then cast into ingots. This procedure produced an iron not only low in carbon, but low in manganese as well. Guillet§ in 1904 made an extensive investigation of the microstructure of the iron-silicon alloys with silicon contents up to 30 per cent. He also studied the mechanical properties of the alloys, but his results, although in general confirming those obtained by his predecessors, are not strictly comparable to them, on account of the high and varying manganese and carbon content. Furthermore, only four of his alloys were forgeable, namely, those containing 5.12 per cent silicon or less. However, Guillet confirmed the results of Hadfield with regard to the effect of carbon upon the limit of forgeability of the iron-silicon alloys. Thus he found that, with a carbon content of 0.94 per cent, the limit lies below 5 per cent silicon. Bisset**

*See: P. Paglianti, Metallurgie, Vol. 9, p. 217, 1912, where references are given to early investigators.

†Report of British Assoc. for the Advancement of Science, 1888.

‡Journ. Iron and Steel Institute, Vol. 36, 1889, II, p. 222.

¶Journ. Iron and Steel Institute, Vol. 64, 1903, II, p. 312.

§Rev. de Metallurgie, Memoirs, Vol. 1, p. 46, 1904.

**Iron Age, Aug. 25, 1910.

states that silicon adds strength to iron comparable to the effect of carbon, but in other respects leaves the iron unchanged. Paglianti,* in 1912 investigated the properties of iron silicon alloys, using as a basis electro-converted iron with a carbon content of 0.10 per cent and with manganese varying from 0.22 to 0.60 per cent. The silicon was added in the form of 50 per cent ferrosilicon, and the specimens were obtained from ingots weighing 80 to 100 kg. The results obtained by these various investigators, as shown in Fig. 1a, b differ considerably in numerical values, probably due partly to the difference in the purity of the iron used as a basis, and partly to the method of melting and the subsequent mechanical-and heat-treatment.

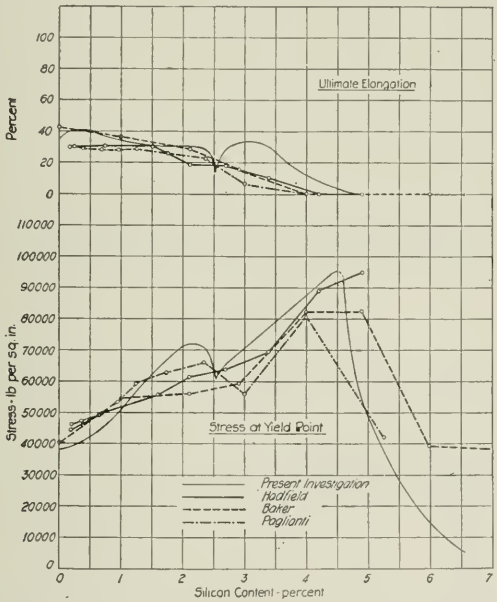

FIG. 1a. MECHANICAL PROPERTIES OF IRON-SILICON ALLOYS, ACCORDING TO VARIOUS INVESTIGATORS. AS FORGED.

*Metallurgie, Vol. 9, p. 217, 1912.
Rev. de Metal. Extracts, Vol. 11, p. 4, Jan., 1914.

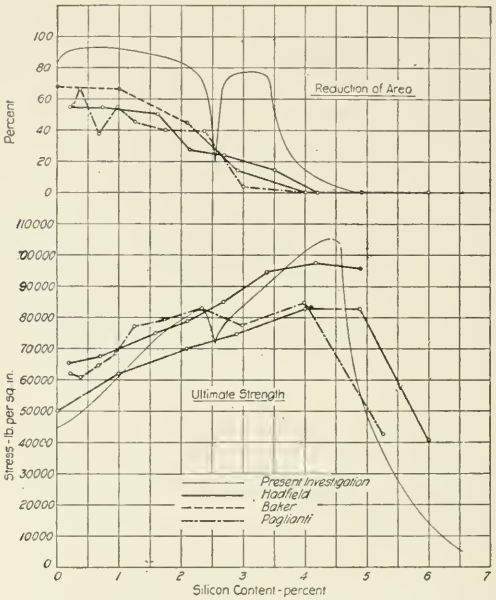

FIG. 1b. MECHANICAL PROPERTIES OF IRON-SILICON ALLOYS, ACCORDING TO VARIOUS INVESTIGATORS. AS FORGED.

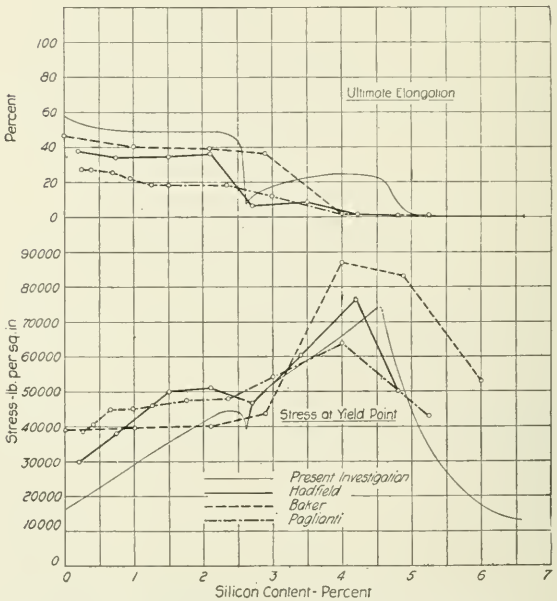

FIG. 2a. MECHANICAL PROPERTIES OF IRON-SILICON ALLOYS, ACCORDING TO VARIOUS INVESTIGATORS. ANNEALED.

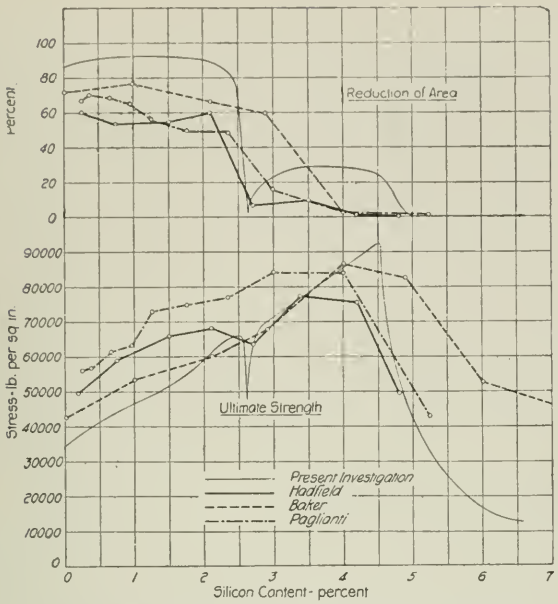

FIG. 2b. MECHANICAL PROPERTIES OF IRON-SILICON ALLOYS, ACCORDING TO VARIOUS INVESTIGATORS. ANNEALED.

TABLE 1.

MAGNETIC AND ELECTRICAL PROPERTIES OF HADFIELD'S IRON-SILICON ALLOYS.

Mark	Silicon %	Max. Induction for H = 45.	Permeability for H = 8	Retentivity in Terms of B	Coercive Force in Terms of H	Energy Dissipated per Complete Cycle. ergs per cc.	Spec. Elec. Resistance. microhms
S. C. I.	.07	.....	....	....	...	.....	10.2
B	.14	16800	1625	9770	1.66	10760	10.9
898 E	2.50	16420	1680	4080	.90	7900	42.1
898 H	5.50	15980	1630	3430	.85	6500	65.2

The results due to Baker, for example, show the tensile test curves to lie considerably to the right of those due to the other investigators. This fact, in conjunction with the appearance of his photomicrographs, suggests that part of the silicon in his alloys may be in the form of silica, caused by the method of melting. In view of these differences it is not strange that the results of the above investigators should disagree to some extent.

3. *Magnetic and Electrical Properties.*—In connection with his investigation of the mechanical properties of iron-silicon alloys in

1889, Hadfield* touched upon their magnetic and electrical properties. The results arrived at were not favorable, as it was found "that the material (containing 4.43 per cent silicon and .18 per cent carbon) had less susceptibility and more retentiveness than good soft iron, and that it had enormously less retentiveness than hard steel suitable for magnet making." However, Hadfield continued his investigations on the magnetic properties of iron-silicon alloys and in 1900, assisted by Barrett and Brown, he was able to give a report that was quite different from the one above referred to.† Among the large number of alloys investigated, the report includes two iron-silicon alloys, containing 2.5 and 5.5 per cent silicon respectively. The results show (see Table 1) that both alloys have a higher maximum permeability and a decidedly lower hysteresis loss than pure iron, and that the electrical resistance increases at the rate of 10 to 12 microhms per c. c. for each per cent of silicon added.‡ This remarkable discovery caused an almost immediate adoption of the silicon-steel for use in electrical machinery, particularly in transformers. It also gave a new impetus to the investigation of iron and steel alloys for electrical purposes, and a large number of investigators have been searching for new treasures in this field. Except for permanent magnets, however, silicon still holds the first place as the alloying element, giving best results for electromagnetic machinery.¶ Among those who have investigated the magnetic and electrical properties of iron-silicon since Hadfield's discovery, only a few will be mentioned in the following review.

Gumlich and Schmidt§ investigated a large number of commercial steels and among these certain iron-silicon alloys, in general confirming the results of Hadfield. Baker,** as already mentioned, used a very pure grade of iron, the carbon content being only 0.04 per cent. His results are summarized in Table 2. From this table it is seen that the permeability increases and the coercive force and hys-

*Journ. Iron & Steel Inst., Vol. 36, 1889, II, p. 237.

†Barrett, Brown & Hadfield; Scient. Trans. Royal Dublin Soc. VII Ser. 2, PaPrt 4, Jan., 1900.
Barrett, Brown & Hadfield: Jour. Inst. of Elect. Engrs. Vol. 31, p. 674, 1901-02.
Reviewed by E. Gumlich: Stahl u Eisen 1902, No. 6, p. 230.

‡The electrical conductivity of iron alloys is described in detail by Barrett: Proc. Royal Soc. Vol. 69, p. 480, 1902.

¶The iron-cobalt alloy Fe_2Co, first investigated by Weiss, has recently been investigated by the writer, and may prove to be a valuable material on account of its high permeability in medium and high fields. See Gen. Elect. Review. Vol. 18, p. 881, Sept., 1915.

§Elektrotech. Zeitsch. Vol. 22, p. 691 1901.

**Journ. Iron and Steel Inst. Vol. 64, 1903, II, p. 312.
Journ. Inst. of Elect. Engrs. Vol. 34, p. 498, 1904-05.

TABLE 2.

MAGNETIC PROPERTIES OF IRON-SILICON ALLOYS, (FROM BAKER).

Silicon %	Max. Induction H = 20	Permeability for H = 4	Retentivity	Coercive Force	Energy Dissipated per Complete Cycle
0.02	16000	2325	8375	1.8	10550 ergs per cc
1.02	16200	2562	8000	1.7	8798 " " "
2.90	15500	2750	7325	1.5	8081 " " "
4.89	14750	2665	7200	1.2	6110 " " "
7.47	14000	2937	9000	1.0	5613 " " "

teresis loss decrease with increasing silicon content up to the limit of forgeability.

Dillner and Engstrom,* in contradiction to other investigators, found silicon to reduce the permeability and increase the hysteresis loss of sheet iron. By using silicon and aluminum together in certain proportions the best results were obtained.

Guertler† investigated the electrical conductivity of numerous alloys of iron with all the more common elements. From the curves given the resistance of the 3.4 per cent iron-silicon alloy is shown to be 50 microhms per cu. cm. confirming the results obtained by Barrett.

Burgess and Aston‡ have investigated a large number of iron alloys, using doubly refined electrolytic iron as a basis. This iron, containing only 0.03 per cent impurities, was melted in an electric resistance furnace together with the alloying element. This method of melting, however, proved to involve some danger, inasmuch as the iron would decompose the carbon monoxide gases in the furnace and combine with small quantities of carbon and oxygen.¶ Burgess and Aston found that silicon improves the iron magnetically, the 4.65 per cent alloy having a decidedly lower hysteresis loss than their pure iron standard.

Gumlich and Goerens§ in 1912 carried out researches on iron-silicon alloys using samples made for them for the purpose by the firm of Krupp at Essen. The results are summarized in Table 3. It is seen from this table that for rods the coercive force and probably also the hysteresis loss is a minimum in the region of 4 per cent silicon, as shown by previous investigators, while for sheets the minimum occurs at 0.50 per cent silicon. This difference between sheets

*Journ. Iron and Steel Inst. Vol. 67, 1905, I, p. 474.

†Zeitschr. Anorg. Chemie. Vol. 51, p. 397, 1906.

‡Met. and Chem. Eng'n. March, 1910.

¶This was shown by the writer to be the case in his own preliminary experiments, described in Bulletin No. 72 of the Eng. Exp. Sta. Univ. of Ill. 1914.

§Trans. Faraday Soc. Vol. 8, p. 98, 1912.
Ferrum 10 (12) p. 33, 1912; Chem. Zentr. Bl. [5] 17 (13) p. 380.

Table 3.

Magnetic Properties of Iron Silicon Alloys (From Gumlich and Goerens.) Samples annealed at 800° C.

Silicon %	Coercive Force		Maximum Permeability		Saturation Value, $4\pi I_s$
	Sheets	Rods	Sheets	Rods	
.06	1.80	1.25	3000	3000	21200
.15	.70	1.10	....	....	21300
.30	.65	1.15	....	....	21200
.40	.54	1.20	11600	2800	21150
.60	.80	1.30	....	2400	21050
.90	.75	1.50	....	....	20950
1.00	.80	1.30	8500	3500	20900
1.65	.70	1.20	....	....	20600
1.95	.75	1.25	....	2900	20400
2.35	.85	1.00	7700	2700	20300
3.70	.85	.65	....	4900	19750
4.00	...	...	9400	4100	.....
4.50	1.05	.65	5500	4400	19100
5.25	...	.75	....	4300	18300
8.60	...	.95	....	3000	16100

These values are only approximate as they are obtained from curves. The flux densities are not given.

and rods could not be accounted for at the time of the report, but was being investigated. For sheets the maximum permeability curve has two maxima, one, of 11600, occuring at 0.40 per cent silicon, and the other, of 9400, at 4 per cent silicon. For rods, only one maximum occurs, namely 4900, at 3.7 per cent silicon. In view of the saturation values, and of the photomicrographs and chemical analysis, the authors are of the opinion, previously suggested by Hadfield and Hopkinson,* that silicon has not directly any improving influence on the iron, but that the improvement is due to the neutralizing effect of silicon upon the carbon contained in the iron. Thus, silicon appears to convert all the carbon from the dissolved form into either cementite or graphite; and even to prevent the formation of hardening carbon by quenching. The correctness of this opinion is apparently borne out by the values for the coercive force before and after annealing for a number of silicon alloys containing relatively large amounts of carbon.

The conclusions arrived at by Gumlich and Goerens have received further confirmation by the results obtained by Paglianti.† In Table 4 the magnetic properties of a few of his alloys have been given. The carbon content is approximately 0.10 per cent. It is seen from this table that quenching increases the coercive force 4 to 7 times for low silicon iron, and only twice for high silicon iron. Similarly the hysteresis loss is increased 2½ to 4½ times for low, and only 1¼

*Journ. Inst. Elect. Engrs. Vol. 46, p. 235, 1911.

†Metallurgie, Vol. 9, p 217, 1912; Rev. de Metall. Extracts, Vol. 11, p. 4, Jan., 1914.

Table 4.

Magnetic and Electrical Properties of Iron-Silicon Alloys
(From Paglianti)

Alloy No.	Silicon %	Heat treatment	Maximum Permeability μ^{max}	Coercive Force for B_{max} = 16000 gilberts per cm.	Retentivity for B_{max} = 16000. gausses	Hysteresis Loss for B_{max} = 16000. ergs per cc. per cycle	Spec. Elect. Resistance microhms
1	.24	A	980	4.75	10075	17865	16.98
3	.67		985	4.25	9100	15700	22.20
7	2.35	As	1200	3.10	7375	13890	42.50
10	5.26	forged	1900	1.75	7900	9650	66.00
1	.24	B	1475	2.37	7200	10700	15.62
3	.67	Annealed	1600	2.42	7650	11550	21.00
7	2.35	at 1100°	3200	.78	4625	3900	39.10
10	5.26	C, cooled in 36 hours	2760	.85	4000	4900	64.50
1	.24	K	500	8.37	7975	28600	16.66
3	.67	Annealed	320	15.87	8975	46100	22.20
7	2.35	at 1100°	1400	1.50	3500	7350	41.50
10	5.26	Slowly cooled to 900°. Quenched in water from 900°	925	1.75	2400	6500	66.00

times for high silicon iron. Again, the retentivity is increased for low silicon but considerably decreased for high silicon. All these results apparently point in the direction suggested by Gumlich. Taking into account both the hysteresis loss and eddycurrent loss as well as the mechanical properties of the alloys, Paglianti concludes that the 2 per cent alloy has the most favorable characteristics for electromagnetic machinery. This conclusion is based on the assumption that the eddycurrent loss varies inversely as the resistance of the material, an assumption which according to recent investigations by Ball and Ruder* is not strictly correct. No law could be found by them for expressing the dependence of eddycurrent loss on resistivity, except that the loss in general decreases with increasing resistance, but at a slower rate. However, this discovery would not change Paglianti's conclusion, as it would favor the low rather than the high silicon content, and a silicon content much below 2 per cent would be out of the question in any case.

4. *Metallography.*—The effect of silicon upon some of the physical properties of iron having been dealt with in the two previous sections, it remains to inquire into the manner in which silicon has been found to influence the structure of the iron, and the relation between the

*Gen. Elect. Review, Vol. 17, p. 487, May, 1914.
Electrician Vol. 74, p. 55, 1914.

structural and physical properties, although this has already been touched upon.

The metallography of the iron-silicon series was investigated by Osmond* in 1890, followed by Arnold† and Baker.‡ By their experiments the polymorphic transformation points, Ar_2 and Ar_3 were established with some degree of certainty for alloys containing up to about 8 per cent silicon. Later, Guertler and Tammann¶ investigated the complete series, but only that part which deals with crystallization from the liquid to the solid state is as yet available, (Fig. 3.)

However, from the data thus obtained Gontermann§ has worked out an equilibrium diagram for the iron-silicon alloys up to the first eutectic point, 21.4 per cent silicon. The diagram is shown in Fig. 4, where the curves that have not been established by experiments are shown broken. According to this diagram, silicon in quantities of about 15 per cent or less remains dissolved in the iron throughout all the allotropic modifications of the latter. If the solution contains more than 15 per cent silicon, two different crystals are formed upon cooling, namely the mixed saturated crystal of iron and silicon, containing about 15 per cent silicon, and a crystal of the composition FeSi.**

The results of the above investigations are confirmed by the photomicrographs published by Baker,‡ Guillet,†† Guertler and Tammann,¶ Gumlich and Goerens,‡‡ and Paglianti.¶¶ These shows that with a silicon content of 15 per cent or less there is only one kind of crystal present in the alloys, with the exception—in the low silicon alloys—of the cementite crystals caused by the carbon present in the iron to a more or less extent. With a higher silicon content, above 3 or 4 per cent, carbon is no longer precipitated as cementite, but as graphite, and appears in the forms of black spots on the surface of the polished specimens.

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*Journ. Iron & Steel Inst. Vol. 37, 1890, I, p. 62.

†Journ. Iron & Steel Inst. Vol. 45, 1894, I, p. 107.

‡Journ. Iron & Steel Inst. Vol. 64, 1903, II, p. 312.

¶Zeitschr. Anorg. Chem. Vol. 47, p. 163, 1905.

§Zeitschr. Anorg. Chem. Vol. 59, p. 384, 1908.
Journ. Iron & Steel Inst. Vol. 83, 1911, I, p. 431.

**According to Guertler and Tammann the 20 per cent alloy forms the compound Fe_2Si, and this in turn forms a eutectic with FeSi when the alloy contains 21.4 per cent silicon.

††Rev. de Metallurgie, Memoirs, Vol. 1, p. 46, 1904.

‡‡Trans. Faraday Soc. Vol. 8, p. 98, 1912.

¶¶Metallurgie, Vol. 9, p. 217, 1912.
Rev. de Metallurgie, Extracts, Vol. 11, p. 4, 1914.

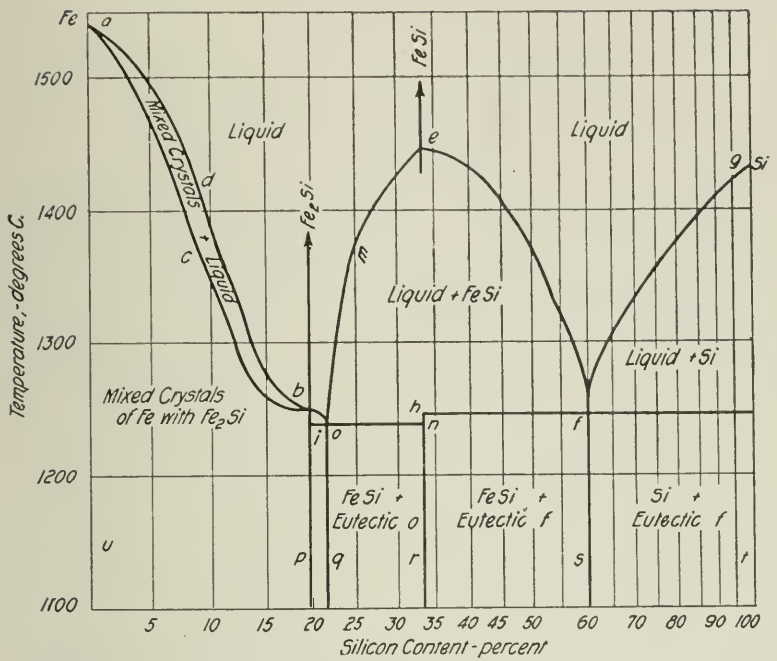


FIG. 3. PHASE DIAGRAM OF IRON-SILICON ACCORDING TO GUERTLER AND TAMMANN.

With regard to the exact manner in which the precipitation of carbon takes place in the presence of silicon, the writer has found no definite explanation. Wüst and Petersen* and Gontermann have made a thorough study of the ternary system, iron-silicon-carbon. The latter suggests that graphite is partly formed by a separation out of supersaturated silico-austenite and partly by the dissociation of silico-cementite.

5. *Summary of Historical Review.*—The information gathered from this review as to the action of silicon upon iron may be summarized as follows:

(a) Silicon is soluble in quantities up to about 15 per cent in all the allotropic modifications of iron. In the presence of silicon, carbon is readily precipitated, and if sufficient silicon is present,—above 4 per cent,—carbon may be completely precipitated as graphite, even though the solution should be rapidly cooled from the molten state.

(b) It has been shown that carbon in the form of graphite has a much less damaging effect upon the magnetic properties of iron than it has in the dissolved form or in the form of cementite. As silicon

*Metallurgie, Vol. 3, p. 811, 1906.

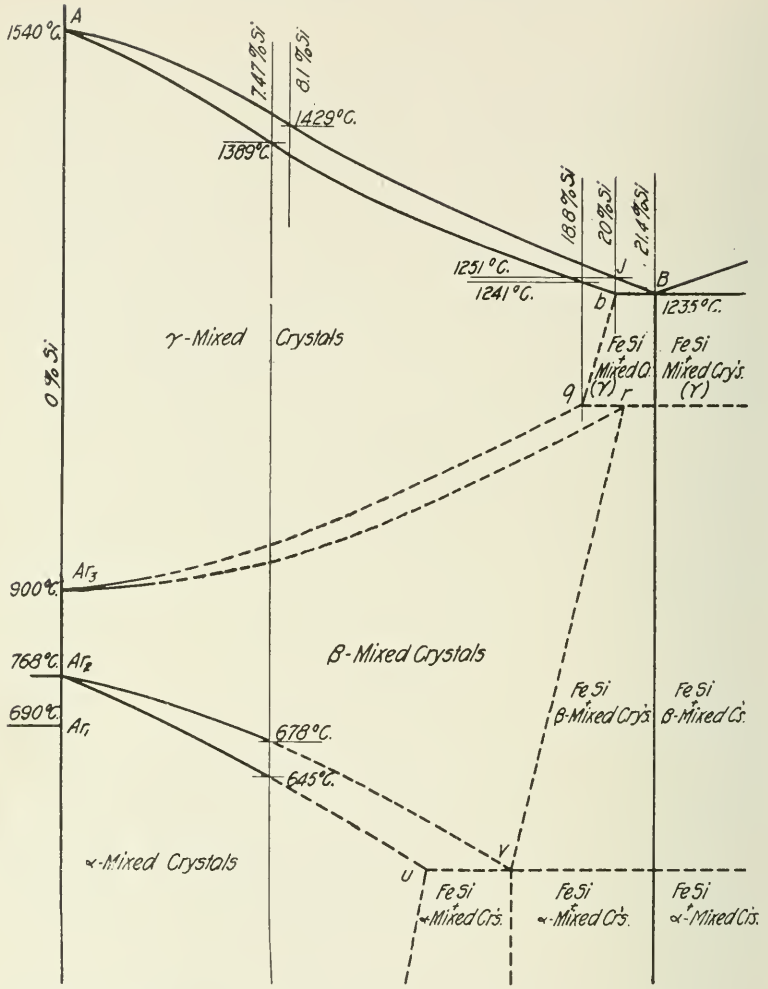


FIG. 4. EQUILIBRIUM DIAGRAM FOR IRON-SILICON ALLOYS ACCORDING TO GONTERMANN.

precipitates carbon as graphite, it is readily seen why silicon indirectly should have a beneficial effect upon the magnetic properties of iron, since even the purest iron contains small amounts of carbon. The most favorable silicon content varies, according to the various investigators, from 2 to 4 per cent.

(c). Silicon increases the electrical resistance of iron by 10 to 12 microhms per cu. cm. for each per cent added. As the eddycurrent loss decreases with an increase in resistance, it follows that the silicon

content should be made as high as possible without loss of the most favorable magnetic properties.

(d). Silicon in quantities up to 4 per cent increases the strength of iron in proportion to the amount added. If more than 4 per cent is added the strength decreases rapidly probably due to the formation of graphite, and the yield point coincides with the breaking point. The ductility is but little affected by the silicon below 2.5 per cent; above this amount the alloys are brittle, and with 4 per cent or more the elongation and reduction of area are nil. With 7 per cent or more silicon, the alloys are not forgeable. Unlike carbon, silicon does not confer upon iron the property of becoming hardened when water-quenched.

III. Material, Apparatus and Methods

Although details with regard to the methods of testing were given in Bulletin No. 72, so many changes have been made since its appearance, that it seems desirable to describe these changes as well as to review briefly the apparatus and methods as a whole. Where details are lacking the reader is referred to Bulletin No. 72.

6. *Iron*—The iron used as the basis of the investigation consisted of doubly refined electrolytic iron, containing 0.006 to 0.01 per cent carbon and 0.01 per cent silicon. Before being used, the iron was thoroughly cleaned with HCl, distilled water and alcohol and then dried by means of ether in vacuo.

7. *Silicon.*—The analysis of the silicon used as the alloying element gave the following result:

Si	92.23 %
Fe	8.50 "
C	.016 "
	100.746 %

8. *Melting.*—About 600 grams of iron and silicon in the desired proportion were placed in a crucible, made from fused magnesia, and covered by means of a magnesia cover. The melting was done in an Arsem type vacuum furnace, Fig. 5, under a finishing pressure of 0.5 mm. of mercury, measured by means of a McLeod gage. To obtain this vacuum it was necessary, after the preliminary low heating, to bring the temperature of the carbon heating element up to as high a temperature as possible before the iron should have time to melt and begin to vaporize. This was accomplished by an input of about 14 KW. By this method large amounts of occluded gases were driven

out of the carbon, that otherwise would have prevented a good vacuum from being reached within a reasonable time. In about 15 minutes the iron would begin to vaporize from the hottest parts of the crucible,

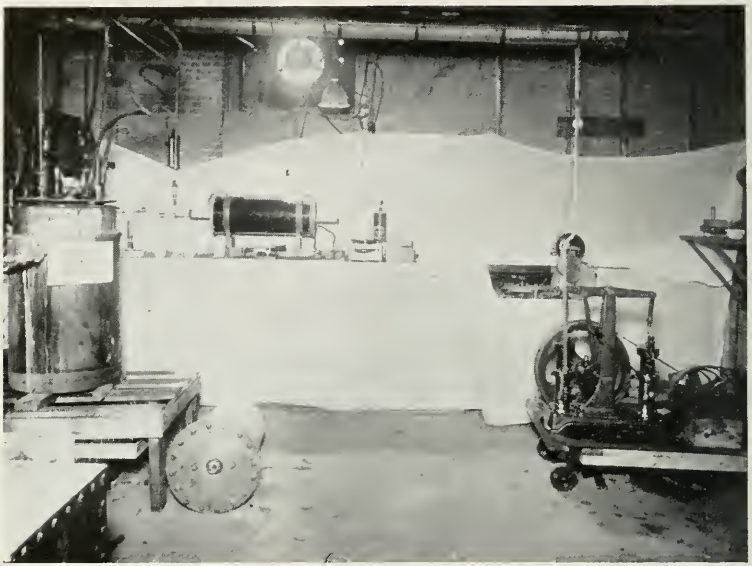


Fig. 5. Furnace Room, Showing Vacuum Furnaces.

and, if the input was maintained, iron vapor arcs were formed that would practically cause a short-circuit of the furnace. The input was, therefore, decreased from 14 to 12 KW and maintained there for about 1 hour. At the beginning of this period the iron would be molten and the pressure would be about 2.0 mm, while at the end of the period the pressure would have decreased to 1.0 mm. To further reduce the pressure the input was decreased to 11 KW, this being just sufficient to keep pure iron molten. The carbon, having been allowed to cool off, would absorb some of the gases in the furnace and at the end of another one-hour period a pressure of 0.5 mm. was obtained. With the high silicon alloys the input could be reduced still further before the iron started to freeze, and consequently lower pressures—as low as 0.3 mm—could be obtained. The ingot was then allowed to cool in the furnace so as to prevent oxydation. When removed from the furnace the tops of the ingots would be perfectly bright, occasionally having the appearance shown in Fig. 6, due to a thin film of slag, probabaly SiO_2.

9. *Forging.*—The forging was done by heating the ingots in an ordinary coke forge to a cherry red, and drawing them out into rods

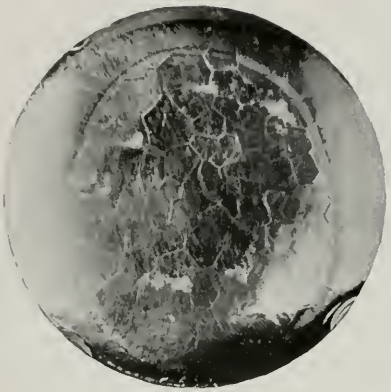


FIG. 6. TOP SURFACE OF A PURE IRON INGOT AS REMOVED FROM FURNACE.

about ½″ (1.25 cm) x 20″ (50 cm) under a steam or other power driven hammer. No difficulty was experienced in the forging of the alloys with exception of those containing 2.55, and 8.55 per cent silicon, both of which fell to pieces by the first blows of the hammer. That the latter should not forge was not surprising, but that an alloy containing 2.55 per cent silicon was non-forgeable was quite new. On that account a second ingot was made, that analyzed 2.57 per cent silicon. It, too, broke down into a mass of large crystals just like the previous ingot. The pieces from the 2.57 per cent alloy are shown in Fig. $30a_2$ showing crystals varying in size from ⅛″ (3.2 mm) to ¼″ (6.4 mm) across. The 8.55 per cent alloy is shown in Fig. 39, exhibiting crystals considerably smaller than the 2.57 per cent alloy. The rest of the alloys forged very well, but as the silicon content increased above 3 per cent the alloys became increasingly harder, and the 6.57 per cent ingot had to be heated repeatedly to a high temperature before it could be drawn out.

10. *Testpieces.*—From the forged rods the following testpieces were prepared:

(a). One testpiece for the magnetic and electrical tests, 0.392″ (0.996 cm) in diameter and 14″ (35.5 cm) long.

(b). Two testpieces for the mechanical tests, having a middle section 0.3″ (0.76 cm) in diameter and 1.5″ (3.8 cm) long, with threaded ends 0.5″ x 0.5″ (1.25 cm). (See Fig. 13.)

(c). One small sample for the metallographic investigation.

The shavings obtained by the process of preparing these testpieces were collected, after first removing the exposed parts of the rods, and used for chemical analysis.

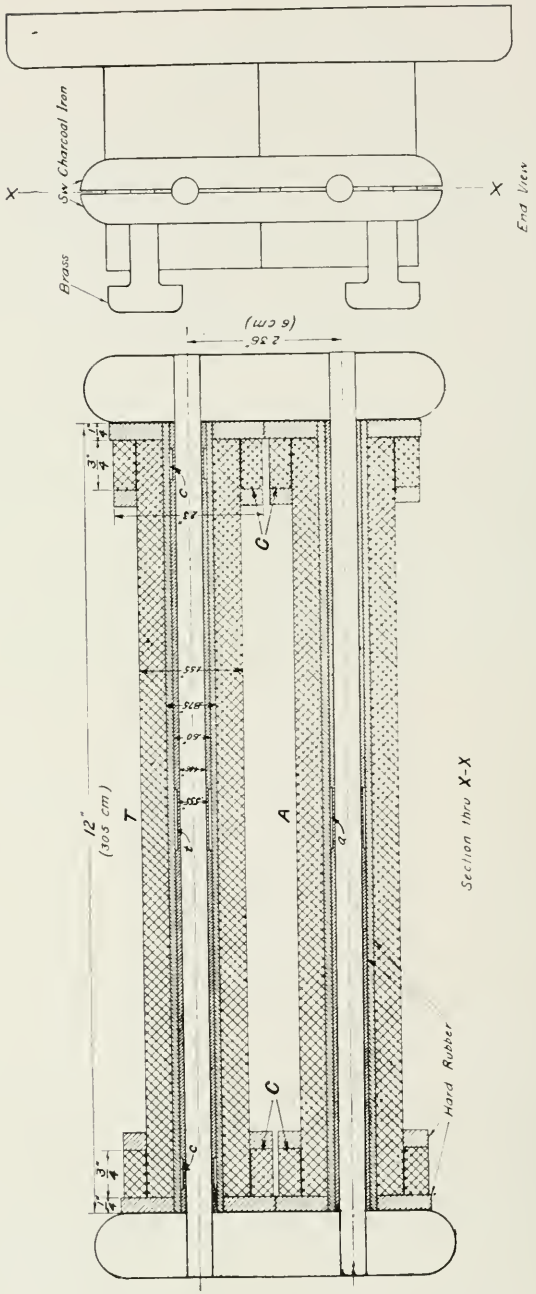


FIG. 7 PERMEAMETER.

Coil Data.

T, *A* and *C* coils—10 layers No. 18 B. & S. enameled copper wire, 20 turns per inch per layer (7.875 turns per cm.)
t and *a* coils—64 turns each of No. 30 B. & S. d. s. c. copper wire; *c* coils 32 turns each of No. 30 B. & S. d. s. c. copper wire—connected in series.

All the rods machined without any particular difficulty, with the exception of the 6.57 per cent rod. This rod broke repeatedly in the lathe in spite of the care exercised by the machinist. The vibration was evidently sufficient to break it. No magnetic testpiece was therefore obtained from this rod, and mechanical testpieces were secured only by omitting the threads at the ends. These testpieces, therefore, had to be tested between flat grips, instead of screw sockets.

11. *Magnetic Testing.*—The magnetic testing was done by means of the Burrows compensated double bar and yoke method,* now generally adopted wherever accurate testing is desired. The apparatus used in the previous parts of the investigation was superseded by one requiring a 14″ (35.5 cm) rod, instead of a 12″ (30.5 cm) rod as previously used, and the distance between the rods was shortened. These changes were made in order to decrease the errors due to the ends of the various coils. With the new apparatus, shown in Fig. 7, and Fig. 8, the error in H, as measured by the magnetizing current in the main coil, and with the same current in all three coils, has been calculated according to the method described in Bull. No. 72, and found to be –0.02 per cent. With the highest permeability rods, however, it

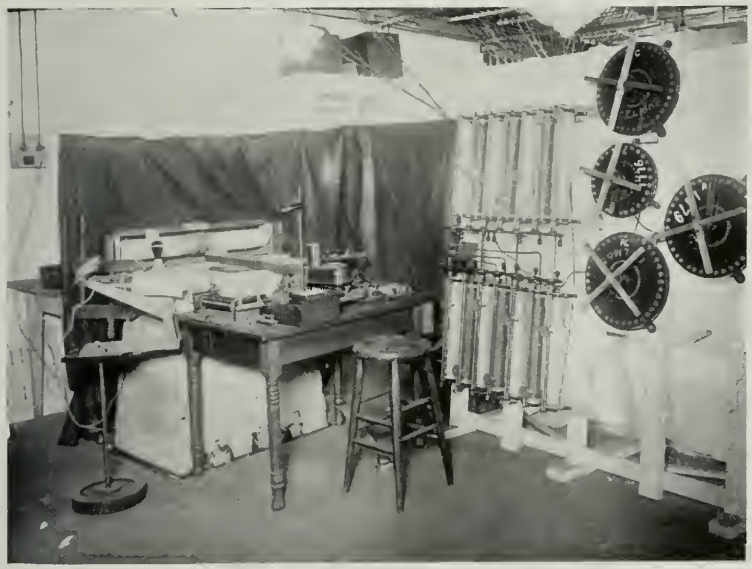


Fig. 8. Magnetic Testing Apparatus.

*Bull. Bureau of Standards. Vol. 6. No. 1. Reprint No. 117.

was found that the current in the compensating coils in exceptional cases had to be 30 times the current in the main and auxiliary coils. Consequently, the maximum error in H is found to be 2.30 per cent, that is, H as measured should be increased by 2.30 per cent. Corrections have been made in the results according to these theoretical calculations, but as will be shown in section III, it is probable that the errors are larger than these would lead to.

A Grassot fluxmeter was used for the measurement of B. In this instrument the deflection is independent of the duration of the magnetic change, a feature which is essential in testing iron of high permeability and low resistance. With the high resistance silicon alloys a long period ballistic galvanometer would probably serve the purpose.

The apparatus was calibrated from time to time by means of an aircoil, and also by means of rods submitted to the Bureau of Standards for standardization. The results obtained by the writer check very well with those obtained by the Bureau and also with results obtained by the calibration laboratories of two of the large manufacturing companies. As will be shown in section III, care has to be exercised in clamping the rods between the yokes, as a very slight strain due to bending has a considerable influence upon the permeability of the high permeability rods.

12. *Heat-treatments.*—The rods were tested magnetically and electrically and photomicrographs were obtained after the following treatments:

(a). As forged
(b). Annealed at 900° C. Cooled at a rate of 30° per hour
(c). Annealed at 1100° C. Cooled at a rate of 30° per hour

The annealing was done in vacuo, by means of the furnace shown in Fig. 5 and Fig. 9. This is a special type Hoskins resistor furnace, fitted with two electroquartz tubes, one inside the other. The test-pieces were placed in the center of the inner tube around the pyrometer protecting tube in such a way that the end of the pyrometer was located at the center of the rods. The containing tube was then filled with magnesia and the ends closed by means of rubber stoppers and sealed with mercury. A vacuum of 0.2 mm to 0.1 mm was maintained during the entire heat treatment. Accidents happened occasionally through the breaking of the tube or leak in the ends, destroying the rods in one case, but ordinarily the rods after removal from the furnace needed very little cleaning. A trial experiment was made using nitrogen by first evacuating the furnace, and then filling it with

nitrogen obtained by drawing air through four gas wash bottles filled with pyrogallic acid and one bottle filled with concentrated sulphuric acid. This experiment gave rods practically free from scale and may prove a convenient method for annealing, but the rods used in this trial were discarded as far as this report is concerned. Practically uniform cooling of the rods was obtained by means of a regulator that automatically inserted resistance in the circuit every half hour.

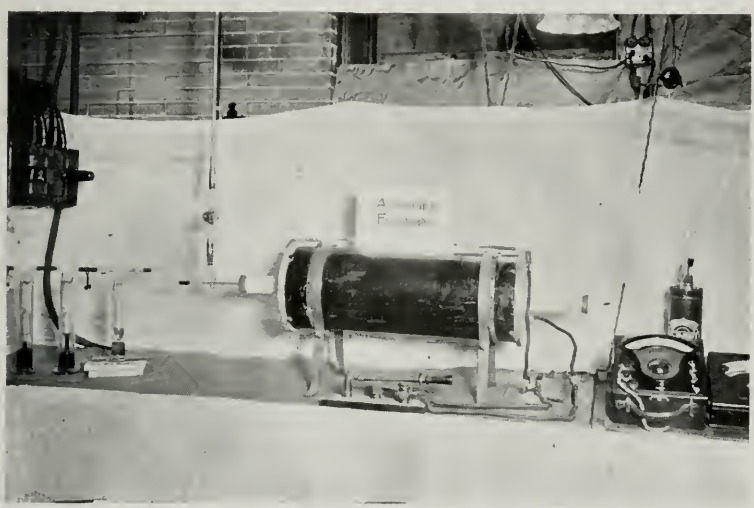


FIG. 9. VACUUM ANNEALING FURNACE.

13. *Photomicrographs.*—These were obtained by polishing in the usual way using jewelers' rouge for the final polishing. Great care had to be used with the higher silicon alloys. Numerous small spots would appear on the surface of some of the specimens, too numerous to be caused by graphite, as the carbon content amounted to only 0.01 to 0.02 per cent. Fig. 10 shows a peculiar formation on one of these specimens before etching. By careful polishing the spots were in most cases removed, as seen by the photomicrographs shown in section IV.

The etching was generally done by means of saturated picric acid in alcohol. In a few cases, however, a 10% solution of nitric acid in alcohol was found to give more satisfactory results.

The highest silicon alloys, containing 4.92 and 6.57 per cent respectively, had to be washed with dilute hydrofluoric acid to remove

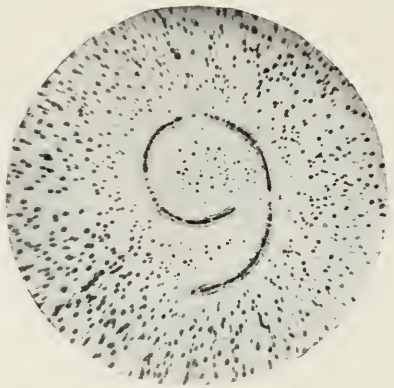


FIG. 10. PECULIAR FORMATION.

Black spots would appear in polishing some of the high silicon specimens, too numerous to be caused by graphite. This photomicrograph shows a peculiar formation of these black spots. By careful and prolonged polishing the spots would nearly all disappear.

Alloy No. 3Si27. 2.73% Si.
20 Diam. Unetched.

the thin film of a silicon composition that invariably formed on the surface after etching.*

14. *Mechanical Testing.*—The mechanical testing consisted in obtaining the yield point, ultimate strength, the elongation before the specimen had commenced to "neck," the ultimate elongation, and the reduction of area at the point of fracture. The testing was done on an Olsen 10000-pound testing machine, in which the test pieces were secured between screw sockets. The load was applied uniformly by means of an electric motor, and in most cases the yield point was detected with certainty. As stated before, the testpieces from the 6.57 per cent alloy could not be threaded on account of their brittleness and had to be tested between flat grips.

IV. MAGNETIC TESTING. EFFECT OF STRAIN AND COMPENSATING CURRENT.

While it is well known that mechanical stress affects the magnetic properties of iron, it was at first considered safe to clamp the magnetic test rods in the permeameter without regard to the method of clamping, on the assumption that the strain due to the clamping would be negligible. It was, however, noticed that in certain cases the perme-

*L. Guillet: Rev. de Metallurgie, Memoirs, Vol. 1, p. 46, 1904.

ability increased materially by turning the rods from one position to another and reclamping, while in other cases the opposite took place. It soon became evident that this change of permeability was due to mechanical strain, in such a way that the least strain gave the highest permeability.* In the construction of the permeameter used in the present investigation, particular attention was given to the yokes to be sure that the holes for receiving the rods were perfectly parallel. However, even with these yokes, inconsistencies were observed that finally were traced to strain. Table 5 gives the results obtained by different methods of clamping. It shows (Table 5a) that by tight clamping the values of B, for H = 0.5, may vary from 8000 to 11000 for the same rod, that is, about 30 per cent, and that it seems to make little difference which rod is used as auxiliary. With the yokes clamped so as to give good contact and yet not strain the rods, the results vary only about 1 per cent, and this may be due to influences other than mechanical strain. The figures for tight yokes show that it is possible to have the yokes tight and yet have little strain on the rods, but there is no way to ascertain whether the rods are strained, if the yokes are tight.

Table 5b shows, perhaps even more convincingly, that the inconsistent results shown in 5a, are due to strain. Starting with no strain on the rods and the circuits balanced, the value for B was found to be 11460, which is considered to be the correct value for the rod used. Without changing any of the magnetizing currents the yokes were then tightened. Under this new condition the contact between the yokes and the rods were better than under the first condition and should require a smaller compensating current to balance the magnectic circuit. In spite of the too high compensating current, B was apparently decreased to 11300. Balancing the magnetic circuit shows the true value of B under this condition to be 10550. Leaving the magnetizing currents unchanged, the yokes were then loosened (condition 4). The compensating current under this condition was naturally too small, as the contact was not as good as before, and yet the apparent value of B was increased to 11460. This is probably the most forceful evidence in favor of the contention. Balancing the circuit (condition 5) shows the true value to be 11460, the same as under condition 4. This is evidently due to the fact that the auxiliary rod had been strained more than the main rod, shown by the necessity of decreasing H_{aux} from 0.8 to 0.7.

*Bulletin No. 72 Eng. Exp. Sta. p. 40.

Table 5.

Effect of Strain and Compensating Current on Magnetic Testing.

a. TEST ROD NO. 3Si14C WITH VARIOUS AUXILLIARY RODS.
INDUCTION B, FOR H = 0.5.

Aux. rod used	Rods more or less strained, due to tight yokes.			Yokes loose, no strain
	Series 1	Series 2 after reclamping	Series 3 after reclamping and cleaning ends of rods	Series 4
3–54B	9330	9800	9250	11700
3Si09C	9900	8240	8040	11600
3Si10C	10110	10250	8940	11800
3Si11C	11060	10250	11160	11800
3Si12C	9650	10670	8290	11700
3Si13C	10850	9230	8600	11700

b. TEST ROD NO. 3Si10C WITH AUXILLIARY ROD NO. 3Si09C.
INDUCTION B, FOR H = 0.5.

Cond. No.	Method of Clamping.	H_{aux} for 3Si09C	Compensating Current, I_c	B for 3Si10C
1	Yokes loose, no strain on rods, no air gap between rods and yokes	0.7	.059	11460
2	Yokes tight, rods strained, magnetizing currents same as above	same	same	11300[x]
3	Yokes tight, rods strained, magnetic circuit balanced	0.8	.042	10550
4	Yokes loose, no strain on rods, magnetizing currents same as above	same	same	11460[t]
5	Yokes loose, no strain on rods, magnetic circuit balanced	0.7	.051	11460
6	Yokes loose, no strain on rods, small air gap between rods and yokes	0.7	.105	12460
7	Yokes tight, rods strained, small air gap between rods and yokes	0.8	.058	10890
8	Yokes loose, no strain, no air gap between rods and yokes	0.7	.059	11460

[x]Magnetic circuit not balanced; on account of better contact between rods and yokes, I_c is too large, and yet B is reduced, showing that strain decreases permeability.
[t]Magnetic circuit not balanced; on account of poorer contact between rods and yokes, I_c is too small, and yet B is increased, showing that removal of strain restores the rod to its normal condition.

Under condition 6 a small airgap was produced between the rods and the yokes by means of tissue paper, necessitating a doubling of I_c. The value obtained for B in this case was 12460, an increase of 1000 over the value obtained with no airgap. According to theoretical calculations, only one-tenth of this increase can be attributed to the increase in I_c, as a compensating current of 0.105 (which is 21 times the main magnetizing current) should increase H as measured, only by about 2 per cent. It is possible that the rest of the increase may be due to the removal of a slight strain that existed with no airgap, but it seems more probable that the compensating current has a larger effect than is shown by theoretical considerations. If it be assumed that the total increase is due to I_c, H as measured is evidently in-

creased 8 per cent due to a compensating current equal to 10 times the main magnetizing current, and correspondingly for other ratios. Should this assumption be correct, the results recorded in this bulletin are somewhat exaggerated, as corrections have been made according to theoretical considerations only. More experimental evidence, is, however, necessary before the final judgment can be passed, and in the meantime the reader, in studying the results, should keep this matter in mind, remembering at the same time that whatever the effect of the compensating current may be, the results are strictly comparable.

Under condition 7 the rods were strained, bringing the value of B down to 10890, nearly the same as under 3. Removing the airgap and testing with no apparent strain, the value found for B was 11460, the same as under similar conditions before. It is interesting to note that I_c under the last two conditions was practically the same, while B with the rod strained was about 600 gausses less than with no strain, another forceful argument showing that the permeability is decreased by a slight strain.

The condition of no strain can only seldom be applied in practice, but it is necessary in an investigation of this kind to make the tests under conditions that can be duplicated and standardized, in order to obtain results that are comparable, and Table 5 shows very plainly that the "no strain" condition is the only one that fulfills this requirement. Another argument for the adoption of the "no strain" condition is that this is the condition that exists in the only other reliable method of magnetic testing that the writer knows of, namely the Rowland ring method.* In the appendix will be found some results obtained with ring specimens together with a discussion of the same.

V. Results

15. *Chemical Properties.*—Table 6 gives a complete list of all the alloys made, together with the chemical analysis for silicon. From the amount of silicon added, the percentage lost in terms of the weight of the ingot has been calculated and listed in the fourth column. The only elements that are present in the alloys as impurities in a measurable quantity are carbon, amounting to about 0.01 per cent, and oxygen.

In bulletins No. 72 and No. 77 of the Engineering Experiment Station it was shown that the electrolytic iron, even after the thorough

*Most of the inconsistencies that appeared in Bulletin No. 72 on the properties of pure iron can no doubt be attributed to strain in view of the results shown here.

cleaning to which it was subjected before melting, contained about 0.4 per cent oxygen in the form of some oxide of iron. It was shown that while carbon will reduce the iron oxide before commencing to combine with the iron, boron will combine with the iron before all the iron oxide is reduced, its affinity for oxygen being about twice its affinity for iron. From Table 6 it is seen that silicon in this respect acts like boron, with the difference, that its affinity for iron is much stronger than its affinity for oxygen. The percentage silicon lost, that is, the percentage that has been oxidized and changed into slag, increases, somewhat irregularly with the silicon added, but reaches a

TABLE 6.

LIST OF ALLOYS.

Specimen No.	Silicon Added per cent	Silicon Content as per Chem. Anal. per cent	Silicon Lost per cent	Remarks
3–51	.000	ab't. .001		These rods were discarded after annealing at 1100° C on account of oxidation due to a leak in the furnace.
3–52	.000	'' .001		
3–53	.000	'' .001		
3–54	.000	'' .001		
3–55	.000	'' .001		
3Si05	.092	.068	.024	
3Si06	.185	.148	.037	
3Si07	.276	.242	.034	The magnetic data for these rods, after annealing at 900° C. were taken before the effect of strain due to tight clamping of the yokes was noticed.
3Si08	.368	.309	.059	
3Si09	.460	.400	.060	
3Si10	.551	.472	.079	
3Si11	.643	.563	.080	It was impossible to re-test these rods annealed at 900° because they were already annealed at 1100°. Data for the 1100° annealing were taken without strain.
3Si12	.734	.673	.061	
3Si13	.825	.698	.127	
3Si14	.916	.822	.094	
3Si15	.138	.064	.074	
3Si16	.046	.010	.036	
3Si17	.230	.230	.000	
3Si18	1.810	1.741	.069	
3Si19	2.690	2.550	.140	Not forgeable. Crushed into mass of crystals.
3Si20	3.560	3.580	−.020	Flaw in center of rod after forging.
3Si21	.092	.048	.044	
3Si22	.185	.091	.094	
3Si23	.276	.205	.071	
3Si24	2.690	2.570	.120	Not forgeable. Same as 3 Si 19.
3Si25	3.560	3.400	.160	
3Si26	2.260	2.180	.080	Contaminated in melting.
3Si27	3.125	2.730	.395	
3Si28	4.410	4.440	−.030	
3Si29	5.230	4.920	.310	
3Si30	8.420	8.550	−.130	Not forgeable.
3Si31	2.260	1.710	.550	
3Si32	6.850	6.570	.280	
3Si33	.459	.420	.039	Not used. Made from Electrolytic Iron that proved to be impure.
3Si34	.915	.700	.215	
3Si35	.276	.193	.083	
3Si36	3.980	3.550	.430	
3Si37	4.810	4.390	.420	
3Si38	2.460	2.260	.200	Used for mechanical tests only.
3Si39	3.560	3.185	.375	Used for rings for comparative magnetic testing.
3Si40	3.560	2.960	.600	
3Si41	2.690	2.360	.330	
3Si42	2.690	2.700	−.010	Used for mechanical tests only.
3Si43	2.770	2.550	.220	
3Si46	2.600	2.530	.070	
3Si47	2.510	2.410	.100	
3Si51	2.660	2.600	.060	

maximum of about 0.5 per cent. As silicon oxidizes to SiO_2 the maximum amount of oxygen absorbed is 0.44 per cent, or the same amount that was found in the two previous investigations, using carbon or boron.

16. *Mechanical Properties.*—The results of the mechanical tests are shown in Tables 7 and 8 and graphically in Figs. 11 and 12. A comparison between these results and those obtained by previous investigators was given in Figs. 1 and 2. Fig. 13 is a photograph of some of the test pieces after being tested, showing very clearly the variation in the elongation and reduction of area caused by silicon.

From the data thus presented it may be stated in general that silicon increases the strength of iron in almost direct proportion to the amount added, until the maximum strength is reached with a silicon content of about 4.5 per cent. From this point on, the elastic limit coincides with the ultimate strength, and both decrease very rapidly. The limit of forgeability lies at about 7 per cent. The values for the forged condition are considerably higher than for the annealed condition, the difference varying between 10,000 and 20,000 lb. per

TABLE 7.

RESULTS OF MECHANICAL TESTS.

AS FORGED.

Number of Specimen	Silicon Content, %	Yield Point, lb. per sq. in.	Ultimate Strength, lb. per sq. in.	Elongation %		Reduction of Area, %	Remarks
				Before "Necking"	Ultimate		
3–39	.001	35,800	44,700	..	39	80.4	
3–53	.001	44,400	46,500	3	24	53.8	
3–55	.001	38,000	40,800	8	40	88.5	
3Si16	.010	41,800	45,200	11	35	78.0	
3Si21	.048	42,850	46,900	5	25	91.6	Failed near Punch Mark
3Si05	.068	36,800	43,800	10	37	92.0	
3Si22	.091	35,600	43,750	12	36	91.7	
3Si06	.148	38,600	45,000	11	42	94.8	
3Si23	.205	42,500	49,700	10	39	93.4	
3Si17	.230	41,300	47,500	10	45	89.7	
3Si11	.563	40,750	51,000	12	41	92.6	
3Si12	.673		58,000	9	30	91.4	
3Si14	.822	45,200	55,800	11	36	93.1	
3Si31	1.71	68,100	76,300	6	29	87.2	
3Si38	2.25	66,550	77,750	12	37	82	
3Si41	2.36	77,200	86,450	8	24	66	
3Si47	2.41	63,500	75,800	9	23	65	
3Si46	2.54	60,300	73,400	14	16	20	
3Si51	2.63	68,700	78,500	13	24	43	
3Si43	2.65	59,000	74,700	14	34	72	
3Si42	2.78	68,250	78,000	6	21	74	Slight Flaw.
3Si25	3.40	74,500	86,300	10	31	74.7	
3Si36	3.55	83,400	99,300	12	23	41.3	
3Si37	4.39	94,000	105,000	6	6	7.5	
3Si29	4.92		50,250	Nil	Nil	Nil	Failed at Base of Head
3Si32	6.57	5,120	5,120	Nil	Nil	Nil	Tested between grips without being machined

sq. in. (7 to 14 kg. per sq. mm.). For the 4.5 per cent alloy "as forged" the ultimate strength is 105,000 lb. per sq. in. (73.5 kg. per sq. mm.) about 8000 lb. (5.6 kg.) higher than the maximum obtained by previous investigators. The practical absence of carbon in the vacuum iron causes the low-silicon alloys to be weaker than the corresponding alloys tested by previous investigators, but this same absence of carbon is evidently a cause for added strength in the 4.5 per cent alloy, in which the carbon exists in the form of graphite.

TABLE 8.

RESULTS OF MECHANICAL TESTS.

ANNEALED.

Number of Specimen	Silicon Content, %	Yield Point, lb. per sq. in.	Ultimate Strength, lb. per sq. in.	Elongation %		Reduction of Area, %	Remarks
				Before "Necking"	Ultimate		
Annealed at 970° C.							
3–39	.001	16,400	36,100	..	61	80.9	
3Si16	.010	16,050	34,900	25	53	81.5	
3Si21	.048	20,100	35,000	23	48	89.3	
3Si15	.064	14,750	34,100	27	29	45.1	Failed near Head Flaw.
3Si05	.068	20,400	34,900	28	64	94.8	
3Si22	.091	14,290	35,400	26	64	91.8	
3Si06	.148	15,890	35,200	29	48	67.0	
3Si23	.205	25,075	38,650	19	50	89.4	
3Si19	.230	14,910	35,500	30	60	84.7	Broke near Head. Flaw
3Si07	.242	18,100	38,400	25	60	91.3	
3Si08	.309	21,650	40,400	25	55	90.0	
3Si09	.400	26,000	42,000	20	55	91.0	
3Si10	.472	17,340	42,750	26	54	91.4	
3Si11	.563	25,700	41,200	08	..	...	Was not Stressed to Failure. Flaw
3Si12	.673	26,550	45,230	21	45	88.2	
3Si13	.698	23,100	43,000	25	57	89.0	
3Si14	.822	26,200	45,150	28	50	91.6	
3Si31	1.710	35,800	54,250	25	50	90.6	
3Si18	1.741	45,750	55,000	14	..	84.7	Failed at Punch Mark.
3Si38	2.28	42,200	63,500	23	50	85.0	
3Si43	2.36	46,200	63,600	17	29	51.0	
3Si41	2.38	43,000	64,200	26	50	84.5	
3Si47	2.38	47,500	68,700	22	45	78.5	
3Si46	2.56	47,250	64,200	14	18	31.5	
3Si51	2.59	42,500	68,200	22	42	82.5	
3Si42	2.70	48,500	61,800	6	6	11.3	
3Si27	2.73	49,600	67,800	18	19	15.5	
3Si25	3.40	57,100	77,400	15	21	28.7	
3Si37	4.39	85,000	85,000	Nil	Nil	1.2	Annealed at 1030° C in Nitrogen
3Si28	4.44	72,900	91,600	14	24	25.1	
3Si29	4.92	47,700	47,700	Nil	Nil	Nil	Head failed
3Si32	6.57	13,000	13,000	Nil	Nil	Nil	Tested between Grips.
Annealed at 1100° C.							
3Si41	2.35	39,400	58,300	8	10	10	
3Si47	2.43	41,700	63,100	24	45	67	
3Si46	2.48	45,200	66,300	21	47	75	
3Si43	2.53	43,300	62,500	20	40	71	
3Si42	2.61	38,900	59,500	17	32	61	Slight Flaw
3Si51	2.63	43,300	45,600	1	2	Nil	

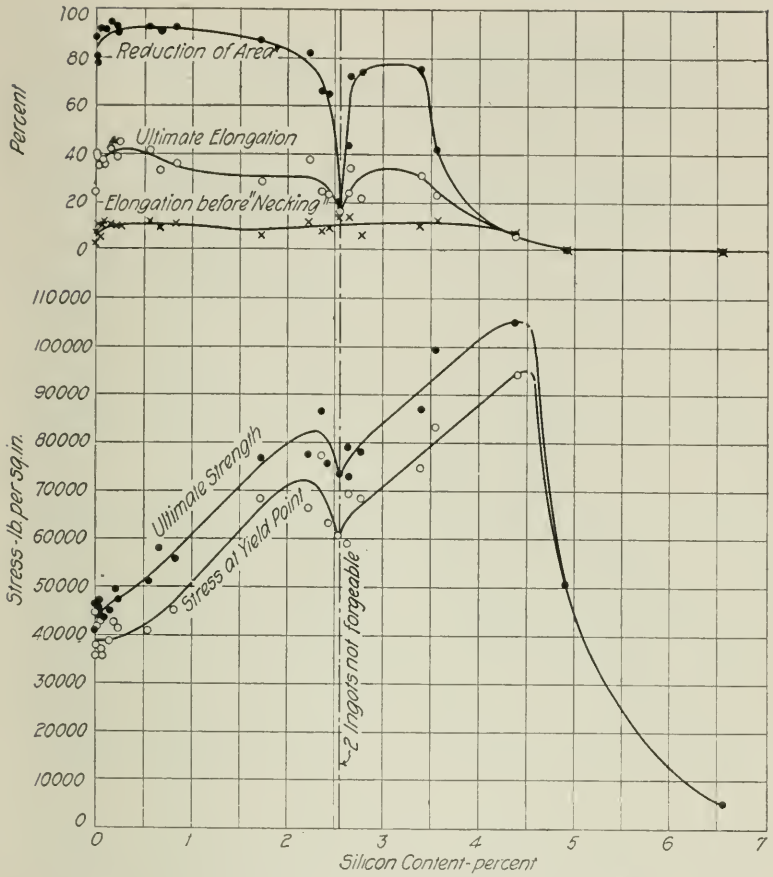


FIG. 11. MECHANICAL PROPERTIES OF IRON-SILICON ALLOYS, MELTED IN VACUO. AS FORGED.

With regard to the elongation and reduction of area, the results in general confirm those obtained by Hadfield, Baker and Paglianti concerning the effect of silicon. However, the vacuum alloys again show the effect of the lack of carbon in being much tougher in the region of low silicon as well as in the region between 3 and 5 per cent. The latter is particularly significant, as it is in this region that the maximum strength occurs. While the strength maximum for alloys containing small amounts of carbon corresponds to zero elongation and reduction of area, the strongest vacuum alloy in the forged condition has a reduction of area of 8 per cent, and an elongation of 7 per cent.

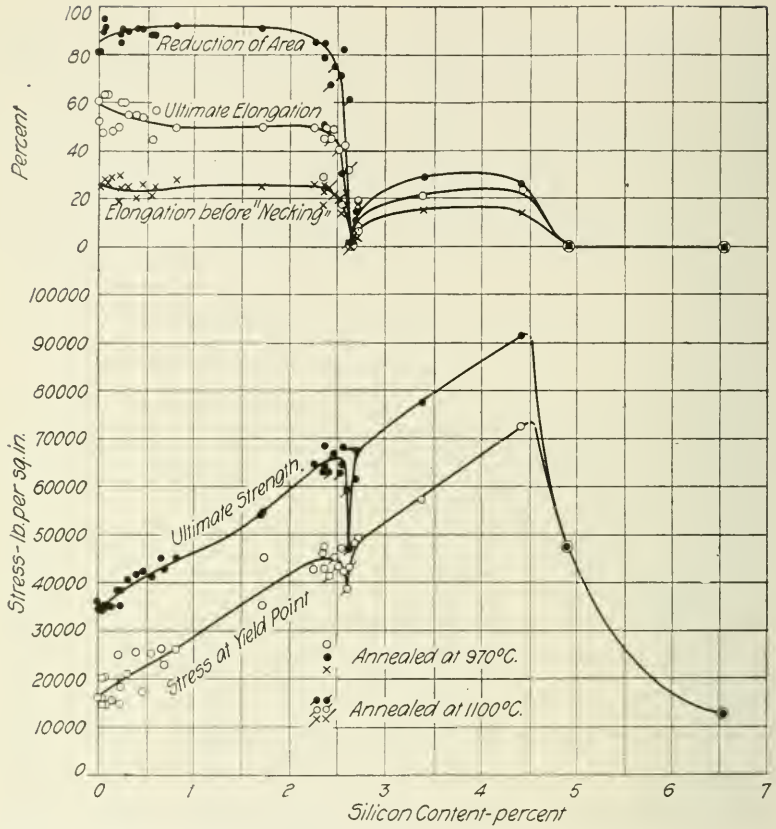


Fig. 12. Mechanical Properties of Iron-Silicon Alloys, Melted in Vacuo. Annealed.

In the annealed state the corresponding figures for the same alloy are 24 and 22 per cent.

A very interesting feature, as shown by the curves of Figs. 11 and 12, is the critical point that occurs with about 2.60 per cent silicon, the characteristic of the alloys in this region being their comparative brittleness. This point was first noticed by the fact that two ingots, containing 2.55 and 2.57 per cent silicon respectively, were not forgeable, but fell into a mass of crystals that apparently had no adhesive strength. One of these alloys is shown in Fig. 30. That this occurrence was not accidental is shown by the fact that the two alloys were made at different times, and they were subjected to forging on different days, in company with other alloys that forged perfectly. The

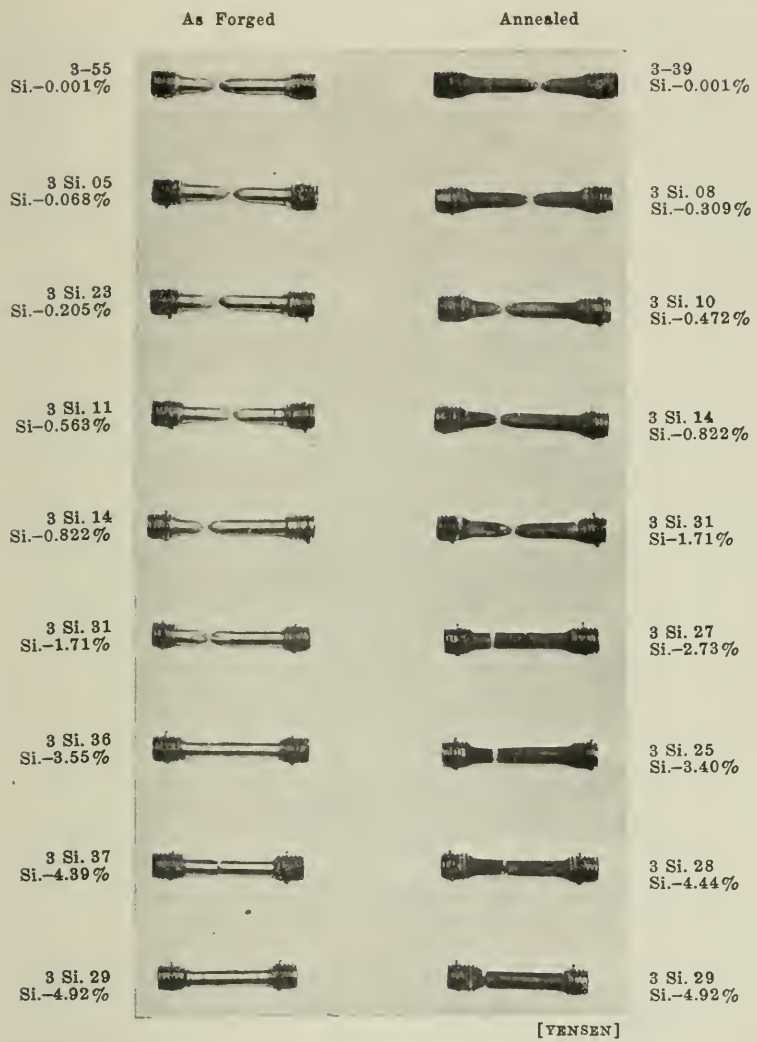


FIG. 13. SOME OF THE MECHANICAL TEST PIECES AFTER BEING TESTED.

structure of the two alloys is identical, consisting of large allotriomorphic crystals ⅛ in. (3 mm.) to ¼ in. (6 mm.) across. Since his first report* upon the iron-silicon alloys, where this critical point was mentioned, the writer has obtained additional experimental data in the region in which this point occurs. These data are included in the present report, making it possible to draw definite curves through

*A. I. E. E. Proceedings, Vol. 34, p. 2455, Oct., 1915.

the critical range. The critical point appears in all the characteristic curves for the mechanical properties, both for the unannealed and for the annealed alloys. For the unannealed specimens there is a sudden drop in the reduction of area and ultimate elongation at 2.25 per cent silicon, the curves reaching their minima at about 2.56 per cent, which is the silicon content of the two non-forgeable alloys. With increasing silicon contents these curves again rise as suddenly as they previously dropped, attaining about the same values as before the critical point was reached. At 3.50 per cent the second and final drop begins for these curves, reaching zero at about 5 per cent silicon.

For the annealed specimens the critical point is even more marked, but the reduction of area and the elongation do not recover as completely after the critical point is passed as was the case with the unannealed specimens. The critical point for the annealed alloys occurs with a silicon content of about 2.65 per cent, slightly higher than for the unannealed alloys. The writer knows of no satisfactory explanation of this critical point, neither has he been able to find any such phenomenon reported by anybody else. From Figs. 1 and 2 it is seen, however, that both Hadfield and Paglianti, and to some extent Baker, indicate irregularities in some of their curves for a silicon content between 2 and 3 per cent. None of the above investigators, however, and, as far as the writer has been able to find out, no other investigator has recorded in the literature the properties of an iron-silicon alloy containing between 2.50 and 2.67 per cent silicon.

As critical points are usually associated with the formation of definite compounds of the elements present, it may be of some interest to note that a compound of the formula $Fe_{19}Si$ would contain 2.56 per cent silicon, and similarly that a compound of the formula $Fe_{19}Si_2$ would contain 4.99 per cent silicon. It was stated above that the first critical point in the present case occurs with a silicon content of 2.56 per cent, and also that there is a sudden change at about 5 per cent silicon. Whether this agreement is a mere coincidence, or whether these compounds, or others, actually exist, the writer is not prepared to say, as conclusive evidence, in the way of cooling curves for these particular alloys, are not available.

17. *Magnetic and Electrical Properties.*—The results of the magnetic and electrical tests are shown in Tables 9 and 10, and in Figs. 14 to 18 incl. Figs. 14 and 15 give at a glance the magnetic and electrical properties of the series, Fig. 14 after annealing at 900°C, and Fig. 15 after annealing at 1100°C. The properties in the forged state have not been thus plotted for the reason that they are of less

interest on account of their inferiority. However, in order to have them included in the report, Fig. 16 gives the flux densities for various magnetizing forces for the alloys as forged; Figs. 17 and 18 give the corresponding values after annealing at 900°C and 1100°C respectively, thus affording means for comparison as to the effect of the various heat treatments. From these three figures it is seen that for H = 50 or above the annealing at 1100° has the effect of decreasing B by 500 to 1000 gausses for low silicon alloys, while for H = 20 and below the forged condition is decidely inferior to the annealed conditions. Comparing the 900° and 1100° annealing it is seen that for H = 20 to H = 0.5 the 900° annealing is superior for decreasingly lower silicon contents only, and for H = 0.3 and below the 1100° annealing is superior for the whole range. In general, it may be said that for high magnetizing forces B decreases with increasing silicon content, no matter what the heat treatment has been. This was expected to be the case from previous researches on the saturation values of iron-silicon alloys. Thus Gumlich and Goerens found that the saturation value $4\pi I_{max}$ is decreased by 500 gausses for each 1 per cent silicon.

TABLE 9.

RESULTS OF MAGNETIC AND ELECTRICAL TESTS
RODS ANNEALED AT 900°C.

Rod No.	Silicon Content, %	Maximum Permeability	Density for Max. Permeability gausses	Permeability for B = 10000	Hysteresis Loss, ergs per cc. per cycle		Retentivity gausses		Coercive Force gilberts per cm		Spec. Elec. Res. at 20° C.-microhms	Remarks
					for B_{max} = 10000	for B_{max} = 15000	for B_{max} = 10000	for B_{max} = 15000	for B_{max} = 10000	for B_{max} = 15000		
3–54	.001	23100	8500	21800	764	1610	9400	14200	.25	.30	9.85	
3–55	.001	22500	11000	21300	875	1790	9100	13800	.29	.36	9.82	
3Si16	.010	25000	10000	25000	795	1770	9480	14600	.28	.35	9.89	
3Si15	.064	22800	10000	21700	782	1738	9100	14500	.26	.35	10.65	
3Si05	.068	37500	9000	36300	405	1210	9480	14520	.12	.23	10.75	
3Si06	.148	47000	8000	42500	396	965	9300	14100	.12	.19	11.8	
3Si17	.230	30000	6000	26300	496	1311	9000	13600	.15	.23	11.60	
3Si10	.472	14000	7000	12700	960	1863	8900	11700	.30	.33	16.2	Rod Strained in Testing
3Si14	.822	13500	9000	13300	1215	2432	9100	12400	.42	.53	21.3	Rod Strained in Testing
3Si31	1.71	18000	7000	15900	800	1541	8400	11540	.26	.32	33.2	
3Si18	1.740	14300	8000	14100	935	2162	9000	12600	.29	.42	31.2	
3Si27	2.73	16800	6000	13300	821	1779	8270	10500	.27	.34	41.8	
3Si25	3.40	20000	6000	15900	560	1390	7900	9570	.20	.23	48.5	
3Si36	3.55	14000	4500	8850	802.5	1812	6900	8100	.28	.34	51.5	Rods Contaminated by Impure Rods.
3Si37	4.39	12750	4500	7500	846.0	1956	6600	7700	.29	.31	58.8	
3Si28	4.44	16100	4800	10100	623	1575	7000	8370	16	.24	57.7	
3Si29	4.92	9100	4500	5330	776	2006	4500	5300	.27	.36	66.5	

Table 10.

Results of Magnetic and Electrical Tests Rods Annealed at 1100°C.

Rod No.	Silicon Content, %	Maximum Permeability	Density for Max. Permeability gausses	Permeability for B = 10000	Hysteresis Loss, ergs per cc. per cycle		Retentivity gausses		Coercive Force gilberts per cm.		Spec. Elec. Resist. at 20° C.-microhms	Remarks
					for Bmax = 10000	for Bmax = 15000	for Bmax = 10000	for Bmax = 15000	for Bmax = 10000	for Bmax = 15000		
3–54	.001	22800	8000	21300	665	1860	9300	13300	.20	.24	9.84	
3–55	.001	25800	9000	25600	707	1451	9300	12700	.23	.28	9.85	
3Si16	.01	29000	9000	28670	707	1604	9600	14300	.21	.31	9.90	
3Si21	.040	27000	10000	27000	700	1660	9440	14480	.23	.32	10.50	
3Si15	.064	36800	9000	36300	502	1336	9500	14300	.16	.25	10.67	
3Si05	.068	44200	9000	43500	407	1214	9480	14200	.13	.22	10.78	
3Si22	.091	45250	9000	43500	394	929	9500	14300	.13	.17	10.96	
3Si06	.148	66500	6500	41700	286	916	9080	12000	.09	.16	11.80	
3Si23	.205	30200	9000	29500	649	1526	9300	14480	.20	.27	12.50	
3Si17												Contaminated by impure Rods
3Si07	.242	36500	7500	33000	436	1346	9700	14500	.13	.21	13.40	
3Si08	.309	44800	9000	43500	445	1412	9600	14500	.13	.24	14.40	
3Si09	.400	22500	9000	22000	725	1820	9440	14480	.21	.32	15.30	
3Si10	.472	31150	6200	25000	535	1358	9300	14200	.16	.21	16.57	
3Si11	.563	25000	9000	25000	601	1624	9200	14320	.20	.28	17.50	
3Si12	.673	28000	7000	24500	468	1636	9200	13670	.13	.23	19.10	
3Si13	.698	20350	8000	19600	780	2220	9300	14400	.25	.40	19.60	
3Si14	.822	30800	9500	30300	542	1765	9200	14100	.18	.35	21.25	
3Si31	1.71	30150	6500	24700	440	1292	8700	12000	.12	.22	33.25	
3Si18	1.740	33000	7000	26300	416	1112	9200	12600	.13	.19	31.00	
3Si27	2.73	46800	9500	46000	404	1260	9100	13300	.13	.23	42.00	
3Si25	3.40	63300	6500	46500	280	1025	9100	12400	.08	.15	48.50	
3Si36	3.55	36000	7500	29500	419	1157	8920	12000	.13	.21	48.50	Rods annealed in Nitrogen
3Si37	4.39	25700	6000	15400	591	1819	8300	10200	.20	.25	56.10	
3Si28	4.44	30200	3000	15900	405	1171	7000	8000	.12	.15	57.40	
3Si29	4.92	12200	5000	7040	780	2620	6300	7100	.26	.35	66.20	

Turning now to Figs. 14 and 15 two maxima appear very distinctly in the curves for maximum permeability, corresponding to two minima in the curves for hysteresis loss and coercive force. The first of these points occurs at a silicon content of 0.15 per cent, the second at a silicon content of about 3.5 per cent for the permeability and about 4.0 per cent for the hysteresis and coercive force. The reason that the second maximum for permeability and minimum for hysteresis loss do not occur at the same silicon content appears very clearly from the retentivity and coercive force curves, when it is remembered that the hysteresis loss primarily depends upon the coercive force and the retentivity and only to a less extent upon the maximum permeability.

That a maximum—or minimum—should occur for a low silicon content was not surprising in view of the results previously obtained

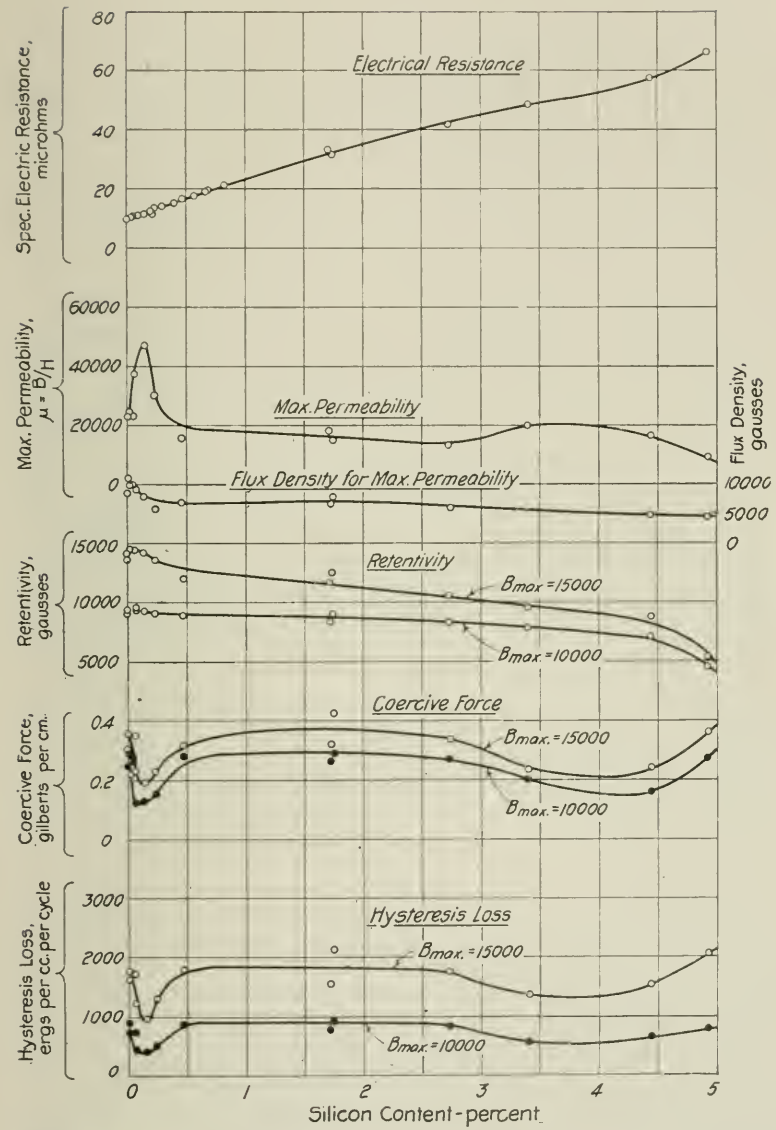


FIG. 14. MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-SILICON ALLOYS MELTED IN VACUO. ANNEALED AT 900° C.

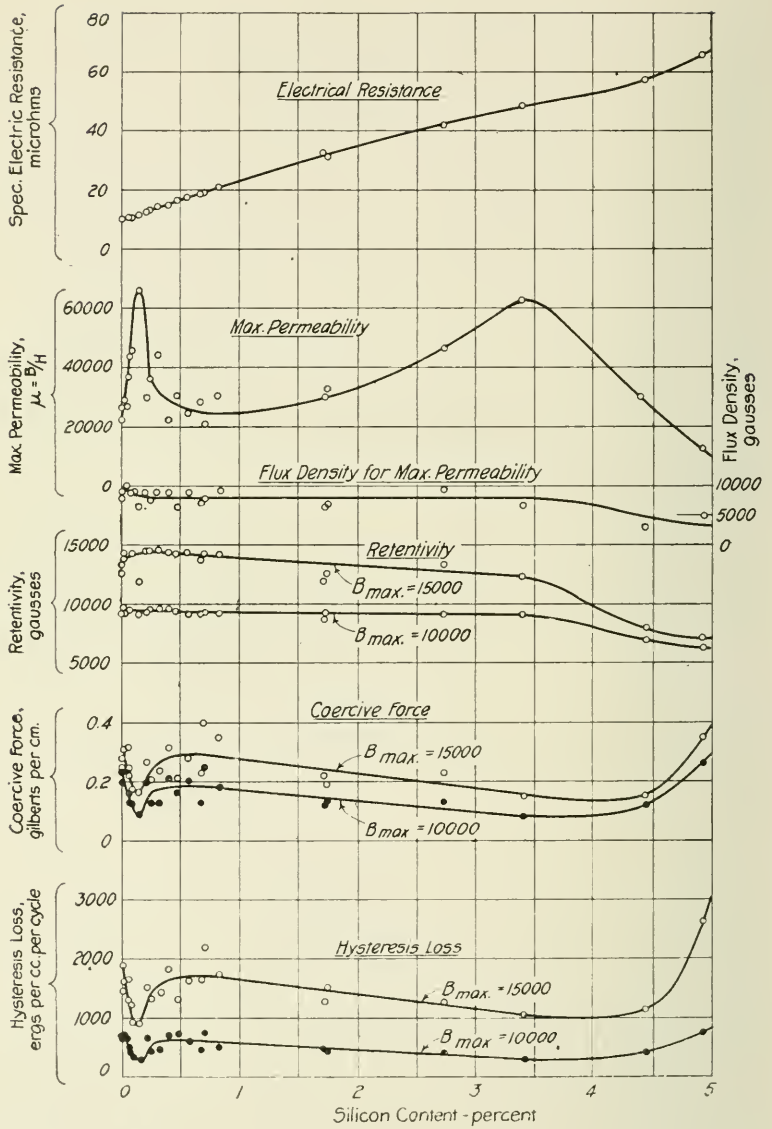


FIG. 15. MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-SILICON ALLOYS MELTED IN VACUO. ANNEALED AT 1100° C.

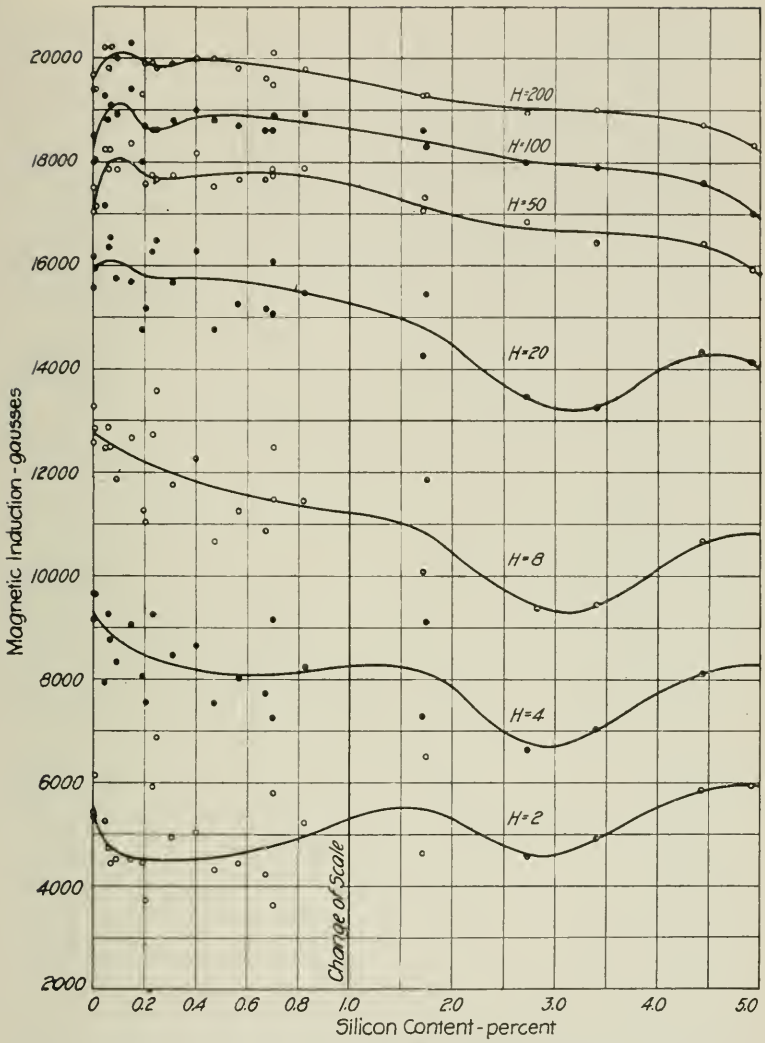


FIG. 16. FLUX DENSITY FOR VARIOUS MAGNETIZING FORCES. AS FORGED.

with pure iron, iron-carbon, and iron-boron alloys. In the latter case a maximum was obtained with a trace of boron, evidently on account of a slight purification of the iron, but as soon as the boron content became measurable the magnetic properties immediately depreciated. The first maximum in the present case can, no doubt, be accounted for in the same way, and consequently this point may be regarded as

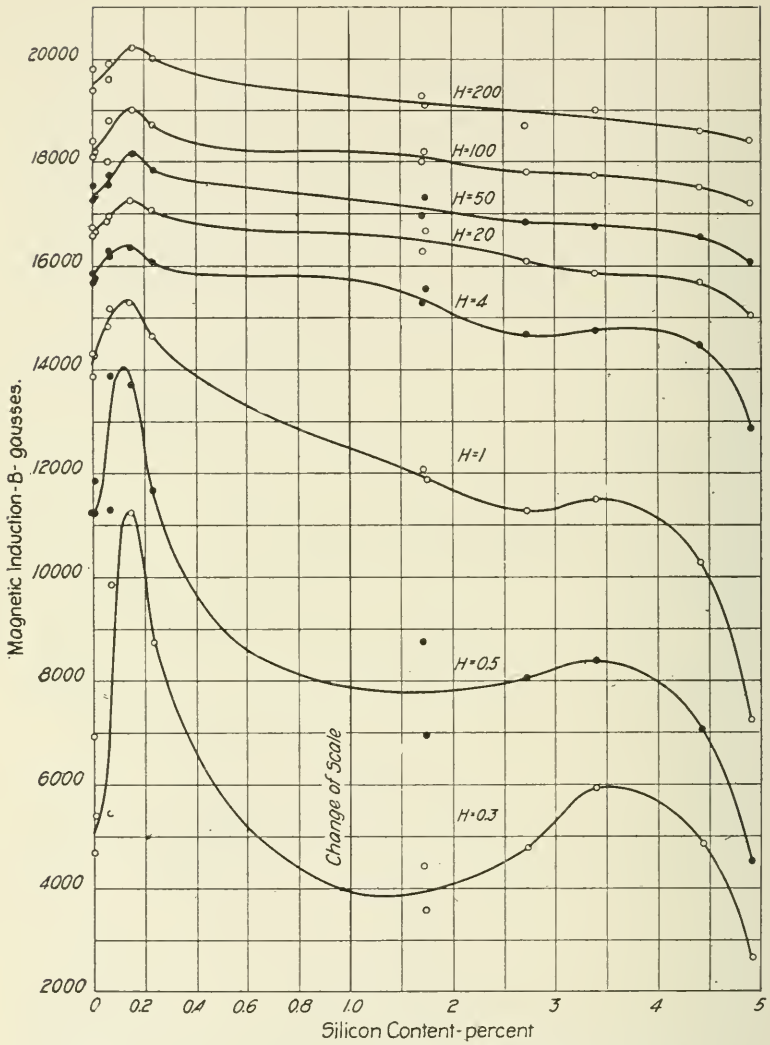


FIG. 17. FLUX DENSITY FOR VARIOUS MAGNETIZING FORCES. ANNEALED AT 900° C

characteristic of the purest iron obtainable under the present conditions, containing 0.15 per cent silicon and a small amount of oxygen in the form of iron oxide. The slight uncertainty existing for silicon contents below 1.0 per cent, as shown by the distribution of the points, is probably due to the varying amounts of oxide left in the iron.

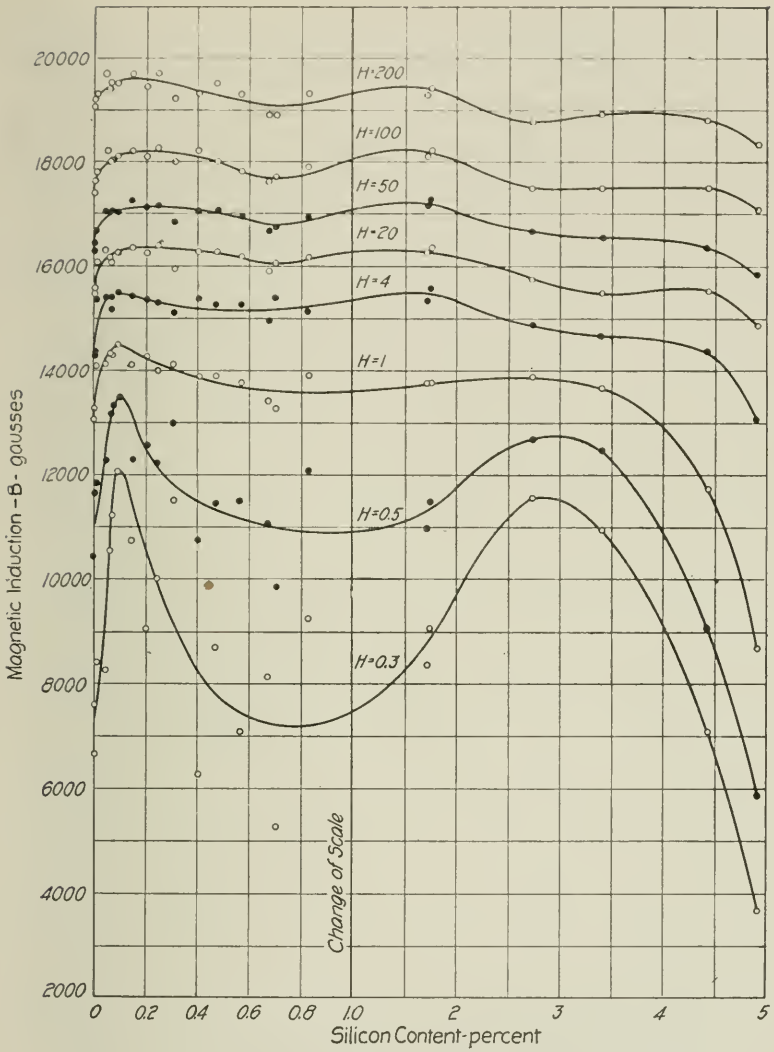


FIG. 18. FLUX DENSITIES FOR VARIOUS MAGNETIZING FORCES. ANNEALED AT 1100° C.

That the oxide in the iron before melting varied to some extent is apparent from Table 6, judging from the amount of silicon oxydized in the high alloys, where probably all the oxide was reduced. This reasoning would consequently also account for the uniform results obtained for the high silicon contents.

The second maximum—or minimum—was wholly unexpected as strength and brittleness are not generally associated with high magnetic quality. It is true that previous investigators have found a maximum between 2.5 to 4.0 per cent silicon, but as was pointed out on pp. 12 and 16 this was thought to be due to the neutralizing effect of the silicon upon the relatively large amounts of impurities, chiefly carbon, present in the iron. In the present case the alloys contained only about 0.01 per cent of carbon, an amount considered too low to be of any consequence, and only minute traces of other impurities were present. Thus it seems improbable that the second maximum in this case can be attributed solely to the conversion of 0.01 per cent of combined carbon into graphite. It seems more probable that the improvements are due partly to this conversion and partly to the complete reduction of iron oxide. If this second maximum should be due entirely to the conversion of combined carbon into graphite, the current ideas regarding the influence of carbon upon the magnetic properties of iron will certainly have to be changed, and it will become desirable to remove from the iron the last trace of carbon. According to these hypotheses, the first maximum is due to pure iron in spite of small amounts of iron oxide and combined carbon, while the second maximum is due to pure iron in spite of a relatively large amount of dissolved silicon. If *none* of the above hypotheses is correct, the only other explanation remaining is that the second maximum is due directly to silicon dissolved in the iron. As an argument against such a theory Hadfield and Hopkinson* in 1910 and Gumlich† in 1912 brought out the fact that silicon reduces the saturation value of iron in direct proportion to the silicon dissolved in the iron, and consequently it did not seem probable that silicon could directly improve the permeability at lower densities. However, it is a curious coincidence that at the same meeting of the Faraday Society at which Dr. Gumlich made the above statement, Dr. P. Weiss‡ read a paper on iron-cobalt alloys, showing that the alloy Fe_2Co has a saturation value 10 per cent higher than that of pure iron. The writer¶ in cooperation with Dr. E. H. Williams§ has recently shown that while the iron-cobalt alloy, Fe_2Co, melted in vacuo has a saturation value 13 per cent higher than that of pure iron melted under identical conditions, its permeability at low densities is much lower. Evidently

*Inst. of Elect. Engrs. Vol. 46, p. 225, 1911.
†Trans. Faraday Soc. Vol. 8, p. 109, 1912.
‡Trans. Faraday Soc. Vol. 8, p. 149, 1912.
¶Gen. Elect. Rev. Oct., 1915, Vol. 18, p. 881, Sept., 1915.
§Phys. Rev. N.S. Vol. 6, p. 404, Nov. 1915.

both the high saturation value and the comparatively low permeability at lower densities of the Fe_2Co alloy must be due to the combination between iron and cobalt, or, in other words, must be attributed directly to the cobalt. There is no reason, then, why the low saturation value and the high permeability at low densities of the 3.40 per cent iron-silicon alloy can not both be due directly to silicon. That is, no foundation exists any longer for assuming that there is a direct connection between the saturation value of a certain alloy and its properties at low and medium densities, and it is consequently possible that the second maximum occuring in the maximum permeability curve for the iron-silicon series may be due directly to silicon. More experimental evidence is needed, however, before it is safe to make a definite statement in this respect.

While silicon is thus, directly or indirectly, engaged in improving the magnetic properties of iron, it serves a very useful purpose by increasing the electrical resistance of the iron enormously, giving the iron the exact characteristics desired for electromagnetic machinery.

The iron-silicon series thus offers two important alloys for electrical purposes, both having high permeability and low hysteresis loss, but differing in that one has a very low, while the other has a very high electrical resistance.

The values obtained are, undoubtedly, without precedent in the annals of the magnetic properties of iron and iron alloys, and it is only after a careful analysis of the apparatus used and the methods employed in testing that the writer feels justified in publishing them. As an extra precaution, however, he wishes to repeat the statement made on p. 27 that experimental evidence seems to point towards a larger percentage error due to the compensating current than theoretical considerations according to Burrows and others would lead to. But even if the maximum error in the results as given should amount to 20 per cent, their significance would not be altered appreciably. Whether the true maximum permeability attained is 66,500 or 53,200 is of little consequence at the present time, as long as it is reasonably certain that it lies in this neighborhood. Should it be definitely established that the value of the required compensating current imposes a limit upon the Burrows double bar and yoke method, beyond which the errors introduced are too large, a new method will undoubtedly be developed to meet this exigency.

It is only a few years ago that a permeability of 6000 was regarded as exceptionally high. This was gradually raised to 8000 and then Terry,* in 1910, obtained a value of 11000 for a ring of electrolytic iron as deposited and annealed. Gumlich, in 1912, also obtained this value for a high grade low silicon alloy in the form of sheets. Last year the writer published 19000 as the maximum found for pure iron melted in vacuo, and this value was regarded as remarkable. The step from 19000 to 66500 indeed seems a long one, but intermediate steps have been taken in the meantime in the laboratory. The values for the hysteresis loss have followed in a similar path, as seen from Table 11.

TABLE 11.

PROGRESS MADE IN RECENT YEARS.

Year	Investigator	Kind of Material used	Maximum Permeability	Coercive Force gilberts per cm.		Hysteresis Loss ergs per cc per cycle	
				for B_{max} = 1000	for B_{max} = 15000	for B_{max} = 10000	for B_{max} = 15000
1900	Hadfield	SW. Char. Iron	4000	0.920	1.00	abt. 2700	abt. 5500
1900	Hadfield	2½% Si–Iron	5100	0.72	0.79	" 2200	" 4700
1901	Gumlich & Schmidt	Wrought Iron	8350		0.60		
1903	Baker	4.9% Si–Iron			1.20		" 6200
1910	Terry	Electrolytic Iron	11000				
1912	Gumlich & Goerens	0.4% Si. Sheets	11600		0.54		
1912	Gumlich & Goerens	4.0% Si. Sheets	9400				
1912	Paglianti	1.75% Si–Iron		0.60	0.75	1650	" 3500
1914	Yensen	Pure Vacuum Iron	19000		0.29	813	1640
1915	Yensen	0.15% Si. " Iron	66500	0.09	0.16	286	916
1915	Yensen	3.40% Si. " Iron	63300	0.08	0.15	280	1025

*Phys. Rev. Vol. 30, p. 133, 1910.

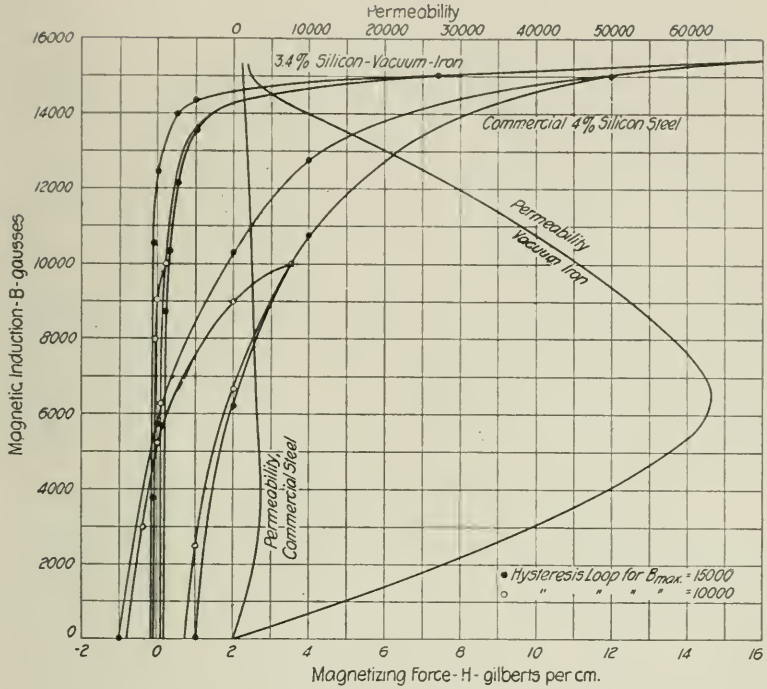


FIG. 19. COMPARISON BETWEEN VACUUM IRON AND COMMERCIAL STEEL, BOTH CONTAINING BETWEEN 3 PER CENT AND 4 PER CENT SILICON—THOROUGHLY ANNEALED.

	Vacuum Iron	Commercial Steel
Hysteresis loss, for B_{max} = 10,000; ergs per cu. cm. per cycle. . . .	280	2160
" " " " = 15,000; " " " " " " " "	1025	4290
Specific electrical resistance, microhms.	48.50	51.15

Fig. 19 illustrates the difference in magnetic quality between a silicon-vacuum-iron and a commercial silicon steel, both containing approximately the same amount of silicon. The maximum permeability is as 20 to 1, the hysteresis loss for $B_{max} = 10000$ as 10 to 1, and for $B_{max} = 15000$ as 3 to 1 in favor of the vacuum alloy. For $B = 15000$ the permeability of the latter is twice the permeability of the commercial iron. The electrical resistance is nearly the same for both alloys, or about 5 times that of pure iron.

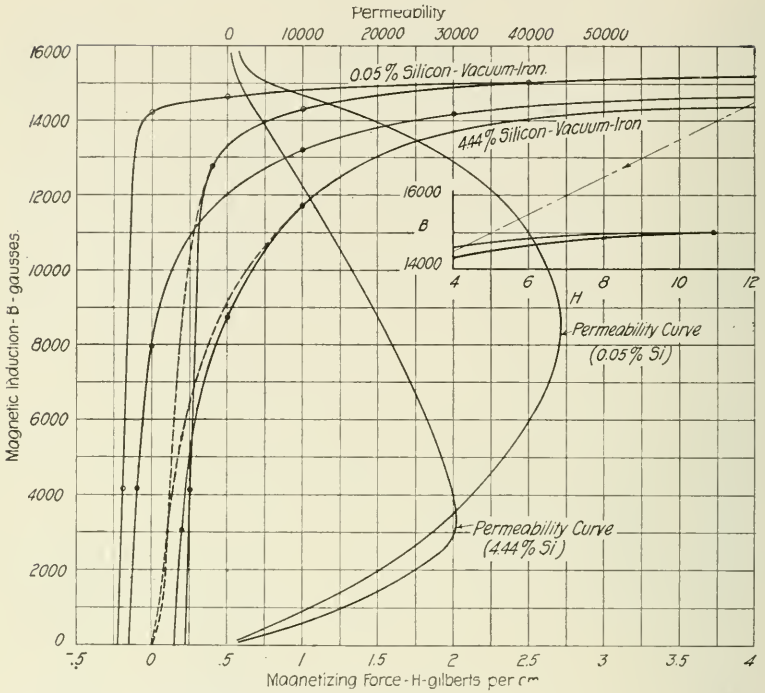


FIG. 20. COMPARISON BETWEEN A HIGH AND LOW SILICON VACUUM IRON—ANNEALED AT 1100°C.

	0.048% Si.	4.44% Si.
Hysteresis loss, for B_{max} = 10,000; ergs per cu. cm. per cycle	407	405
" " " " = 15,000; " " " " " "	1214.5	1171
Specific electrical resistance, microhms	10.78	57.40

Fig. 20 shows to a large scale the difference between a high and a low silicon-vacuum-iron, having about the same hysteresis loss. The characteristics of the low silicon alloy are seen to be: very high retentivity and high permeability for medium and high densities, while the high silicon alloys have a low retentivity, and the maximum permeability occuring at a low density. The chief advantage of the high silicon alloy lies in the high electrical resistance amounting to more than 5 times that of the low silicon alloy.

An attempt was made to obtain a few more points for the high silicon alloys, and two alloys, Nos. 3Si36 and 3Si37 were melted and testpieces prepared. Unfortunately, however, these rods were annealed first at 900° and then at 1100°, in company with three rods made from impure iron and were evidently contaminated by them.

This was discovered by the behavior of rod No. 3Si17 that had previously been annealed at 900° under normal conditions. After annealing at 1100° in company with the contaminated rods, its magnetic quality was sadly impaired, while the five other rods all had improved. As a matter of fact, it was the first occurence of a rod depreciating during the 1100° annealing, and the only cause that could be found was contamination by the impure rods. One rod being thus affected it seemed natural to conclude that Nos. 3Si36 and 3Si37 were similarly affected.* The results for these two alloys are shown in the tables, but the values have not been plotted in the figures, although the points would have deviated but little from the curves. Another reason for not plotting these values is that the 1100° annealing was done in an atmosphere of nitrogen. While the nitrogen did not seem to act differently from the vacuum, the method has not been tested sufficiently to warrant including the results thus obtained without question. However, these results, although not wholly satisfactory, serve as a check upon the results previously obtained.†

18. *Photomicrographs.*—In the following pages a large number of photomicrographs are reproduced representative of the alloys tested. They are arranged in order of their silicon content, the pure iron appearing first and the highest silicon alloy last. With only a few exceptions the "As forged" condition occupies the upper part of the pages, the 900° annealed condition the middle part, and the 1100° annealed condition occupies the lower part. The magnifications used are either 40 diameters or 10 diameters. In one case recourse was had to 7 diameters. For the high silicon alloys both the 40 and the 10 diameter magnifications are shown in most cases. The lower magnifications give a general view of the structures, as they cover about ⅛ of the surface of the specimens, while the higher magnifications give more detail. Even with the information thus at hand perfectly definite conclusions can not be made with regard to each alloy taken separately, as the specimen may not in every case be strictly representative of the particular alloy from which it was taken. Furthermore, in comparing the photomicrographs after the different heat treatments, it should be remembered, that the specimens had to be repolished after each treatment, and the photomicrographs do not, therefore, represent the same spot in each case. However, general conclusions can be drawn by considering the series as a whole.

*That such contamination takes place during annealing was shown in Bulletin No. 72, in the case of five rods that were annealed in company with a rod that contained 0.18 per cent carbon.

†In the tests comparing the Burrows method and the ring method a rod was obtained, containing 3 per cent silicon, that according to the Burrows method of testing has a maximum permeability of 72000. (For details see Appendix.)

From these photomicrographs it is seen very clearly that the iron-silicon alloys, in the range here investigated, consist of only one kind of crystals, thus confirming the results obtained by previous investigators that iron and silicon, for silicon contents below about 15 per cent, form a solid solution everywhere between the freezing point and ordinary temperature.

Below 1 per cent silicon appears to have no marked effect upon the structure of the iron, either as forged or annealed. Annealing at 900°C does not seem to change the structure of the alloys in this range, but annealing at 1100° produces numerous small crystals inside the larger ones, giving an appearance of a very fine structure.*

There is no sign of foreign substances in these structures, other than those arising from imperfect polishing. After passing the 1 per cent mark silicon begins to show its effect. The crystals are generally larger than for the pure iron, and are readily polished in relief, showing that they are not of uniform hardness. There is no breaking up of the crystals by the 1100° annealing as in the case of the low alloys. The 1.71 per cent and the 2.73 per cent alloys, as seen in Figs. 29 and 31, show very irregular structures as forged, and after the 900° annealing, but these give way to structures of more regularity by the 1100° annealing. In the region between the last two alloys is found the nonforgeable alloys, Nos. 3Si19 and 3Si24, containing 2.55 to 2.57 per cent silicon respectively. The structure of these alloys is shown in Fig. 30, exhibiting very large, uniform crystals, measuring ⅛″ (3 mm.) to ¼″ (6 mm.) across. In the first specimen used for the 3.40 per cent alloy the crystals are of about the same size and shape as the ones for the nonforgeable alloys, as seen in Fig. 35. In order to investigate this singularity further, another specimen from the same alloy was prepared, and this showed a much more normal structure (Fig. 32). Specimens were also investigated for two other alloys with very nearly the same silicon content, and these also showed structures that were quite normal, (Figs. 33 and 34) so that the large crystals, shown in Fig. 35, are evidently freaks, caused by

*This phenomenon is explained by Stead and Carpenter, Journ. Iron & Steel Inst. Sept., 1913, as follows: Upon heating the iron above the Ac_3 point (900° C for pure iron) and holding it in the region of the gamma modification sufficiently long, both the alpha crystals and their nuclei will be destroyed, giving place to gamma crystals. If now the iron is cooled, numerous alpha nuclei will be formed simultaneously on passing through Ar_3 and the resulting crystals are consequently small.

See also Storey: AM. Electroclem Soc. Apr. 18, 1914.

peculiar conditions. It may be that the specimen was taken at one end of the forged rod. Nevertheless, the occurence of these enormous crystals is very interesting, not only on account of the enormous size, but also because it shows that the size of the crystals in and of itself does not prevent the material from being forgeable. The remainder of the photomicrographs exhibit quite normal structures with no marked change caused by the two heat treatments.

The principle difference between these photomicrographs and the ones published by previous investigators is the absence of foreign matter in the structure of the vacuum alloys. Even Baker, whose alloys contained only 0.04 per cent carbon, shows in his photomicrographs besides small amounts of pearlite or graphite, some other foreign substance that could not be explained at the time. It seems probable, in view of what has been said earlier in this report regarding the method used for melting and the mechanical properties of his alloys, that these other foreign substances may be oxides. The other investigators invariably show comparatively large amounts of pearlite for low alloys and patches or spots of graphite for high alloys. The size of crystals for the vacuum alloys, excluding the abnormal cases, is very much larger than for the less pure alloys. This is true both for low and high silicon contents.

As Forged.

FIG. 21a. 40 DIAM. PICRIC ACID.

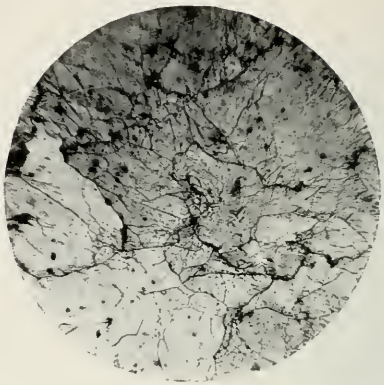


FIG. 22a. 40 DIAM. PICRIC ACID.

Annealed at 900° C.

FIG. 21b. 40 DIAM. PICRIC ACID.

FIG. 22b. 40 DIAM. PICRIC ACID.

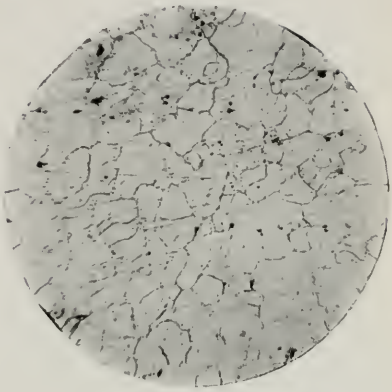
Annealed at 1100° C.

FIG. 21c. 40 DIAM. PICRIC ACID.

ALLOY NO. 3—39. 0.001% SI.

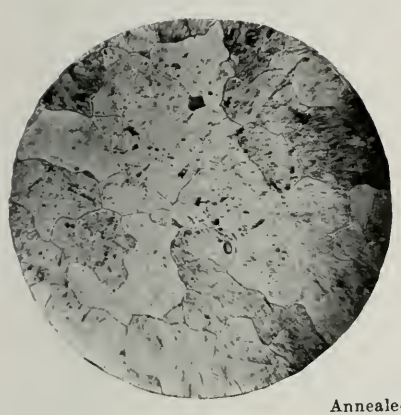
FIG. 22c. 40 DIAM. PICRIC ACID.

ALLOY NO. 3—54. 0.001% SI.



As Forged.

FIG. 24a. 40 DIAM. PICRIC ACID.



Annealed at 900° C.

FIG. 23b. 40 DIAM. PICRIC ACID.

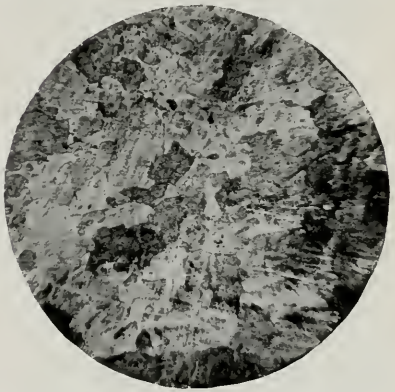


FIG. 24b. 40 DIAM. PICRIC ACID.

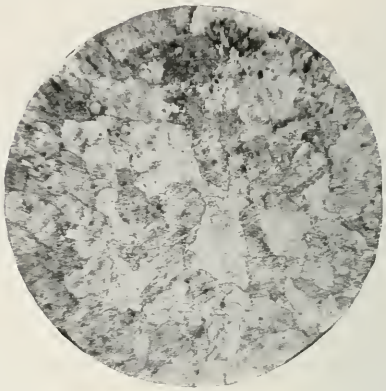
Annealed at 1100° C.

FIG. 23c. 40 DIAM. PICRIC ACID.

ALLOY NO. 3SI05. 0.068% SI.

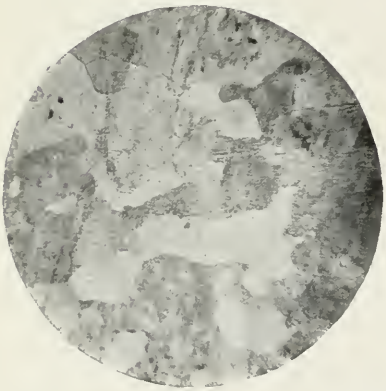
FIG. 24c. 40 DIAM. PICRIC ACID.

ALLOY NO. 3SI06. 0.148% SI.



As Forged.

FIG. 26a. 40 DIAM. PICRIC ACID.



Annealed at 900° C.

FIG. 25b. 40 DIAM. PICRIC ACID.

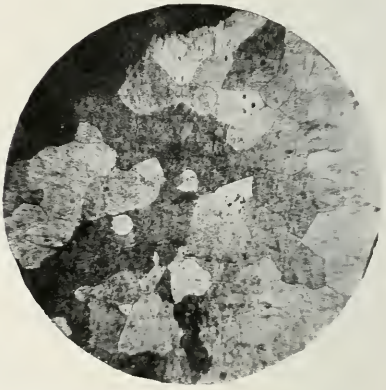
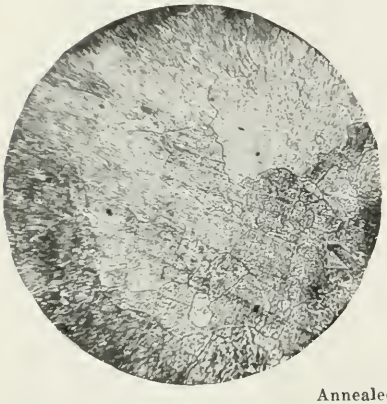


FIG. 26b. 40 DIAM. PICRIC ACID.



Annealed at 1100° C.

FIG. 25c. 40 DIAM. PICRIC ACID.

ALLOY NO. 3SI08. 0.309% SI.

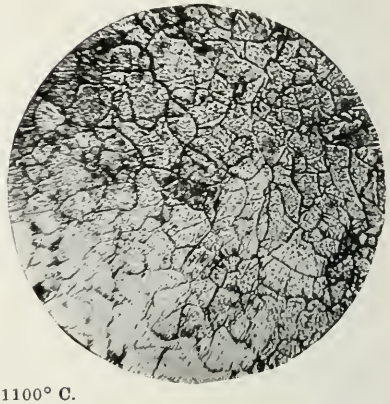
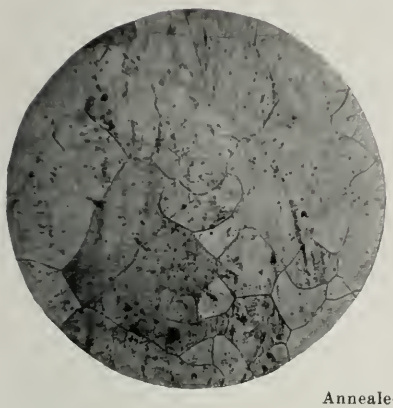


FIG. 26c. 40 DIAM. PICRIC ACID.

ALLOY NO. 3SI10. 0.472% SI.



Annealed at 900° C.

FIG. 27b. 40 DIAM. PICRIC ACID.

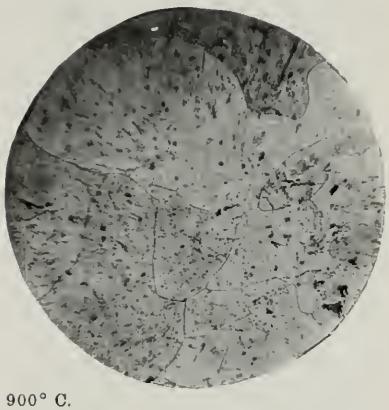
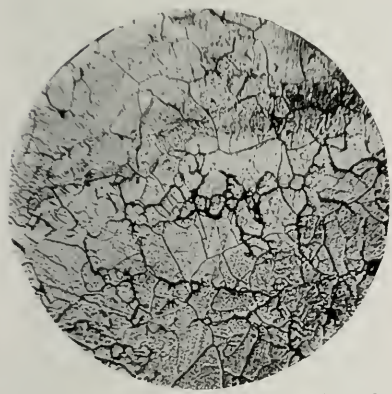


FIG. 28b. 40 DIAM. PICRIC ACID.



Annealed at 1100° C.

FIG. 27c. 40 DIAM. PICRIC ACID.

ALLOY NO. 3SI12. 0.673% SI.

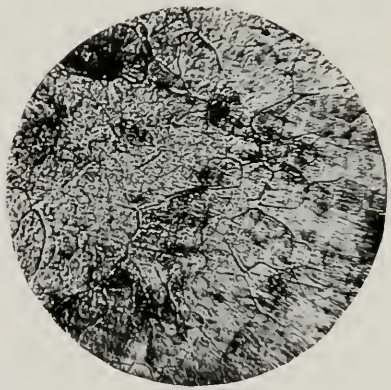
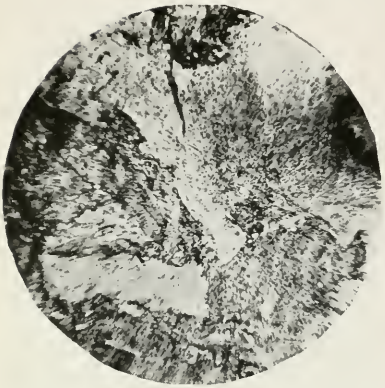


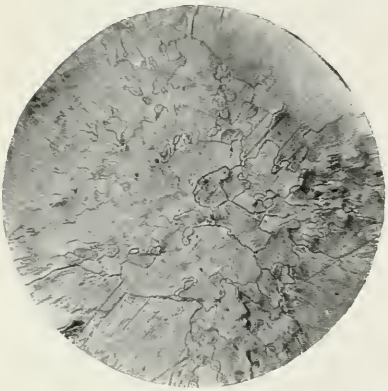
FIG. 28c. 40 DIAM. PICRIC ACID.

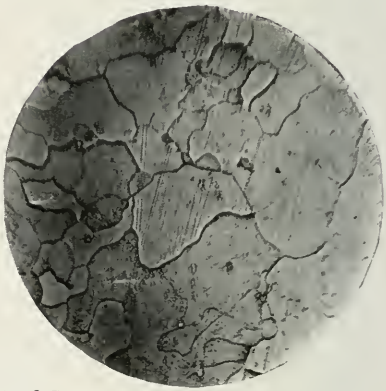
ALLOY NO. 3SI14.. 0.822% SI.



As Forged.

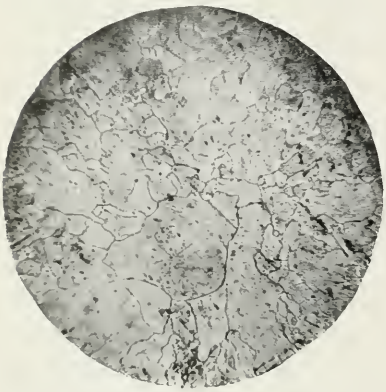
FIG. 29a. 10 DIAM. PICRIC ACID.





Annealed at 900° C.

FIG. $29b_1$. 10 DIAM. PICRIC ACID. FIG. $29b_2$. 40 DIAM. PICRIC ACID.

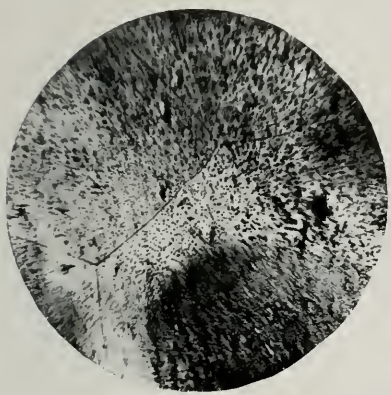


Annealed at 1100° C.

FIG. $29c_1$. 10 DIAM. PICRIC ACID. FIG. $29c_2$. 40 DIAM. PICRIC ACID.

ALLOY NO. 3Si31. 1.71% Si.

As cut from Ingot before Forging
FIG. 30a$_1$. 10 DIAM. PICRIC ACID.



Annealed at 900° C.
FIG. 30b. 10 DIAM. PICRIC ACID.

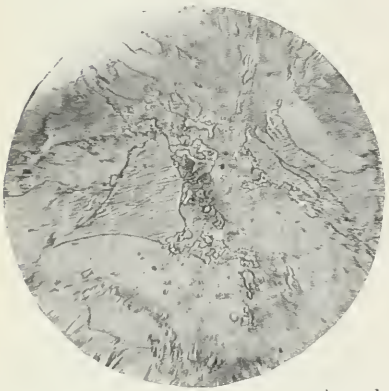
Annealed at 1100° C.
FIG. 30c. 10 DIAM. PICRIC ACID.

Ingot after Forging.
FIG. 30a$_2$. NEARLY FULL SIZE.

THE NON-FORGEABLE ALLOY NO. 3Si24. 2.57% Si.

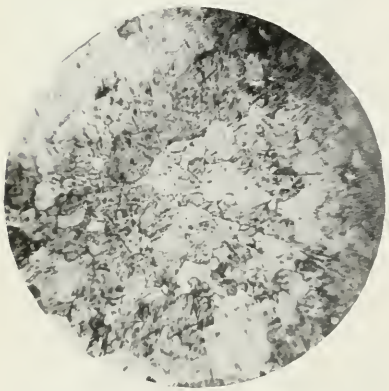
As Forged.

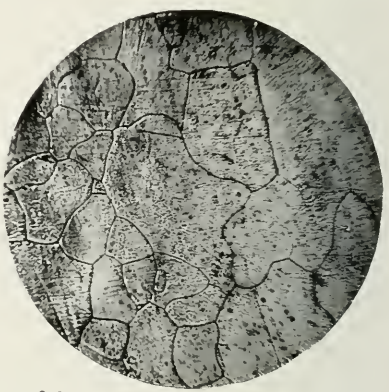
FIG. 31a. 10 DIAM. PICRIC ACID.



Annealed at 900° C.

FIG. $31b_1$. 10 DIAM. PICRIC ACID. FIG. $31b_2$. 40 DIAM. PICRIC ACID.

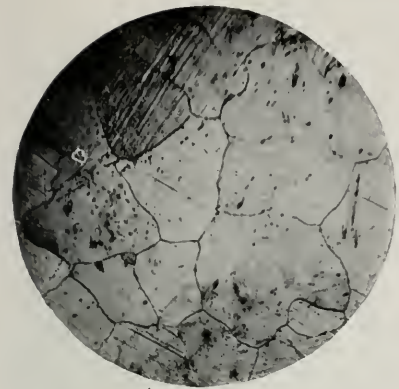




Annealed at 1100° C.

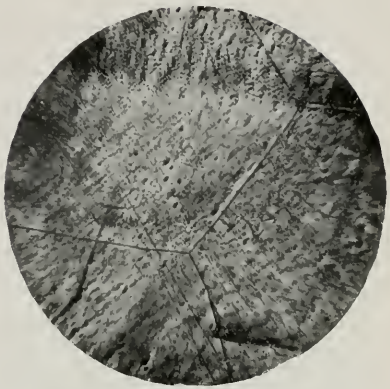
FIG. $31c_1$. 10 DIAM. PICRIC ACID. FIG. $31c_2$. 40 DIAM. PICRIC ACID.

ALLOY NO. 3Si27. 2.73% Si.



As Forged.

FIG. 32. 40 DIAM. NITRIC ACID.
ALLOY NO. 3Si25 (Spec. 1) 3.40% Si.



As Forged.

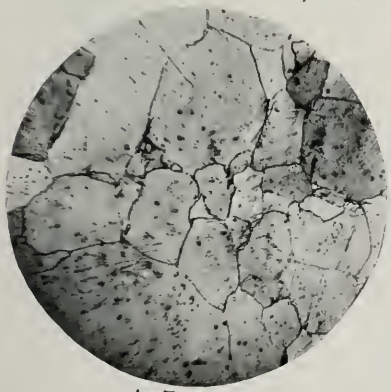
FIG. 35a. 10 DIAM. PICRIC ACID.

As Forged.

FIG. 33. 40 DIAM. NITRIC ACID.
ALLOY NO. 3Si20. 3.58% Si.

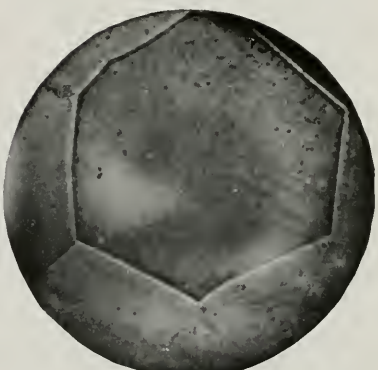
Annealed at 900° C.

FIG. 35b. 10 DIAM. PICRIC ACID.



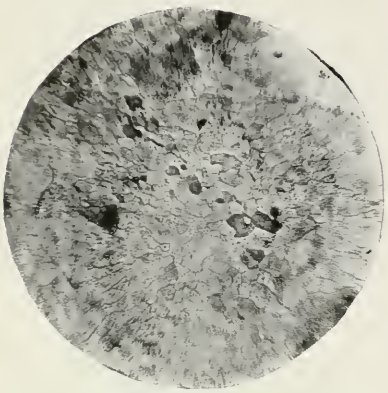
As Forged.

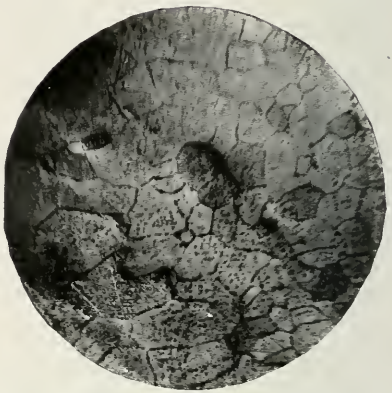
FIG. 34. 40 DIAM. NITRIC ACID.
ALLOY NO. 3Si36. 3.55% Si.



Annealed at 1100° C.

FIG. 35c. 7 DIAM. PICRIC ACID.
ALLOY NO. 3Si25 (Spec. 2). 3.40% Si.





Annealed at 900° C.

FIG. 36b$_1$. 10 DIAM. PICRIC ACID. FIG. 36b$_2$. 40 DIAM. PICRIC ACID.

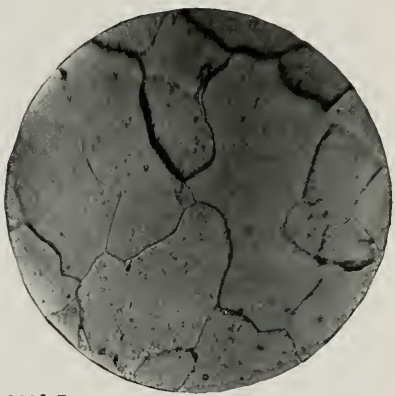
Annealed at 1100° C.

FIG. 36c$_1$. 10 DIAM. PICRIC ACID. FIG. 36c$_2$. 40 DIAM. PICRIC ACID.

ALLOY NO. 3Si28. 4.44% Si.

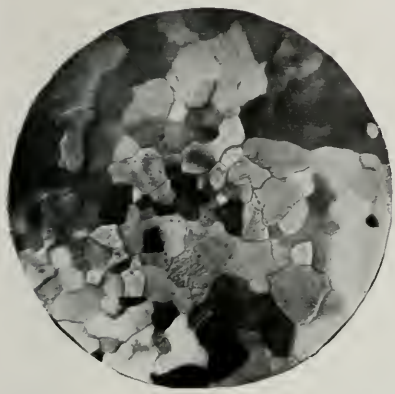
As Forged.

FIG. 37a. 40 DIAM. PICRIC ACID.



Annealed at 900° C.

FIG. 37b. 40 DIAM. PICRIC ACID.

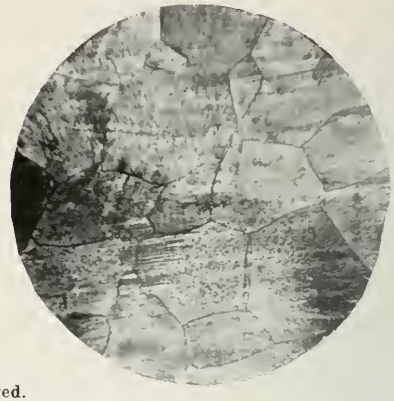


Annealed at 1100° C.

FIG. $37c_1$. 10 DIAM. PICRIC ACID. Washed with 10% HF Sol.

FIG. $37c_2$. 40 DIAM. PICRIC ACID. Washed with 10% HF Sol.

ALLOY NO. 3Si29. 4.92% Si.



As Forged.

FIG. 38a. 40 DIAM. PICRIC ACID.

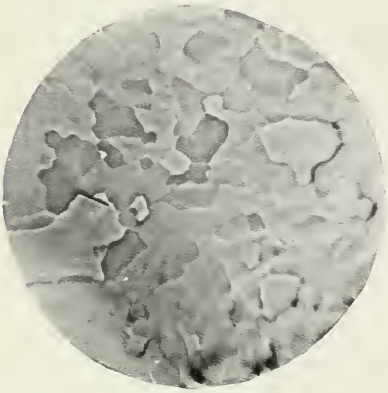


FIG. 38b$_1$. 10 DIAM. PICRIC ACID. Slightly repolished.

Annealed at 900° C.

FIG. 38b$_2$. 40 DIAM. PICRIC ACID. Slightly repolished.

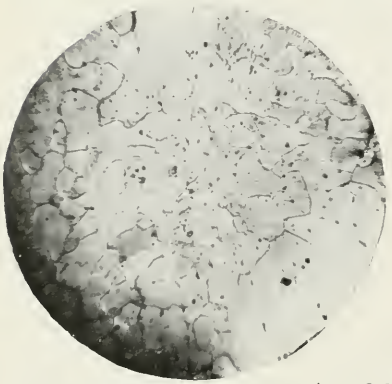
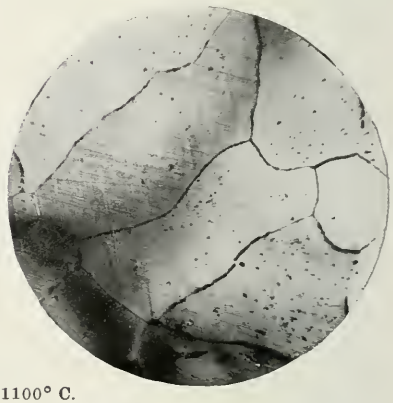


FIG. 38c$_1$. 10 DIAM. PICRIC ACID. Washed in 10% HF Sol.



Annealed at 1100° C.

FIG. 38c$_2$. 40 DIAM. PICRIC ACID. Washed with 10% HF Sol.

ALLOY NO. 3S132. 6.57% SI.

The Non-forgeable Alloy 3Si30. 8.55% Si.
Fig. 39. Nearly Full Size.

V. Summary and Conclusions.

The results recorded in the previous pages may be summarized as follows:

(a). By means of the vacuum method of melting it is possible to obtain a decidedly purer product than has thus far been obtained in any other manner. Consequently by the use of this method more definite conclusions can be drawn with regard to the effect of silicon upon iron, than has hitherto been possible.

(b). Silicon, like boron, has a double effect upon iron. Part of it combines with the iron and remains in solid solution throughout the cooling of the alloy, while a smaller part reduces the iron oxide present.

(c). The tensile strength of the vacuum product follows in general the same law as alloys made under ordinary conditions, but the ductility of the former is much greater, particularly below 2 per cent and above 3 per cent, probably due to the absence of carbon. The maximum tensile strength of 105000 pounds per square inch occurs with a silicon content of 4.5 per cent.

(d). The limit of forgeability lies between 7 and 8 per cent silicon. A critical range occurs between 2.55 and 2.60 per cent, in which the alloys are exceedingly brittle, in some cases being not even forgeable.

(e). With regard to the magnetic properties the vacuum alloys exhibit most remarkable characteristics. The best alloys are obtained with about 0.15 per cent and 3.40 per cent silicon after annealing at 1100°C. The maximum permeability for both of these alloys is above 50,000, and the hysteresis loss for $B_{max} = 10{,}000$ and 15,000 is about 300 and 1000 ergs per cu. cm. per cycle respectively. This hysteresis

loss is $\frac{1}{8}$ and $\frac{1}{3}$ of the corresponding loss for commercial silicon steel. The most favorable annealing temperature is in every case 1100°C.

(f). The specific electrical resistance increases about 13 microhms for the first per cent silicon added. For each additional per cent added the increase is about 11 microhms. Consequently, the 3.40 per cent alloy mentioned under 5 has a resistance nearly 5 times that of the 0.15 per cent alloy.

By the vacuum process two silicon alloys have thus been produced that have very valuable characteristics; one, low in silicon, not very strong, but extremely ductile, of high permeability, low hysteresis loss, and of low electrical resistance; the other high in silicon, very strong, moderately tough, of high permeability, low hysteresis loss and of high electrical resistance. The properties of these two alloys are summarized in Table 12. The first is evidently suitable for use in places where high permeability and low hysteresis loss are the chief requirements, while the second alloy is suitable for electromagnetic machinery, principally transformers, where a low eddy current loss is

TABLE 12.

PROPERTIES OF THE TWO BEST IRON-SILICON VACUUM ALLOYS.

Silicon Content %	Stress at Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.	Elongation %	Reduction of Area %	Maximum Permeability	Density for Max. Permeability	Hysteresis Loss ergs per cc. per cycle		Spec. Elect. Resistance microhms
							for $B_{max} = 10000$	for $B_{max} = 15000$	
0.15	18500	37000	56	90	66500	6500	286	916	11.80
3.40	58000	76500	21	28.5	63300	6500	280	1025	48.50

an additional requirement. The mechanical properties of the second alloy makes possible its use also in dynamo machinery, where the present commercial silicon steel can not be used on account of its brittleness.

It should be pointed out, that electrolytic iron is not essential to the attainment of high magnetic quality. Any low carbon iron that is practically free from phosphorus, sulphur and manganese, *when melted in vacuo,* will give magnetic properties approaching very closely to those obtainable with electrolytic iron. There are a number of commercial grades of iron coming within these specifications, that are obtainable at the present time.

The writer fully realizes the difficulties that have to be overcome, before the vacuum iron can be used in the industries in competition with the materials available at the present time. Suggestions have, nevertheless, already come from several sources for its employment in

places, where its high cost of production is not of vital importance, and there is no doubt but that its usefulness in limited fields will increase as time goes on. However, the writer would like to suggest that the results here given be partly considered as a new indication of the possibilities obtainable in the realm of magnetic properties, rather than as the final word regarding certain properties of iron-silicon alloys, that can be turned into commercial use at the present time.

VII. Appendix.

Results obtained with Ring Specimens.

In view of the unprecedented results obtained with regard to the magnetic properties of the alloys described in this bulletin, it was deemed desirable to test a few alloys in the form of rings. As the ring method is the old established method of magnetic testing, and as this method requires no compensating or other auxiliary windings to cause uncertainties, the evidence obtained with such testpieces would naturally be more convincing than results obtained by any other method. On the other hand, it has been shown by Richter* and Lloyd† that the magnetic induction in a ring specimen is not uniformly distributed, but that it is crowded towards the inside of the ring. The variation is greatest near the steepest part of the magnetization curve, where the maximum permeability occurs, and may here amount to as much as 100 per cent for high permeability material. Consequently it is impossible with ring specimens to measure the maximum permeability, and this should be borne in mind when comparing the results obtained by the two methods.

The dimensions of the rings used are shown in Fig. 40.

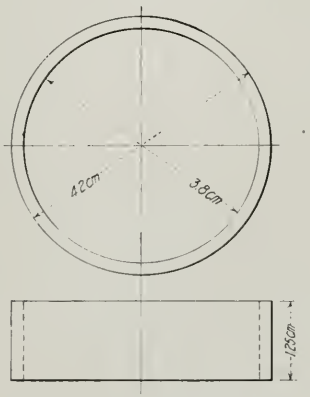


Fig. 40. Ring Specimen.

*Electrotech. Zetischr. Vol. 24, p. 710; 1903.
†Bull. Bureau of Standards, Vol. 5, No. 3, Reprint No. 108, 1909.

With these dimensions the true magnetizing force, as shown by Lloyd and Richter is

$$H_o = 1.0009 \frac{2NI}{10R_o}$$

Where R_o = mean radius = 2.0 cm.
N = total number of turns = 100
I = magnetizing current in amperes
Hence by using $H_o = 10\ I$
the error introduced is less than 0.1 per cent.

The secondary winding consisted of 100 turns of No. 30 B & S wire wound next to the ring and connected directly to the Grassot fluxmeter. With this arrangement

$$\triangle B = 400\ D$$

where $\triangle B$ = change of average flux density in ring in gausses
D = deflection of fluxmeter.

Three rings were prepared all containing approximately 3 per cent silicon. From No. 3Si39, one ring was machined directly out of the ingot as it came from the melting furnace, while another was made from the remainder of the ingot after forging it. The latter was imperfect, however, and was discarded. The third ring was made from No. 3Si40 after forging the ingot, and the remainder of the latter was then forged into a rod to be tested by the Burrows method.

The specimens were first tested in the original state, unannealed. The windings were then removed and the rings and the rod placed in the annealing furnace and annealed in vacuo, the rod occupying the space along the axis of the tube. The maximum temperature to which the rod was subjected was 1100° C, but the rings were heated to a somewhat higher temperature as they occupied a space nearer the heating element, their axes coinciding with that of the rod. The specimens were cooled at the standard rate, namely 30° C per hour.

The results are shown in Table 13. In this table is included the results for rod No. 3Si25 from the main part of the investigation.

With regard to permeability the results show that in the unannealed state the two methods of testing agree very well. In the annealed state, however, the maximum permeability obtained for 3Si40 by the Burrows method is twice that obtained by the ring method. The latter method, however, as has been stated above, does not measure the maximum permeability, on account of the nonuniformity of the flux distribution, and in view of the results shown in Table 5, it is probable that the Burrows method gives too high a maximum perme-

ability. Making allowance for possible differences due to material and heat treatment, as well as mechanical treatment, it seems probable from these results that the true maximum permeability is in the neighborhood of 50,000. That the permeability for B = 15,000 is so low for the ring specimens must be due to the fact that the rings were annealed at a higher temperature than was the rod. It was shown in Figs. 17 and 18, that with a silicon content of 3 per cent for H = 20 the flux density after annealing at 900° was 16,000, while after annealing at 1100° it was only 15,600. A corresponding decrease in this region may consequently be expected also for higher annealing temperatures. This point is further borne out by the results obtained with the ring made from the ingot 3Si39 without forging. This ring may be considered as having been annealed at the melting point of the alloy. Its permeability at B = 15,000 is only 583 as compared with 962 for the ring made from 3Si40 after forging.

Turning now to the hysteresis loss it is found that in the unannealed state the loss in ring No. 3Si40 is less than half the loss in the rod. As the two methods are known to be equally reliable in this case the difference must be attributed to the fact that the rod was forged much more than was the ring. That this is the cause becomes evident from the result obtained for ring No. 3Si39 that was not forged at all, as this shows a decidedly lower hysteresis loss even than ring No. 3Si40. After annealing, the loss for B = 10,000 is found to be less for the rod than for the ring, while for B = 15,000 the reverse is the case. The coercive force for B = 15,000 is 60 per cent larger for the rod than for the ring, while the retentivity is also lower for the ring.

These results seem to point towards the following conclusions: For such high quality material as that described in this bulletin the Burrows method gives too high maximum permeability and too high retentivity. For low densities the hysteresis loss obtained is low, while for medium and high densities the coercive force and consequently also the hysteresis loss is too high.

While these rings were prepared for the purpose of verifying the results given in the main part of the bulletin, the results can not be dismissed without calling especial attention to the results obtained with the ring made from the ingot No. 3Si39 without forging. Unannealed this ring has a retentivity of only 1000 and the hysteresis loss for B = 15,000 is only 1130 ergs per cc per cycle, not much above the value found for the best rod after annealing at 1100° C. After

annealing the ring at 1100° the hysteresis loss for B = 15,000 drops to 577 ergs. This value is less than ⅕ of the corresponding value for the commercial 4 per cent silicon steel. The low hysteresis loss for this ring shows the advantage, as far as this characteristic is concerned, of annealing at as high a temperature as possible, as this ring may be considered as having been annealed at the melting point of the alloy. On the other hand, the results also show that the permeability for high magnetizing forces decreases with increasing annealing temperatures. That this is the case for temperatures between 900° and 1100° C has previously been established. That it is equally true for temperatures above 1100° C is now indicated by the results obtained for this ring.

Table 13.

Comparison between the Ring Method and the Burrows Method of Magnetic Testing.

Test Piece	Silicon tent Con- %	Maxi- mum Perme- ability	Density for Max. Perme- ability gausses	Permeability for B = 10000	Permeability for B = 15000	Hysteresis Loss ergs per cc per cycle for Bmax 10000	Hysteresis Loss ergs per cc per cycle for Bmax 15000	Coercive Force gilberts per cm for Bmax 10000	Coercive Force gilberts per cm for Bmax 15000	Retentivity gausses for Bmax 10000	Retentivity gausses for Bmax 15000	Spec Elec. Resis- tance mic- rohms	Remarks
Unannealed													
3Si39 Ring–Made from Ingot	3.19	2000	4000	1600	405	955	1130	.45	.45	1000	1000		Tested by Ring Method
3Si40 Ring–Made from Forging	2.96	3000	3000	1470	485	1305	1540	.50	.50	2500	2500		" " " "
Rod–Made from Forging	2.96	2350	4000	1210	545	3130	4450	1.00	1.00	4800	4800	44.75	" " Burrow's "
3Si25 Rod–Made from Forging	3.40	3000	3000	1065	476	.....	.....	Not	Tested	.....	.....	48.5	" " " "
Annealed													
3Si39 Ring–Made from Ingot	3.19	32000	8000	28600	583	340	577	.10	.10	7320	9600		Tested by Ring Method
3Si40 Ring–Made from Forging	2.96	36200	8000	31200	962	337	757	.10	.10	7600	11000		" " " "
Rod–Made from Forging	2.96	72600	9000	71400	2500	254	926	.085	.16	9400	13600	44.75	" " Burrow's "
3Si25 Rod–Made from Forging	3.40	63300	7000	46500	2030	280	1025	.085	.15	9000	12400	48.5	" " " "

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

Circular No. 3. Fuel Tests with Illinois Coal (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907), by L. P. Breckenridge and Paul Diserens. 1909. *Thirty cents.*

Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

Bulletin No. 5. Resistance of Tubes to Collapse, by Albert P. Carman and M. L. Carr. 1906. *None available.*

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Bulletin No. 7. Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr, and Henry B. Dirks. 1906. *None available.*

Bulletin No. 8. Tests of Concrete: I, Shear; II, Bond, by Arthur N. Talbot. 1906 *None available.*

Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition 1912. *Fifty cents.*

Bulletin No. 10. Tests of Concrete and Reinforced Concrete Columns. Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 11. The Effect of Scale on the Transmission of Heat Through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

Bulletin No. 12. Tests of Reinforced Concrete T-Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 13. An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *None available.*

Bulletin No. 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 15. How to Burn Illinois Coal Without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*

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**Bulletin No. 39.* Tests of Washed Grades of Illinois Coal, by C. S. McGovney. 1909. *Seventy-five cents.*

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*A limited number of copies of those bulletins which are starred are available for free distribution.

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**Bulletin No. 41.* Tests of Timber Beams, by Arthur N. Talbot. 1910. *Twenty cents.*

**Bulletin No. 42.* The Effect of Keyways on the Strength of Shafts, by Herbert F. Moore. 1910. *Ten cents.*

Bulletin No. 43. Freight Train Resistance, by Edward C. Schmidt. 1910. *Seventy-five cents.*

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**Bulletin No. 50.* Tests of a Suction Gas Producer, by C. M. Garland and A. P. Kratz. 1912. *Fifty cents.*

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TESTS OF REINFORCED CONCRETE FLAT SLAB STRUCTURES

BY

ARTHUR N. TALBOT

And

WILLIS A. SLATER

ONTARIO RESEARCH FOUNDATION
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AND

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TESTS OF REINFORCED CONCRETE FLAT SLAB STRUCTURES

I. Introduction.

1. *Purpose and Scope.*—It is the purpose of this bulletin to present the results of certain tests made on four reinforced concrete buildings and one reinforced concrete test structure. These tests were made with a view of getting experimental information on the action of the concrete and the reinforcing bars in floor slabs of the flat slab type of building construction. Data were obtained also on the bending action of the supporting columns. Efforts were made to find the distribution of stress in the bands of reinforcement both laterally and longitudinally, and that of the compressive stresses in the concrete on the opposite face of the slab; these in the regions of both the negative bending moment and of the positive bending moment.

The description of the methods used in making the test is limited to those features which are different from the methods used in the tests described in Bulletin No. 64 of the University of Illinois Engineering Experiment Station, "Tests of Reinforced Concrete Buildings Under Load", and from the methods described in a paper on "The Use of the Strain Gage in the Testing of Materials."*

It will be appreciated that the circumstances surrounding the floor test of a building are unfavorable to securing definite and uniform quantitative results. The structure is not homogeneous. There is a distribution of the resistance afforded by the structure to parts beyond the portion which is loaded. Effects of changes in temperature are troublesome. The stresses developed in the steel and in the concrete are small and there is considerable variation between parts which are supposedly similar in action. The conditions under which the measurement of deformation must be made are unfavorable to securing exactness. The location and presence of the loading material also add to the difficulties of the work.

It will be seen that it is impracticable to obtain complete information or to formulate conclusions which are entirely definite. Only general results and conclusions of a qualitative character may be

*Proceedings of the American Society for Testing Materials, vol. 13, p. 1019.

expected. However, it is believed that the tests herein recorded bring out information of value on the action of reinforced concrete flat slabs and of the supporting columns. Since among engineers there is such a marked variation of opinion concerning the action of the flat slab and since there is such uncertainty in the analysis of the flat slab, it is believed that the information given will be regarded as adding to the general knowledge of this subject, and that it will be useful in considering many features of the design of buildings of the type tested.

2. *General Statement of Tests.*—The tests were largely cooperative work. The interests of engineers and contractors in learning the properties of the flat slab and their willingness to bear expenses of the tests made the test work possible. In two cases the tests were made by engineering firms, Mr. Slater being connected with the work, and the data of the tests were placed at the disposal of the Engineering Experiment Station. Mr. Slater was an observer in all of the tests. In every case the data have been carefully worked over in the office of the Engineering Experiment Station and the results discussed and put into form. In the long work of studying the data, many inconsistencies and uncertainties were found, and the presentation of many matters on which it was hoped that the tests would give information had to be abandoned because the results of the tests were indefinite or inconclusive. Results from which at least qualitative conclusions may be drawn have been recorded in the bulletin.

The structures tested and the arrangements for the tests were as follows:

(a) Shredded Wheat Factory, Niagara Falls, N. Y. Flat slab floor with two-way reinforcement. Designed by Corrugated Bar Company, Buffalo, N. Y. Building built by Braas Bros., contractors, Niagara Falls, N. Y. Tested by Corrugated Bar Company.

(b) Soo Line Freight Terminal, Chicago, Ill. Flat slab floor with four-way reinforcement. Designed and built by the Leonard Construction Company, engineers and contractors, Chicago. Tested by cooperation between Leonard Construction Company, Central Terminal Railway Company, and the Engineering Experiment Station of the University of Illinois.

(c) Schulze Baking Company Building, Chicago, Ill. Flat slab floor with four-way reinforcement. Designed by Lieberman and Klein, engineers, Chicago. Built by McLennan Construction Company, contractors, Chicago. Tested by Mr. Slater for American System of Reinforcing. The contractors placed and removed the loading material.

(d) Worcester Slab Test, Worcester, Mass. A sixteen-panel slab having four different designs of reinforcement. Constructed especially for the test. Built according to plans prepared by B. S. Brown, consulting engineer, Boston. Tested by cooperation between Mr. Brown, Worcester Polytechnic Institute, and the Engineering Experiment Station of the University of Illinois.

(e) Curtis-Leger Company Building, Chicago, Ill. Flat slab floor having four-way reinforcement at interior of panel, and two-way reinforcement in region of columns.

Designed by Barton Spider Web System, Chicago, and built by the Simpson Construction Company, contractors, Chicago. Tested by the Engineering Experiment Station of the University of Illinois with the assistance of the engineers and contractors.

Table 1 gives general data concerning the tests.

3. *Acknowledgment.*—The test of the Shredded Wheat Factory Building was conducted by Mr. F. J. Trelease, Chief of the Research Department of the Corrugated Bar Company, and Mr. Slater. The Corrugated Bar Company bore the expense of the test. Acknowledgment is made to this company for the courtesy in permitting the use of the test data.

The technical part of making the test of the Soo Terminal Structure was done as the work of the Engineering Experiment Station. The Leonard Construction Company bore the expense of the preparation for the test and of matters connected with the application of the load. Mr. A. R. Lord, consulting engineer, represented the Leonard Construction Company in planning and carrying out the test. The Central Terminal Railway Company placed the track for the test, and applied and removed the test load. Mr. Slater was in immediate charge of the preparations for the test and of its conduct. The following observers and recorders from the University assisted in the test: Messrs. H. F. Moore, D. A. Abrams, N. E. Ensign, H. F. Gonnerman,H. R. Thomas, G. A. Maney, and M. Abe. Messrs. Meyer, O. R. Erickson, C. J. Nelson, and O. R. Kellner of the force of the Leonard Construction Company also assisted in the test.

The floor test of the Schulze Baking Company Building was conducted by Mr. Slater for the American System of Reinforcing, Chicago. This company, and the contractors, McLennan Construction Company, bore the expense of the test. Acknowledgment is made to the American System of Reinforcing for permission to use the data for publication.

The Worcester slab test structure was conceived and planned by Mr. B. S. Brown, consulting engineer, of Boston, and the expense of carrying out the test was borne principally by Mr. Brown. The materials, labor, and supervision of the construction were furnished by the Allentown Portland Cement Company, Boston, Carnegie Steel Company, Boston, Varnum P. Curtis Gravel Company, Worcester, Aberthaw Construction Company, Boston, and Mr. Brown. The plans for the test were laid out by Mr. Brown, Professor French, of Worcester Polytechnic Institute, and the writers. The conduct of the test was directed by Mr. Slater. Professor H. F. Moore of the Engineering Experiment Station cooperated in the work.

The expense of the Curtis-Leger Building test was borne by F. M. Barton, engineer and architect, Chicago, and the Simpson Construction Company, contractors, Chicago. Mr. Slater conducted the test.

II. The Shredded Wheat Factory Building Test.

4. *The Building.*—The building of the Shredded Wheat Company is of reinforced concrete construction of the flat slab type, three stories in height. The main portion of the building is 265 ft. 4 in. long and 81 ft. wide. The panels are 20 ft. by 22 ft. The floor is the type of flat slab designated by the trade name, Corr-Plate Floor. The floor on which the test load was applied is the first floor above the basement. This floor is nominally 7 in. thick in the central portion of the panel and 9 in. thick throughout an area 8 ft. 6 in. square (the depressed head) surrounding each column. It was designed for 125 lb. per sq. ft. live load. The interior columns have a pyramidal capital of octagonal form 42 in. in diameter at the top and sloping 45° with the horizontal. The slab has two-way reinforcement, designed to resist negative moment at all points across the edges of the panels and positive moment across the center lines of the panels. Corrugated bars were used for reinforcement. Fig. 1 and 2 show the distribution of the reinforcing bars in the test floor of this building. Fig. 2 contains information on the bending and supporting of bars and on other details of the slab. Readings with an engineer's level at numerous positions on the floor gave an average floor thickness of 9.13 in. at the columns and 7.29 in. midway between columns. The average measured depths from the compression surface of the concrete to the center of gravity of the reinforcement of the central panel of the test area were 6.82 in. and 4.95 in. for positions of negative moment in the depressed head and in the thinner portions of the slab

TABLE 1.

GENERAL DATA OF TESTS.

Structure	Type	Test Area		Loading Material	Design Load lb. per sq. ft.	Test Load lb. per sq. ft.	Load Handled tons	No. of Gage Lines	No. of Observers	Days required for	
		In square feet	Per cent of Total Area							Preparation	Test
Shredded Wheat Factory	Two-way reinforcement; depressed head	3960	18.4	Gravel	125	(c) 243	(g) 390	288	2		7
Soo Line Terminal	Four-way reinforcement; depressed head	2304	0.38	Ore cars loaded with broken stone	(b)	700	1006	816	6	13	8
Schulze Baking Company Building	Four-way reinforcement; depressed head	1400	2.95	Brick stacked in piers	300	(d) 722	437	263	1	7	6
Worcester Slab	Four-way reinforcement; no depressed head	3280	100.0	Gravel		(e) 328	459	320	2	11	6
Curtis-Leger Company Building	Four-way reinforcement; no depressed head	1355	11.1	Bags of Cement	200	(f) 500	(h) 237	142	1	7	8

(a) 4-way in central portions of panel, 2-way at column.
(b) Designed for E-50 Wheel-loading.
(c) 191 lb. per sq. ft. over entire 9-panel area.
243 lb. per sq. ft. over only one panel.
(d) 722 lb. per sq. ft. over south two panels, and part of north two panels; 435 lb. per sq. ft. over remainder of 4-panel test area.
(e) 328 lb. per sq. ft. over about 11 panels.
215 lb. per sq. ft. over about 5 panels.
(f) 500 lb. per sq. ft. over 678 sq. ft.
200 lb. per sq. ft. over 1355 sq. ft.
(g) Includes the rehandling of 3 per cent of the material.
(h) Includes the rehandling of 28 per cent of the material.

respectively, and 6.16 in. for positions of positive moment. The columns are octagonal, 25 in. in short diameter for the basement columns, and 22 in. in short diameter for the first story.

The concrete in the building was of excellent quality. Gravel was used as aggregate. At the time of making the test the concrete in the floor was about 80 days old. Four test cylinders, made at the time of pouring the test floor, gave an average strength of about 3500 lb. per sq. in. at an age of 115 days in tests made at the University of Illinois, none falling below 3200 lb. per sq. in.

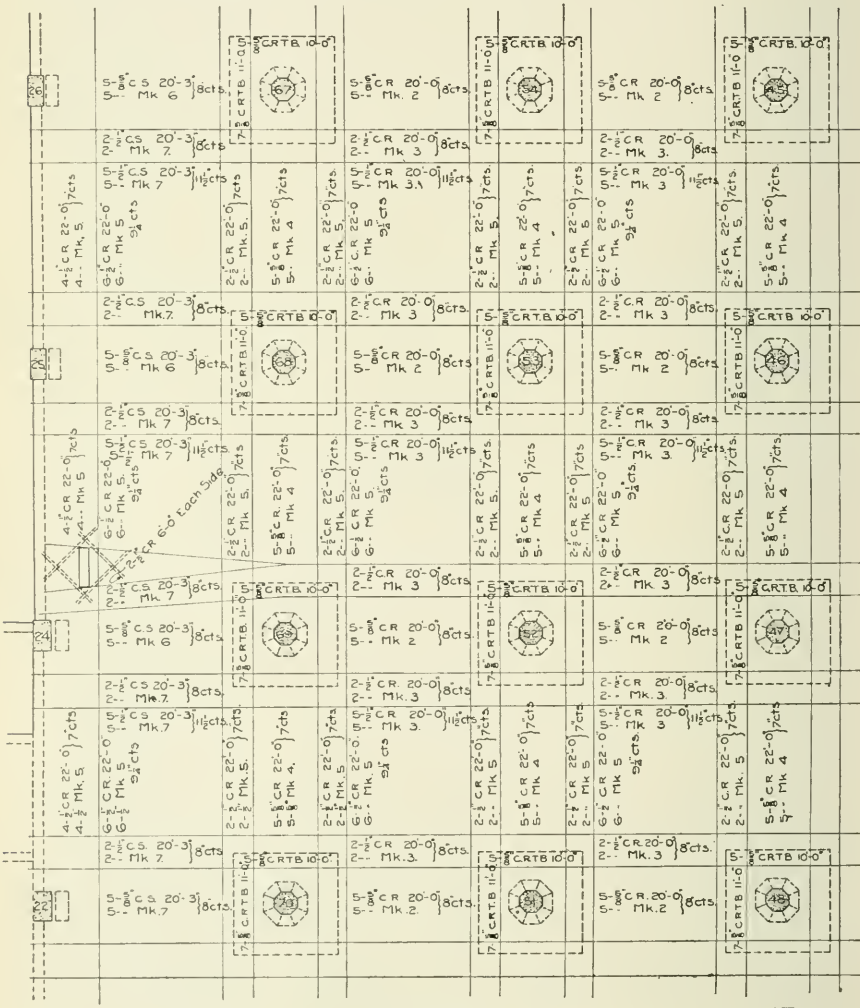
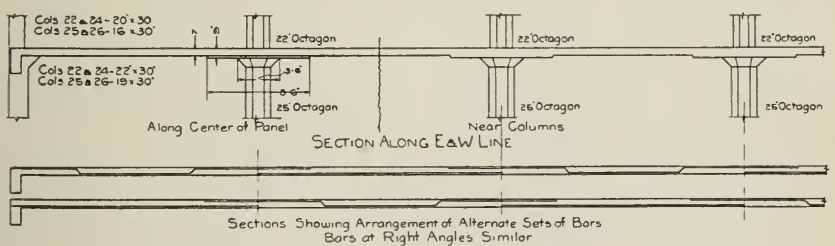


FIG. 1. DISTRIBUTION OF REINFORCEMENT FOR TEST FLOOR OF SHREDDED WHEAT FACTORY.

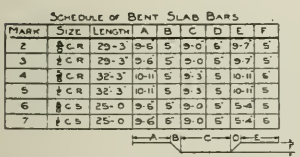
5. *Testing.*—The deformation measurements were taken on 137 gage lines on the reinforcement and 151 gage lines on the concrete. Fig. 3 and 4 show the location of the gage lines, and Fig. 5 shows the key to the grouping of the gage lines for purposes of comparison. Deflection readings were taken at 33 points (see Fig. 6).

The loaded area is shown in Fig. 6. Gravel was used as the loading material. It was raised by means of a concrete hoist to the second floor and there deposited into the hopper of a concrete chute. By moving the chute the gravel was distributed as desired. Fig. 7 shows the load in position. The load covered the entire panel areas except for aisles 2 ft. wide extending from column to column and boxes about 3½ ft. square placed at the center of each panel. These areas were left uncovered to afford access to the gage lines. Each increment of load was leveled carefully. At the load of 191 lb. per sq. ft., in order to ascertain the total weight per cubic foot of gravel as compacted, a bottomless box was sunk through the gravel after the fashion of an open caisson. The gravel was shoveled out from the inside of the box, measured and weighed. The weight per cubic foot of loose gravel was found to be 113 lb. per cubic foot, and of the compacted gravel, 134½ lb. per cubic foot. As only 79 per cent of the floor area was covered, the corresponding average load per square foot for the total test area was 106½ lb. per ft. of height of compacted gravel. The exact degree of compactness of the gravel at all times was not known, but it is thought that the values used are representative of the test load.



SCHEDULE OF BENT SLAB BARS

MARK	SIZE	LENGTH	A	B	C	D	E	F
2	⅝ C.R.	29-3	9-6	5	9-0	6	9-7	5
3	¾ C.R.	29-3	9-6	5	9-0	6	9-7	5
4	⅝ C.R.	32-3	10-11	5	9-3	5	10-11	6
5	¾ C.R.	32-3	10-11	5	9-3	5	10-11	5
6	⅝ C.S.	25-0	9-6	5	9-0	5	5-6	5
7	¾ C.S.	25-0	9-6	6	9-0	6	5-6	6



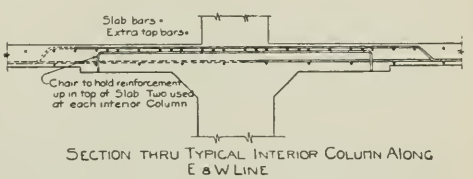


FIG. 2. BENDING OF BARS FOR TEST FLOOR OF SHREDDED WHEAT FACTORY.

Nine panels were first loaded (see Fig. 6). Strain gage observations were made at loads of 56, 120, and 191 lb. per sq. ft. The 191-lb. test load was allowed to remain in position two and one-half days. The load was then removed from the three wall panels and from the three panels farthest from the wall, leaving three panels loaded as shown in the figure. Strain gage observations were taken with this load in place. Next, the two outer panels of the three were unloaded and the load on the center panel was increased to 243 lb. per sq. ft.

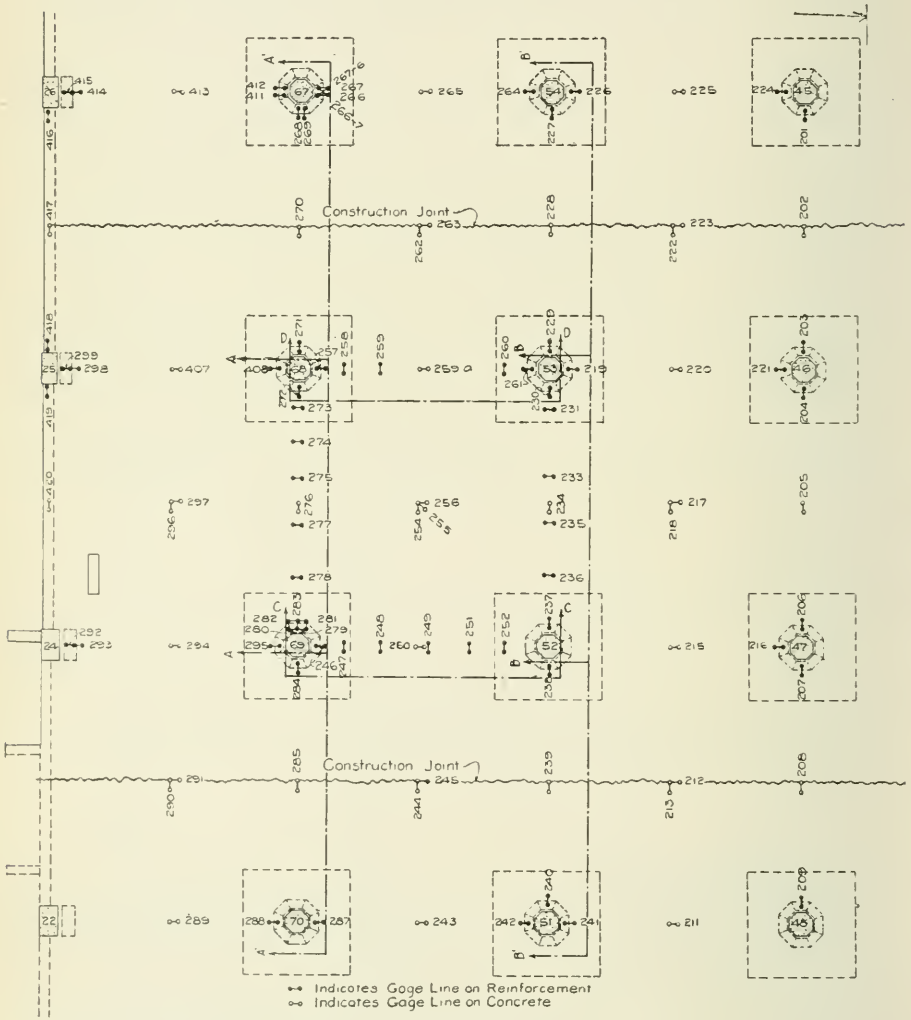


FIG. 3. LOCATION OF GAGE LINES ON UPPER SURFACE OF TEST FLOOR OF SHREDDED WHEAT FACTORY.

At each stage of the test the load was allowed to remain in position at least 12 hours before the final strain gage readings were taken. In order to obtain information on the effect of time on deformation under load, readings were taken at the 191-lb. load immediately after completing the loading operation and again after the load had been in place about 60 hours.

6. *Load-deformation Diagrams.*—The load-deformation diagrams have been plotted in Fig. 8 and 9, the grouping being such as to

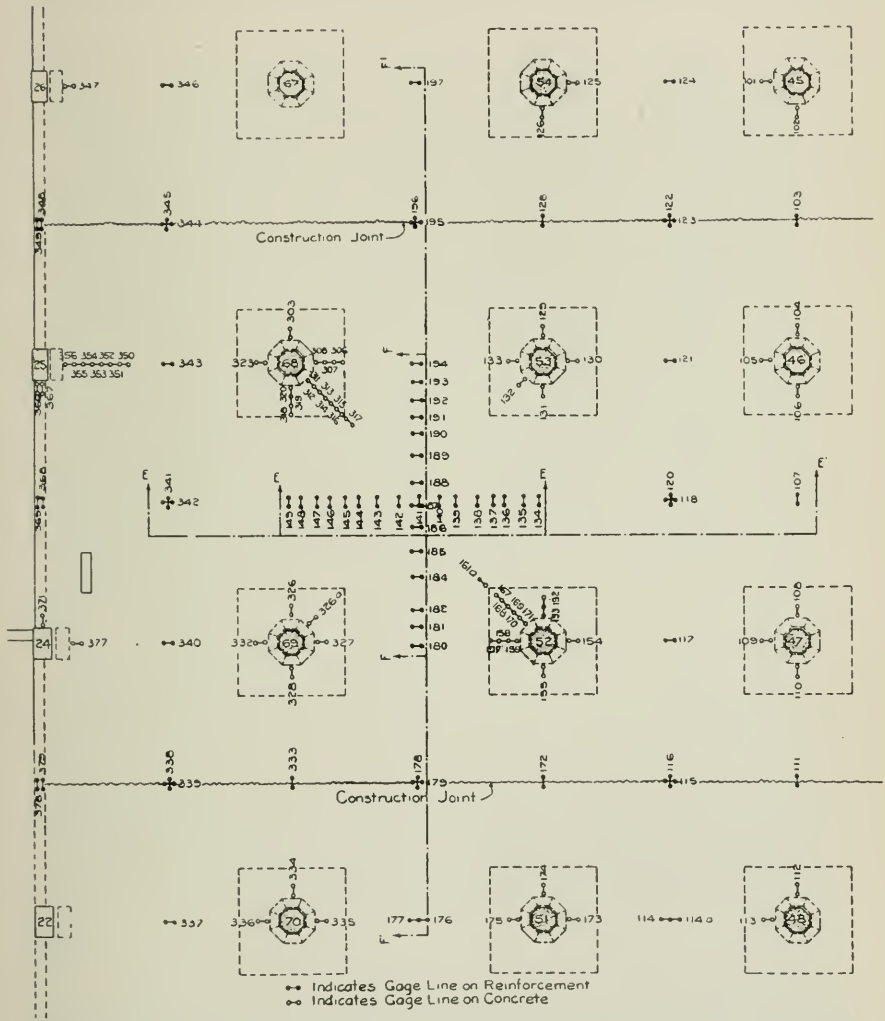


FIG. 4. LOCATION OF GAGE LINES ON UNDER SURFACE OF TEST FLOOR OF SHREDDED WHEAT FACTORY.

place close together the diagrams for gage lines located in similar positions on the floor. In these diagrams the values from the zero load to the first point plotted for the load of 191 lb. per sq. ft. are for nine panels loaded (test area No. 1, Fig. 6). The second point at load of 191 lb. per sq. ft. in each diagram is for three panels loaded (test area No. 2). The point corresponding to 243 lb. per sq. ft. is for the final one-panel load (test area No. 3). The number of panels loaded at various stages of the test is indicated for the diagram marked (1) a, Fig. 8. This is typical of all the diagrams in Fig. 8

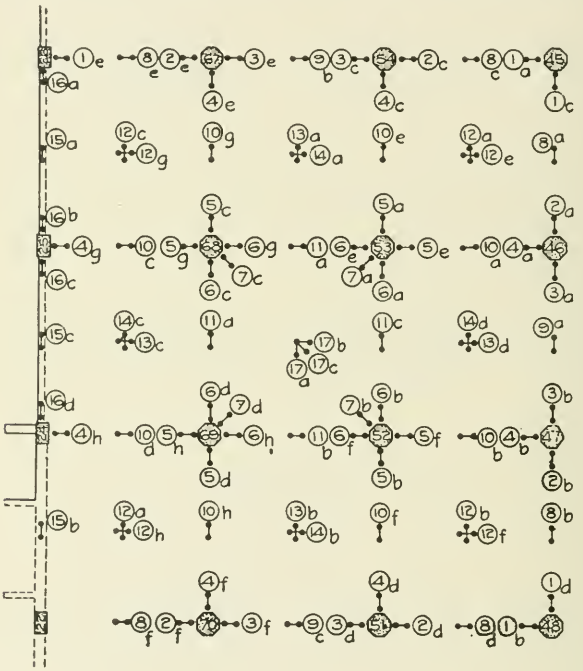


FIG. 5. KEY TO GROUPING OF GAGE LINES; TEST FLOOR OF SHREDDED WHEAT FACTORY.

and 9. The numbers on the diagrams correspond to the gage line numbers given in Fig. 3 and 4. The grouping of the diagrams is indicated by the numbers which correspond to those given in the "Key to Grouping of Gage Lines," (Fig. 5).

In some cases the load-deformation diagrams give peculiar results; however, the similarity of the deformations found in positions remote from each other but similarly situated is marked and tends to give confidence in the results. An example of this is found in Groups 1, 2, and 3 in which, in several instances, slight tension was found where

compression would be expected. It is quite possible that these apparently erratic results may be due to general changes in temperature in the concrete.

Examination of these diagrams shows that in all but a small number of cases the removal of load from six panels caused the kind of change in stress which would be expected from the nature of the change in loading.

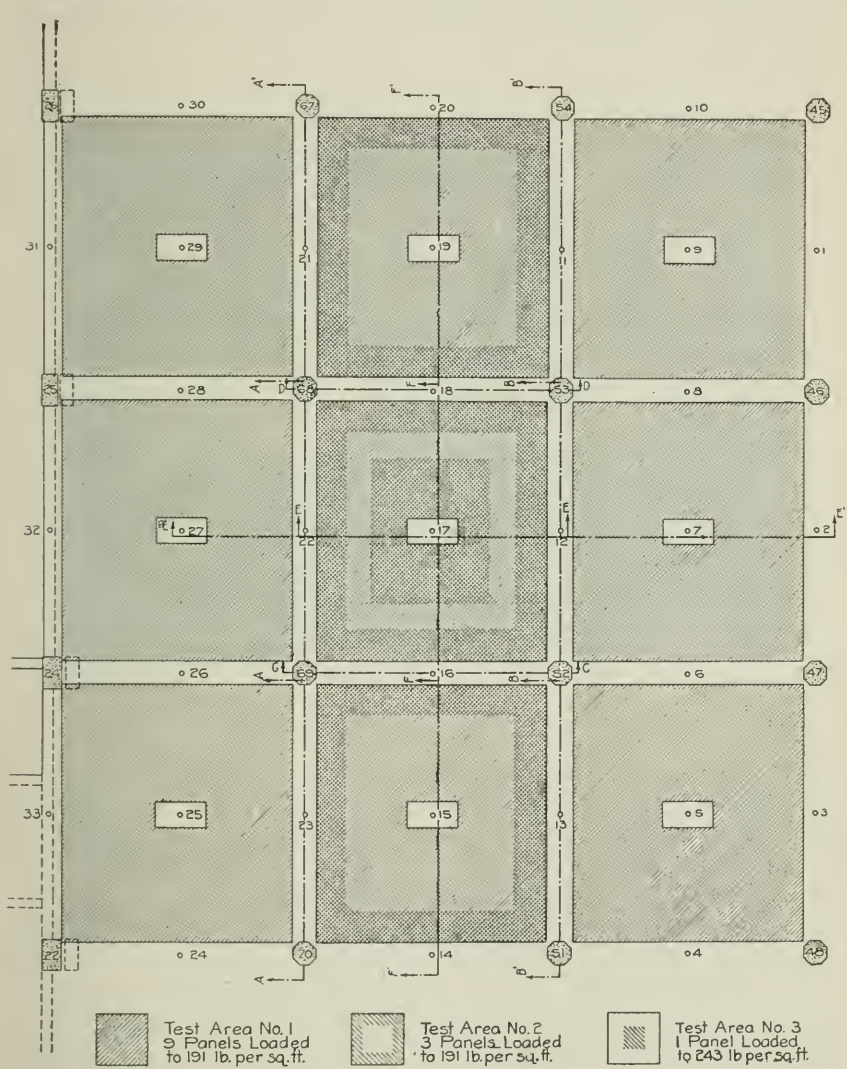


FIG. 6. LOCATION OF DEFLECTION POINTS AND PLAN OF LOAD DISTRIBUTION AT THREE SUCCESSIVE STAGES OF TEST OF SHREDDED WHEAT FACTORY FLOOR.

7. *Effect on Deformations of Standing Under Load.*—With 191 lb. per sq. ft. on the nine-panel area, strain gage readings were taken immediately after placing the load in position and again after it had been in place for about 56 hours. With few exceptions there was a material increase in the deformation during this time, and the increase for the gage lines on the concrete nearly always was greater than for the gage lines on the steel. An examination of 66 gage lines indicates an average increase in unit-deformation approximating 30 per cent for the concrete and 20 per cent for the steel over that which existed

FIG. 7. VIEW OF TEST LOAD IN SHREDDED WHEAT FACTORY.

just after the load was applied. There was no apparent systematic difference in the amount of increase between positions at the columns and positions near centers of panels and none between gage lines at the edge of the loaded area and those in the interior portions.

With the load of 243 lb. per sq. ft. on one panel only, the measurements indicate little or no general increase in deformation, but most of the gage lines on which these observations were taken were so situated that they were affected by the removal of load from the outer panels, and it seems probable that the recovery may have been progressing at the same time that the deformations were increasing due to standing under load.

TABLE 2.

EFFECT ON UNIT-DEFORMATION OF CHANGES IN LOAD DISTRIBUTION.

Plus indicates increase and minus indicates decrease in deformation

Position of Gage lines	Direction of Measurement	Average Change in Unit-deformation in per cent of Unit-deformation before Moving Load	
		Change from 9 panels to 3 panels both at 191 lb. per sq. ft.	Change from 3 panels at 191 lb. per sq. ft. to 1 panel at 243 lb. per sq. ft.
Across center line of panel, Section F′–F′; on steel	E–W	+66	
Across center line of panel, Section F–F; on steel	E–W	+56	0
Across center line of panel, Section F′–F′; on concrete	E–W	+116	
Across center line of panel, Section F–F; on concrete	E–W		—33
Across edges of panel, Sections A′–A′ and B′–B′; on steel	E–W	—19	
Across edges of panel, Sections A–A and B–B; on steel	E–W		+32
Across center line of panel, Section E–E; on steel	N–S	—5	+23
Across center line of panel, Section E–E; on concrete	N–S	—31	
Across edges of panel, Sections C–C and D–D; on steel	N–S	—22	—5

8. *Effect of Change in Number of Panels Loaded.*—Table 2 gives a summary of the results of an examination of the change of unit-deformation caused by the change of loading from nine panels to three panels, and from three panels to one panel (see also Fig. 10 and 11). The areas occupied by these loads and their intensities are indicated as test areas No. 1, No. 2, and No. 3, Fig. 6. In this table increases in the deformation are given a positive sign and decreases are given a negative sign. The most striking features of this table are the great increases in deformation in gage lines which cross the north and south center line of the panel (section F′-F′) in both the concrete and the steel. It is of note also that, in the three instances in this table in which changes in deformation in steel and concrete on the same section can be compared, the changes in concrete are much greater than those in the steel.

The change from nine panels to three panels caused an average increase in deformation of 66 per cent in gage lines which cross section F′-F′. Taking only the gage lines of the middle panel of the three, the

average increase was only 56 per cent. (See Fig. 11). The greatest changes of all were in gage lines 179 and 195 both of which lay close to and parallel with construction joints. It is not apparent that the

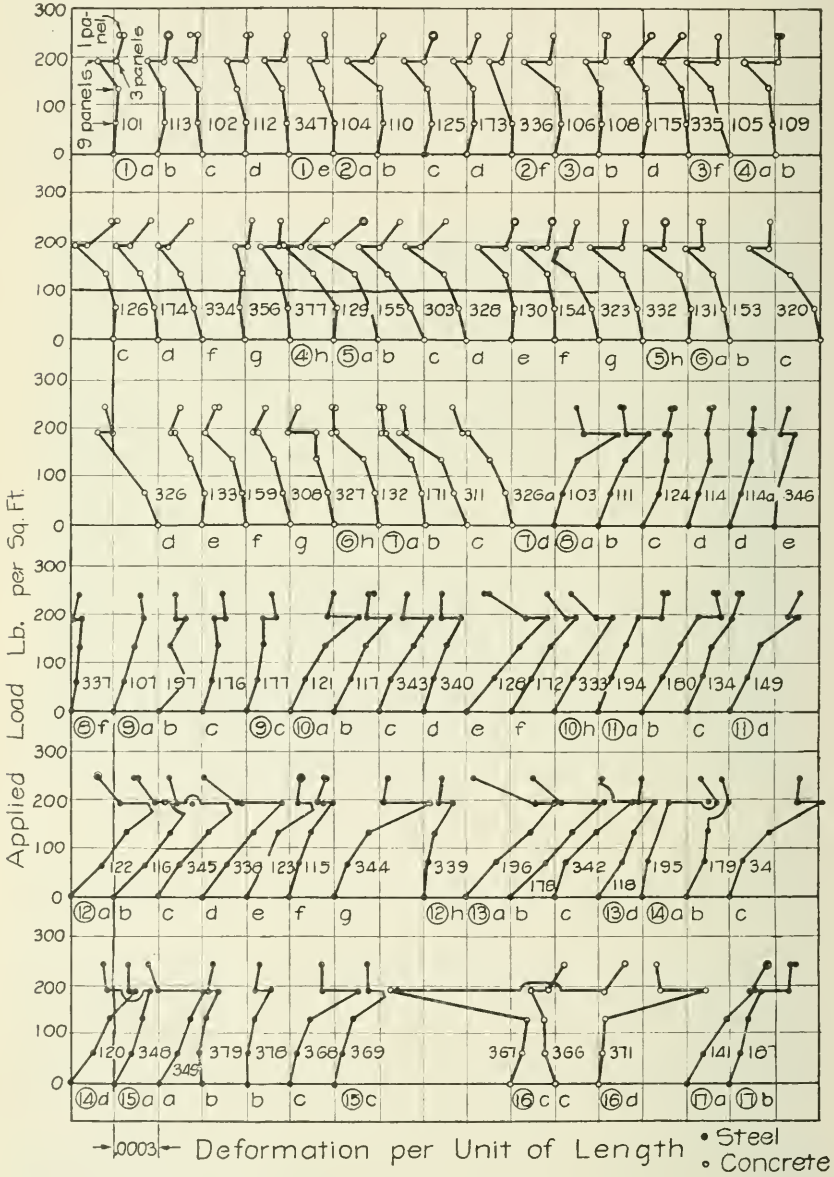


Fig. 8. Load-deformation Diagrams for Gage Lines on Upper Surface of Test Floor of Shredded Wheat Factory.

proximity of the construction joint should influence the change since the gage lines do not cross the joint. Gage lines 176 and 177 (at the edge of the test area) showed very large increases also, but that in 197 was below the average.

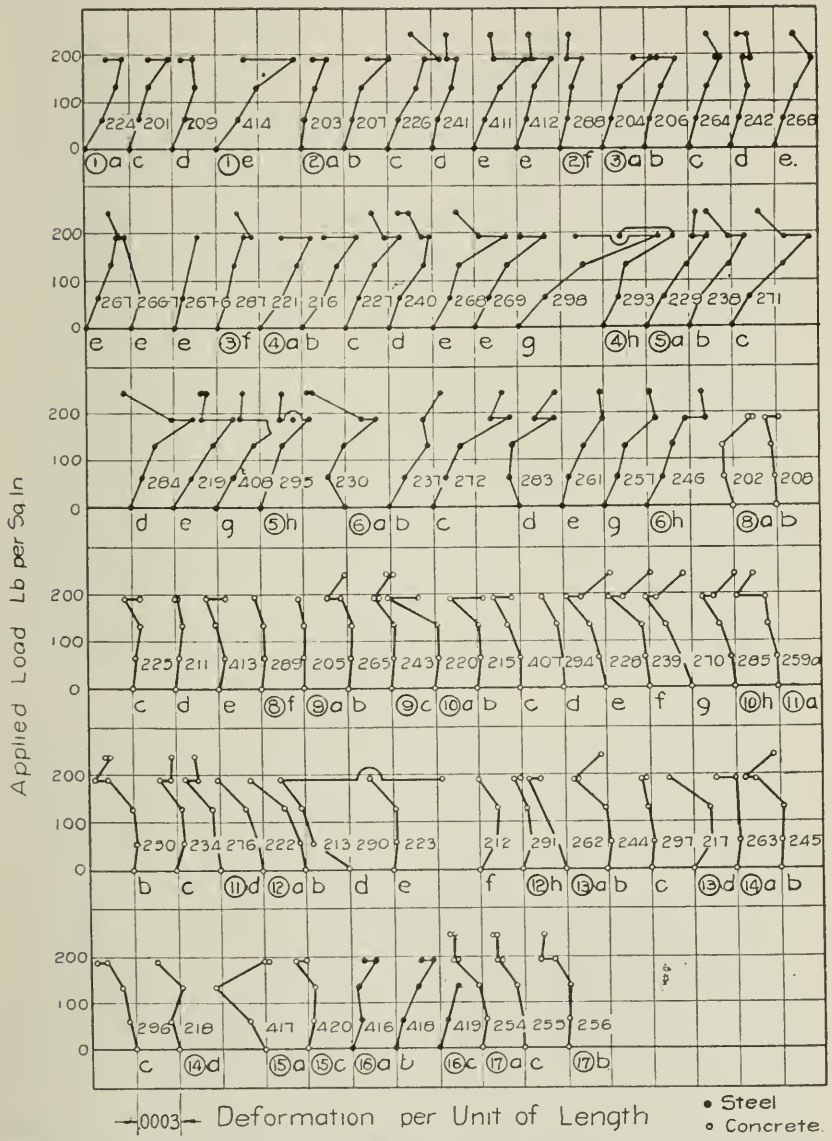


FIG. 9.—LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES ON UNDER SURFACE OF TEST FLOOR OF SHREDDED WHEAT FACTORY.

With few exceptions the deformations in the reinforcement across the edges of the panel (that is, across sections A-A and B-B) decreased with the change of load from nine panels to three panels. These exceptions all are in the case of bars which pass through a column or over a column capital. In all cases the bars which cross the panel edge at points intermediate between the columns lost part of their stress on changing the load from nine panels to three panels. These phenomena would indicate that if, for purposes of comparison, the slab were to be conceived of as a double system of strip beams, the strips passing through the columns could be considered as fixed or nearly fixed at their supports while those passing between columns must be considered to have appreciably less end restraint. The average for all gage lines crossing sections A-A and B-B shows a decrease in deformation of 19 per cent due to the change from nine panels to one panel loaded.

Gage lines were located on the under side of the slab close to the columns in the center panel and in an outer panel. On the removal of the load from the outer panels, the load of 191 lb. per sq. ft. being left on the row of three panels, the deformation in the concrete on the

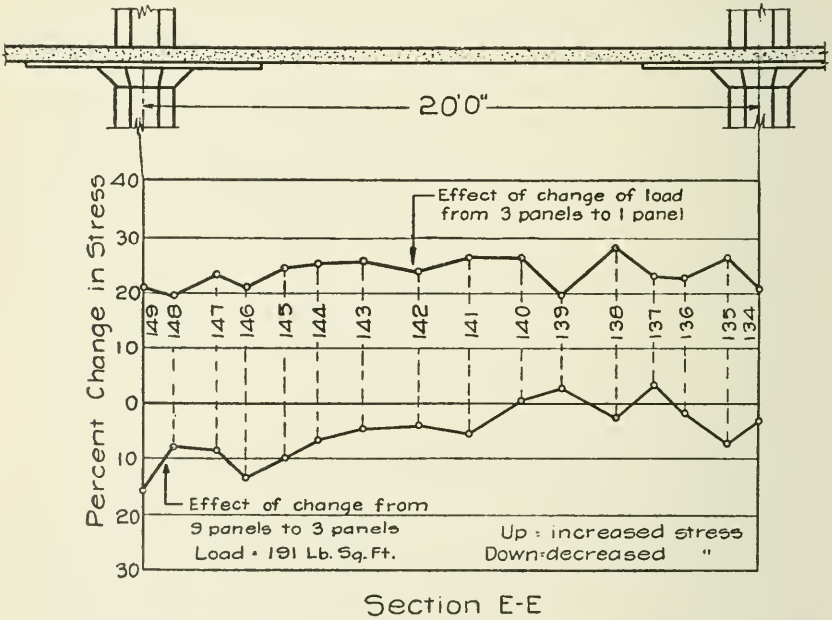


Fig. 10. Change in Stress Accompanying Changes in Loading for Gage Lines across East-west Center Line of Central Panel, Shredded Wheat Factory.

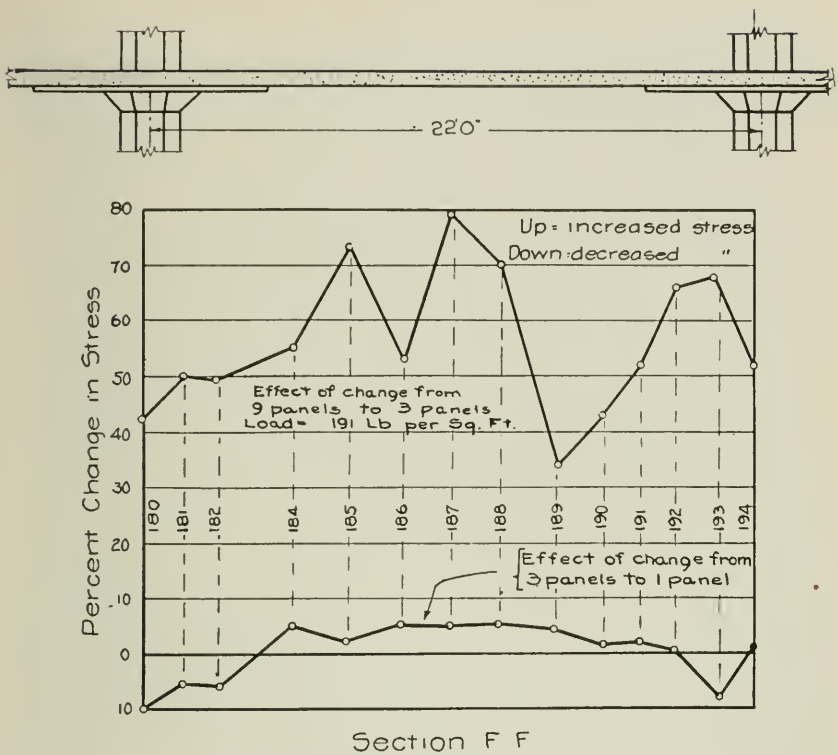


FIG. 11. CHANGE IN STRESS ACCOMPANYING CHANGES IN LOADING FOR GAGE LINES ACROSS NORTH-SOUTH CENTER LINE OF CENTRAL PANEL, SHREDDED WHEAT FACTORY.

under side of the slab at the unloaded side of the column (gage lines 323, 332, 154, and 130) decreased markedly, and the deformation in the concrete on the loaded side of the column (gage lines 327, 159, 133, and 308) increased somewhat. Assuming a modulus of elasticity of 3,000,000 lb. per sq. in. for the concrete, the amount of the compressive stress in the concrete on the loaded side amounted to about 800 lb. per. sq. in., and on the unloaded side to about 300 lb. per. sq. in.

Fig. 13 and 15 show the deformations in the gage lines which cross the center lines of the central panel (sections F-F and E-E) for the three panels loaded and for the one panel loaded. Fig. 10 and 11 show the change in stress in these sections caused by the changes in loading in per cent of the stress present before moving the load.

The decrease in the deformation in bars lying under the load but near the panel edge was more marked than that in bars parallel to them under the central part of the load. Taking into account the

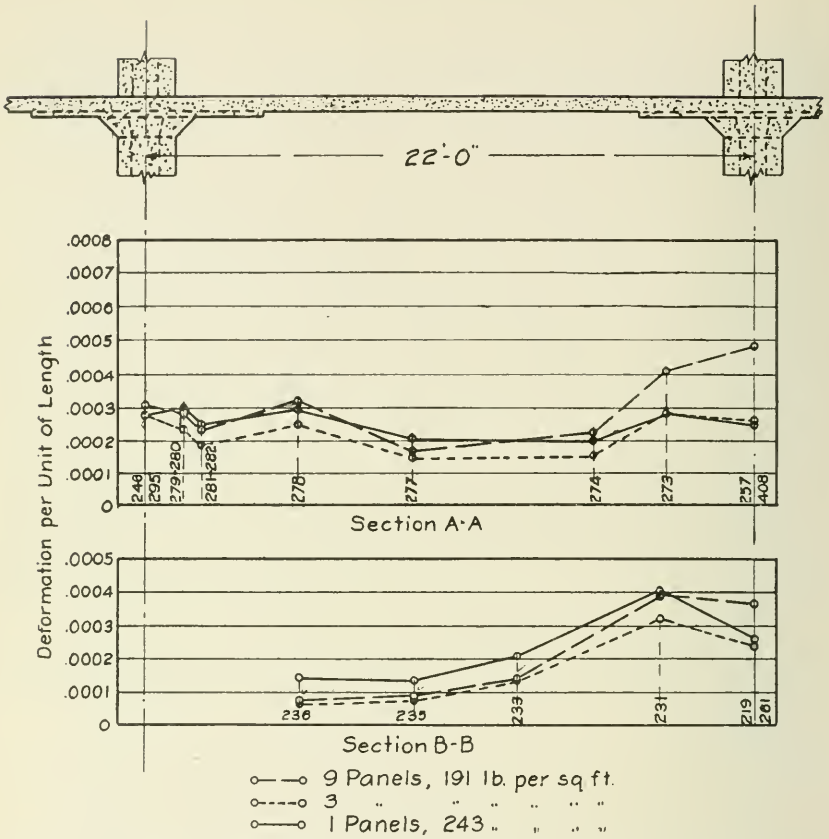


Fig. 12. North-south Distribution of Deformation in East-west Bars across Edge of Central Test Panel in Shredded Wheat Factory.

increase in the intensity of the load simultaneously with the removal of load from the two outer panels, leaving only the central panel loaded, it seems that there must have been a distribution of stress laterally to bars outside the loaded area (or assistance given by the development of stress in the reinforcement in the unloaded panels adjacent to the loaded panel). The effect seems to have disappeared at a distance of about one-quarter panel width from the edge of the loaded area.

For some reasons it might be expected that the change from the three-panel load to the one-panel load would cause an increase in deformations on gage lines which cross section E-E (see Fig. 10). However, the increase in deformation on section E-E (23 per cent)

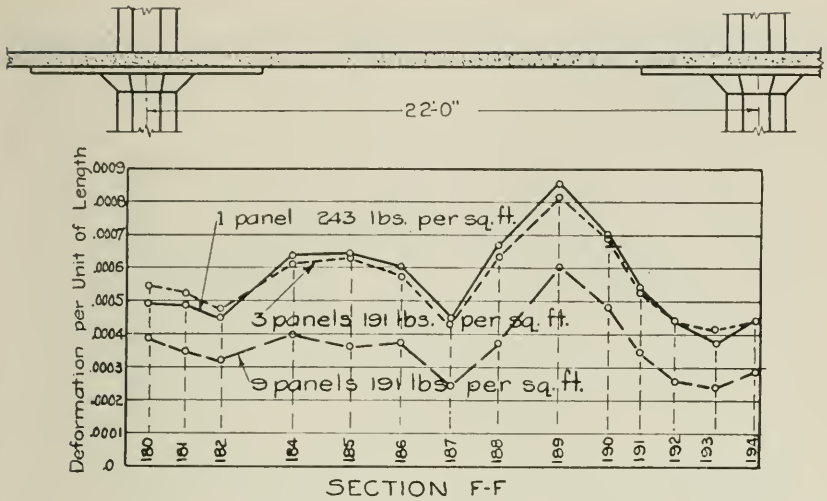


FIG. 13. NORTH-SOUTH DISTRIBUTION OF DEFORMATION IN EAST-WEST BARS ACROSS CENTER LINE OF CENTRAL TEST PANEL IN SHREDDED WHEAT FACTORY.

following the change from three panels at 191 lb. per sq. ft. to one panel at 243 lb. per sq. ft. was less than the increase in intensity of the load (27 per cent).

9. *Distribution of Stress and of Moments.*—In the design of the floor slab the reinforcing bars were distributed across the panel width in accordance with a method which the Corrugated Bar Company derived from the test made by Mr. Trelease on a small rubber slab.* To show the distribution of stress over the width of the slab, as measured in the test, the deformations developed in the reinforcing bars at the load of 191 lb. per sq. ft. over nine panels have been plotted in Fig. 12, 13, 14, and 15, using deformations as ordinates and distances at right angles to the direction of the length of bar as abscissas (width of slab). It will be seen that rather large differences were found in the deformations in bars which were close together. That these differences are not generally errors of observation is indicated by the fact that a set of check readings on a large number of gage lines gave stresses practically identical with those of previous observations at the same load.

Fig. 16 gives moment factors showing the distribution of the positive resisting moment across the width of the slab based upon the measured deformations in the reinforcing bars, and Fig. 17 gives similar

*See Proceedings National Association of Cement Users, vol. VIII., p. 218.

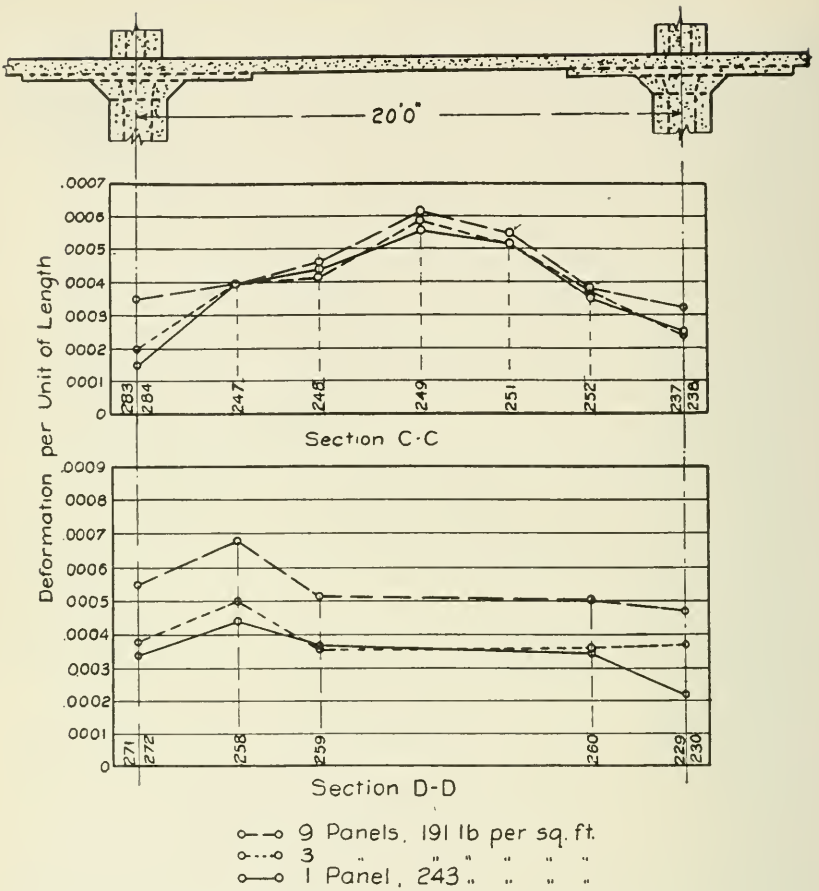


FIG. 14. EAST-WEST DISTRIBUTION OF DEFORMATION IN NORTH-SOUTH BARS ACROSS EDGE OF CENTRAL TEST PANEL IN SHREDDED WHEAT FACTORY.

moment factors for the negative resisting moment based upon the measured deformations in the steel. These moment factors represent the coefficients by which wl^2 must be multiplied to obtain the positive (or negative) resisting moment per unit of width of section developed by the stress found in the steel, w being the load per unit of area and l the panel length center to center of columns in the direction of the stress considered.

In making up these two diagrams the width of the slab was divided for convenience into several portions, and for each portion the average stress in the reinforcement was multiplied by the effective moment arm times the cross-sectional area of the reinforcement. The moments for the several portions were divided by the

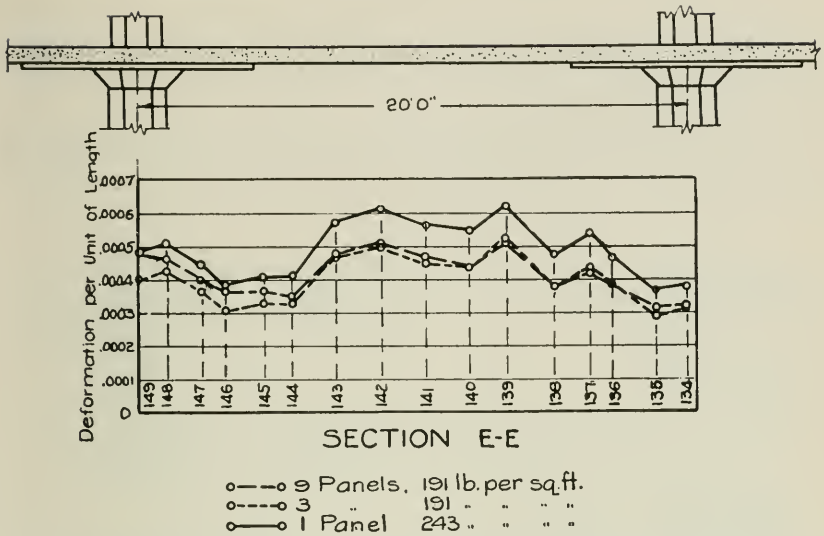


Fig. 15. East-west Distribution of Deformation in North-south Bars across Center Line of Central Test Panel in Shredded Wheat Factory.

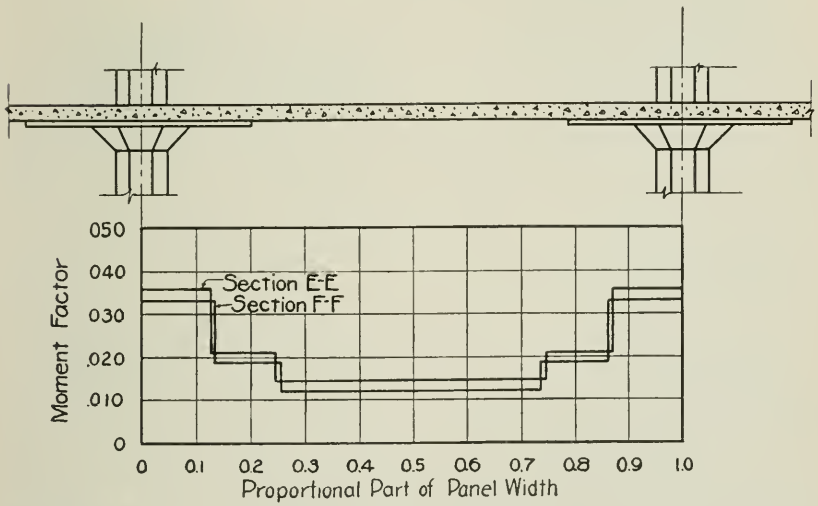


Fig. 16. Moment Factor Diagram for Sections of Positive Moment (Sections E-E and F-F) of Central Test Panel, Shredded Wheat Factory.

widths and by the quantity wl^2. Calculations were thus made for two sections (E-E and F-F) at positions of maximum positive moment and for four sections (A-A to D-D) at positions of negative moment. The stresses used were taken from Fig. 12, 13, 14, and 15 for the load

of 191 lb. per sq. ft. over nine panels in such a way as seemed best to represent the stresses over the entire section. In some cases a uniform stress represented the distribution as accurately as any symmetrical curve which could be used.

It will be noted that the distribution of the values of the moment factors in Fig. 16 and 17 is largely dependent upon the distribution

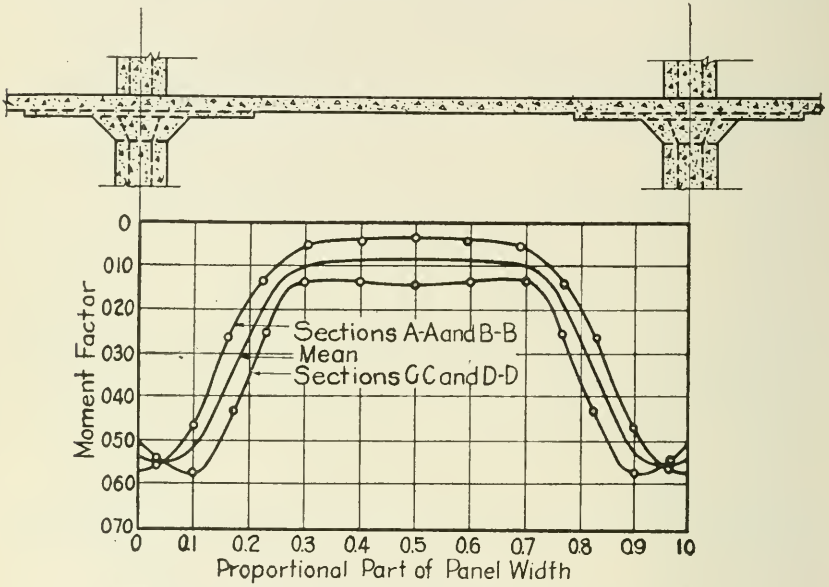


FIG. 17. MOMENT FACTOR DIAGRAM FOR SECTIONS OF NEGATIVE MOMENT (SECTIONS A-A TO D-D), SHREDDED WHEAT FACTORY.

of the reinforcing bars over the width of the section. If the bars had been grouped less closely over the column head there probably would have been less variation in the values of the moment factors.

By integration of the areas between the moment factor curve and the axis, the value for the positive resisting moment due to the measured stresses in the steel for a width equal to the width of the panel is found to be 0.021 Wl and that for the negative resisting moment 0.026 Wl, where W is the total load on the panel and l is panel length. These moments are the averages of the values obtained from the two curves for the positive moment and from the two curves for negative moment. These moments represent the value of the resisting moment developed by the steel for one direction only, as determined by average unit-deformations over 8-in. gage lengths. With the increased deformations at certain places when only three panels were loaded the average positive moment would be found to

be larger than the value given here. As the tensile resistance of the concrete may be expected to be considerable for beams or slabs having as low percentages of reinforcement as these and as low deformations as were found in the steel, the moments given above can not be taken to be the actual resisting moments developed in the slab.

10. *Stresses and Moments in Columns.*—In case not all of the panels are loaded, the bending moment developed in the slab at the

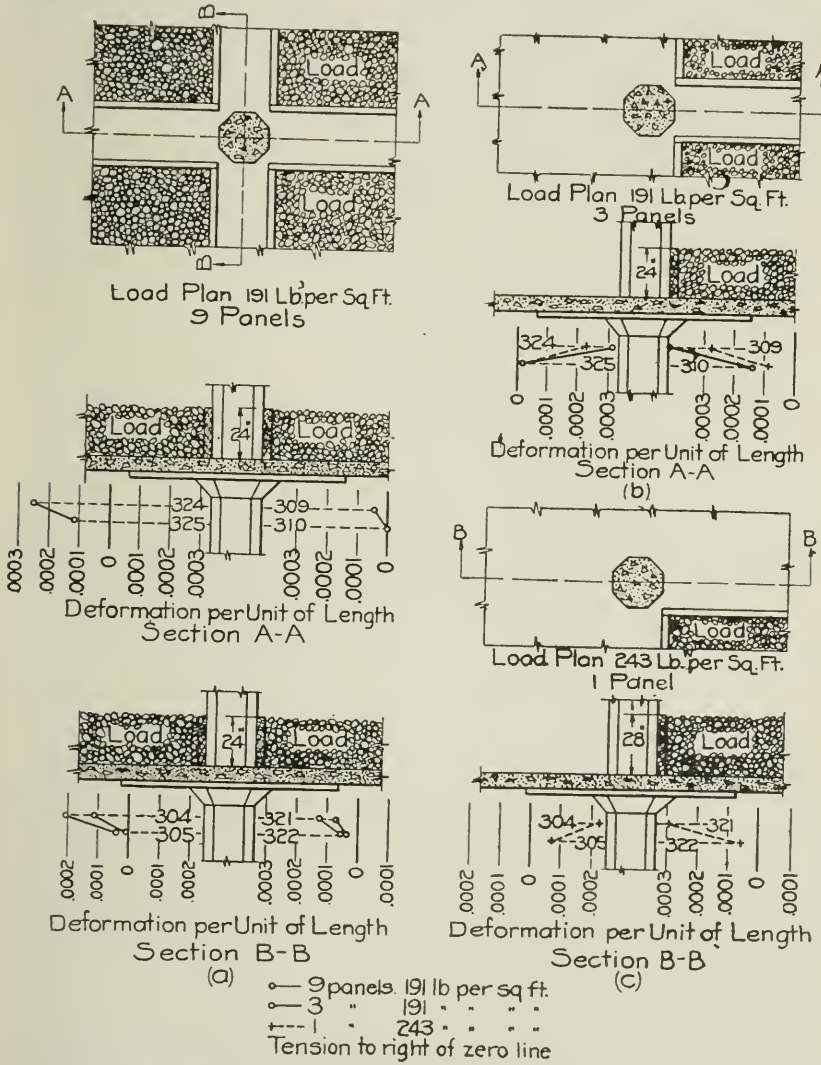


FIG. 18. DEFORMATIONS IN INTERIOR COLUMN NO. 68 OF SHREDDED WHEAT FACTORY.

edge of the loaded panels is resisted by a restraining moment taken by the columns at the edge of the loaded area and by the slab of the adjacent unloaded area. The division of this restraining moment into the three restraining moments, that taken by the column above the floor, that taken by the column below the floor, and that taken by the adjacent unloaded slab, is dependent upon the relative stiffness of these members (represented by the moments of inertia of the members and by the relative length of the members). As the modulus of elasticity of the concrete in the structure is not known and as the effect of the tensile strength and stiffness of the concrete is quite uncertain, it is not practicable to make a definite statement concerning the amount of bending stresses in the columns or the exact relative bending moments taken by the columns and by the unloaded floor. It may be interesting, however, to note the phenomena as they were observed (see Fig. 18).

Gage lines 309, 324, 304, and 321 were located on the four sides of a basement column just below the column capital. The deformations

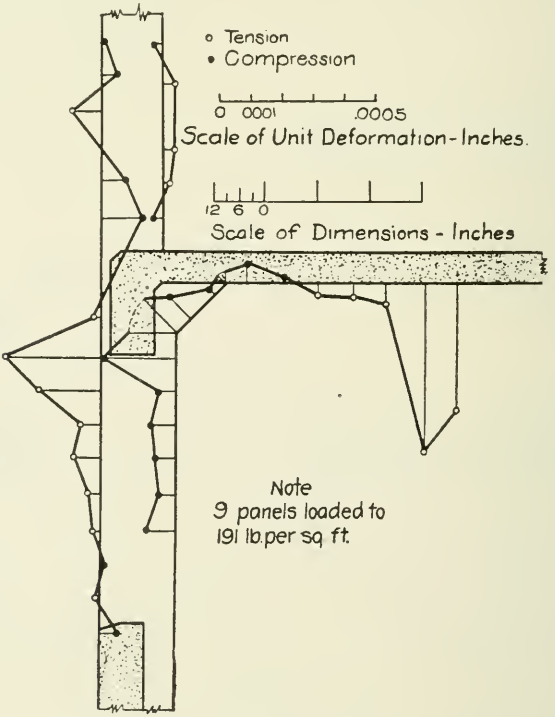


Fig. 19. Deformations in Wall Column No. 25 and Point of Zero Unit-deformation on Under Surface of Test Floor near Wall Column, Shredded Wheat Factory.

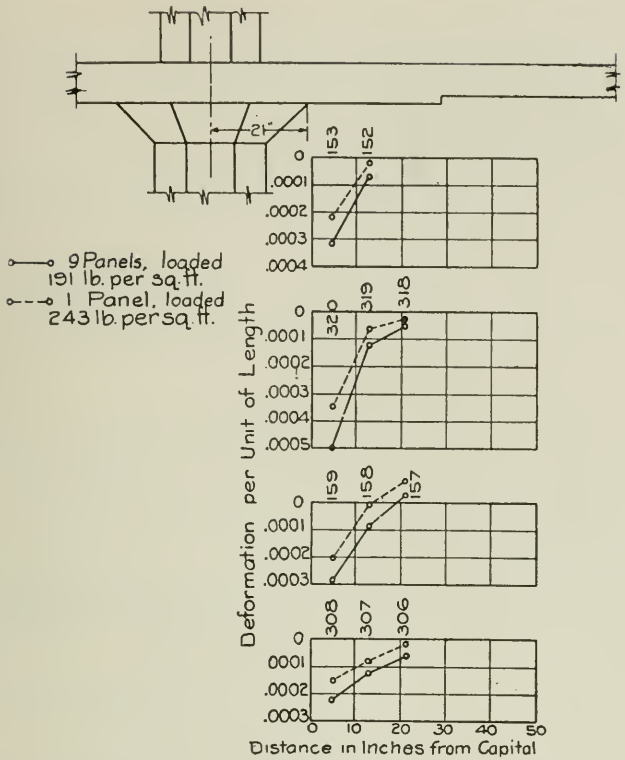


FIG. 20. POINTS OF ZERO UNIT-DEFORMATION ON UNDER SURFACE OF TEST FLOOR NEAR INTERIOR COLUMNS OF SHREDDED WHEAT FACTORY.

in this column show (see Fig. 18) that the partial loading of the slab developed a severe bending in the columns of the basement story. No measurements were taken on the columns of the first story.

11. *Point of Inflection.*—With a view of determining the position of the point of inflection, several series of measurements of deformations in the concrete on the under side of the slab were taken. The results are shown in Fig. 19, 20, and 21. Reference to Fig. 4 will show the location of gage lines. In Fig. 20 only one series of measurements is carried far enough to cover a change from compression to tension. The tendency of these curves to change their slope abruptly makes it impracticable to find the point of zero stress by producing the line till it crosses the axis. Some floor tests have shown the position of zero-deformation of the under surface of the slab to be closer to the column than that for the upper surface, while other tests are quite the opposite. No reason is known for such variation in

results. It seems unwise to base conclusions as to the positions of the point of inflection upon data obtained on the under surface only. The curves in Fig. 20 and 21 show a tendency for the point of zero-stress to move toward the column when the load is changed from test area No. 1 to test area No. 2.

12. *Lintel Beams.*—During the progress of the tests, minute cracks were found in the concrete in various places in the structure,

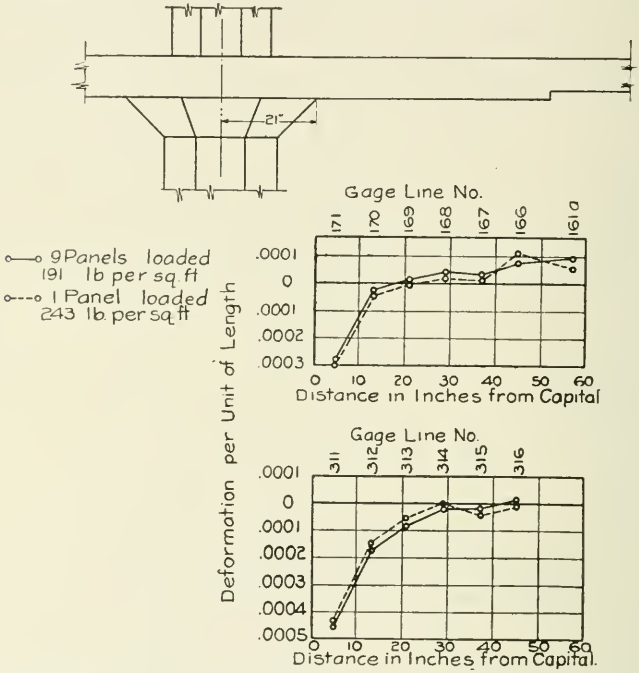


FIG. 21. POINTS OF ZERO UNIT-DEFORMATION ON UNDER SURFACE OF TEST FLOOR NEAR INTERIOR COLUMNS OF SHREDDED WHEAT FACTORY.

and the presence of these cracks confirmed the results of the strain gage measurements. The most important indications of this kind were cracks which were found on the interior side of the lintel beams near their ends. The cracks did not appear on the exterior of these beams and except for this, the cracks, by their position and direction, resembled the diagonal tension cracks ordinarily found in beams subjected to high shearing stress. The web reinforcement in the lintel beams consisted of both inclined members and vertical members which were wrapped about the horizontal bars. The presence of cracks on the interior surface and the absence of cracks on the exterior surface may indicate that this phenomenon was due to torsion. The

slab and the lintel beams were built monolithically, and the bending moment developed in the slab at its end by the load on the wall panel would produce torsion in the beam. The formation of the cracks shows that stresses exist in the lintel beam which are not ordinarily considered. The effect of the bending of the slab on the lintel beam to which it is attached should be considered.

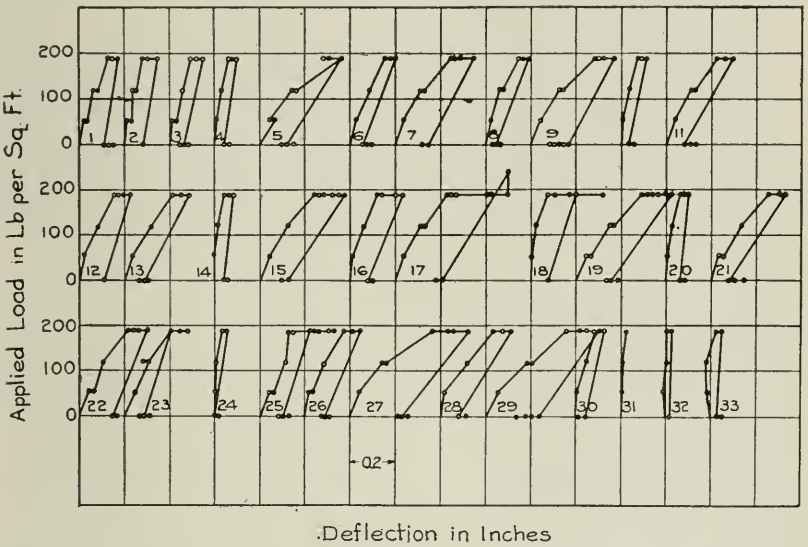


Fig. 22. Load-deflection Diagrams for Test Floor of Shredded Wheat Factory.

13. *Deflections.*—Little use has been made in this report of the deflections which were measured in the tests, but since in many cities the building regulations make certain requirements for deflection under load, it is believed that the presentation of deflection data may serve a useful purpose. In Fig. 22 are given diagrams in which the ordinates represent the load in pounds per square foot, and the abscissas represent the deflection in inches. Accompanying these diagrams are the numbers of the deflection points, the locations of which are shown in Fig. 6.

14. *Summary of Results.*—The principal results brought out in the foregoing discussion are as follows:

1 There was a considerable increase in the deformations in both steel and concrete during the fifty-six hours of retention of the load.

2 Upon the removal of the load from the six panels, there was an

increase in the deformations across the center line of the three panels which remained loaded (section of positive bending moment). There was a decrease in the deformations across the side edges of the area remaining loaded (sections of negative bending moment). There was also a decrease in the deformations in the direction of the side edge of the loaded area in those bars under the load which lay near this edge.

3 The positive bending moment for a panel width corresponding to the deformations measured in the reinforcing bars in one direction was found to average 0.021 Wl for a panel width; the negative bending moment found in the same way was 0.026 Wl. These values may be of interest in comparing the results of this test with the results of other tests. It must be understood, however, that these do not represent values of the bending moment coefficients which should be used in design.

4 The distribution of stresses in the reinforcement across panel edges and across panel center lines was substantially uniform, taking averages of the several sections. The variation from uniform distribution of the moment factors for these sections corresponds closely to variation in the slab thickness and in the distribution of the reinforcement.

5 The measurements show that a large bending moment was developed in the basement columns under a partial loading of the slab.

6 In the lintel beams cracks were found on the interior side near the ends which probably were caused by the twisting action produced by bending moment developed in the slab at its edge by the load on the wall panel.

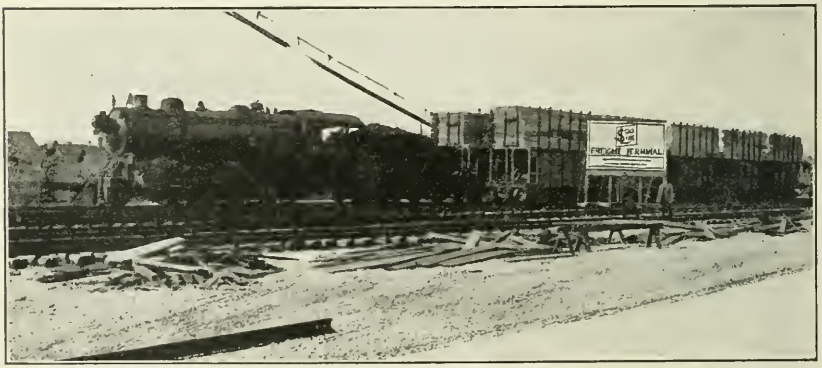


FIG. 23. VIEW OF TEST LOAD, SOO TERMINAL STRUCTURE.

III. The Test of the Flat Slab of the Soo Line Freight Terminal.

15. *The Structure.*—The Soo Line Freight Terminal of Chicago, comprises a freight yard and a central receiving and distributing building for the Central Terminal Railway Company of the Soo Line. The freight yard is elevated above the ground so that the intersecting streets cross underneath the terminal without a depression of their grades. Both across these streets and between them for a distance of more than half a mile the yard is on a reinforced concrete slab structure which is supported on columns. The space under the slab not occupied by streets is arranged to be utilized for storage purposes.

A more complete general description of the structure is given in Engineering Record, August 16, 1913 (Vol. 68, No. 7), in

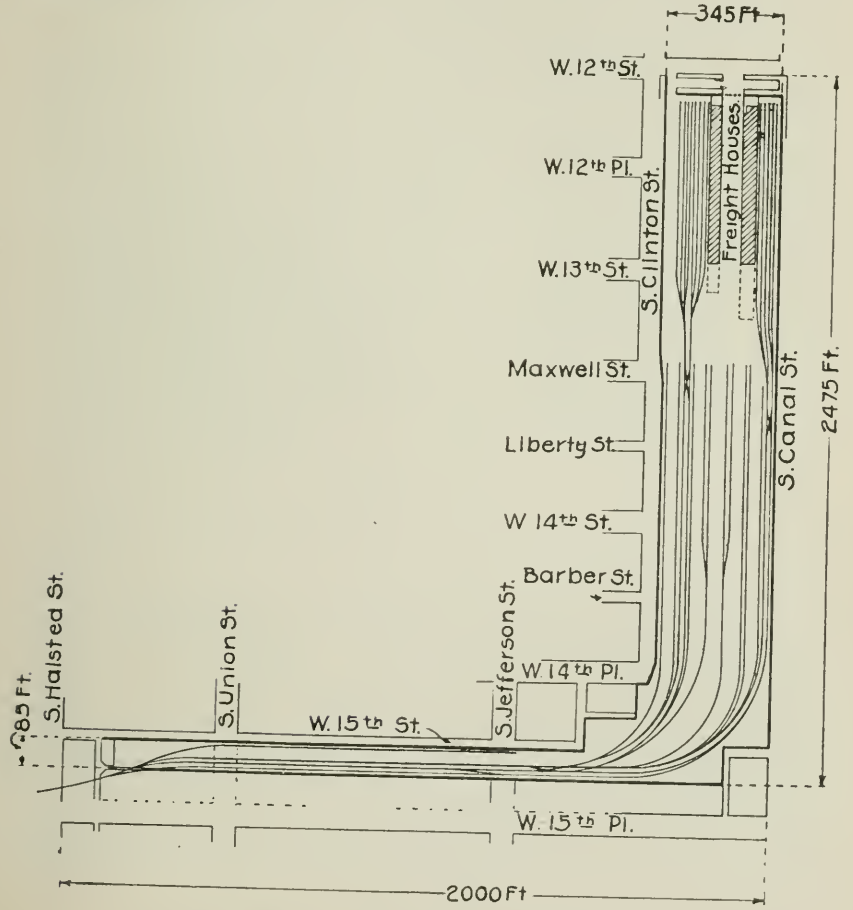


Fig. 24. General Plan of Soo Terminal Structure.

Engineering News, August 21, 1913 (Vol. 70, No. 8), and in Railway Age Gazette, August 22, 1913.

The general plan of the structure is shown in Fig. 24 and the design of a typical panel is shown in Fig. 25. For the test a four panel area was chosen which represents the typical flat slab of this structure, which is an unusually heavy one. The test area was located between West Fourteenth Street and Barber Street,

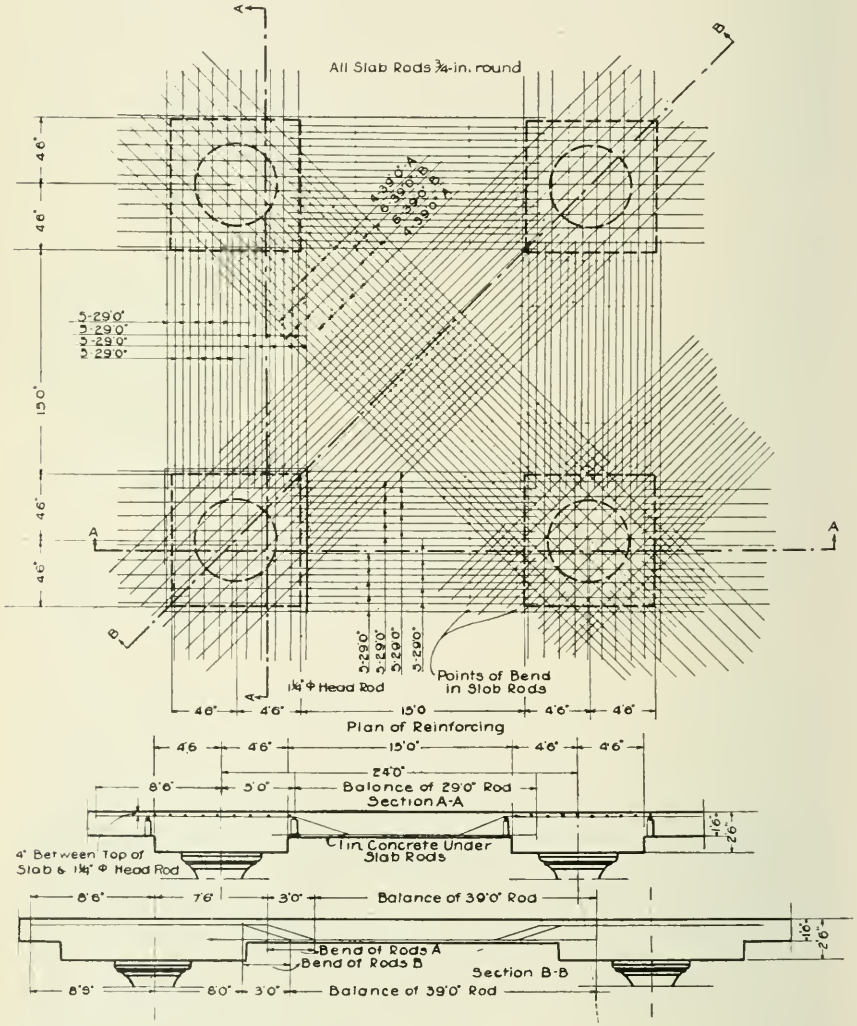


Fig. 25. Dimensions and Reinforcement of Typical Panel of Soo Terminal Test Floor.

Chicago. The test panels are 24 ft. square. The floor slab in the central portion of the panel is nominally 18 in. thick. Its nominal thickness is 30 in. throughout the area of a 9-ft. square surrounding each column, this portion of the slab being referred to as the depressed head. The necessary slope for drainage purposes was obtained by increasing the thickness of the slab to form alternate ridges and valleys. The thickness of the slab along the ridges was in some places as much as 2 in. greater than the nominal thickness. The additional thickness did not affect the position of the reinforcing bars. In regions of negative moment it is possible that an additional thickness of concrete affected the amount of the contribution of the tensile strength of the concrete to the resisting moment. In regions of positive moment an additional depth would increase the moment arm of the reinforcement stress.

The slab contains four-way reinforcement, as shown in Fig. 25. In slabs of ordinary thickness and span it has been a common practice to depend upon the weight of the reinforcing bars to pull the bars down into place in the center of the span, bars of one-half inch diameter or less being used. In a floor built for very heavy loading as in the case of the terminal slab under consideration, it is necessary to bend the bars to a definite shape. This bending was done after the steel had been placed in position. The average distance from the compression surface of the slab to the center of gravity of the reinforcement at the edge of the capital of column N63 was 26¼ in. The distance to the center of gravity of the reinforcement midway between columns varied somewhat due to the presence of the ridges previously referred to. The greatest and least depths found were 20¾ in. and 16¾ in., the least being fully as great as the depth which would be expected from the nominal thickness given above.

The columns are cylindrical, 32 in. in diameter, and end above in bell-shaped heads or capitals 5 ft. 6 in. in diameter at the top. The columns are reinforced with a spiral of ½-in. round steel, having a mean diameter of 28½ in. and a pitch of 2 in. In addition, 20 1⅛-in. diameter longitudinal reinforcing bars are placed just within the spiral and wired fast to it. The spiral is carried from a point 16 in. below the level of the basement floor to a point well into the bottom of the depressed head. Bearing and anchorage of the longitudinal rods are afforded by right angle hooks 8 in. long at the bottom and at the top. For the portion of the structure tested, the length of the columns from the top of the basement floor to the bottom of the depressed head averages about 12 ft. 10 in.

The footings are of reinforced concrete 14 ft. square with a concrete pier 4 ft. square extending from the top of the footing upward to a point 18 in. below the top of the basement floor.

16. *The Test.*—The concreting of the panels on which the test was made was done on June 12, 1913, the building being 112 days old at the time of the test, and the four-panel test load was applied on October 2, 1913. Ore cars having bins built on top to an average height of 6 ft. 2¼ in. above the regular height of the car were filled with broken stone and run upon the test area to furnish the load. Fig. 23 shows the loaded cars in position for the five-panel test (Load C).

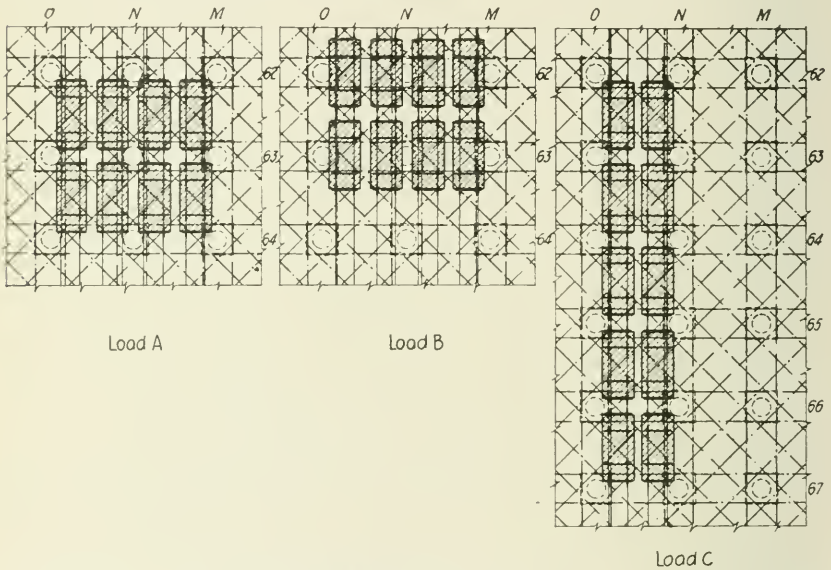


Fig. 26. Plan Showing Positions of Cars for Loads A, B, and C on Soo Terminal Test Floor.

Portions of the ballasted and of the uncovered slab are seen in the foreground. The cars held on an average 1740 cu. ft. each., and the average weight including the car was 200,800 lb. The intensity of this load is considerably greater than the Terminal Railway Company expects the slab to be subjected to in service. Table 3 gives data of the loaded cars. It may be said in anticipation of results reported hereinafter that this load caused stresses in the slab so small that their interpretation is difficult.

The three different positions of load, designated as A, B, and C, are indicated in Fig. 26. Load A was designed to give the representative stresses around a central column approaching the condition of

all panels loaded. The movement into position B was designed to determine the effect of a change of a half panel length in the position of the load. Load C was designed to find the effect of loading adjacent tracks lying within a row of panels.

The preparation for the test covered the time from Sept. 17 to Sept. 29, inclusive. As showing the extent of the preparations, it may be stated that during this time from eight to twelve laborers were employed in cutting or drilling into the concrete to expose the steel

TABLE 3.

WEIGHTS OF LOADING MATERIAL AND DIMENSIONS OF CARS.

Average weight of cars alone	32,000 lb.
Average weight of stone in cars normally	74,600 lb.
Average additional height of bin	6 ft. 2¼ in.
Average length of bin inside	18 ft. 7½ in.
Average width of bin inside	8 ft. 5¼ in.
Average additional capacity	970 cu. ft.
Average unit weight of stone used	97 lb. per cu. ft.
Average additional weight of stone per car	94,200 lb.
Average total weight of car and stone	200,800 lb.
Number of cars loaded	10
Maximum variation of total weight of any car from average	400 lb.
Length of car center to center of coupling	24 ft.
Center of coupling to center of first wheel	2 ft.
Center of first wheel to center of second wheel	5 ft.

and to place gage lines for measurements on the concrete. The test occupied the time from Sept. 30, 1913 to Oct. 8, 1913. Observations were taken on 816 gage lines, of which 338 were on the upper surface and 478 on the lower surface of the slab. Locations of slab gage lines are shown in Fig. 27, and 28 and locations of deflection points in Fig. 30. Locations of gage lines on columns may be determined by a study of Fig. 37 to 41 inclusive. In the carrying out of the test considerable delay was experienced because of rains which flooded the gage holes and necessitated a large amount of tedious work in draining and cleaning.

In order to lessen the chances of error in readings, two complete sets of strain gage readings were taken before applying a load, and also two complete sets under Load A (see Fig. 26). The load was then moved to the position indicated as load B, Fig. 26, and two series of readings were taken in only the significant positions. The cars were then removed from the test area and a complete series of no-load readings was taken. With Load C in position readings were taken at the most important positions.

17. *Settlement and Deflections.*—Levels were taken on the slab at several of the columns and at deflection points before loading,

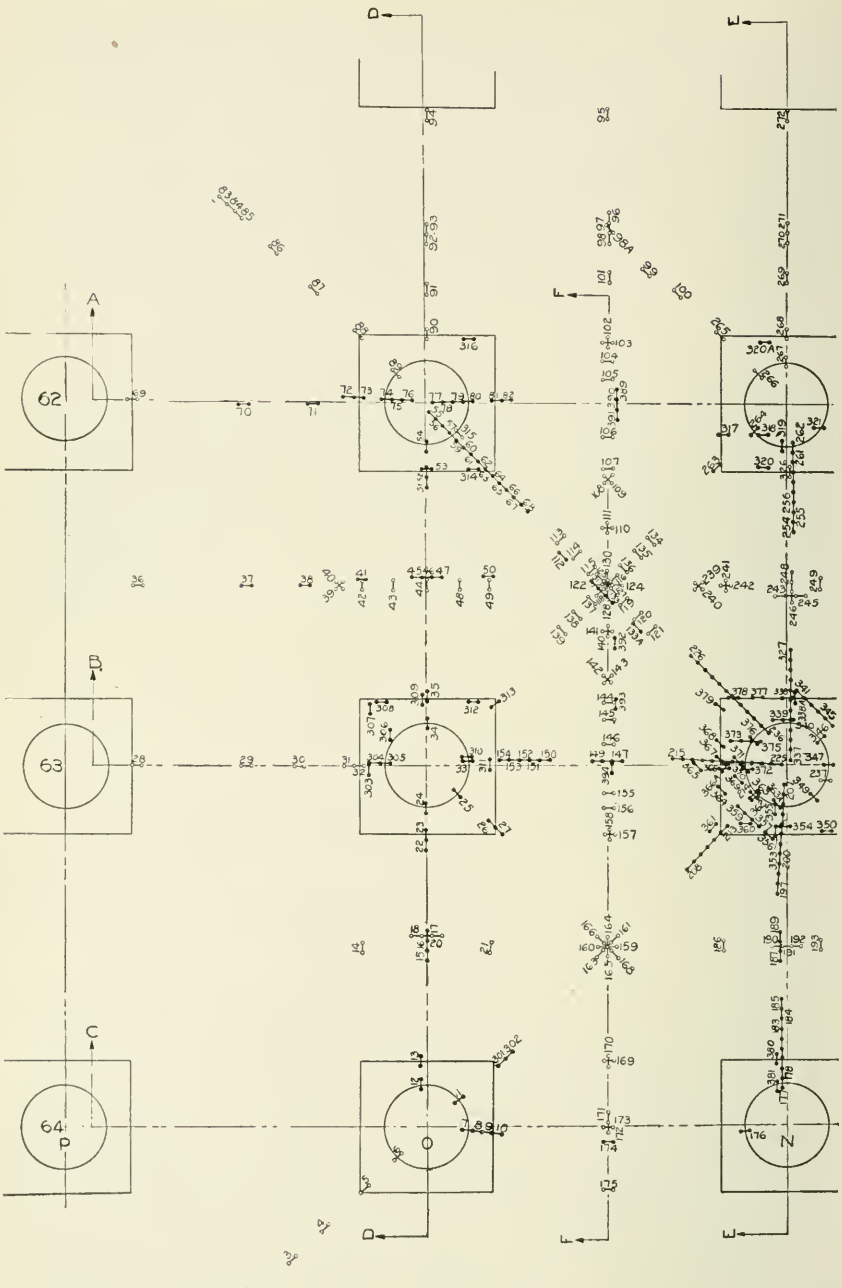
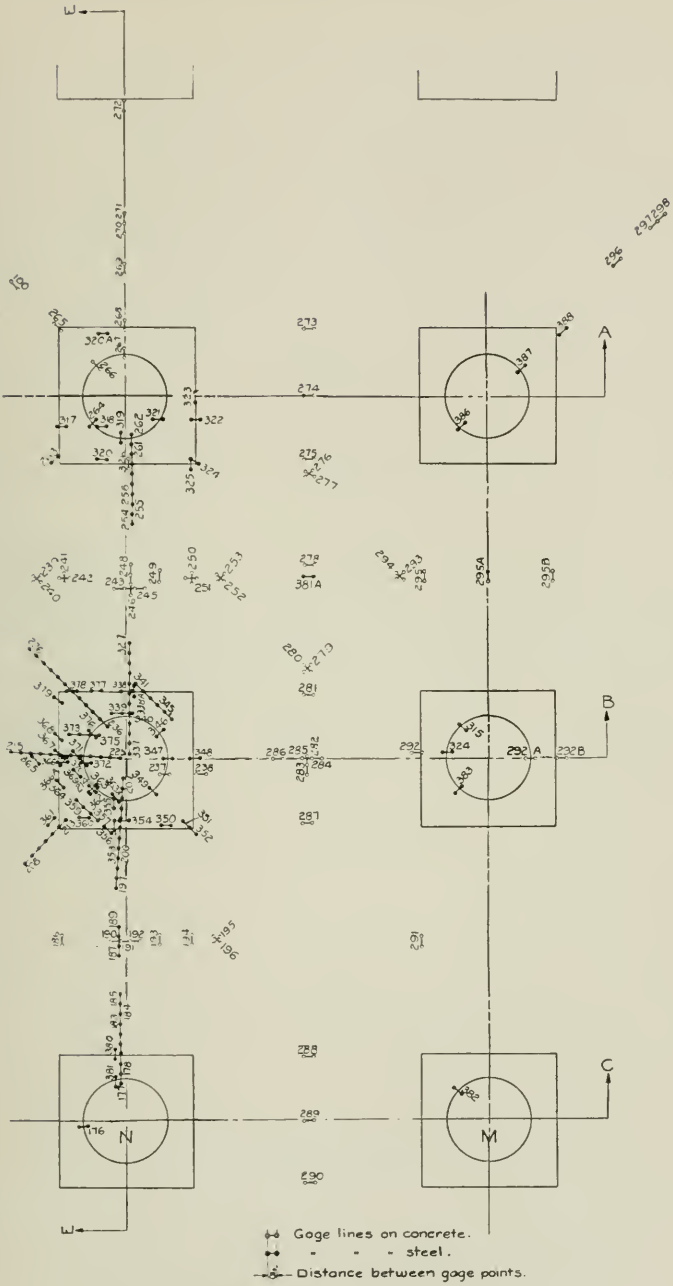


FIG. 27. LOCATION OF GAGE LINES



ON UPPER SURFACE OF SCO TERMINAL TEST FLOOR.

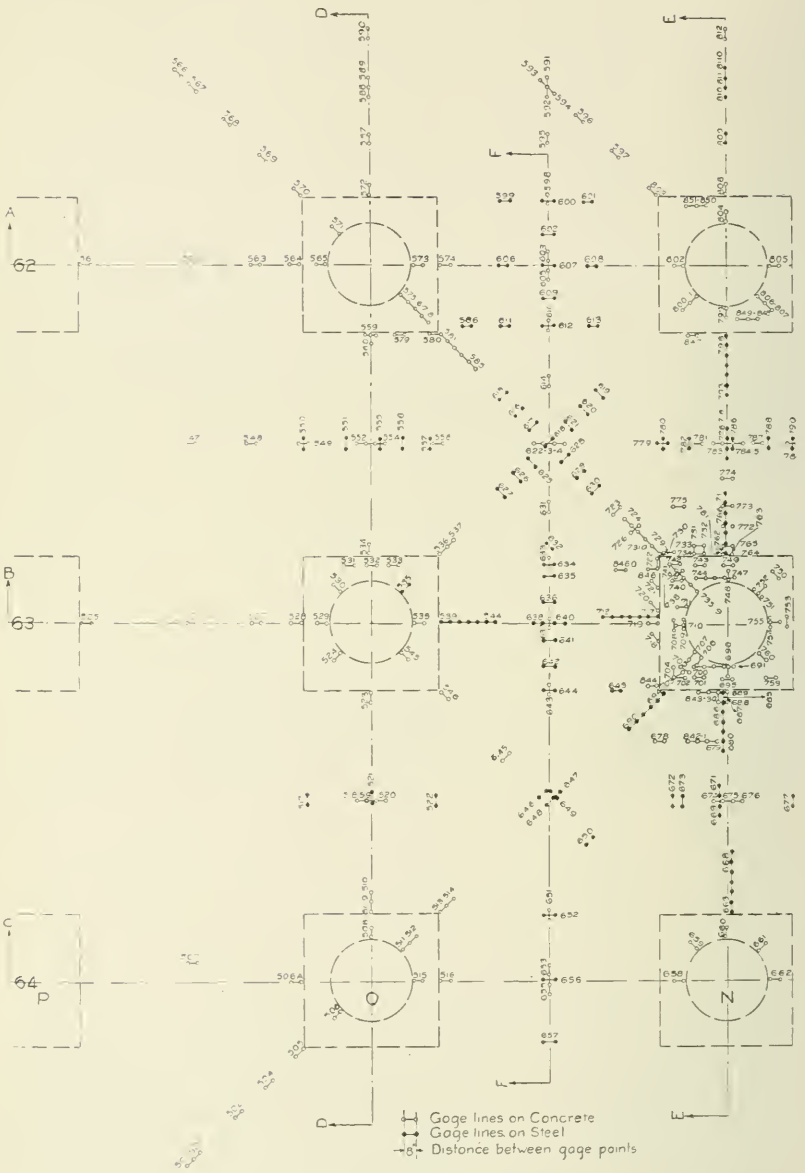
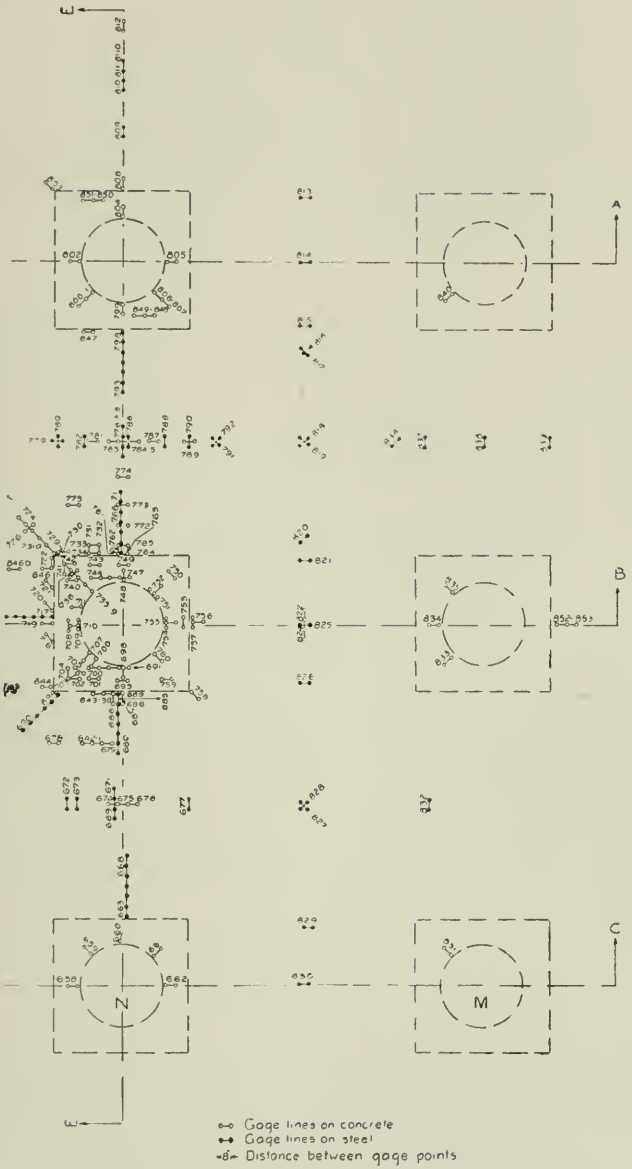


FIG. 28. LOCATION OF GAGE LINES



ON UNDER SURFACE OF SOO TERMINAL TEST FLOOR.

with load on, and with load removed. The levels were taken by engineers connected with the construction and the work is known to have been carefully done, but the results in some respects seem conflicting and it is quite possible that they do not show the actual changes which took place. It is possible that the bench mark was affected by the movement of the structure under load. As the measurement of deflections of slabs was taken from the floor below, uncertainties about the settlement of the structure will also affect the observations on deflections.

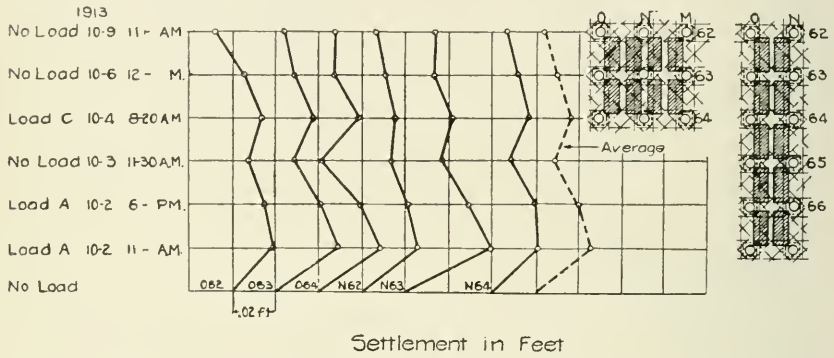


Fig. 29. Settlement of Columns Supporting Test Floor of Soo Terminal Structure.

The data of settlement and deflections are given in Fig. 29 and 30. The maximum settlement indicated by the level notes was 0.04 feet which appears to have occurred at Column N63, the central column of the four-panel test area. The almost complete recovery from this settlement on removal of the load and the large amount of recovery during the time from 11:00 a. m. to 6:00 p. m., October 2, while the full load was in place, seem improbable and hence raise serious question as to whether as much settlement as that indicated above was present at any time. Besides, the maximum measured deflection of 0.14 in. is so small as to make this value also seem not to represent the actual deflection. However the settlement is not large for buildings on Chicago soil. There are many inconsistencies between the deflections and settlements observed at different places and under different loads, and the uncertainty of the conditions makes it futile to attempt to account for the recovery from settlement or otherwise to interpret the data. However, Fig. 29 and 30 are given for the purpose of record. It is evident that even slight settlement under load will modify the distribution and

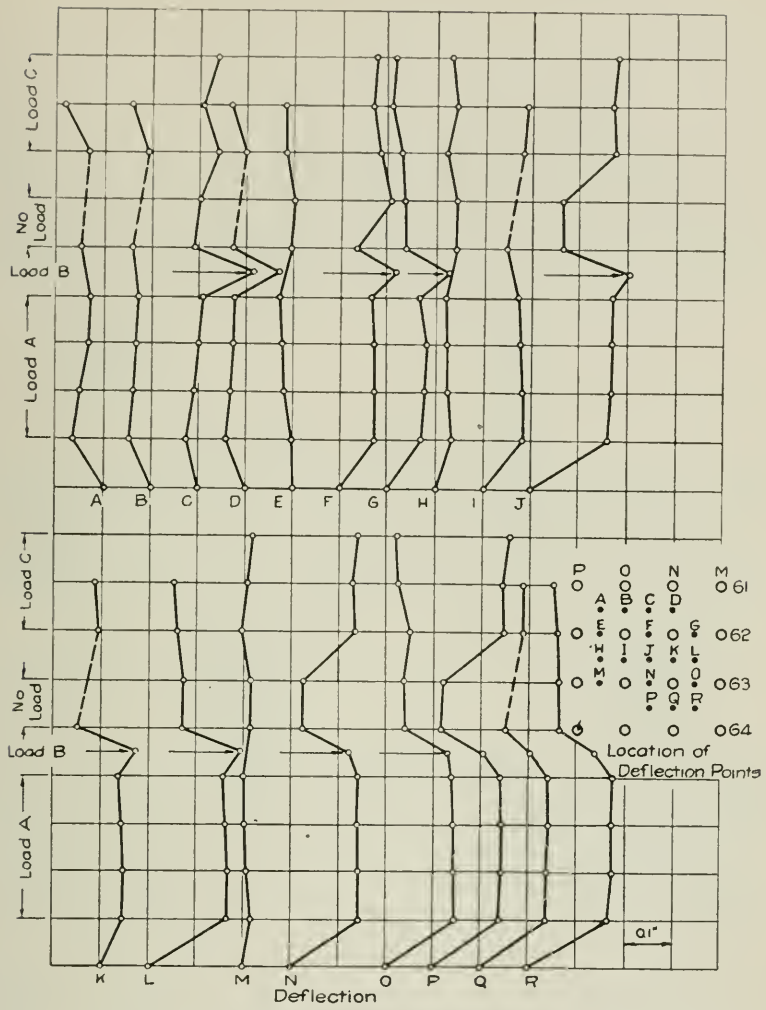


FIG. 30. DEFLECTION OF TEST FLOOR AT POINTS MIDWAY BETWEEN COLUMNS OF SOO TERMINAL STRUCTURE.

the amount of the stresses in the structure. The reference to settlement is given in the discussion as an aid in interpreting the observed phenomena and not in any way as having a bearing on the stability of the structure.

18. *Deformation in the Slab.*—The diagrams in Fig. 31 to 34 give the results of measurements taken to determine the distribution of deformations in the slab along the edges of panels, the measurements being taken in the direction of these edges, that is, along or parallel

to the sections shown in these figures. The diagrams in Fig. 35 and 36 give the results of measurements taken to determine the distribution of deformations across the center line of panels (section F-F) on the top and on the bottom of the slab. In this case the measurements were taken normal to the sections shown.

The deformations measured in the slab for the several loads were very small, in general, much smaller than were anticipated. As a result the data are not such as to throw much light on several of the questions on which information was sought, as for example, the effect

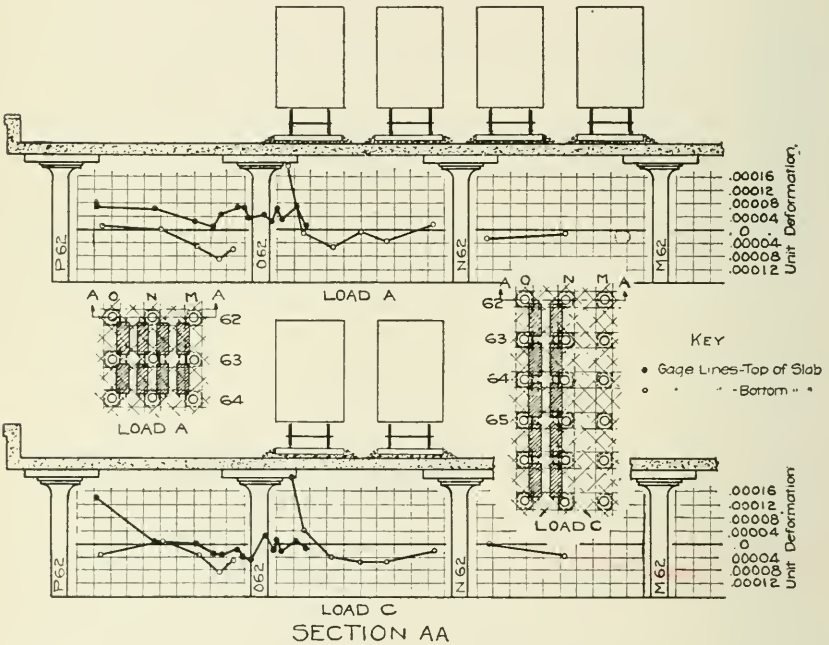


Fig. 31. Deformation along Section A-A (Normal to Tracks) of Soo Terminal Test Floor.

of shifting the load a short distance. In many cases the changes in length were smaller than the possible errors of observation, and such results are therefore meaningless. Besides, with the low stresses found, the tensile strength of the concrete must have played an important but uncertain part in the resistances developed. To complicate the matter further, unequal settlement of the footings evidently modified the action of the structure. Altogether, it may be said that on account of the smallness of the deformations and complications with the tensile resistance of concrete and the settlement of the foot-

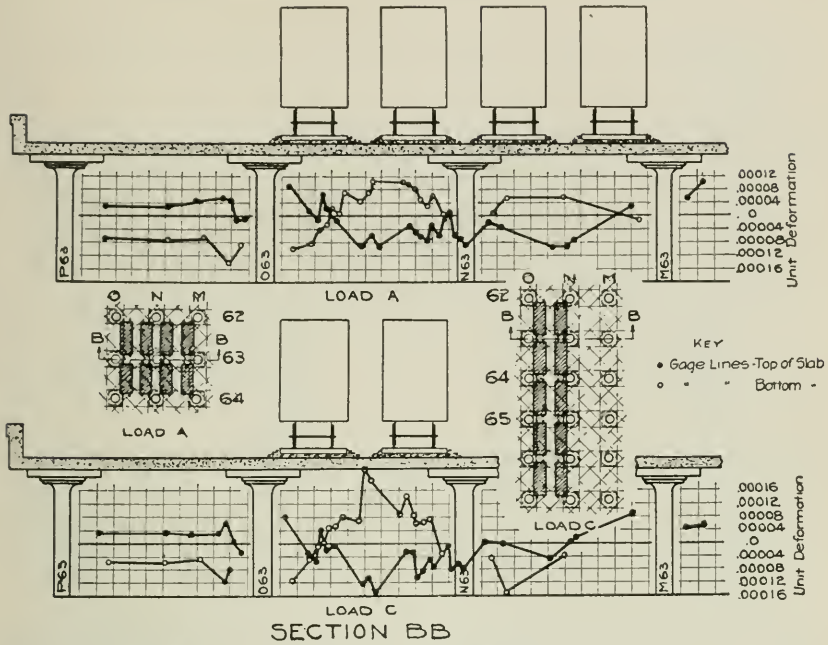


FIG. 32. DEFORMATION ALONG SECTION B-B (NORMAL TO TRACKS) OF SOO TERMINAL TEST FLOOR.

ings little conclusive information was obtained on the main questions connected with the action of the slab, except that the smallness of the stresses developed may be considered as important in judging of the action of such flat slab structures. In some respects the observations give indications of the way in which the stresses are distributed, but frequently they are so masked by uncertainties that comparison cannot be made with any degree of confidence. However, comment will be made as best it can.

Some of the difficulties may be seen from the following examples. For the negative bending moment around the central column (column N63, Fig. 32), with four panels loaded, the compressive stresses in the concrete due to both dead and live loads would not be expected, from calculations made according to current practice, to exceed 500 lb. per sq. in. If the concrete on the tension side remained intact, the tensile stresses in the concrete in this region would be less than this amount and the stresses in the reinforcement would be correspondingly low. As no cracks were observed in these positions and as the observed tensile deformations here were very small, it seems probable that the tensile strength of the concrete was sufficient

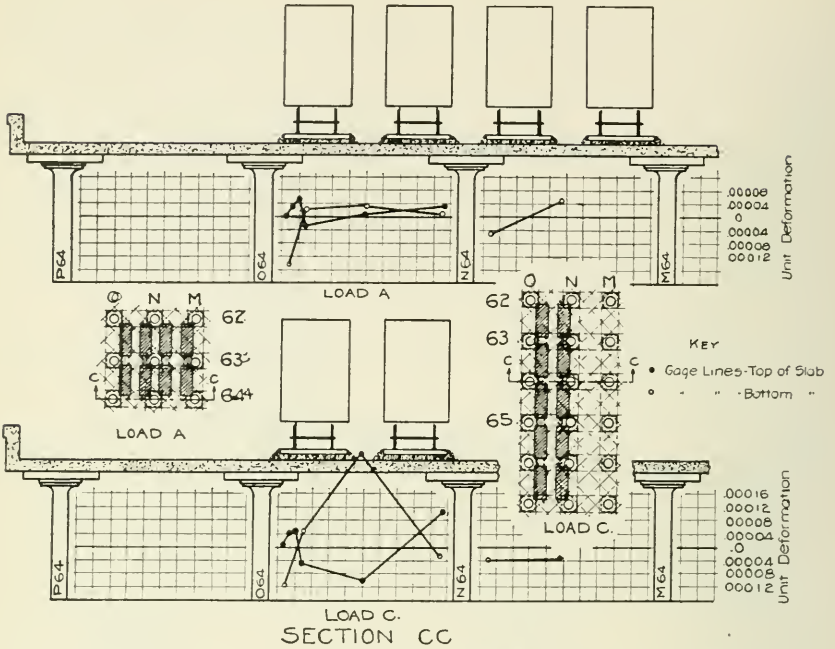


FIG. 33. DEFORMATION ALONG SECTION C-C (NORMAL TO TRACKS) OF SOO TERMINAL TEST FLOOR.

to prevent cracking in regions of negative moment even without relief of the stresses here by settlement of the column. The differences in depth of embedment of the reinforcing bars in the different bands also added to the uncertainties. In view of these difficulties, no quantitative values of the negative bending moment developed in the slab around the central column can be given. Again, for the four-panel loading, the highest deformation measured in any of the reinforcing bars of the bands at the central column corresponded to 2700 lbs. per sq. in. tension in the bar and to, say, 270 lb. per sq. in. tension in the concrete on the upper surface of the slab and the range was from this value to a small compressive deformation. It is evident that with such stresses as these it would be of little value to try to make comparison of stresses in the different bands or in different bars of the same band.

The deformations in the reinforcing bars of the rectangular bands at the bottom of the slab half way from the central column to the adjoining column were larger than those in the top of the slab near the central column (see Fig. 32). The settlement of columns may have had some influence on this. Even here the highest deformation

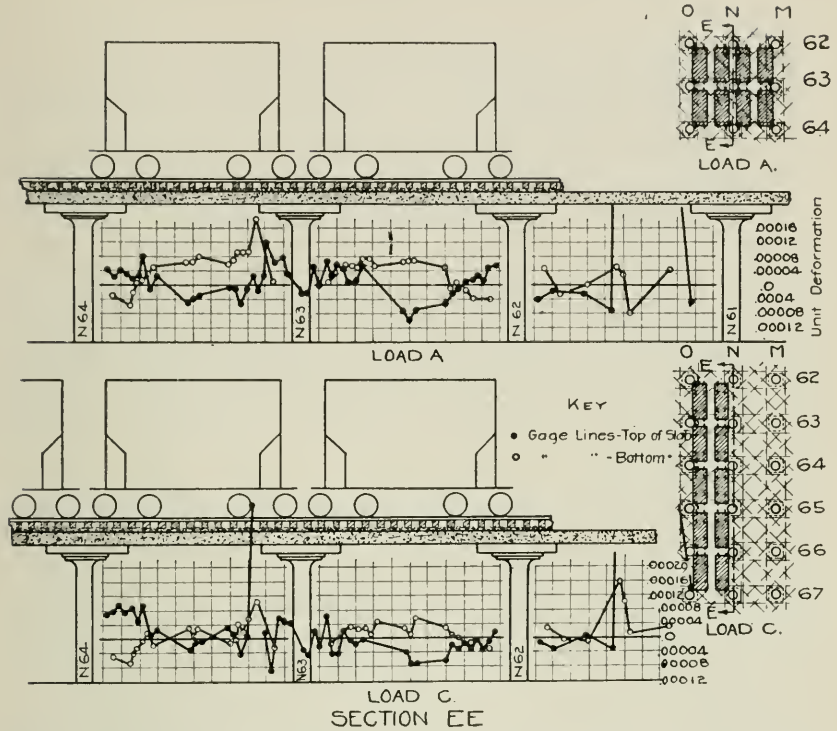


FIG. 34. DEFORMATION ALONG SECTION E-E (PARALLEL TO TRACKS) OF SOO TERMINAL TEST FLOOR.

measured in any bar under load A corresponds to a stress of less than 4000 lb. per sq. in., and most of the bars show values much smaller than this.

In the four-panel loading (see plan of load A, Fig. 26) the bars in a band of reinforcement which has load on both sides of it (as the rectangular band from Column N62 to N63 and on to N64, and also the rectangular band from Column M63 to N63 and on to O63) may be expected to take their full share of the stresses due to the load. In bands at the edge of the loaded area (as in the band extending from Column O62 to O63 to O64) the bars at the edge of the band which lie within the loaded area will be stressed higher than the bars at the other edge of the same band which lie outside the loaded area, but the latter bars will carry a considerable proportion of stress, and the stresses developed in the former bars will be materially less than if the latter bars were not present. This condition will need to be taken

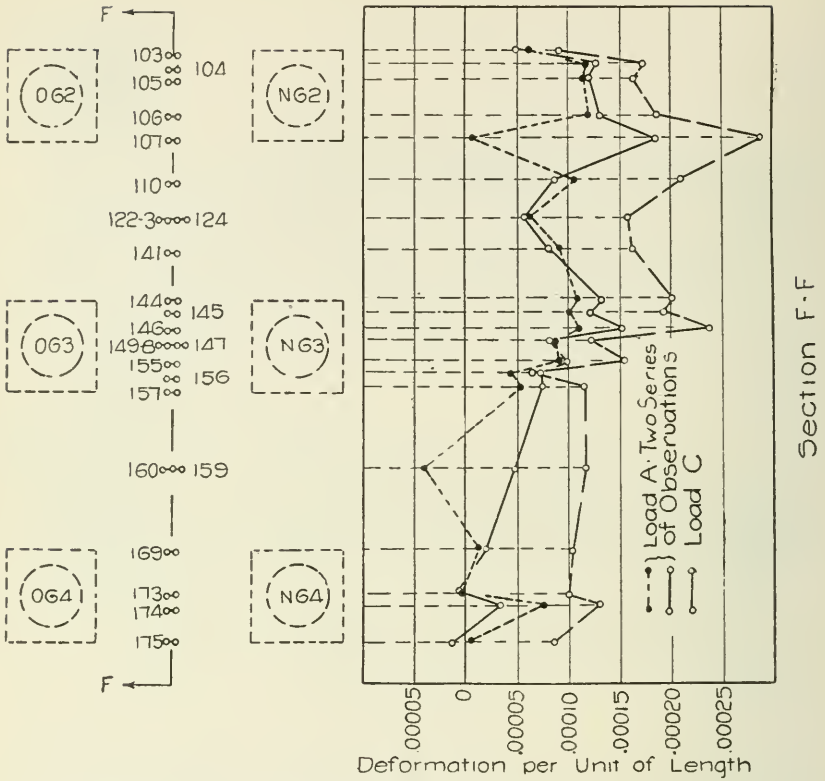


Fig. 35. Effect of Change from Load A to Load C on Deformation in Gage Lines Crossing Section F-F on Top of Soo Terminal Test Floor.

into consideration in comparing the deformations found when the load is removed from a panel but is left on the adjoining panel.

In the change from a four-panel loading (load A) to a load on a row of panels (load C), besides the condition just referred to, a change may be expected in the stresses in the bands which run across the row of loaded panels. Due to the bending of any column under the new arrangement of load (like Column N63, Fig. 32, which in the four-panel loading was surrounded by a symmetrical load) and with the resulting change in slope at the top of the column, and due also to whatever reverse bending occurs in the slab outside the loaded area, it may be expected that for a band such as the one running from Column N63 to O63 the stresses at Column N63 (due to negative bending moment) would be less for the row of panels loaded than it

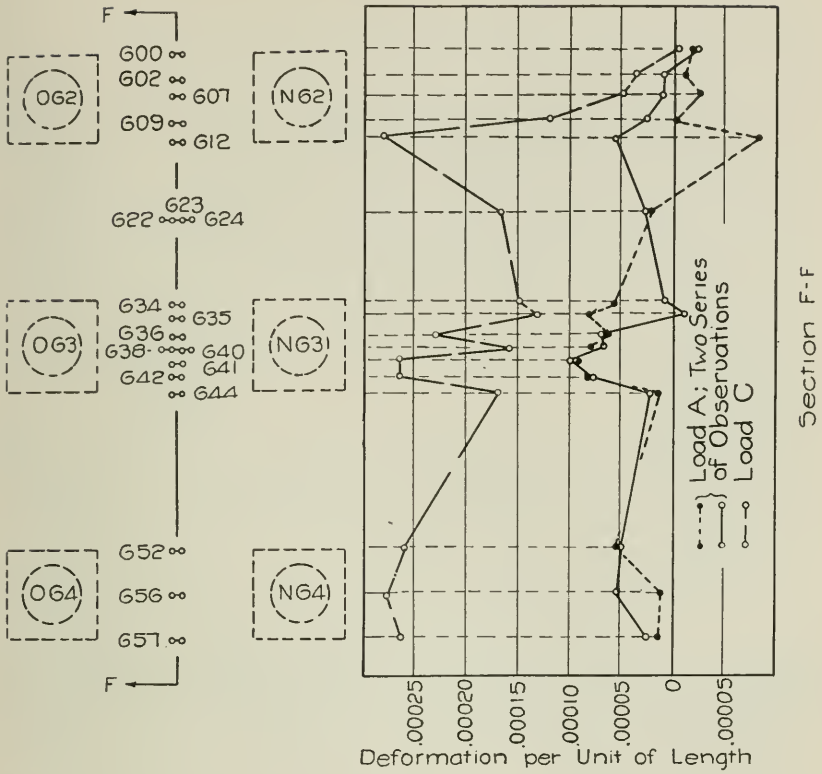


FIG. 36. EFFECT OF CHANGE FROM LOAD A TO LOAD C ON DEFORMATION IN GAGE LINES CROSSING SECTION F-F ON BOTTOM OF SOO TERMINAL TEST FLOOR.

had been for the four-panel load, and that at points half way between Columns N63 and O63 the stresses (due to positive bending moment) would show an increase over those for the four-panel load. Section C-C, Fig. 33, shows the effect of loading a row of panels and of extending the load entirely across a section instead of loading on only one side of the section. This effect is seen in the considerable increase in deformation midway between Columns N64 and O64 (see also Fig. 35 and 36.)

When the loading was changed from load A to load C the measurements at Column N63 showed very little change in the stresses in the bars of the band which extends from Column N63 to O63 (see Fig. 32). At a point half way between these columns the stresses in the bars increased, averaging about 1500 lb. per sq. in. with load A and 6000 lb. per sq. in. with load C (see Fig. 32 and 36.) Taking into

account the stresses produced by the dead load, it seems that the tensile resistance of the concrete must have been exceeded with load C, and this makes impracticable the use of the resulting stresses for comparing quantitatively the bending moments developed by the two loads.

For the band running from Column O62 to O63 the average stress observed in the reinforcement at mid-span under the four-panel loading (load A) was only 500 lb. per sq. in. and under the loading of a row of panels (load C) 900 lb. per sq. in. For the band running from Column N62 to N63 (see Fig. 34), the average stress observed at mid-span under load A was 2000 lb. per sq. in. and under load C 500 lb. per sq. in.

It will be seen that for load A, there is considerable similarity in the distribution of stresses along section B-B (Fig. 32) and section E-E (Fig. 34).

19. *Columns.*—Columns located at the edges of the loaded area showed considerable bending stress. The stresses in columns were

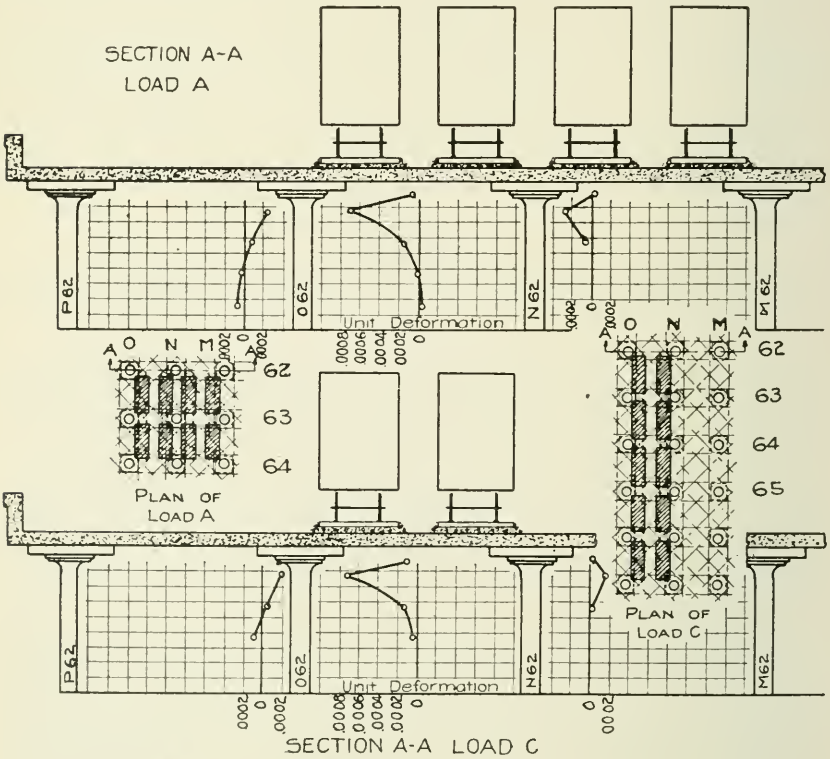


Fig. 37. Deformation in Columns Cut by Section A-A (Normal to Tracks) of Soo Terminal Structure.

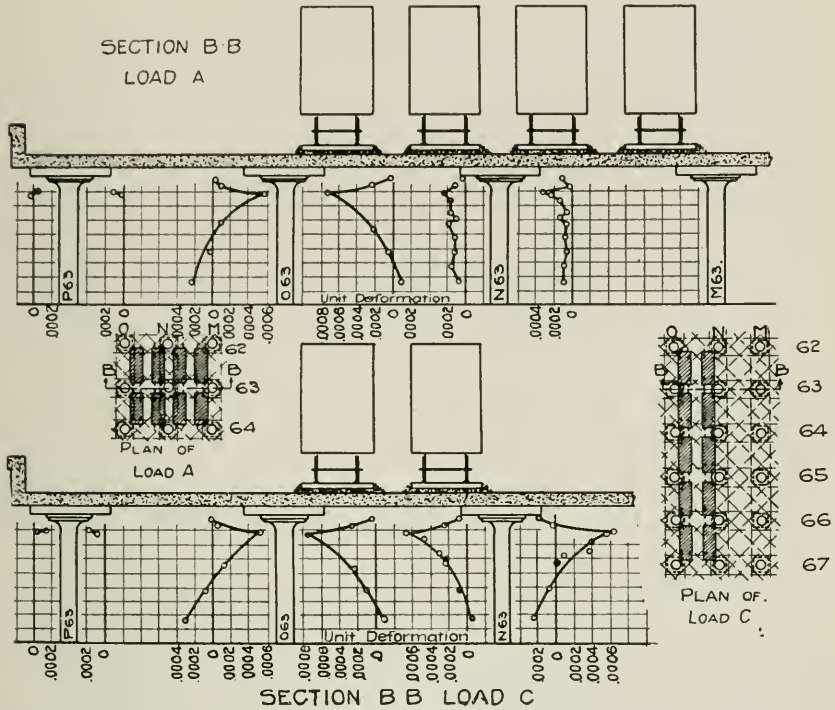


FIG. 38. DEFORMATION IN COLUMNS CUT BY SECTION B-B (NORMAL TO TRACKS) OF SOO TERMINAL STRUCTURE.

higher and more definite than the stresses in the slab. The amount of the deformations in the concrete on opposite sides of the column is shown graphically in Fig. 37 to 41. The large tensile deformations in the columns show that the reinforcing bars were subjected to considerable tensile stress, and even considering the compression due to dead load the tensile strength of the concrete must have been exceeded. This action is also shown by the formation of cracks on the tension side of the columns. Measurements made on the reinforcing bars (not plotted in the figures) show tensile deformations smaller than those observed in the concrete, but still considerable. The distribution of the flexural deformations along the column length is also shown in Fig. 37 to 41. In general the point of zero deformation is lower on the compression side than on the tension side of the column, a difference which may be due in part to the direct compression caused by the test load. Table 4 gives the distances from the bottom of the depressed head to the point of zero deformation on the two sides of their columns and their averages. The distances are

TABLE 4.

POSITIONS OF POINTS OF ZERO DEFORMATION ON COLUMNS.

Column No.	Load	Distance in Inches from Depressed Head to Point of Zero Deformation			Ratio of Average to Column Length
		Loaded side	Unloaded side	Average	
063	A	East 115	West 81	98	.64
063	C	East 105	West 85	95	.62
N63	C	West 106	East 78	92	.60
O62	A	East 103	West 103	103	.67
O62	C	East 98	West 78	88	.57
O64	A	East 108	West 106	107	.70
O64	C	East 108	West 100	104	.68
O64	A	North 106			
O62	A	South 103			
O62	C	South 105			

expressed also as proportional parts of the total distance between the bottom of the depressed head and the upper surface of the basement floor. The average of the ratios is 0.64. If the case of a column

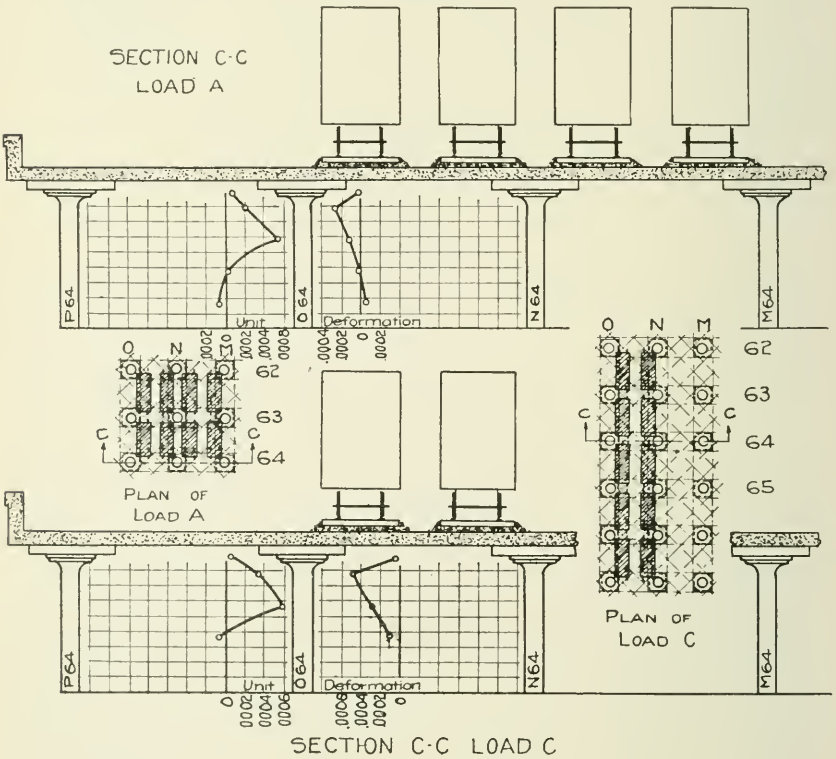


FIG. 39. DEFORMATION IN COLUMNS CUT BY SECTION C-C (NORMAL TO TRACKS) OF SOO TERMINAL STRUCTURE.

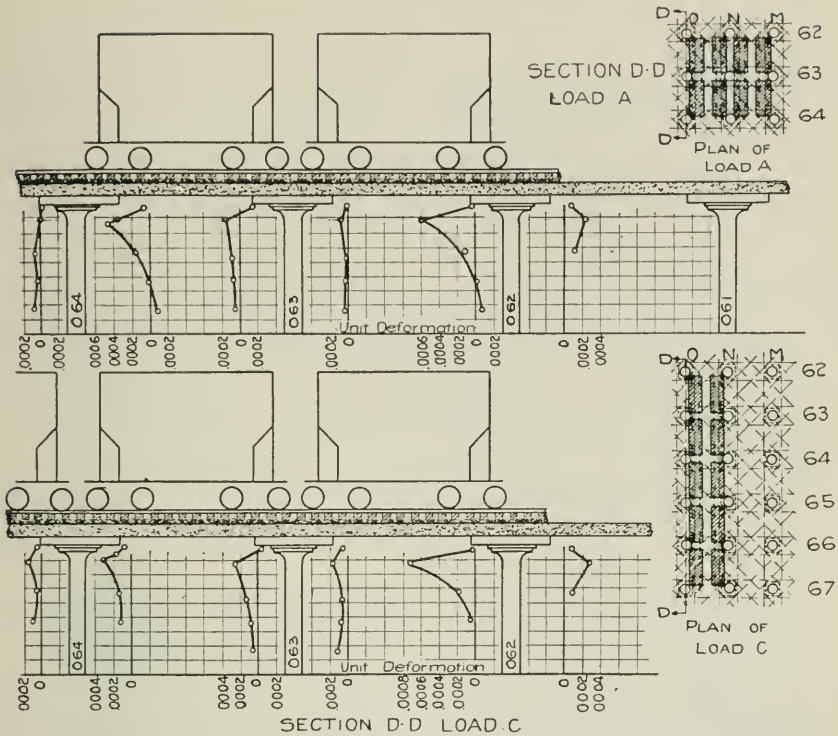


FIG. 40. DEFORMATION IN COLUMNS CUT BY SECTION D-D (PARALLEL TO TRACKS) OF SOO TERMINAL STRUCTURE.

fixed at one end and having a rigid connection with the slab or beam at the other end be subjected to analysis, the point of inflection will be found to be two-thirds of the distance from the point of rigid connection to the point of fixity. The results are in fair agreement with the analysis if the column be considered to be fixed at the basement floor.

Because of the uncertainty of the effect of tensile stresses in the concrete and because the value of the modulus of elasticity of the concrete of the columns is unknown, it is not feasible to calculate the bending moment produced in the columns. It can easily be seen, however, that the bending moment developed in this case is very large, as may be expected from an analysis of a structure made up of thick slabs, and not having the columns continued upward to other stories. The fact that such bending is developed in columns and is shown by measurements may be worth recording.

20. *Cracks.*—The location of small cracks which were observed

in the slab when the four-panel load (load A) was in position is shown in Fig. 42. All these cracks were found on the bottom of the slab. No cracks were found on the top. It is seen that in general they extend in a direction parallel with the rows of columns, displaying also a slight tendency to extend along the diagonal of the panel. Portions of the upper side of the slab were covered with ballast and no examination for cracks could be made in these places, but as the ballasted regions occupied the central portions of the panels where compression should be found cracks would not be expected at these places. The top of the slab was available for examination over all columns, and no cracks could be found in such positions although a careful search for them was made. It is seen that the cracks referred to are in regions where compression would be expected. Their appearance may be explained by even a slight settlement of columns; those on the under side of the slab close to the depressed heads of some of the columns at the edge of

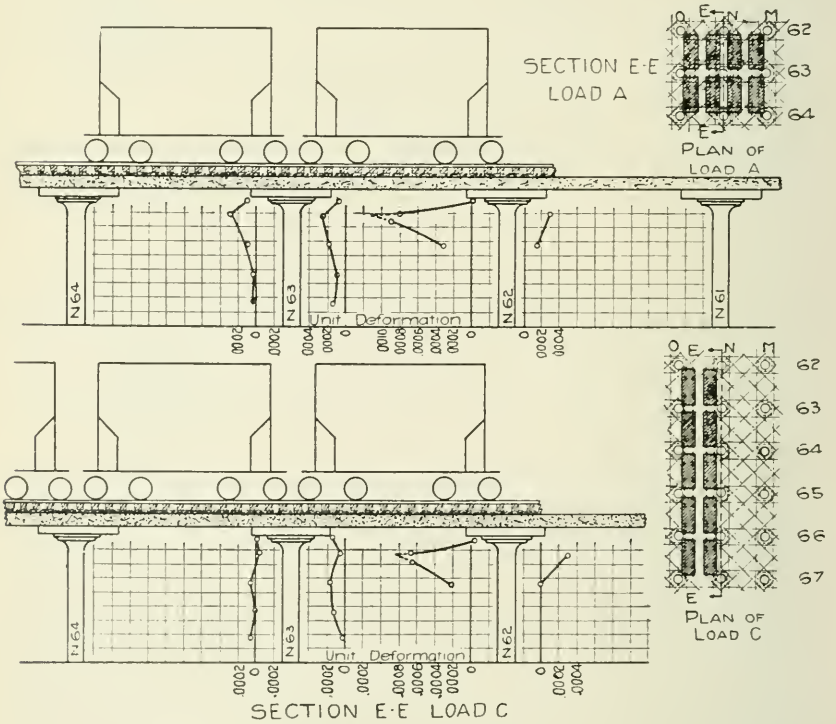


FIG. 41. DEFORMATION IN COLUMNS CUT BY SECTION E-E (PARALLEL TO TRACKS) OF SOO TERMINAL STRUCTURE.

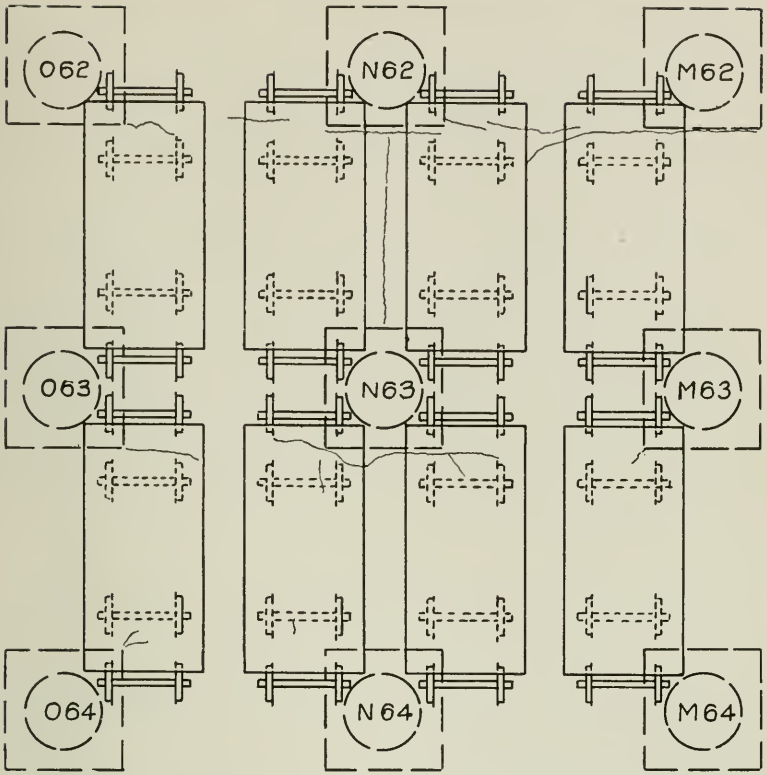


FIG. 42. LOCATION OF CRACKS IN SOO TERMINAL TEST FLOOR.

the loaded area may be explained if there were settlement of these columns, and those which were found on the under side of the slab close to the depressed head of the central column if there were a settlement of this column larger than that of any other.

Under Load A, cracks were observed in several columns located on the edges of the loaded area. These were found on the column capital a little above its bottom edge and on the side away from the loaded area. It was not easy in every case to distinguish between these cracks and other cracks found on the columns which evidently were due to expansion and contraction of the floor caused by changes in temperature, but a survey of the temperature cracks disclosed that they were systematically arranged and could be distinguished from the cracks which were due to eccentricity of loading and which were indicative of bending moment developed in the columns as already discussed under 19. *Columns.*

21. *Measurement of Dead Load Stress.*—An effort was made to obtain information on the amount of stress developed in the slab by the weight of the floor upon the removal of the forms. The test was made upon a 24 by 24-ft. panel. As the floor slab is very thick it cannot be expected that the stresses developed by the dead load would be very high, but the test indicates that it is practicable to measure the effect produced when the forms are removed.

The forms were left standing until the time of the test. Gage lines were placed where readings were desired and initial or zero readings were taken. The forms were then removed and other sets of readings were taken when the slab was supporting its own weight. In order to place gage lines on the bottom of the slab before the forms were removed, it was necessary to cut holes in the forms. As the deformations were expected to be small, special care was taken to insure correctness of the observations. Both initial readings and final readings on all gage lines were taken twice by each of two observers.

TABLE 5.

AVERAGE UNIT-DEFORMATION UNDER DEAD LOAD.

Plus indicates extension and minus indicates shortening.

Location	Average Observed Unit-deformation
Bottom; outside of capital; rectangular direction	—.000042
Top; outside of capital; rectangular direction	+.000013
Bottom; outside of capital; diagonal direction	—.000024
Top; outside of capital; diagonal direction	+.000005
Bottom; outside of capital; Depressed head, rectangular direction	—.000013
Top; outside of capital; Depressed head, rectangular direction	+.000005
Bottom; midway between columns; rectangular direction	+.000013
Top; midway between columns; rectangular direction	—.000048
Bottom; midway between columns; diagonal direction	+.000014
Top; midway between columns; diagonal direction	—.000032

The averages of unit deformations found are given in Table 5. It is seen that for measurements at gage lines located near the column capital the deformations were greater on the bottom of the slab than on the top, while for the locations midway between columns, the deformation was greater on the top of the slab than on the bottom; in other words, the deformation was greater in the regions of compression than in the regions of tension. From the amounts of the deformations measured it may not be expected that the concrete had failed in tension. In judging of the amount of these deformations it must be considered that there is a possibility that some of the dead weight is assumed by the slab in advance of the

removal of the forms because of the shrinkage of the forms or of very slight settlement. The fact that the compressive deformations are higher than the tensile deformations may indicate that arch action played a part in the support of the dead load.

22. *Summary of Results.*—The following comments may be made on the test of the Soo Terminal Building:

1 The deformations measured in the steel and in the concrete of the slab were very small, in many cases smaller than the possible errors of observation. The tensile stresses developed in the reinforcement being so small, the tensile strength of the concrete must have played a very large part in the bending resistance of the slab. It appears also that uneven settlement of the footings under the applied load modified the action of the structure.

2 With the development of such low stresses and the uncertain action due to uneven settlement of the footings, the results of the test may not be used to throw light on the mechanics of the slab and on the distribution of stresses over the slab in the way it was hoped they could be used. As would be expected, an increase in the stress in a cross band under the loaded area was found when the load was changed from four panels in the form of a square to five panels in a row.

3 Marked bending was found in the columns at the edge of the loaded area. The point of inflection of the elastic curve of flexure of the columns was about two-thirds of the distance from the bottom of the depressed head to the upper surface of the basement floor, which is the location to be expected for a column fixed at the bottom and having a rigid connection with the slab at the top.

4 The location of the cracks found on the under side of the slab indicates that stresses in a structure subject even to slightly uneven settlement of footings may be of different character from those found by the ordinary assumptions of design.

5 The measurements made to determine the stresses produced in the slab by dead load upon striking centers indicate that while the deformations were small in the test made, it is practicable to measure the deformations in a reinforced concrete structure due to dead load.

IV. The Schulze Baking Company Building Test.

23. *The Building.*—The building of the Schulze Baking Company is located at 55th St. and Wabash Ave., Chicago. It is five stories

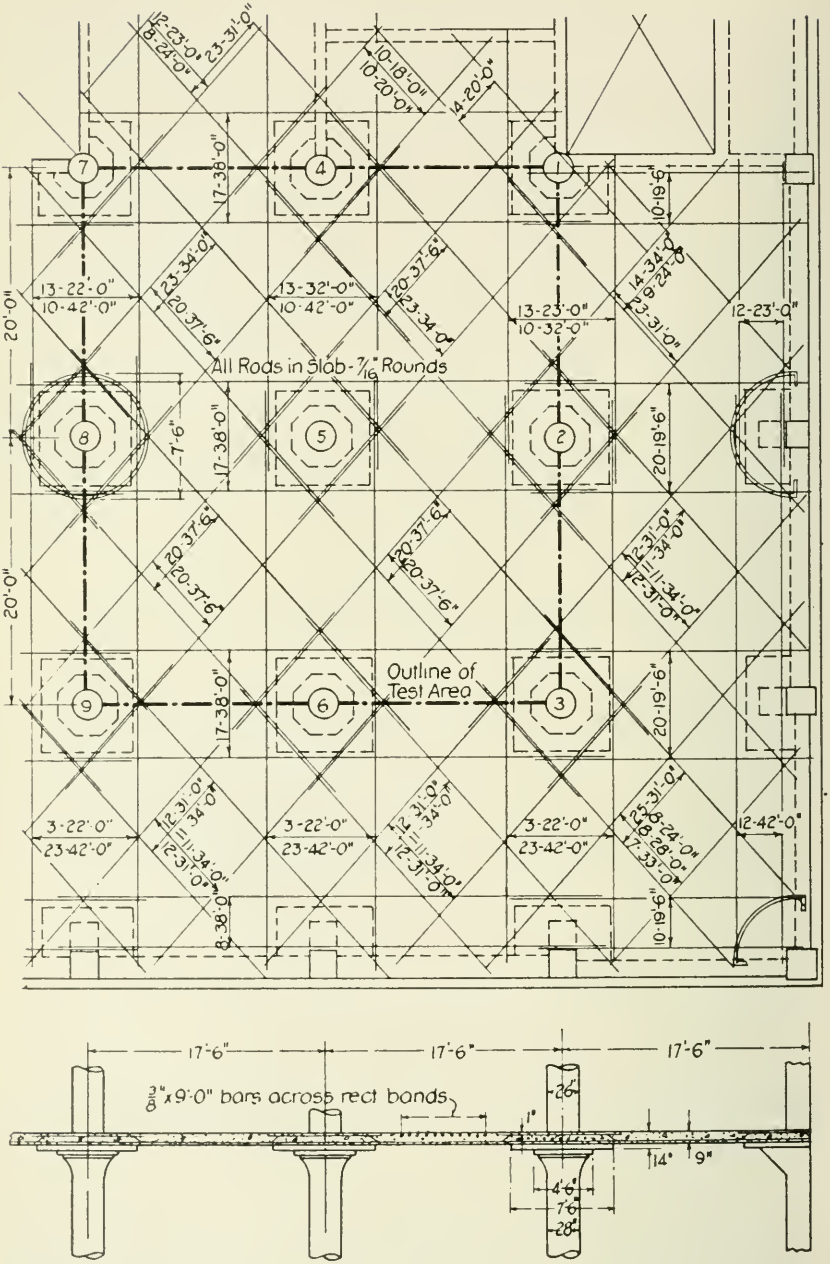


FIG. 43. DIMENSIONS AND REINFORCEMENT PLANS OF TEST FLOOR OF SCHULZE BAKING COMPANY BUILDING.

in height and covers an area 298 ft. 4 in. by 160 ft. The floor space is divided into panels 17 ft. 6 in. by 20 ft. The test load was applied to panels of the second floor (one story above sidewalk level). The floor construction in this part of the building consists of a four-way reinforced concrete slab nominally 9 in. thick in the central portion of the panel and 14 in. thick throughout the area of a 7-ft. 6-in. square surrounding each column. It was designed for a live load of 300 lb. per sq. ft.

The columns below the second floor are circular, 28 in. in diameter, and terminate at the top in octagonal bell-shaped heads

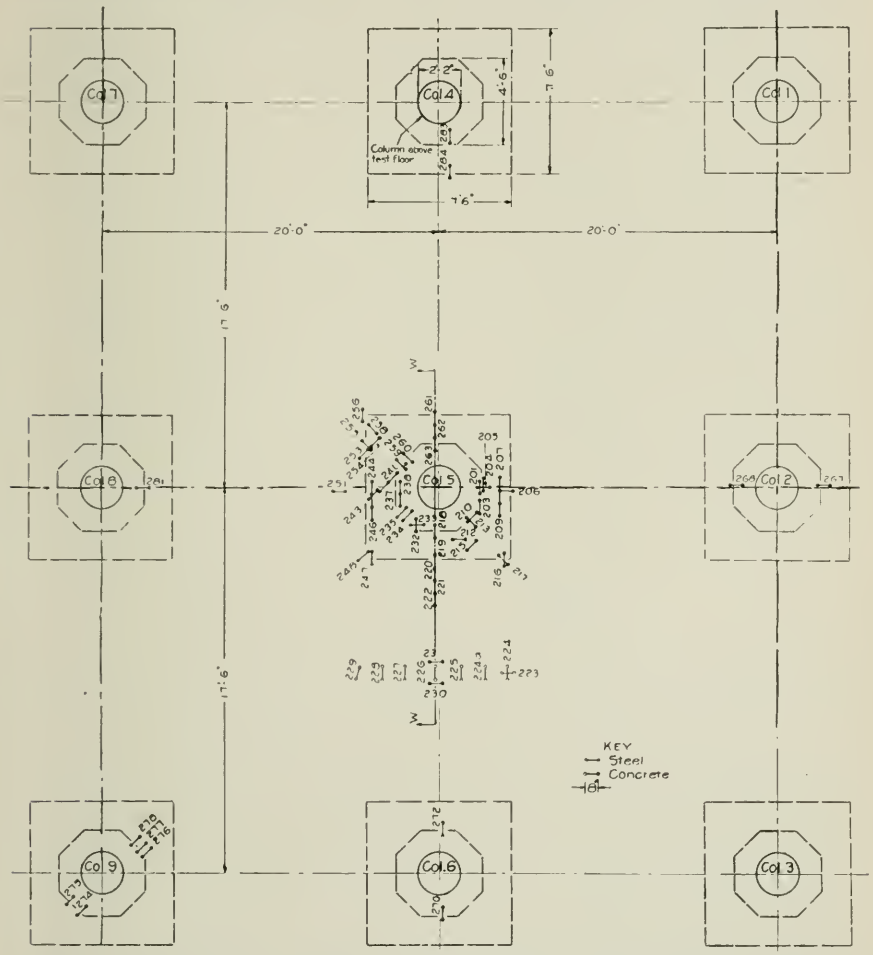


FIG. 44. GAGE LINES ON UPPER SURFACE OF TEST FLOOR OF SCHULZE BAKING COMPANY BUILDING.

54 in. in diameter at their upper ends. The columns between the second and third floors are circular, and are 26 in. in diameter.

The distribution of the reinforcement in the portion of the floor to which the test load was applied is shown in Fig. 43.

Two-thirds of the slab rods of the rectangular bands are located in the top of the slab where they pass over the columns. These rods drop down to the bottom of the slab at a point about two-tenths of the panel length from the center line of the columns. At the corresponding point relative to the next column the rods are bent up again and pass over the column in the top of the slab, ending 9 in.

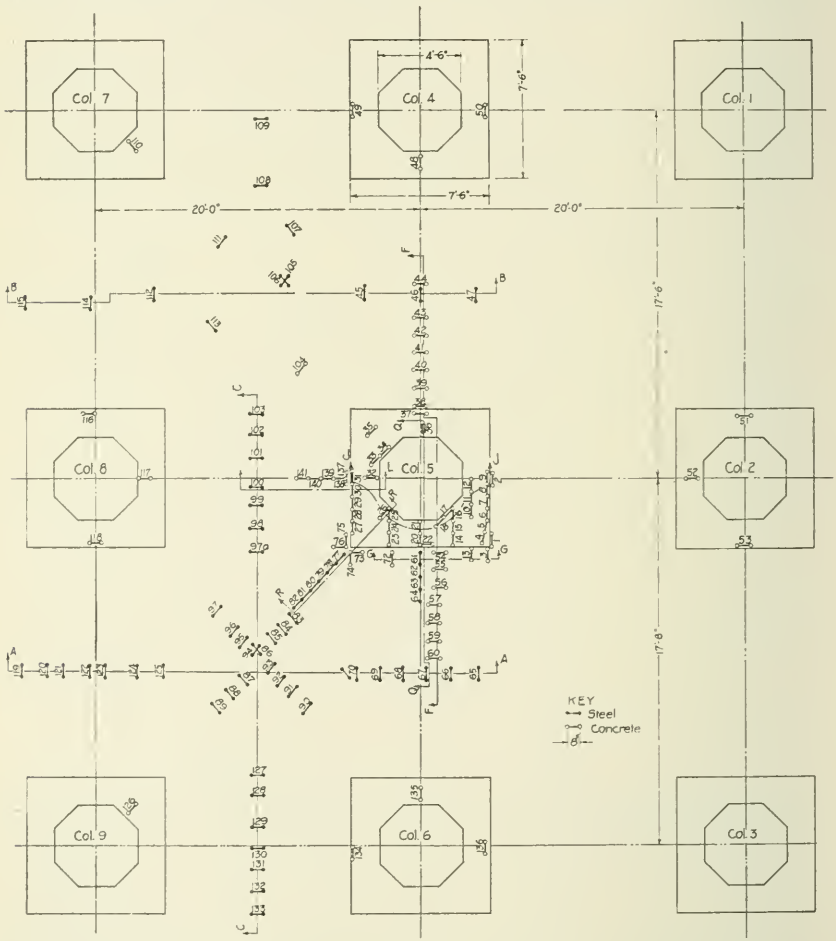


Fig. 45. Gage Lines on Under Surface of Test Floor of Schulze Baking Company Building.

to 18 in. beyond the center line of columns. The remaining one-third of the bars of the rectangular bands extend through the bottom of the slab throughout the length of the bars. All diagonal rods bend up to pass over the column in the top of the slab, and extend 5 ft. 3 in. beyond the center lines of columns. Measurements taken at the time of the test indicate an average depth from the compression surface to the center of gravity of the reinforcement of 10.15 in. for positions of negative moment and 7.45 in. for positions of positive moment.

To prevent concentration of cracks along the center line of the band of reinforcement from column to column, ⅜-in. bars 9 ft. long spaced about 12 in. apart were placed across the boundary line between panels.

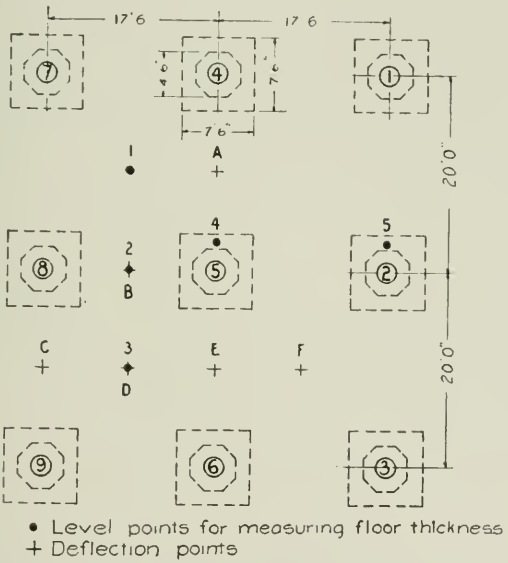


FIG. 46. LOCATION OF POINTS FOR MEASUREMENT OF THICKNESS AND OF DEFLECTION OF TEST FLOOR IN SCHULZE BAKING COMPANY BUILDING.

The concrete was placed late in October, 1913, and there was considerable cold weather during the time of hardening. For this reason it was expected that the concrete would not show up as well as concrete poured and set under the more favorable conditions of summer, and because of the unfavorable conditions a little more time than usual was allowed before making the test. At the time of the test it appeared that there still was considerable moisture in the concrete, but no unfavorable indications were found in the concrete.

24. *The Test.*—The period of preparation for the test covered the time from January 13 to January 19, 1914. Deformations were measured in 123 gage lines on the reinforcement of the slab, 82 gage lines on the concrete of the slab, and 58 gage lines on the concrete of the columns. The gage lines on the upper and under surfaces of the slab are shown in Fig. 44 and 45. All the strain gage readings were taken by Mr. Slater. Deflections and floor thicknesses were measured in five places, the locations of which are shown in Fig. 46. The floor thicknesses for the various points in order were as follows: (1), 9⅛ in.; (2), 8 3/16 in.; (3), 8 11/16 in.; (4), 14⅛ in.; (5), 14 7/16 in. This gives an average thickness for the thin portion of the floor of 8.87 in. and for the thick portion of 14.28 in.

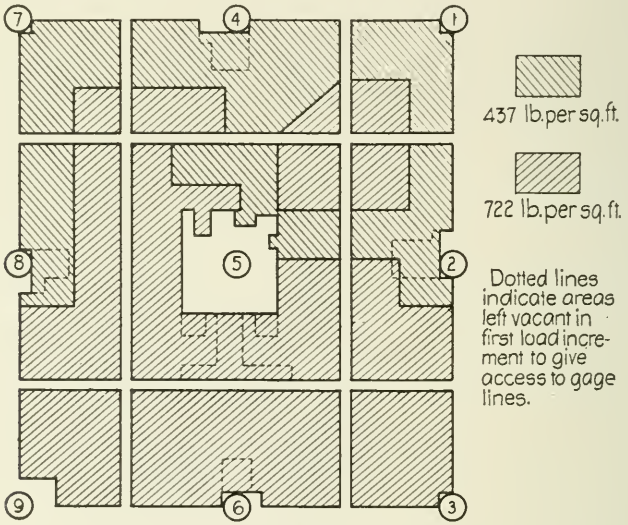


Fig. 47. Plan Showing Distribution of Load on Test Floor of Schulze Baking Company Building.

Brick was used as the loading material. Previous to the test its weight was determined by weighing a number of wheelbarrow loads, noting both the number of bricks and measuring the cubic contents when stacked. The weight per cubic foot was found to be 96 lb. The load was applied to four panels as shown in Fig. 47, the brick being stacked in piers. Fig. 48 is a view of the load in place. On account of shortages of brick the load was not always uniformly distributed over the floor at the time of taking the readings. Table 6 shows the loads at which strain gage readings were taken and indi-

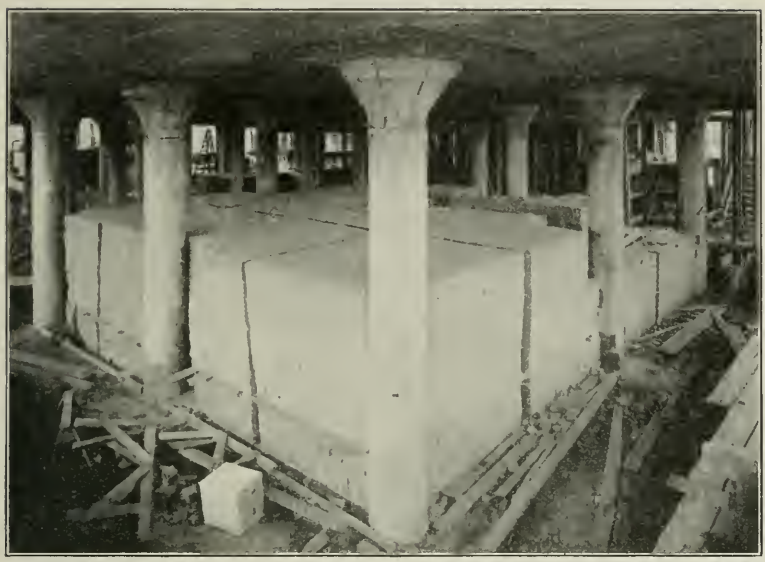


FIG. 48. VIEW OF TEST LOAD ON FLOOR OF SCHULZE BAKING COMPANY BUILDING.

cates the nominal loads used in plotting the load-deformation curves.

TABLE 6.

LOADING OF FLOOR IN SCHULZE BAKING COMPANY BUILDING.

Stage of Loading	Height in inches	Courses of Brick	Load lb. per sq. ft.
1	29	12	189
2	48¼	20	319
3	65	27	437
4	106	44	722

The maximum load was 722 lb. per sq. ft. (twice the design live load plus the dead load), except for a portion of the two north panels (Fig. 47) which had 437 lb. per sq. ft. The deficiency was due to a shortage of material, but the load on these panels was distributed with a view to making the material at hand as effective as possible in producing bending moment. It is believed that the moment in the fully loaded panels, both at the central column and midway between columns was not less than 90 per cent of the moment which would have been developed if the load had been 722 lb. per sq. ft. over the entire area.

Expansion and contraction of the steel and concrete with change in temperature made complications which partially obscure the results of the test. In an effort to eliminate possible errors from this source, corrections were determined by taking readings on an unstressed gage line in the floor. In spite of the indications that the expansion and contraction of the concrete go through the same cycles of changes as the temperature of the air, corrections made on the basis of these observations do not remove all the inconsistencies from the load-deformation curves. However, it is believed that such errors have been greatly reduced by the corrections used.

25. *Tension in Slab.*—The measured unit-deformations observed in this test were unusually small, both in the steel and in the concrete. Very few unit-deformations were more than 0.0002 and the majority were less than this at the maximum load of 722 lb. per sq. ft. Assuming a modulus of elasticity of 30,000,000 lb. per sq. in. for steel, the stress at a unit deformation of 0.0002 is 6,000 lb. per sq. in.

TABLE 7.

AVERAGE STRESSES IN TENSION REINFORCEMENT AT MAXIMUM LOAD.

Gage Lines	Location and Direction of Gage Lines	Average Stress lb. per sq. in.
203, 208, 216, 232, 237	Long rectangular direction at central column	7200
205, 212, 233	Short rectangular direction at central column	3900
215, 210, 234, 235, 242, 253	Diagonal across loaded area at central column	5400
272, 283	Long rectangular direction at columns at edge of loaded area	4300
268, 281	Short rectangular direction at columns at edge of loaded area	3300
276, 277, 278	Diagonal at column at corner of loaded area	7100
65 to 70, inclusive	Long rectangular direction midway between columns	8700
97a to 103, inclusive	Short rectangular direction midway between columns	6300
90 to 97, inclusive	Diagonal across loaded area midway between columns	3900
83 to 89, inclusive	Diagonal across corner of loaded area midway between columns	3000
127 to 133, inclusive	Long rectangular direction midway between columns on edge of loaded area	4900
119 to 125, inclusive	Short rectangular direction midway between columns on edge of loaded area	3600
267	Short rectangular direction just outside of column at edge of loaded area	1200
274, 275	Diagonal just outside of column at corner of loaded area	2800

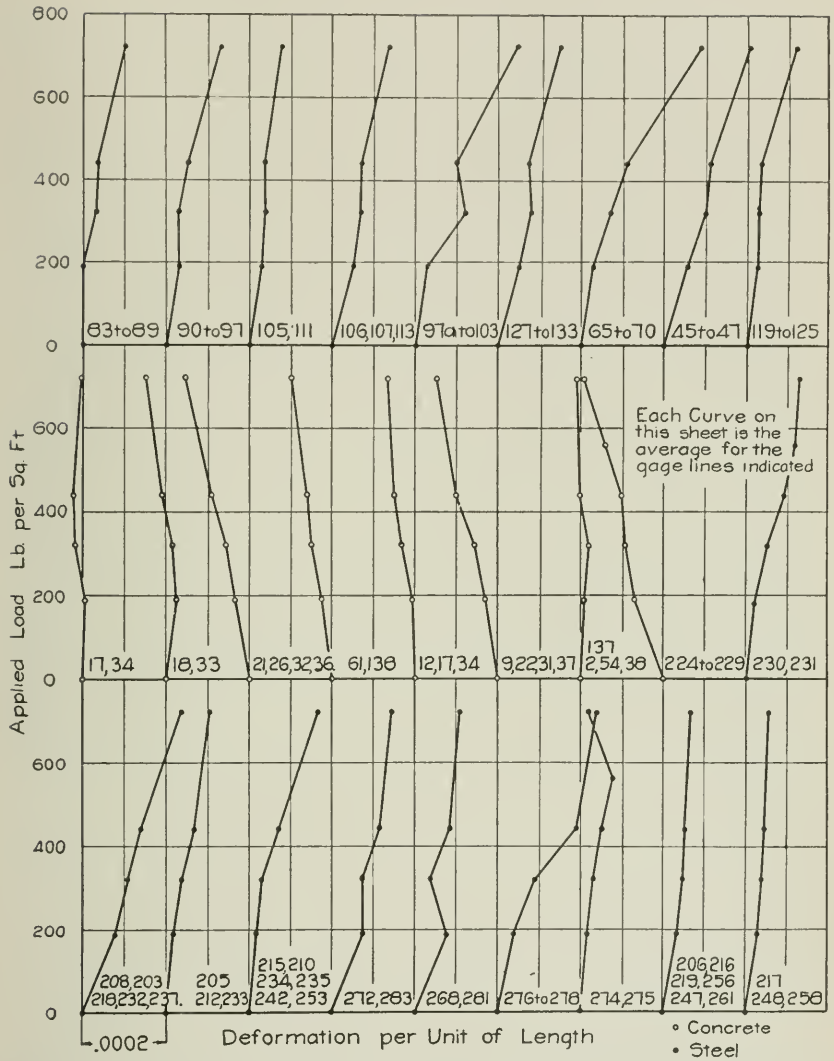


FIG. 49. AVERAGE LOAD-DEFORMATION DIAGRAM FOR TEST IN SCHULZE BAKING COMPANY BUILDING.

There were only two cases in which a value as high as 10,000 lb per sq. in. was reached.

In Table 7 the average stresses in steel for groups of gage lines at various locations are shown. While these figures may indicate correctly the relation of the stresses of the several groups, it is recognized that in a test where stresses generally were so small,

unknown variations in conditions in different parts of the slab, such as the formation of cracks at certain parts, may influence the distribution of stresses more than some of the known elements of the design.

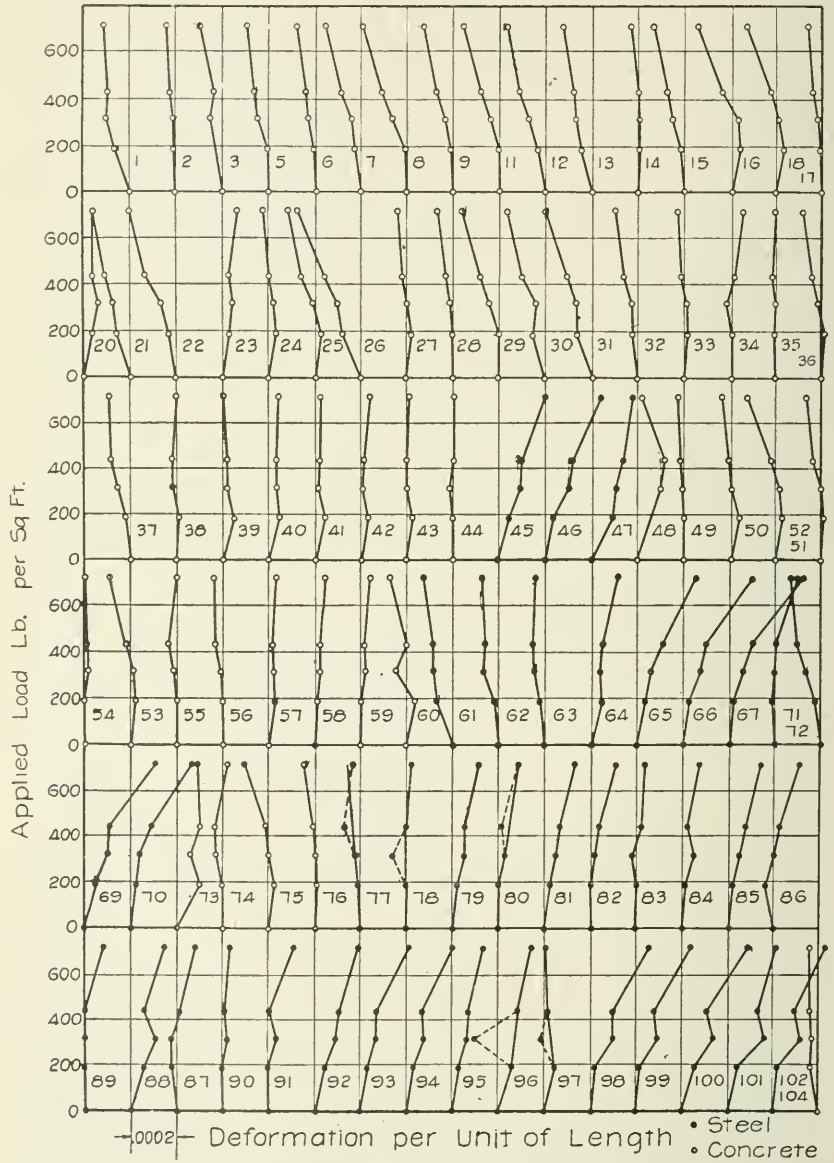


Fig. 50. Load-deformation Diagrams for Gage Lines 1 to 102 of Test in Schulze Baking Company Building.

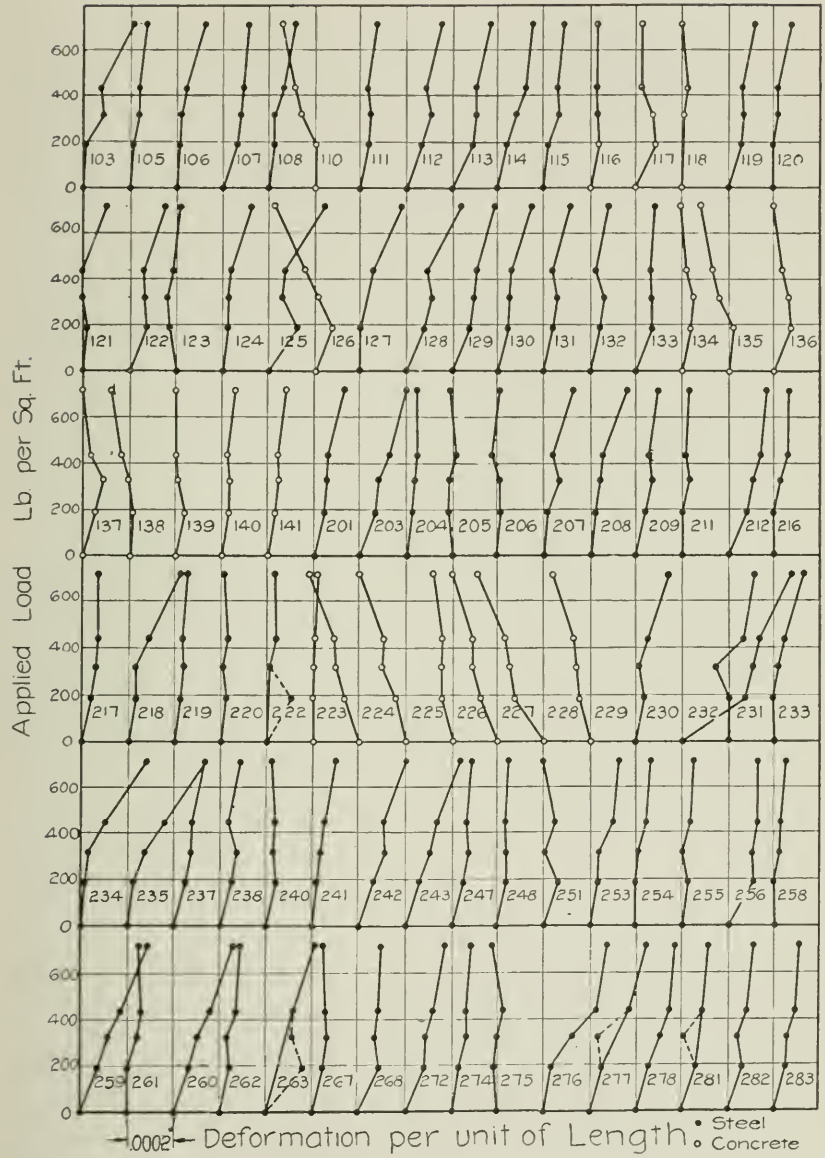


FIG. 51. LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES 100 TO 283 OF TEST IN SCHULZE BAKING COMPANY BUILDING.

For instance, the formation of cracks at certain positions may be a more important factor in stress distribution than the difference in panel length in the two directions. In the use of this table such limitations must be kept in mind. Load-deformation diagrams, averages for groups of gage lines, are given in Fig. 49 and the diagrams for the individual gage lines are given in Fig. 50 and 51. Unit-deformations across or along various sections through the floor are shown in Fig. 52 to 55. The locations of the sections are shown in Fig. 44 and 45.

Table 7 indicates that the stress at the central column and the stress at the column on the edge of the loaded area were greater in the direction of the longer side of the panel than in the direction of

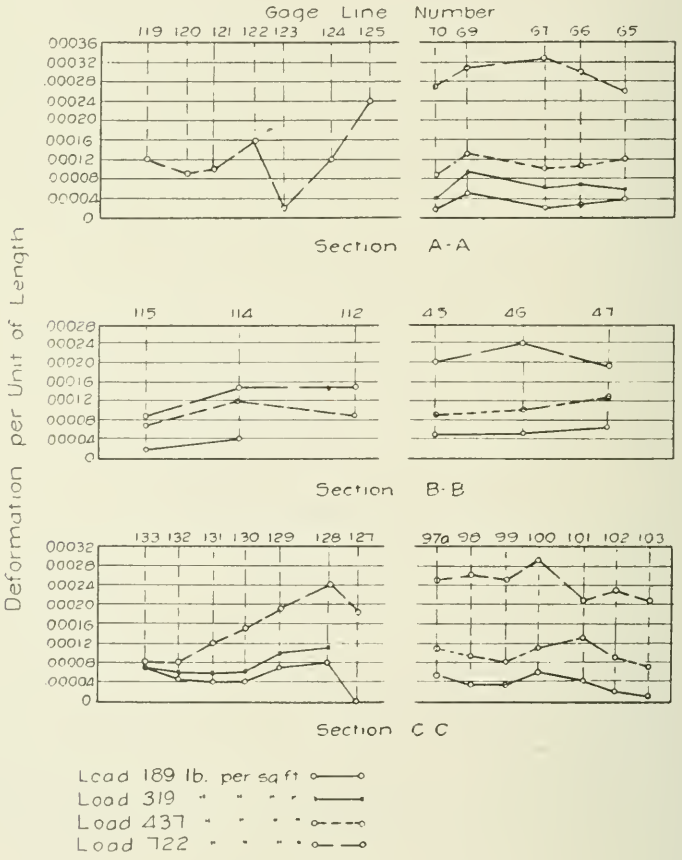


Fig. 52. Deformation in Reinforcement in Rectangular Bands Midway Between Columns in Test Floor of Schulze Baking Company Building.

the shorter side. For the center of the span a similar relation is shown.

A comparison of the deformations at the left hand portions of the sections shown in Fig. 52 with those at the right hand portions shows that the average stress in the rectangular band at the edge of the loaded area was considerably less than that in the corresponding band at the central portion of the loaded area. There is also an indication that the stress increased (somewhat irregularly) from the outer portion of the outer band at the edge of the loaded area toward the inner edge of this band.

In the diagonal bands of reinforcement the stress was about the mean of the average stresses in the two rectangular bands for positions at the central column and at the center of the panel (see Table 7). However, the stress in the diagonal band at the corner of the loaded area was greater than that in the rectangular bands at the columns on the edges of the loaded area. Only a few gage lines were read at the positions near the edge of the loaded area and the stresses found there may not represent the normal conditions for such positions.

The measurements on gage lines 230 and 231 (taken on the bars placed across the boundary line between two panels) indicate an average stress of about 3000 lb. per sq. in. at the maximum load (see Fig. 51). This would indicate that such rods may be effective in accomplishing the purpose for which they were designed, namely, the distribution or prevention of cracks which are likely to occur along the line from column to column. It indicates also that reinforcing bars in such positions may be of considerable value as reinforcement for negative moment. Of course, the reinforcing bars should be carried well beyond the usual position of the point of inflection to provide sufficient anchorage for cases of partial loading.

26. *Compressive Stresses and Unit-Deformations.*—Table 8 gives average values of the compressive unit-deformation in the concrete for representative positions. In tests of girderless slabs high unit-deformations are expected on the bottom of the depressed head close to the capital, and those on the bottom of the thin portion of the slab close to the edge of the depressed head have been found to be nearly as high. Examination of the data of this test shows that in the rectangular directions the deformations in these two places are not far different from each other.

The largest compressive unit-deformation, 0.00028, was found in the diagonal direction at the central column on gage line 26. Using a value of 3,000,000 lb. per sq. in. for the modulus of elasticity of

TABLE 8.

AVERAGE COMPRESSIVE UNIT-DEFORMATION AT MAXIMUM LOAD.

Gage Lines	Location of Gage Lines	Unit-deformation
8, 11, 16, 21, 25, 31	Long rectangular direction on bottom of depressed head	.00016
22, 32, 37	Short rectangular direction on bottom of depressed head	.00013
13, 61, 72	Long rectangular direction on slab near depressed head	.00012
21, 32, 36	Average of long and short rectangular directions near column capital	.00011
61, 138	Average of long and short rectangular directions near depressed head	.00013
9, 22, 31, 37	Across panel lines, bottom of depressed head near edge	.00015
*224 to 229 inclusive	Long rectangular direction midway between columns	.00015

*On upper surface of slab; all others on under surface.

concrete (frequently assumed in such cases), the corresponding compressive stress is 840 lb. per sq. in. A relatively large unit-deformation is found at the other end of the same diagonal in gage line 126. All other compressions in the diagonal direction were small.

In the long rectangular direction it is found that the maximum deformations would be cut by a curved section through gage lines 8, 11, 16, 21, 25, and 31 (see section J-J, Fig. 45). The average unit-deformation across this section was 0.00016 (see Table 8). In the short rectangular direction the average unit-deformation on gage lines 22, 32, and 37 was 0.00013.

It is seen that in general the deformations in the concrete were low. With the concrete intact in tension the neutral axis would be farther from the compression face than it would be otherwise. For the same resisting moment this would result in a smaller compressive

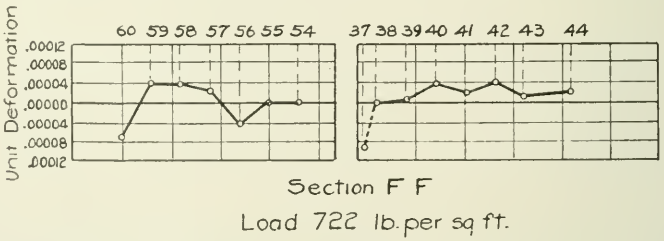


FIG. 53. LATERAL DISTRIBUTION OF COMPRESSIVE DEFORMATION ON BOTTOM OF TEST FLOOR NEAR DEPRESSED HEAD IN SCHULZE BAKING COMPANY BUILDING.

stress in the concrete than if the concrete had cracked generally on the tension side.

27. *Lateral Distribution of Compression.*—Little compression was detected in the two groups of gage lines 54 to 60 and 38 to 44 (see section F-F, Fig. 45 and 53). In fact, some of the measurements indicate a slight tension, but this is so small that the evidence should not be taken as conclusive. This indicates that the moment arm of the tensile stresses in the concrete in the thin portion of the slab may be determined as much by the thickness of the slab within the depressed head as by the thickness of the thinner portion of the slab. The tensile resistance of the concrete in the thinner portion of such a slab may thus give an increased resisting moment over that for a slab without the depressed head. This result suggests also that with a depressed head of the thickness here used the neutral axis is likely to be so close to the level of the bottom of the thin portion of the slab that the section of the slab outside of the depressed head may not be considered to contribute very much to the compressive stresses of the negative resisting moment. Tests to be used for comparing slabs having depressed heads with slabs which do not have them should employ loads large enough to eliminate effects of the kind here discussed.

Fig. 54, section G-G, shows the distribution of the compression

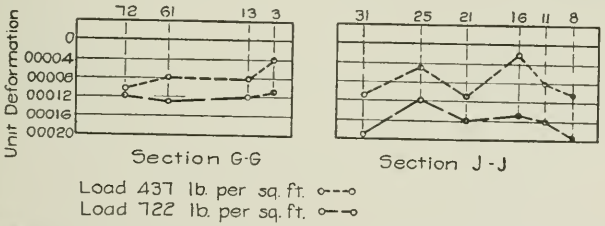


Fig. 54. Compressive Deformation on Bottom of Test Floor near Depressed Head, and near Column Capital in Schulze Baking Company Building.

which is developed across a section close to the capital, and J-J shows the distribution of compression which is developed across a section in the thin portion of the slab close to the edge of the depressed head.

28. *Moment Coefficients.*—Assuming that all the load is carried by flexure, it is apparent that moment coefficients calculated on the basis of the stress in the steel only, will not represent the full resisting moment if the concrete assists in carrying the tensile stresses. In this test the tensile stresses were so low that it is certain that the concrete must have assisted greatly. However, while values of the

moment coefficient obtained by using the observed stress in the steel cannot be used for design, a comparison of the coefficients obtained for different parts of the panel may be of interest.

The resisting moments of the stresses in the reinforcement as given by calculations for the load of 722 lb. per sq. ft. are 0.0050 Wl for the positive and 0.0097 Wl for the negative moment, in which W is the total applied load on the panel and l is the average of the long and the short panel length center to center of columns. These are the components, in a direction parallel with a panel side, of the moment resisted by the stresses in all the reinforcement cut by a section across the entire panel at a position of maximum negative moment and at one of maximum positive moment. The values given are averages of the moment found for the long direction and the short direction of the panel.

Since no cracks could be found in the concrete it is certain that the tensile stresses in the concrete must have assisted very greatly in resisting the bending moment. The bending moment coefficients given, therefore, are not values which may be used for design purposes, and as the concrete may have given more assistance relatively at one of these positions than at the other, the values may not even give the correct ratio between the negative bending moment and the positive bending moment.

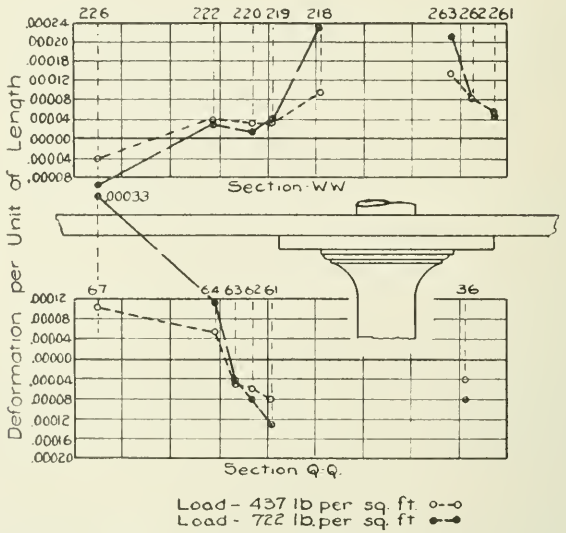


FIG. 55. LOCATION OF POINTS OF ZERO UNIT-DEFORMATION ON UPPER AND UNDER SURFACES OF TEST FLOOR OF SCHULZE BAKING COMPANY BUILDING.

29. *Point of Inflection.*—Points of zero unit-deformation on the upper and under surfaces of the slab in the direct lines between columns may be taken from the diagrams showing unit-deformation along sections W-W, Q-Q, L-L, and R-R, Fig. 55 and 56. These data

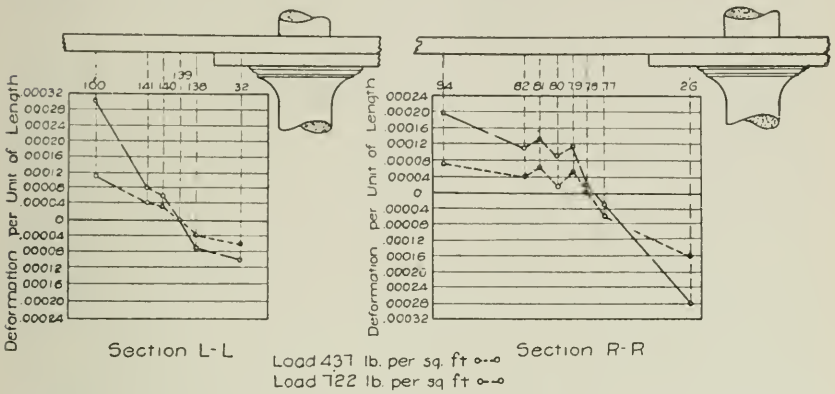


FIG. 56. LOCATION OF POINTS OF ZERO UNIT-DEFORMATION ON UNDER SURFACE OF TEST FLOOR OF SCHULTZE BAKING COMPANY BUILDING.

are inconclusive, but the indications are that as in other tests reported in this bulletin the point of zero unit-deformation for the under side of the slab was closer to the column than that for the upper side; or, in other words, the total tensile stress on any section seems to be greater than the total compressive stress. The presence of arch action would imply an excess of compression. This phenomenon has been observed in several tests and its occurrence need not be doubted. No explanation of this matter is attempted.

30. *Columns.*—The deformations in the columns were rather erratic and allow interpretations of only the most general sort. The differences in amount and character of deformations on opposite sides of the same column were sufficient to make it clear that there was considerable bending of the columns. The largest compressive deformation was found in column 2 which is on the edge of the loaded area, and the largest difference between deformations on opposite sides of any column is found for column 9 at the corner of the loaded area.

The significance of these observations seems to lie in their indication that the most important element of the deformation developed in the columns by load tests is that caused by the moment of unbalanced loads and that there is a larger amount of flexure in the corner columns than in the columns at the side of the loaded area. The latter is indicated also by the observation that the stress was greater

in the diagonal band of slab reinforcement at the corner of the loaded area than in the rectangular bands at the edge of the loaded area.

31. *Examination for Cracks.*—A careful examination of the surfaces of the slabs and columns disclosed no cracks. This was unexpected and it is the only case known personally to the writers in which a floor loaded to twice the design live load plus the dead load did not develop cracks which were large enough to be found by a reasonably careful examination. The absence of visible cracks on the shaft of the columns may be due to the influence of the dead weight of the floors above in overcoming a portion of the flexural tension. However, at certain portions of the slab it seems probable that there were very minute cracks because in several instances steel stresses in the neighborhood of 10,000 lb. per sq. in. were found. Even though there may have been very minute cracks a considerable portion of the concrete must have been intact, and this undoubtedly had a great deal to do with keeping down the steel stresses.

32. *Deflections.*—Load-deflection diagrams are shown in Fig. 57.

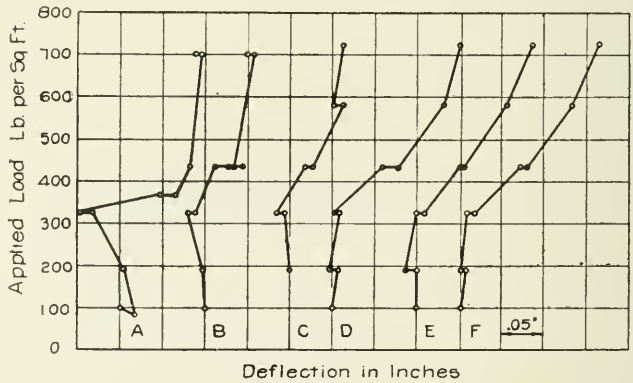


Fig. 57. Load-deflection Diagrams for Floor Test in Schulze Baking Company Building.

Distance to the right of the zero line represents downward deflection. The first set of readings was not obtained until a load of about 100 lb. per sq. ft. was on the floor and at deflection point C the apparatus was disturbed after the first set of readings had been taken; hence the deflection cannot be shown below 190 lb. per sq. ft. for C nor below 100 lb. per sq. ft. for all other points. The indications are that the failure to get readings for zero load makes little difference in the appearance of the curves. The small deflections and the tendencies toward upward deflection for the loads below 319 lb. per sq. ft. are ascribed to the

rather extreme temperature changes. The air temperature rose from 20° F. at the time the load of 319 lb. per sq. ft. was placed to 50° F. at the time the load of 437 lb. per sq. ft. was placed, and then fell again to 25° F.

33. *Summary of Results.*—The results of the test here reported are conditioned upon a correct interpretation of the effect of the rather extreme temperature variation during the time of the test and of the assistance given by the strength of the concrete in tension. A continuation of the test to a point at which the concrete in tension had cracked generally, probably would modify many of the conclusions.

The main results pointed out in the foregoing paragraphs or shown in the diagrams are:

1 Very few steel stresses higher than 6000 lb. per sq. in. were found. On only two gage lines did the observed deformations indicate steel stresses as high as 10,000 lb. per sq. in. The highest of these was 14,400 lb. per sq. in., but the form of the curve indicates that the initial reading may have been in error and that 10,000 lb. per sq. in. is a more probable value.

2 The averages of the stresses in the bands of reinforcement passing under the central portion of the loaded area were higher than averages in the bands under the edges of the loaded area. In the latter also the stresses in the inner bars of the band (the bars on the side toward the center of the loaded area) were larger than the stresses in the outer bars (lying outside the loaded area).

3 The stresses in the reinforcement of the diagonal bands fell between the averages for the two rectangular bands for positions around the central column and midway between columns. The stresses were larger in a diagonal band at the corner of the loaded area than in a rectangular band where it crosses the edge of the loaded area.

4 The stresses in the short bars placed across the panel boundary lines were low but large enough to indicate that the bars may be effective in distributing or preventing cracks along the edge of the panel.

5 The compressive unit-deformations were low.

6 The compressive deformations across a section of the slab for gage lines as near as possible to the edge of the depressed head were nearly as high as those across the section of the depressed head near the edge of the capital.

7 The largest compressive unit-deformation was found in the diagonal direction at the central column.

8 The portions of the slab beyond the edge of the depressed head did not develop compressive stresses on the under side in a direction parallel to that edge.

9 Moment coefficients calculated on the basis of the steel stresses developed are exceedingly low. That for a position of maximum negative moment is about twice as large as that for a position of maximum positive moment.

Especial emphasis should be placed on the fact that these coefficients cannot be taken as indicating the total resisting moment developed.

10 The indications are that the bending of the columns was an important feature of the action of the structure. The largest bending apparently occurred in a column at the corner of the loaded area.

V. The Worcester Slab Test.

34. *The Test Structure.*—The structure on which the test was made was built especially for the test and was located near Worcester, Mass. The slab was designed with the object of obtaining information on the effect of (1) different methods of arranging and distributing the reinforcement, and (2) variation in size of column capital. Fig. 58 gives the general design of the structure.

In order to avoid as far as possible lack of uniformity in conditions of building and testing the different parts and in order to reduce the proportion of the number of wall panels to interior panels, the four types of design used were placed in the four quadrants of a single slab four panel lengths (56 ft.) square. This gave a group of four panels to each of the four designs and a column in the center of each group. The details of the slab are shown in Fig. 58. The arrangements of slab reinforcement at the column capitals for the various groups were as follows:

Group I All tension reinforcement was placed in the diagonal bands. The rectangular bands lay in the bottom of the slab and afforded compression reinforcement at the column capitals.

Group II Both rectangular and diagonal reinforcement were in the top of the slab. There was no reinforcement in the bottom of the slab at the columns.

Group III All tension reinforcement was placed in the diagonal bands. The bars of rectangular bands did not pass over the column capitals. There was no reinforcement in the bottom of the slab at these positions.

Group IV Reinforcement was the same as in Group II but the column capital was smaller.

The amount and distribution of the tension reinforcement at the column capitals in each of the four groups were such that if planes were passed cutting each band at right angles outside the column capital the total area of steel in the top of the slab so cut in an angular distance of 180° around the column was 4.86 sq. in., the same for all groups. It should be noted, however, that the effect of the reinforcement in producing resisting moment across a panel edge will not be exactly the same for all groups. In the group having only diagonal bands a calculation of the rectangular component of the resisting moment (the component in a direction at right angles to the edge of the panel) will be about one-sixth greater than the rectangular component of the resisting moment of the reinforcement in the groups having both diagonal bands and cross bands. This area includes the section of tension reinforcement which in Groups II and IV consists of two rectangular bands and two diagonal bands; in Groups I and III this area is the section of two diagonal bands.

At points midway between columns, both in the rectangular and in the diagonal directions, the amount and distribution of the reinforcement were the same for all groups. In Group III the bars in the rectangular bands ended near the probable points of inflection and were not bent up in any way. The total area of cross-section of two rectangular bands and two diagonal bands was 4.2 sq. in.

Midway between columns two bars ⅜ in. in diameter, 6 ft. long, were placed in the top of the slab across the panel boundary, that is, normal to the direction of the rectangular bands of reinforcement.

The panel length was 14 ft. center to center of columns in all panels. The diameter of the top of column capital was 4 ft. 6 in. for Groups I, II, and III, and 2 ft. 9 in. for Group IV. The ratios of these capital diameters to the panel length were 0.321 and 0.196 respectively. The average of all the measured thicknesses of the slab was 4.93 in. The average measured depth to the center of gravity of

TABLE 9.

CALCULATED SOIL PRESSURE AND MEASURED SETTLEMENT FOR UNIFORM LOAD OF 215 LB. PER SQ. FT.

Column Numbers	Soil Pressure lb. per sq. ft.	Average Settlement in.
E1 A1 E5 A5	1600	.11
E2 D1 B1 A2 E4 D5 B5 A4	2300	.235
C1 A3 C5 E3	2000	.27
D2 B2 D4 B4	3000	.39
C2 B3 C4 D3	2700	.27+
C3	2400	.37

the bands of reinforcement was 1.04 in. midway between columns and 1.63 in. near the columns for the four groups.

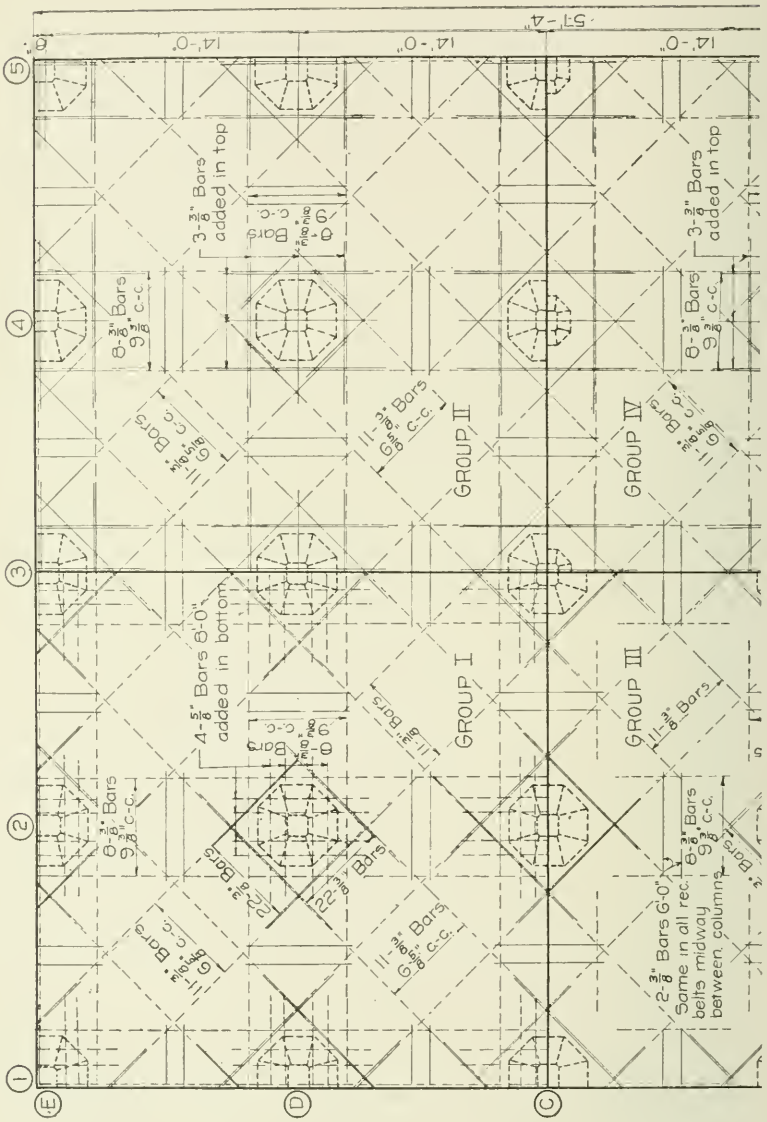
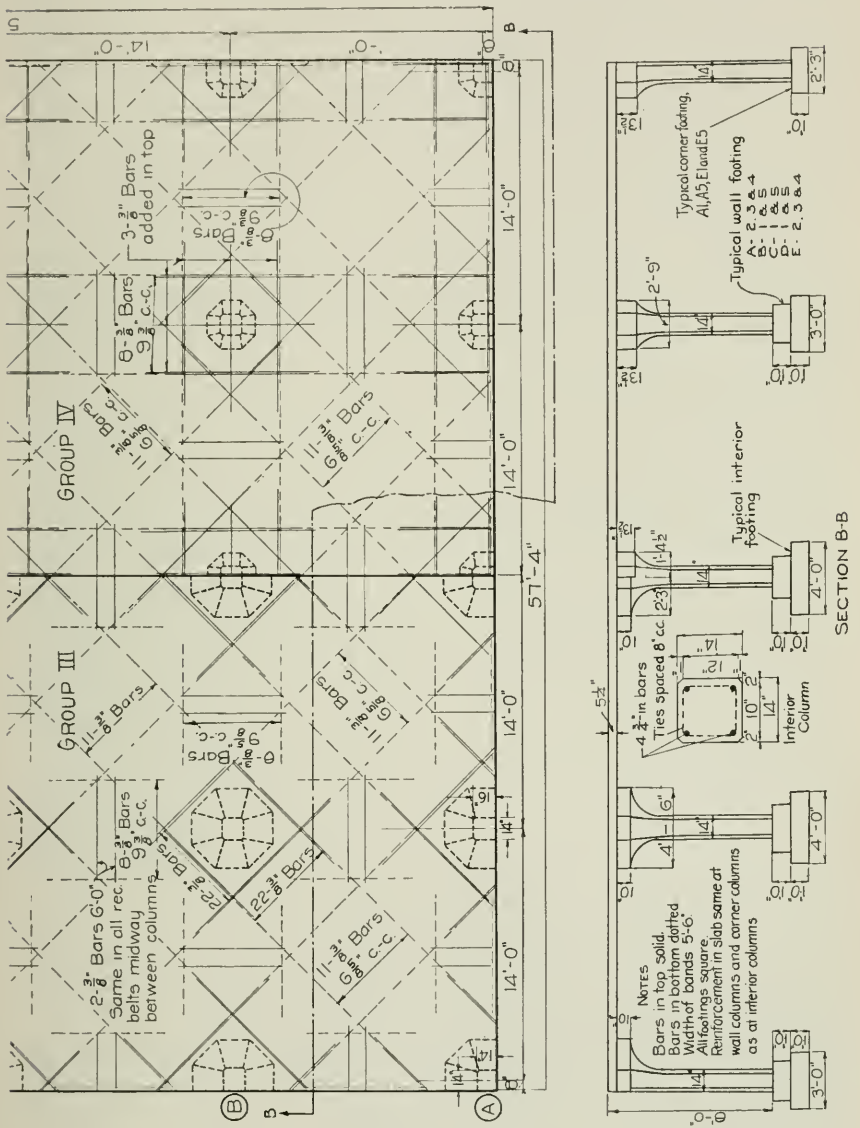


FIG. 58. DIMENSIONS AND REINFORCING

The footings for the interior columns were 5 ft. square, for the corner columns 3 ft. square, and for the columns at the edges 4 ft.



Plan for Worcester Test Floor.

square. It seems likely that the unit pressure on the soil was greater for the interior footings than for the footings at the corners and edges of the slab. Table 9 gives soil pressure calculated on the assumption that the total footing pressures were equal to the sums of the reactions of two systems of four-span, freely supported, continuous beams crossing each other at right angles.

35. *The Test.*—Preparations for testing were started on July 17, 1913, the loading began on July 28, 1913, and the last readings were taken on August 2, 1913. At the time of the test the concrete was about 3½ months old.

Gravel from the bank close at hand was used as loading material. The location of the slab on the side of the gravel hill allowed convenient access to loading material. A runway was built from the test slab to a point on the gravel bank somewhat above the elevation of the slab so that even at the higher loads little elevating of the loading material was necessary.

As explained in the following article, it was necessary to take strain gage readings early in the morning. This made it difficult to obtain readings with the load uniformly distributed over the slab. It was planned to make each increment of load such that it could be completed in a day, but these plans were interfered with by

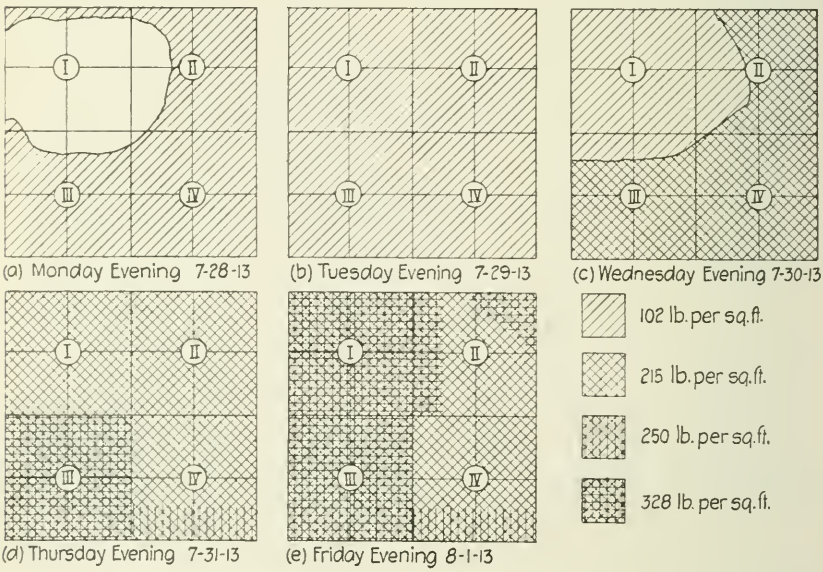


FIG. 59. PLAN SHOWING DISTRIBUTION OF LOAD OVER WORCESTER TEST FLOOR AT SUCCESSIVE STAGES OF THE TEST.

rainy weather, by labor troubles, and by the failure of a portion of the slab at a load between 215 and 250 lb. per sq. ft., a smaller load than it was expected to carry. Fig. 59 indicates the intensity of the load upon the various portions of the slab at various times. After taking the readings at the maximum load the test was discontinued, the slab being left with the load upon it.

Measurements were taken of deformation in the steel and in the concrete, of deflections at the centers of the four interior panels, and of settlement of footings.

Measurements of deformation were taken on 313 gage lines (see Fig. 60, 61, and 72). Of these 281 were on the slab and 32 were on the columns. Of the gage lines on the slab 113 were on the concrete and 168 on the steel. The strain gage observations were made by Professor H. F. Moore of the University of Illinois and Mr. Slater. Great care was exercised in obtaining initial or zero readings. On a predetermined set of gage lines each observer took two independent sets of strain gage readings. Then instruments and assignments of gage lines were exchanged and a third set of zero-load observations was taken. In this way three independent observations were taken on each gage line. Time was taken then to compare the corrected observations before proceeding with the loading. It was found that there was serious lack of agreement not only between the check readings taken by the two observers but also between check readings taken on the same gage line by either observer, the amount of the discrepancy depending somewhat on the time of day the observations were taken.

Evidence was found indicating that temperature variation throughout the day was responsible for the discrepancies between check readings and that the discrepancies could be reduced greatly by taking readings early in the morning. Accordingly, the first three series of zero-load readings were disregarded and one new series was taken by each observer on all the 313 gage lines, beginning at 4:20 a. m. August 28 and finishing a little before 9 a. m. of the same day. The readings thus obtained gave satisfactory checks on accuracy of observation. All subsequent strain gage readings except those used for special purposes were taken between about 4 a. m. and 7 a. m. This experience is given in some detail because it suggests some of the difficulties attending the carrying out of experimental work of this character.

In order to obtain information which might be expected to have the widest range of applicability without undue expenditure of time,

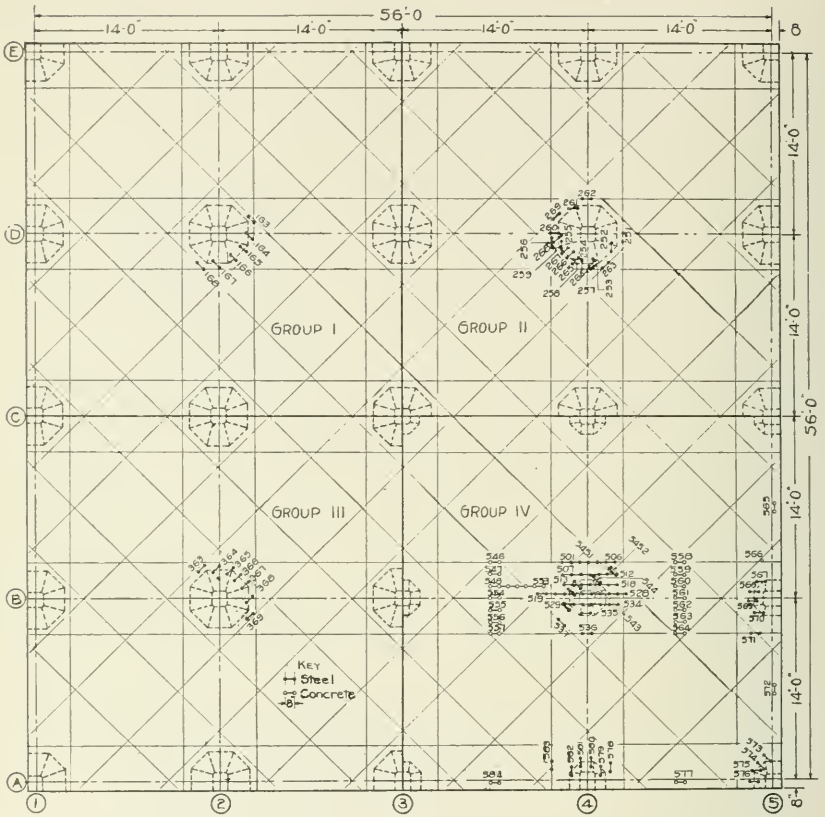


FIG. 60. LOCATION OF GAGE LINES ON UPPER SURFACE OF WORCESTER TEST FLOOR.

it was decided to use a large number of gage lines in one group and a smaller number in each of the other groups. It was expected that the ratios of stresses in interior panels to those at similar positions in wall panels would be about the same for all groups so that a detailed study of one panel would allow the missing parts of the data of the other panels to be filled in. Group IV was chosen as being the most suitable for this purpose because with its smaller capital it would be expected to show higher stresses and because the size of its capital seemed to be nearer general practice than was that of the other capitals. The results of the test seem to indicate that the assumption as to comparative values of stress in similarly located positions was not justified. This probably was due largely to the complications caused by the settlement of columns.

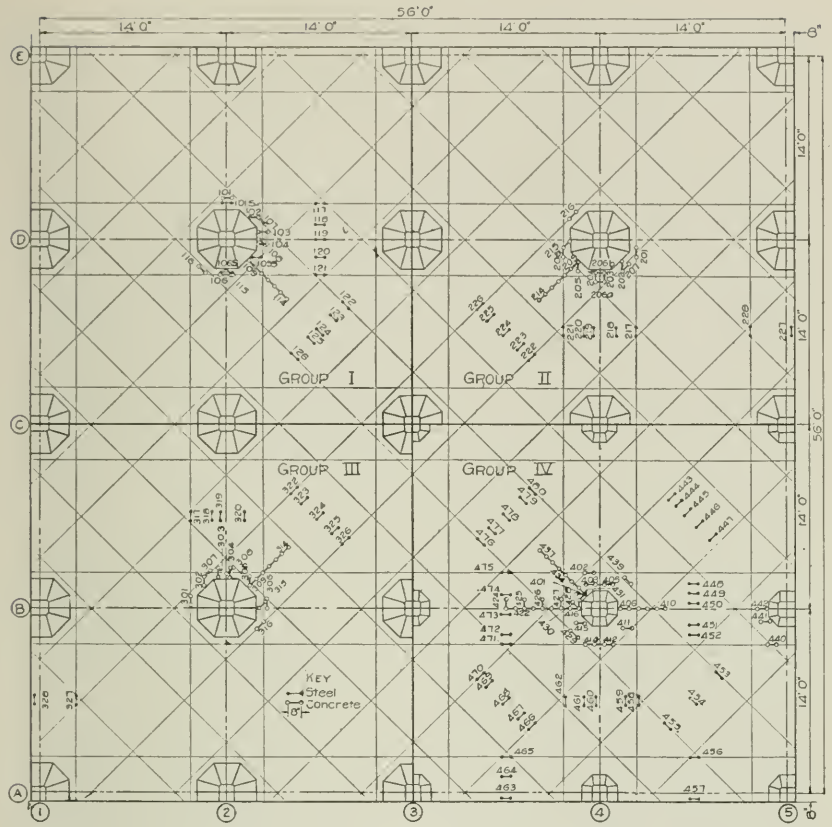


FIG. 61. LOCATION OF GAGE LINES ON UNDER SURFACE OF WORCESTER TEST FLOOR.

Deflections were measured at the center of the inner panel of each of the four groups, but these are so affected by the large settlement of column footings that slight importance is attached to them.

Settlement of footings was measured. A large amount of settlement, of course, was not anticipated and it was thought at first that measurement of settlement of the footings at the corners and at the centers of each of the four groups would be sufficient. These measurements were taken by means of a deflectometer (see Fig. 17a, Bulletin 64 of the University of Illinois Engineering Experiment Station). When a load of 100 lb. per sq. ft. had been applied to the slab it was found that the settlement was so large as not to justify so great refinement of measurement, and the difficulty of keeping the apparatus in adjustment in the presence of so many laborers led to the abandonment of this method and to the use of an engineer's level as a means of

measuring further settlement. Average amounts of settlement are shown in Table 9.

36. *Phenomena of the Test.*—The application of load was begun on Monday morning, July 28, 1913. The first increment of load, approximately 100 lb. per sq. ft. over the entire slab, was completed on July 29. Strain gage readings for the load of 100 lb. per sq. ft. were taken from 4:00 to 8:00 a. m. on July 30. At this load a crack was observed across gage line 581 (at the edge of the capital of the wall column), the stress at this place being 9000 lb. per sq. in. No other cracks were recorded for this load, but stresses in the reinforcement were nearly 10,000 lb. per sq. in. in several gage lines at the center column of Group IV on the diagonal band; also at the corner and wall columns of the same group. Therefore, it seems nearly certain that other cracks were present at such places.

At the load of 100 lb. per sq. ft. it was found that there was considerable settlement of the footings.

On July 30 additional load was applied to Groups II, III, and IV, bringing the load on this area up to 215 lb. per sq. ft. while Group I still had only 100 lb. per sq. ft. Strain gage readings were taken on July 31 from 4:00 to 7:00 a. m. with this load in position. This load is referred to later as the load of 215 lb. per sq. ft. and its distribution is shown in Fig. 59.

At this stage of loading cracks appeared on the outer surface of column D5 across gage lines 8 and 9, Fig. 73, which were on the column reinforcement at the lower part of the bell-shaped portion of the column. The unit-deformation in gage line 9 across the lower crack corresponded to a stress of 46,000 lb. per sq. in. In gage line 10, which was at the bottom of the bell-shaped portion of the column and below the cracks, so great an elongation had taken place that the reading was beyond the range of the instrument. A rough measurement indicated a unit-deformation at this place of 0.006. These high elongations indicate that at this stage of the test, however much slip of column bars there may have been, the cracks were in part due to the very high stresses in the reinforcement.

At the same stage of the test, at the central columns of the groups and midway between the columns the stresses in the slab reinforcement quite generally had reached 15,000 to 20,000 lb. per sq. in. The stresses in the slab reinforcement at the wall columns and at the outer corner columns of Group IV were much higher. The deformations at the latter places corresponded to from 40,000 to 50,000 lb. per sq. in. in several instances, a value beyond the yield-point of the

steel. The highest deformations in the slab reinforcement were in the diagonal band at column A5, and scaling of one of the bars at this place was noted, indicating that the bar had been stressed to the yield point.

At a load of a little more than 100 lb. per sq. ft. a crack had appeared on the bottom of the slab of Group IV, describing approximately a quadrant of a circle having a radius of about 6 ft. with column A5 in the center of the circle. This crack was fairly large when first noticed and increased in size quite rapidly as the loading progressed. After completing the strain gage readings for the load of 215 lb. per sq. ft. on Group II, III, and IV more gravel was placed upon Group I, bringing the load up to 215 lb. per sq. ft. over the entire four groups. No readings were taken at this stage but another increment was begun by applying load to Group IV in the position indicated in Fig 59d. About one-fourth of Group IV had been loaded to an intensity of 250 lb. per sq. ft. when failure of the slab in this group occurred, allowing the slab at the center of the corner panel to deflect about 8 in. and to settle down upon the 4 by 4-in. post which had been used as a datum for the measurement of deflections. Except for the presence of this post it seems likely that the slab in Group IV would have fallen at this time. Large tension cracks appeared at the middle of the exterior corner panel and over the edges of several of the column capitals of this group, and there were appearances of diagonal tension failure around column B4, but it seems likely that this was a result, or at most a secondary cause, of failure. That this is true is indicated by high deformations at various places under the load of 215 lb. per sq. ft. Some bending of column A5 was visible, although the column had shown no structural defects. Column B5 showed structural defects previous to applying the load and high stresses were developed in the reinforcement of the slab near this column, but the column did not fail. The defects appeared to be due mostly to the first pouring of the concrete having been carried too high on the column. A portion of the bell was filled with the first pouring, and the inability of the concrete in the expanded portion of the column to follow down into the shaft of the column, as shrinkage developed with setting, appears to have left voids in the concrete at the place where the bell begins. Some other columns showed the same features, but to a smaller extent than in the case of column B5.

The area and intensity of the load on Group IV at the time of its failure is shown in Fig. 59d. No more load was applied to this part of the slab and with continued application of load in the other groups

the signs of distress in column D5, noted in a previous paragraph, increased. Slipping of the concrete at the top of the column along the column reinforcing bar took place and the crack across gage line 8 opened greatly. At the same time bending in column E5 near the junction of the shaft with the bell became visible and a construction joint opened (see Fig. 73). When about three eighths of Group II was covered with 328 lb. per sq. ft., the remainder of the slab having only 215 lb. per sq. ft., the condition of this portion of the structure became critical and loading was discontinued.

The distribution of the maximum load over the entire slab is shown in Fig 59e. Yielding of column D5 seems to have brought about the critical condition existing in Group II at the maximum load. This column did not fail completely but it had yielded to such an extent that a heavy stress was thrown into the reinforcement extending from this column to column D4.

The settlement of the footings, which was observed to have begun before the completion of the 100-lb. per sq. ft. load, continued, and at the higher loads the difference in elevation of certain parts of the slab could be observed by sighting along the under sides of the column capitals. At the maximum load the settlement of column D4 was 2 1/16 in. Other columns had settled appreciably but no other so much as column D4, and there was no uniformity in the amount of settlement for the various columns.

37. *Usefulness of Results of Test.*—The large amount of settlement of the footings and its unevenness throw serious complications into the interpretation of the results of the test. It seems probable that the distribution of stresses may have been dependent as much upon the relative amounts of settlement of the various footings as upon the variation in distribution of the reinforcement. In the following paragraphs comparisons are given of the stresses and deformations found at various positions on the slab and the discussion of the effect of certain features of the design on the stresses. Such comparisons must be considered to be qualitative and not to show quantitative variations, and further tests may show errors in conclusions based upon such comparisons.

Notwithstanding the limitations to the usefulness of the data it is believed that a presentation of the observed deformations is of value. These are shown in Fig. 62 and 63.

38. *Effect of Variation in the Distribution of Reinforcement.*—An examination of Table 10 shows the same average steel stress in the slab at the central column of Group I as that at the central column of

Table 10.

Average Stresses in Tension Reinforcement; Comparison of Groups.

These stresses are based upon an assumed modulus of elasticity of 30 000 000 lb. per sq. in.

Group	Location and Direction of Gage Lines		Gage Lines	Stress in Reinforcement lb. per sq. in.	
				Load 102 lb. per sq. ft.	Load 215 lb. per sq. ft.
I	Midway between Columns	Rect.	117 to 121	5800	24700
		Diag.	122 to 126	1000	5200
	At Central Column	Rect.			
		Diag.	163 to 168	4300	8500
II	Midway between Columns	Rect.	217 to 221	5100	25700
		Diag.	222 to 226	2800	7300
	At Central Column	Rect.	251 to 256 and 257 to 262	3400 2400	7500 4500
		Diag.	263 to 268	4500	8500
III	Midway between Columns	Rect.	317 to 320	6700	14600
		Diag.	322 to 326	4000	8500
	At Central Column	Rect.			
		Diag.	363 to 368	5600	15400
IV	Midway between Columns	Rect.	*471 to 475	4000	18000
		Diag.	476 to 480	3900	10500
	At Central Column	Rect.	503, 509, 514, 522, 529, 535, 536	4200	23700
		Diag.	537, 538, 539, 540, 542, 542a, 543, 544, 545.1, 545.2	7200	26500
Ratio of stresses in group IV to average stresses in groups I, II, and III	Midway between Columns	Rect.		0.70	0.83
		Diag.		1.53	1.50
	At Central Column	Rect.		1.45	3.86
		Diag.		1.51	2.41

*Area immediately over these gage lines not fully loaded. See observation boxes, Fig. 74.

Group II at 215 per sq. ft., and nearly the same in the two locations at 102 lb. per sq. ft. This does not indicate that the difference in distribution of reinforcement had much effect on the steel stress. Likewise a comparison of the slab stresses and deformations on the upper and under sides of the slab at the columns of Groups I, II, and III (see Tables 10 and 11) shows little difference which it is thought can be attributed to the fact that Group I had compression reinforcement while Groups II and III had none.

39. *Effect of Variation in Size of Capital.*—A comparison of the stresses in the steel and the deformations in the concrete at the central capital of Group II with those at corresponding positions in Group IV may be expected to show the effect of the difference in the size of capi-

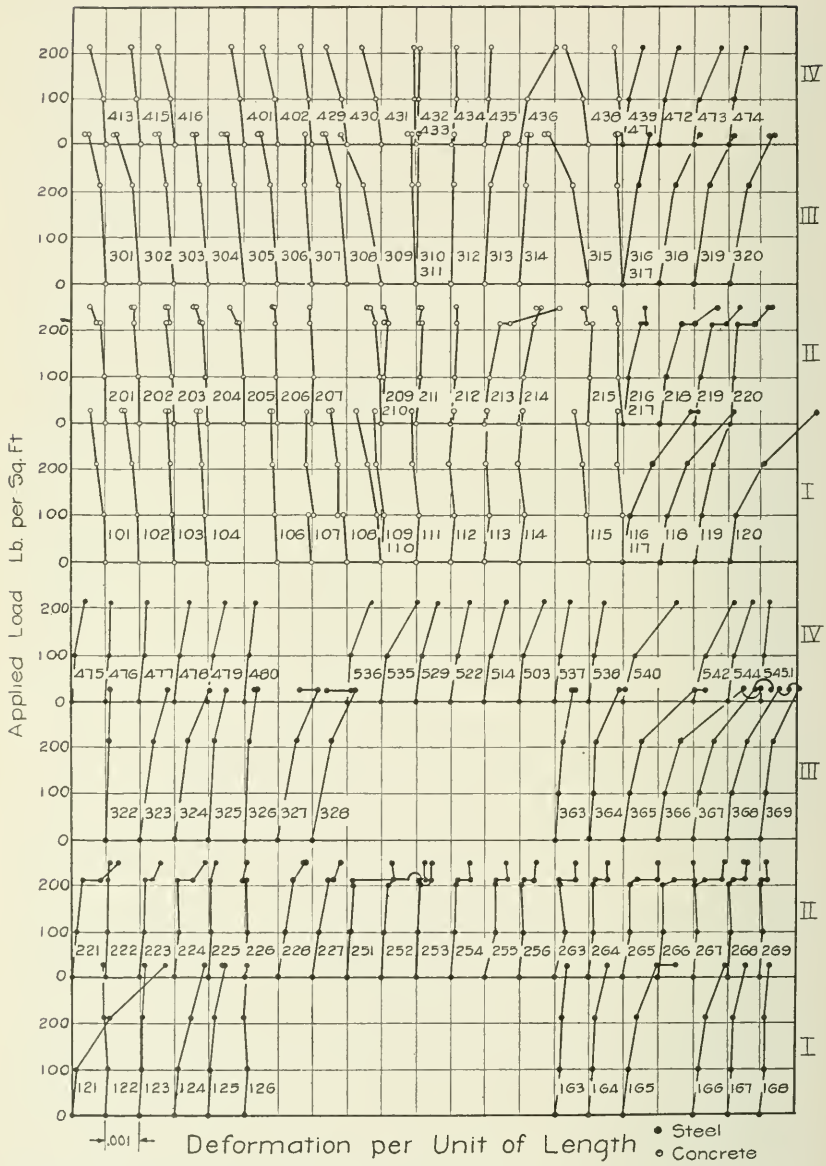


FIG. 62. LOAD-DEFORMATION DIAGRAMS ARRANGED FOR COMPARISON OF GROUPS IN WORCESTER TEST FLOOR

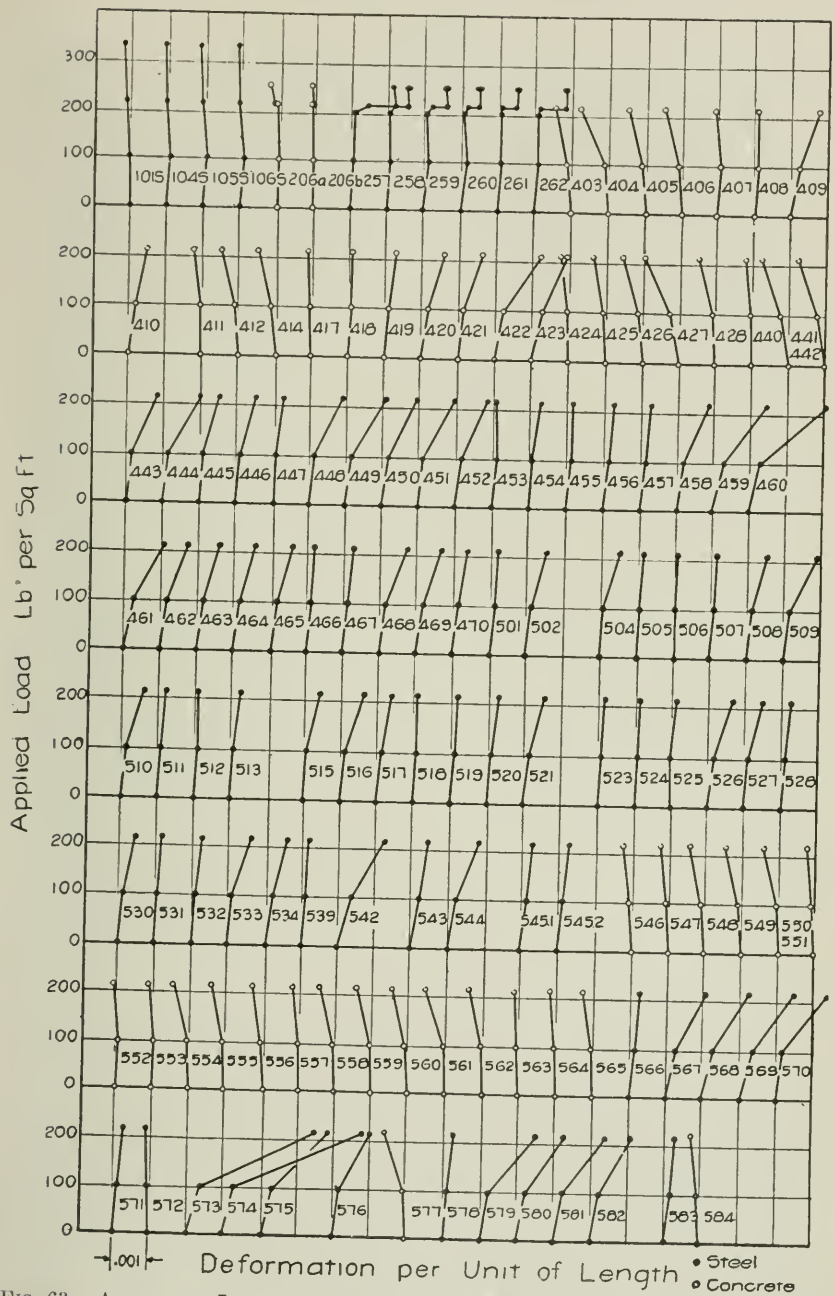


FIG. 63. ADDITIONAL LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES OF WORCESTER TEST FLOOR.

TABLE 11.

AVERAGE COMPRESSIVE UNIT-DEFORMATIONS IN CONCRETE; COMPARISON OF GROUPS.

Group	Location and Direction of Gage Lines		Gage Lines	Unit-deformation	
				102 lb. per sq. ft.	215 lb. per sq. ft.
I	At Central Column	Rect.	101 to 106	.000048	.000187
		Diag.	107, 108, 109, 115, 116	.000045	.000258
II	At Central Column	Rect.	201 to 206	.000038	.000197
		Diag.	207, 208, 209,* 216	.000063	.000099
III	At Central Column	Rect.	301 to 306	.000133	.000287
		Diag.	307, 308, 309, 315, 316	.000210	.000429
IV	Midway between Columns	Rect.	546 to 557	.000067	.000333
	At Central Column	Rect.	404, 401, 416, 415, 413	.000135	.000461
		Diag.	429, 430, 431, 438, 439	.000188	.000568

*215 indicated tension and was omitted.

tals since the design of these groups was the same in other respects. Such a comparison shows that the stresses and deformations were considerably larger in the case of the smaller capital (see Tables 10 and 11 and Fig. 62). A comparison of Groups II and III with Group IV shows nearly the same results.

Attention may well be called to the width and thickness of the large capitals which approach the dimensions sometimes given to the depressed head used over column capitals. It seems evident that the conditions are different from those of the ordinary column capital.

It is seen that the stresses in the steel in Group IV at the 102-lb. load were about 50 per cent greater than the corresponding average stresses in the other groups except at the center of the rectangular band in which case it was 30 per cent less.

The load of 102 lb. per sq. ft. was used for this comparison because at the higher load (215 lb. per sq. ft.) Group IV was on the point of failure, and the excessive stresses at this load are believed to be due partly to a yielding of certain parts of the slab, which apparently had already taken place.

40. *Comparison of Interior Panels with Exterior Panels.*—An examination of Table 12 indicates a smaller stress in the diagonal at the center of the corner panel of Group IV than at the center of the interior panel. In this connection it will be recalled that the corner

TABLE 12.

AVERAGE STRESSES IN TENSION REINFORCEMENT MIDWAY BETWEEN COLUMNS IN GROUP IV; COMPARISON OF INTERIOR PANELS WITH EXTERIOR PANELS.

By term corner panel is meant a panel having two exterior edges.
By term wall panel is meant a panel having one exterior edge.
By term interior panel is meant a panel having no exterior edges.

Band	Location and Direction of Gage Lines	Gage Lines	Stress in Reinforcement lb. per sq. in.	
			Load 102 lb. per sq. ft.	Load 215 lb. per sq. ft.
Diagonal	Corner panel	453, 454, 455	3100	5600
Diagonal	Wall panel	443 to 447 466 to 470	3700 4600	18600 14900
Diagonal	Interior panel	476 to 480	3900	10400
Rectangular	Between corner panel and wall panel	*448 to 452 458 to 462	3800 8100	29200 40800
Rectangular	Between wall panel and interior panel	*471 to 475	4100	18000
Rectangular	Exterior edge of wall panel	463, 464, 465	5900.	17900
Rectangular	Exterior edge of corner panel	456, 457	4500	8500

*Area immediately over these gage lines not fully loaded. See observation boxes, Fig. 74.

panel was the first to show distress by the appearance of a large crack across the diagonal band of reinforcement. The average of the steel stresses in the diagonal bands at the center of wall panels of Group IV was larger than the corresponding stresses in the interior panel for loads of 102 and 215 lb. per sq. ft. In the rectangular bands lying between wall panels and corner panels at points midway between columns the steel stresses were considerably greater than the stresses in the bands between the wall panels and interior panels. These comparisons are for cases where bending of columns at the edge or corner of the slab may be expected to influence the amount of the moment carried by the slab at the center of the span. The conclusion seems justified that the bending of the wall columns was sufficient to allow a material increase in the moment near the center of the rectangular span. The position and size of the cracks indicate that the proportional increase in the moment near the center of the diagonal in the corner panel was still larger though the location of the gage lines was such as not to show that this was true.

41. *Stress at Exterior Edge of Slab.*—Tables 13 and 14 were prepared to show a comparison of stresses and unit-deformation at the exterior edge of a panel with those at similar positions on an interior

TABLE 13.

AVERAGE STRESSES IN TENSION REINFORCEMENT; COMPARISON OF EXTERIOR RECTANGULAR BANDS WITH INTERIOR RECTANGULAR BANDS.

Group	Location of Gage Lines	Gage Lines	Stress in Reinforcement lb. per sq in.	
			Load 102 lb. per sq. ft.	Load 215 lb. per sq. ft.
II	Outer edge wall panel	227, 228	6800	17500
II	Between wall panel and interior panel	217 to 221	5100	26200
III	Outer edge corner panel	327, 328	*1800	18000
III	Between wall panel and interior panel	317 to 320	*3600	14500
IV	Exterior edge corner panel	456, 457	4500	8500
IV	Exterior edge wall panel	463, 464, 465	5900	17900
IV	Between corner panel and exterior panel	448 to 452 458 to 462	3800 8100	29200 40800
IV	Between wall panel and interior panel	471 to 475	4100	18000

*No values tabulated; these obtained from load-deformation curves.

TABLE 14

AVERAGE UNIT-DEFORMATION IN CONCRETE; COMPARISON OF EXTERIOR EDGES WITH INTERIOR EDGES OF PANELS

Plus indicates extension and minus indicates shortening

Location of Gage Lines	Gage Lines	Unit-deformation	
		Load 102 lb. per sq. ft.	Load 215 lb. per sq. ft.
Exterior edge of wall panel	565	—.00002	—.00031
	583	+.00107	+.00328
Between wall panel and interior panel	546 to 557	—.00007	—.00033
Exterior edge of corner panel	572	—.00002	—.00016
	577	—.00005	—.00064
Between corner panel and wall panel	558 to 564	—.00006	—.00041

edge of the panel. The conditions at an exterior edge of the panel are not shown to be more severe than at an interior edge. This fact is of importance in its bearing on the action of a flat slab built without beams at the exterior edges.

42. *Point in Inflection.*—In Group IV, for purposes of detecting the point of inflection, gage lines were placed on the upper and under surfaces of the slab along the line joining the centers of the columns B4 and B3. Fig 64 shows the positions of the gage lines and the variation in unit-deformation along this line. In using the figure it should be

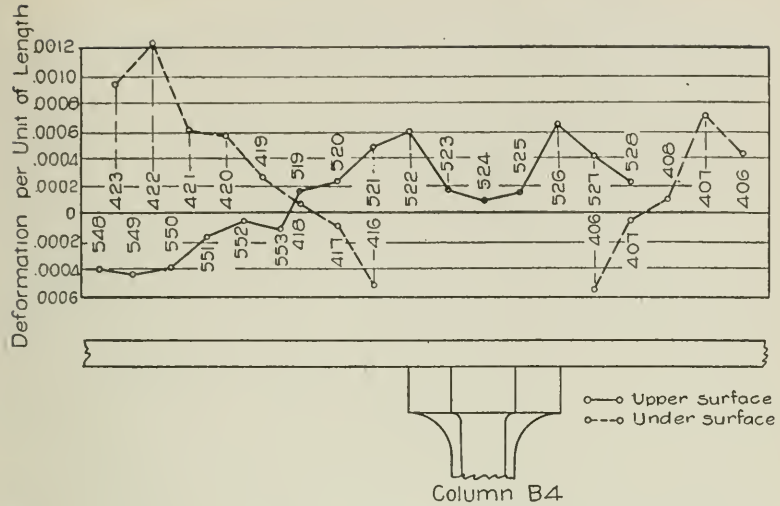


FIG. 64 LOCATION OF POINTS OF ZERO UNIT-DEFORMATION FOR UPPER AND UNDER SURFACES IN GROUP IV OF WORCESTER TEST FLOOR.

borne in mind that settlement of footings probably had much to do with the location of the point of inflection. It appears that the point of zero deformation on the upper side of the slab was farther from the column than that on the under side for both the interior panels and the wall panel. A probable position for the point of inflection, as shown by the intersection of the curves in Fig. 64, is about 33-in. from the column center for both wall panel and interior panels. This corresponds to 17/100 of the panel length from the edge of the column capital and to 20/100 of the panel length from the center of the column.

43. *Locus of Highest Stress.*—In Group IV gage lines were laid out on upper and under surfaces of the slab with a view to determining the locus of the highest stress developed in the elements of the slab parallel to rows of columns. The results for the upper surface are shown in Fig. 65. This diagram indicates that the position of highest stress in a bar crossing the capital is approximately at the edge of the capital. For bars not crossing the capital, the highest stress appears to be at the intersection of the bar with the center line of the row of columns. The locus of highest tensile stress appears to follow rather closely the outline of the capital through 180° and to turn off rather abruptly at the intersection of the capital with the line joining centers of columns. Although it is not possible to get a gage line short enough or close enough to the column to determine what the maximum compressive unit-deformation is, the results of the test indicate that the locus of highest compressive stress is in about the same position as that for tension except that it probably turns off less abruptly at the inter-

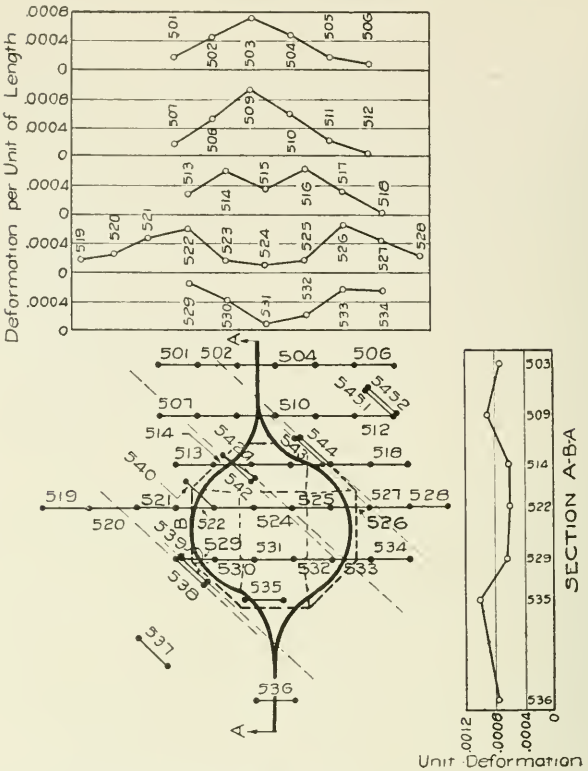


FIG. 65. LOCUS OF POINTS OF HIGHEST STRESS IN A RECTANGULAR BAND OF REINFORCEMENT IN TOP OF SLAB AT COLUMN D4, OF WORCESTER TEST STRUCTURE.

section with the line joining centers of columns. It will be noted that the compressive deformation in gage line 206b is less than that in 206a (Fig. 63). The same phenomenon has been observed in other tests, and is what may be expected.

It may be added that at the load of 215 lb. per sq. ft. the compressive deformations in the section of maximum negative moment were distributed along the section for the full panel width.

44. *Distribution of Stress over Cross Section of Bands.*—Fig. 66 and 67 show the variation of deformation across the width of several bands of reinforcement at sections midway between columns and close to the column capitals.

Some previous tests have indicated that midway between columns the stress distribution in the diagonal bands may be different from that in the rectangular bands, the unit-deformation being greatest in a bar at the edge of the rectangular bands and at the center of the diagonal bands. Fig. 66 indicates that in general the bars in the central por-

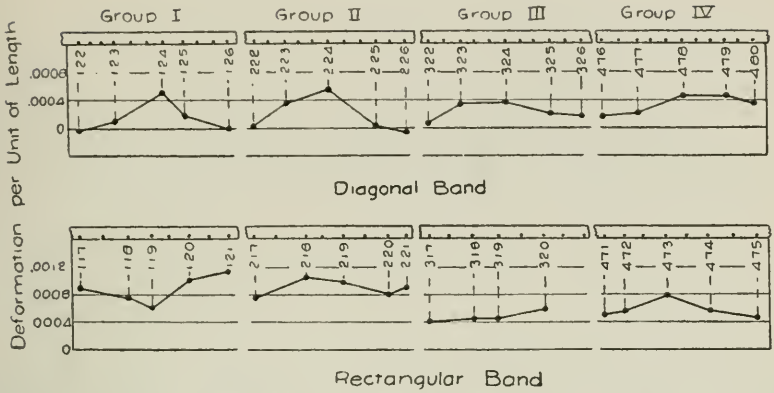


FIG. 66. DISTRIBUTION, MIDWAY BETWEEN COLUMNS, OF DEFORMATION IN DIAGONAL AND RECTANGULAR BANDS OF REINFORCEMENT OF WORCESTER TEST FLOOR FOR LOAD OF 215 LB. PER SQ. FT.

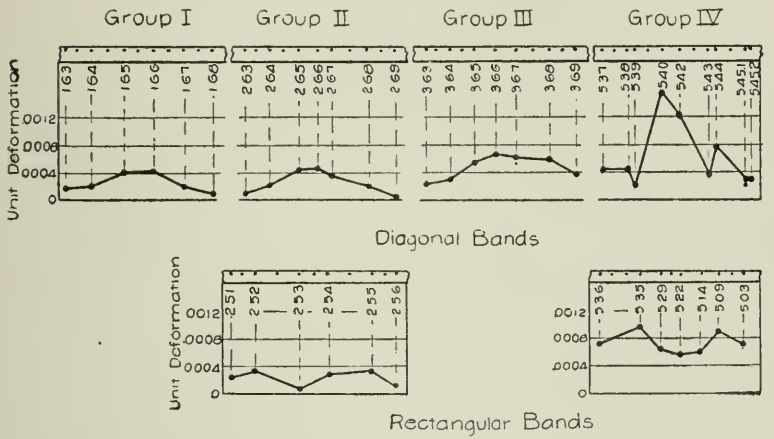


FIG. 67. DISTRIBUTION, NEAR COLUMN CAPITALS, OF DEFORMATION IN DIAGONAL AND RECTANGULAR BANDS OF REINFORCEMENT OF WORCESTER TEST SLAB FOR LOAD OF 215 LB. PER SQ. FT.

tion of the bands show higher stresses than those at the edge. There are one or two cases in which the reverse is true, but there is not sufficient regularity in this to justify the conclusion that this test shows a difference in stress distribution for rectangular and diagonal bands of reinforcement.

45. *Slip of Bars.*—The diagonal reinforcement was lapped at the column capital. The bars of each length extended 12 to 15 in. (32 to 40 diameters) beyond the center line of the column. At several places where the bars were lapped strain gage measurements were taken from one bar across to another for the purpose of determining the

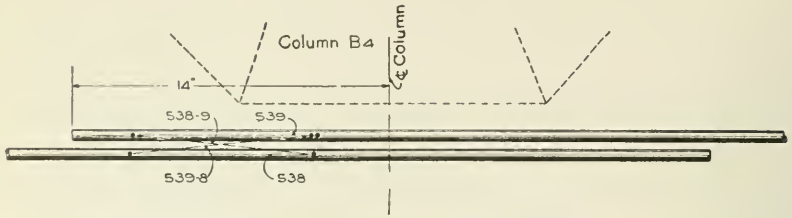


FIG. 68. ARRANGEMENT OF GAGE LINES ON WORCESTER TEST FLOOR FOR MEASUREMENT OF SLIP OF BARS.

amount of slip of the bars relative to each other. Fig. 68 shows the arrangement of gage lines 538, 539, 538-9, and 539-8, used for this purpose, and shows the position of the ends of the bars on which the gage lines were placed. The total change in gage length has been plotted in Fig. 69 as abscissas and the load as ordinates. The movements of the

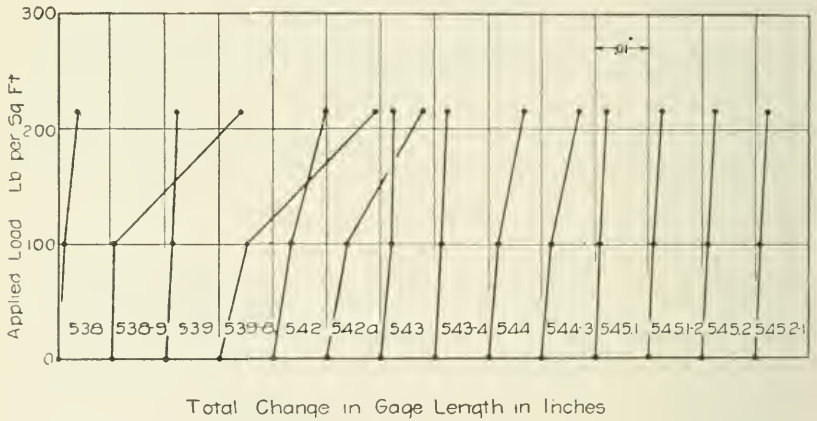


FIG. 69. LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES USED FOR DETERMINING SLIP OF BARS IN WORCESTER TEST FLOOR.

gage points on the two bars relative to each other for loads of 100 and 215 lb. per sq. ft. have been plotted in Fig. 70. In fitting probable curves to the plotted points it was assumed that the curves for the two bars had the same slope at the point where they cross the line joining the centers of the columns A3 and B4 (marked symmetrical axis across bars). This is the same as assuming that the tensile stresses in the two bars are equal at this point, and follows from the symmetry of the arrangement of the bars. If there had been no slip of the bars relative to each other, the curves would touch at the point on the diagram corresponding to the place where the bars cross the line joining the centers of columns A3 and B4. The vertical distance between

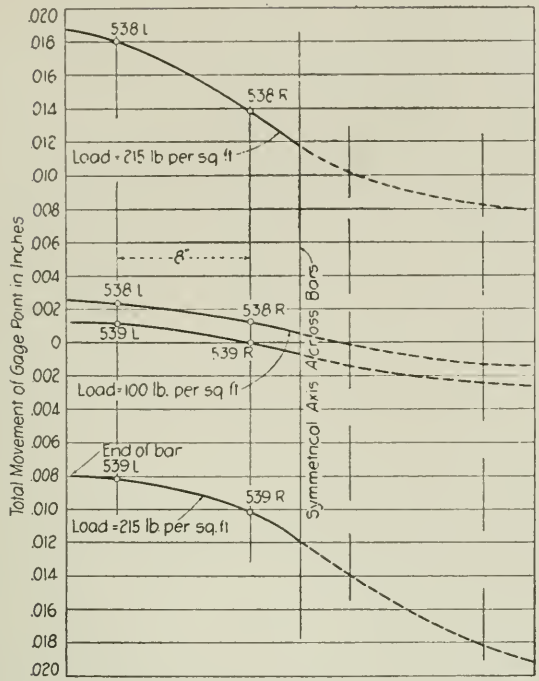


FIG. 70. DIAGRAM INDICATING SLIP OF BARS AT GAGE LINES 538 AND 539 IN WORCESTER TEST FLOOR.

the curves at this point represents the slip of the bars relative to each other.

Assuming that at this place the two bars slipped the same amount relative to a fixed point, but in opposite directions, the zero-line for slip measurements will bisect the vertical distance between curves at their intersection with the symmetrical axis as shown in Fig. 70. Ordinates to points on the two curves indicate the movements of the corresponding points on the bars relative to the line joining columns A3 and B4. Differences between ordinates at any two points on the same curve represent the total deformations taking place in the bar between the points considered. The stress in the bar at any point is proportional to the slope of the curve at the point.

The diagram indicates that at the load of 100 lb. per sq. ft. there was a slip of bars relative to each other of about 0.0013 in. At a load of 215 lb. per sq. ft. the slip of the bars relative to each other had increased to 0.024 in. at the same place. If the two bars did not act in the same way the slip at the place where they cross the center line of the columns may have been all in one bar. This would give a more

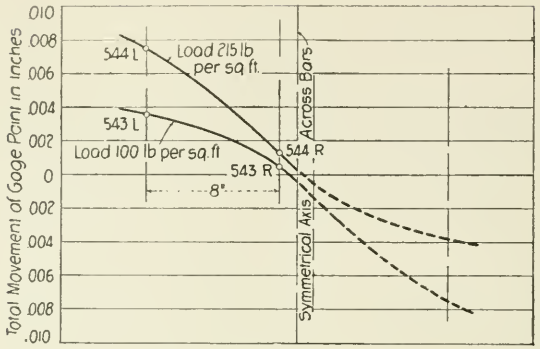


Fig. 71. Diagram Indicating Slip of Bars at Gage Lines 533 and 534 in Worcester Test Floor.

dangerous condition than that in which the slip is the same in both bars at this point.

Fig. 71 was constructed from measurements on gage lines 543, 544, 543-4, and 544-3 in the same manner as Fig. 70 was constructed. It indicates that at a load of 215 lb. per sq. ft. the slip of the bars relative to each other was only about 0.0006 in. It may be due to the smallness of this slip that the difference in stress in gage lines 543 and 544 is so much greater than the difference for gage lines 538 and 539. The difference in stress in the two bars is indicated by the difference in slope of the two curves at the point considered.

From measurements taken in the same way between gage lines 545.1 and 545.2 no slip of the bars relative to each other could be detected at the position of these gage lines, the centers of which were almost on the diagonal passing through the center of the column. At points on the bars near the ends it is likely that there was slip of the bars relative to each other. Measurements on gage lines 542 and 542a indicate at the right hand gage point a slip of one bar past the other of about 0.0078 in. at a load of 215 lb. per sq. ft. Whether this slip extended as far as the center line of the column would be of importance to know, but the observed data do not indicate whether it did or not.

It is not of special importance that these bars showed slip at the ends. This is to be expected if the design includes bars which end in a region of high stress. The significance of the measurements lies in the indications that there was slip at the point where the bar should develop high tensile stress, in the fact that this slip more than doubled with a doubling of the load although the tensile stress in the bar had

not reached the yield point, and in the large amount of slip which occurred in some cases. The danger from a small initial slip has been brought out in a series of tests on beams subjected to continued load (*unpublished*).

After the failure of the slab, end slip varying from ½ in. to 1 in. was found in the bars designated by gage lines 539, 542a, and 543, while in gage lines 545.1 and 545.2 no end slip was apparent. It may be of significance that the bars which showed a large amount of slip at failure were the same as those which showed early slip at the point of highest tensile stress, and that the bars (designated by gage lines 545.1 and 545.2), which showed no early slip at the point of highest tensile stress, likewise at the maximum load did not show a slip which was large enough to be observed by the unaided eye.

That the bond stress was high in other bars crossing the column capital is brought out by an inspection of Fig. 64 and 65 which show the varying unit deformations along the lengths of bars. Within a distance of 8 in. (between gage lines 525 and 526) a change in stress of 15,500 lb. per sq. in. took place. This corresponds to a bond stress of 187 lb. per sq. in. if the bond stress be considered to be uniformly distributed. The shape of the curve indicates that the intensity of bond stress at the edge of the capital must have been nearly twice as much as this and it may have been more. This will be well brought out by a consideration of the slopes at various points of any smooth curve which may be passed through this series of points (gage lines 522 to 526 Fig. 64).

46. *Moment Coefficients.*—With the large and irregular settlement of the footings it is not to be expected that the moment coefficients calculated from the observed stresses will have much quantitative value, but a study of the character of the results may be of use.

In a beam or slab loaded in any manner, the total bending moment at any section may be expressed as kWl in which k is a coefficient, W the load on one panel and l the panel length. Values of this coefficient for the positive moment and the negative moment of the reinforcement stresses in Group IV have been computed in the same manner as was done for the Schulze Baking Company Building (see

TABLE 15

CALCULATED MOMENT COEFFICIENTS FOR GROUP IV

Location	Applied Load, lb. per sq. ft.	
	102	215
Positive	.011	.019
Negative	.015	.030
Ratio	.73	.63

article 28). Table 15 gives these coefficients, each value being proportional to the resisting moment of the stresses in one rectangular band of reinforcement plus the components (in a direction parallel to that rectangular band) of the moment of the stresses in one diagonal band. Table 15 shows that these coefficients are much higher for the 215-lb. load than for the 100-lb. load, both around the columns and at sections midway between columns. The difference may be due partly to uneven settlement of footings and partly to partial failure of concrete in tension. In considering the relative values of the negative and the positive resisting moments developed, it should be borne in mind that the slip of the bars at the column would cause a smaller negative bending moment and a larger positive bending moment than would otherwise be the case.

It is seen that the coefficients given in Table 15 are less than those generally used in designing and still less than those found by the more conservative analyses. For the load of 102 lb. per sq. ft. the coefficient of bending moment calculated from the observed stresses was even less than that for 215 lb. per sq. ft. It is probable that even at the higher load the tensile strength of the concrete has played a considerable part.

In Groups I, II, and III, not enough gage lines were read to permit

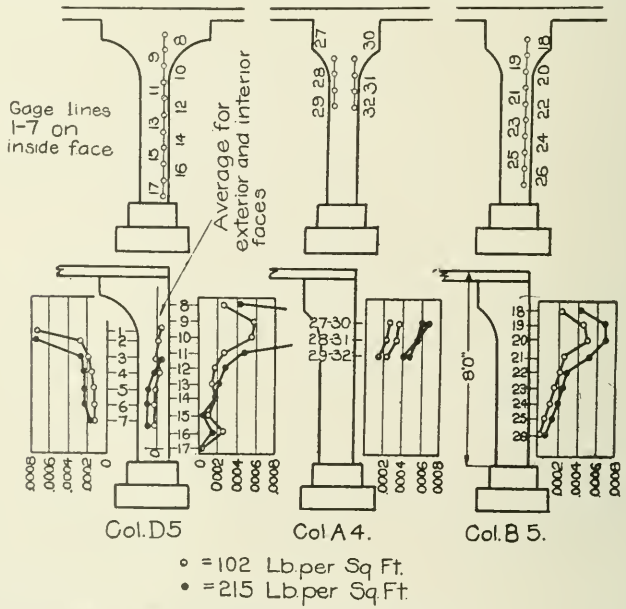


Fig. 72. Diagram Showing Deformations in Columns A4, B5, and D5 of Worcester Test Structure.

the calculation of moment coefficients, but it has been shown that the stresses in these groups were lower than those in Group IV.

47. *Deformation in Columns.*—Fig. 72 shows unit-deformations in the reinforcement and in the concrete of columns A4, B5, and D5. These measurements were made for the purpose of detecting bending in the wall columns. The gage lines on the reinforcement were on the bars nearest the outer surface of each column; those on the concrete were on the inner surface of Column D5, that is on the face toward the interior of the panel.

On column D5 (see Fig. 72) deformations were observed on opposite sides of the column. In the lower part of the column where there were no cracks the neutral surface (so far as flexure is concerned) would lie on the center of the column and the average of the unit-deformations measured on opposite sides of the column should be the unit-deformation due to the direct compression. It is found that this average unit-deformation was about 0.00004 at a test load of 215 lb. per sq. ft.

Near the upper end of the column the tension was so large as to

FIG. 73. VIEW OF COLUMNS D5 AND E5 OF WORCESTER TEST STRUCTURE WITH MAXIMUM LOAD IN POSITION.

cause large cracks (see Fig. 73) and slipping of the outer vertical bars, thus throwing the neutral axis closer to the compression than to the tension side of the column. Averaging the measured deformations for this region of the column, therefore, does not eliminate the flexural deformations. A resultant elongation at the center line of this part of the column should be expected and Fig. 72 indicates that elongation occurred at that place.

For a column fixed at the top of a fixed footing and rigidly connected with the slab at the top of the column but having no lateral movement there, the point of inflection is at a point one-third the column height above the top of the footing. The measurements did not extend down far enough on this column to determine the location of the point of inflection of the column, although the series of gage lines covered 0.93 of the distance from the slab down to the top of the footing. This indicates that there must have been sufficient tilting of the footing to modify the condition of restraint of the column at the top of the footing.

Although no strain gage readings were taken on a corner column, bending of the column at the outer corner of Group IV was observable to the unaided eye at a comparatively early stage in the loading, and at the maximum load bending was pronounced (see Fig. 73). The indications were that bending was more severe in the corner columns than in the columns at the side of a loaded area.

An examination of Table 16 indicates that in Group IV at a load of 102 lb. per sq. ft. the stress in the outer reinforcing bars of the wall columns near the bottom of the capital (12000 lb. per sq. in.) was higher than the average stress in the slab reinforcement where it crossed the edge of the capital of the same wall columns (8000 lb. per sq. in). In Group II, which had larger capitals, no measurements were taken of deformation in the slab reinforcement at a wall column, but the stress in the reinforcement of the wall column, D5, at a load of 102 lb. per sq. ft. was higher (16000 lb. per sq. in.) than that in either wall column of Group IV. However, for the load of 215 lb. per sq. ft. the table indicates that in Group IV the average stress in the slab reinforcement at the wall columns (35000 lb. per sq. in.) had become larger than the stress in the outer reinforcing bars of the columns (19000 lb. per sq. in.). At the same time the stress in the reinforcement of a wall column of Group II (Column D5) had passed the yield point.

In Group II the larger capitals afforded additional stiffness to the slab but very little more to the columns. In this group a very severe

TABLE 16.

STRESSES IN TENSION REINFORCEMENT; COMPARISON OF SLAB AND COLUMN AT WALL.

Location of Gage Lines	Gage Line	Stress in Reinforcement lb. per sq. in.		Average Stress lb. per sq. in.	
		Load 102 lb. per sq. ft.	Load 215 lb. per sq. ft.	Load 102 lb. per sq. ft.	Load 215 lb. per sq. ft.
GROUP IV On Slab at Column A4	567	8100	30600	8000	36700
	568	8300	38800		
	569	11200	34200		
	570	4500	43200		
GROUP IV On Slab at Column B5	579	8000	44300	8800	40800
	580	9500	37200		
	581	8900	41000		
GROUP IV On Column A4	27	10700	18700	9200	18800
	30	7600	19000		
GROUP IV On Column B5	20	14200	20000	14200	20000
GROUP II On Column D5	9	16000	Beyond yield point	16000	Beyond yield point

bending of Column E5 and high bending stresses in D5 followed by complete failure of the latter column occurred before the stress in the slab reinforcement at the central column had reached the yield point. In Group IV no column failed completely, although the bending in the slab was marked, the stress in the slab reinforcement at the central column having reached the yield point.

The results of the investigation of columns are not extensive enough or definite enough to furnish a basis for conclusions on the amount of moment carried by the columns. However, using as a criterion the ultimate loads carried and the manner of failure for Groups II and IV, the behavior of the columns relative to the slab was such as would be expected.

48. *Cracks.*—Fig. 74 gives the location of cracks on the under surface of the slab as shown by an examination at the maximum load and also those found on the accessible portions of the upper surface. The first large crack which was observed was in the bottom of the slab in the outer corner panel of group IV (see Fig. 74). It took approximately the form of a quadrant of a circle having a radius of about 6 ft. with column A5 at the center of the circle. This crack appeared soon after the load of 102 lb. per sq. ft. was applied and it opened rapidly as the loading continued. At a load of 215 lb. per sq. ft. the panel failed with the continued opening of this crack. Probably sev-

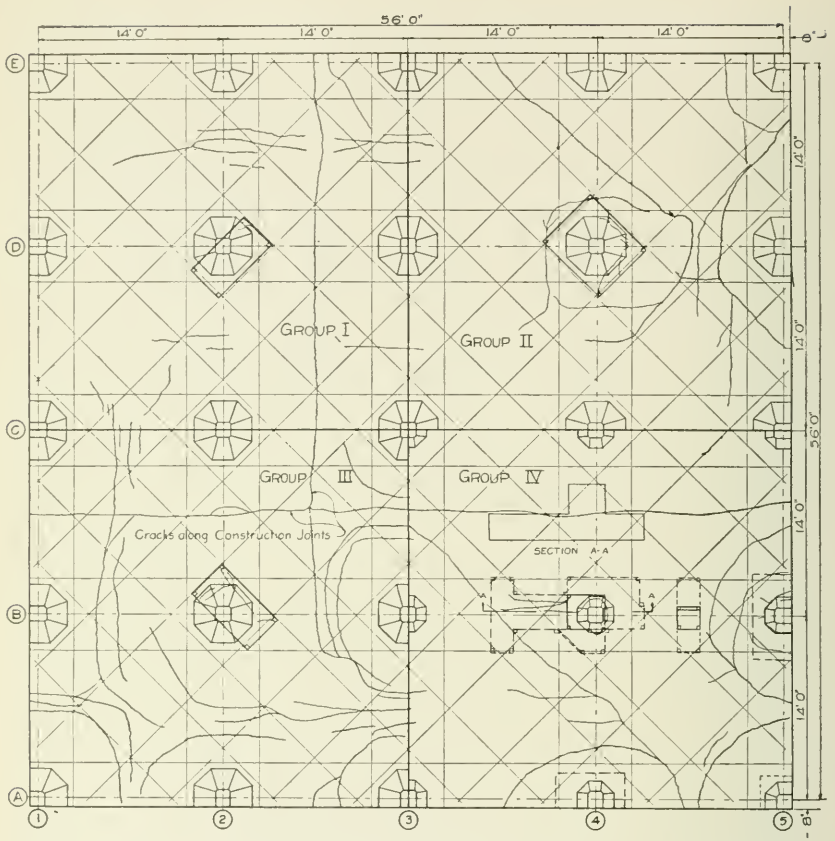


FIG. 74. CRACKS FOUND ON UNDER AND UPPER SURFACES OF WORCESTER TEST FLOOR AT MAXIMUM LOAD.

eral causes contributed to the location of the crack at this place. The unbalanced load caused a bending in Column A5, and it may be expected that this threw the point of maximum positive bending moment outside of the center of the diagonal span. At the short distance from the column at which the crack was located the reinforcement of the diagonal band may not have reached its position in the bottom of the slab. It should be borne in mind that high stresses were observed over the edge of the capital of column A5 in the bars of the diagonal band and that considerable bending of column A5 was visible but that the column did not fail.

In the corner panel of Group II the first prominent crack found took approximately the form of a circle around the corner column

(Column E5) much the same as the crack in Group IV, and a large amount of bending occurred in the corner column, but failure in this group seems to have been due to a weakness in the wall column, D5. In this column the concrete slipped past the upper ends of the vertical reinforcing bars, allowing a crack to form on the outer surface of the column somewhat above the bottom of the column capital, and the column failed in flexure.

The top of the slab was covered with loading material so that the general location of cracks could not be observed. At four interior columns (B2, B4, D2, and D4) the boxes used to give access to gage lines permitted the inspection of the upper surface of the slab over a portion of the column head. Well defined cracks just inside the outline of the capital, as shown within the boxes, were found. At two wall columns (A4 and B5) similar cracks were found.

Uneven settlement may have influenced the formation of cracks. It will be seen from Fig. 74 that many of the cracks are in locations where the reinforcing bars may be expected to be not close to the lower surface. In Group I, having the bars of the rectangular bands carried at the bottom of the slab through to its boundaries, the cracks were comparatively few and small. In this portion of the slab the settlement of the footings was relatively small and regular.

49. *Summary of Results.*—Although settlement of footings puts a serious limitation on the general applicability of the results of the test the following summarized statement of the information obtained is made:

1 In groups having capitals of the same size the variation in stress due to the variation in arrangement and distribution of reinforcement was less, apparently, than that due to uneven settlement of columns.

2 At a load of 102 lb. per sq. ft. the steel stress at the small capital (Group IV) averaged about 50 per cent greater than the stress at the larger capitals. The diameters of the capitals were respectively 0.196 and 0.321 times the panel length. The large stress in Group IV may be due partly to other causes, but it is believed that the small capital is the most important cause.

3 The wall panels and corner panels showed higher steel stresses and generally more pronounced cracks than did the interior panels.

4 This test does not indicate that the wall panels (not the corner panels) suffered because of the absence of wall beams.

5 The point of inflection in Group IV (the group having small capitals) was about two-tenths of the panel length from the center

of the central column, but its exact location is uncertain, since the point of zero unit-deformation on the under surface of the slab was closer to the column than that on the upper surface. The location of the point of inflection probably was influenced by the uneven settlement of the columns.

6 The locus of highest stresses in the bars of a band of reinforcement at a column head followed fairly closely the outline of the column capital through 180°, then branched off and followed the line joining centers of columns. The locus for the compressive stresses on the under side of the slab parallel to a given band occupied a corresponding position so far as may be determined from the data of the test.

7 In few of the bands of reinforcement in which measurements were taken was the stress higher in the bars on the edge of the band than in the central bars. In most cases the stress was highest in the central bars.

8 In cases where bars were lapped as much as 50 diameters beyond the point of maximum stress slipping at that point occurred without the stress having passed the yield-point strength of the steel. The slipping of these bars supports the ruling frequently made that bars should not be spliced at regions of maximum stress. The slipping of bars evidently affected the action of the slab and may have induced failure.

Bars which did not slip were found to have developed a bond stress averaging over the entire gage length 187 lb. per sq. in. At portions of the gage length the bond stress must have been much higher than this.

9 Moment coefficients calculated on the basis of the measured stress in the steel were materially higher at the higher load. Though even at the higher load the coefficients were low, the rapid increase with increased load confirms the view that there is danger in placing reliance on the stresses in the steel measured at ordinary test loads as a basis for determining moment coefficients to be used in design.

10 The bending of corner columns and wall columns was an important feature of the action of the test structure. In certain instances this bending was apparent to the eye.

11 The first large crack on the under surface of the slab was in the corner panel of Group IV near where the diagonal bars were carried from the top of the slab to the bottom.

The location of cracks in the other groups seems to have been influenced by the settlement of the footings, the bending of the outer

columns, and the position of the point of carrying the reinforcing bars from the top to the bottom of the slab.

VI. The Test of the Factory Building of the Curtis-Leger Fixture Co.

50. *Description of the Building.*—The floor loaded is in the addition to the Curtis-Leger Fixture Company's factory near Van Buren and Aberdeen Streets, Chicago. This addition is 53 ft. 6 in. by 57 ft. The floor is an 8-in. flat slab of reinforced concrete and is supported on reinforced concrete columns placed 19 ft. and 17 ft. 10 in. apart center to center in the two rectangular directions. It was designed for a live load of 200 lb. per sq. ft. and a dead load of 100 lb. per sq. ft.

A distinctive feature of the Barton Spider Web System of Reinforcement is the use of unit mats as reinforcement for the negative moment at the columns, the mats being independent of the reinforcement for positive moment in the slab. The bars in these mats extend in two directions only, parallel to the sides of the panel. The mats in this building were made up in the shop and consist of ⅝-in. square Havemeyer bars placed as shown in Fig. 75. Certain loose bars were added in this case, and these extend beyond the outlines of the mat. For the positive moment at the center of the panel, four-way reinforcement of ½-in. square Havemeyer bars was used. These bars were not bent up to the top of the slab, but extend along the bottom of the slab

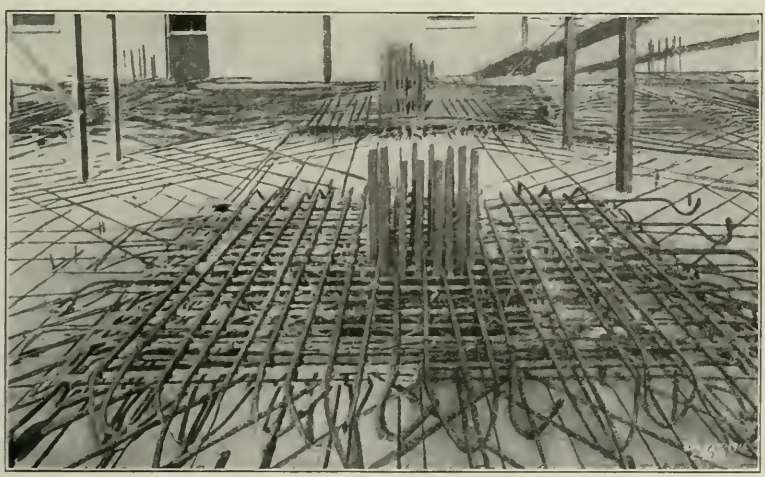


Fig. 75. View Showing Slab Reinforcement in Place in Test Floor of Curtis-Leger Company Building.

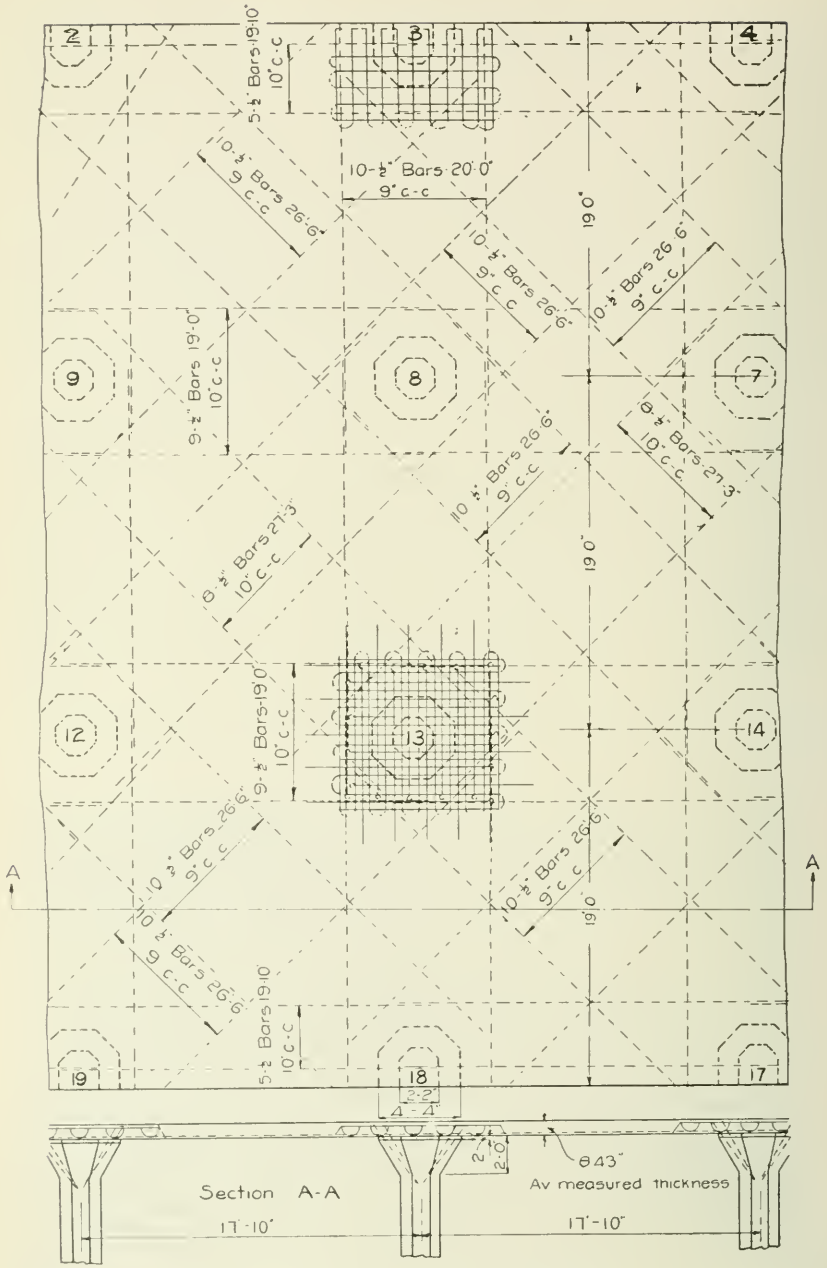


FIG. 76. PLAN SHOWING DIMENSIONS AND REINFORCEMENT OF TEST FLOOR OF CURTIS-LEDGER COMPANY BUILDING.

into the region of compression. They end 6 in. and 7 in. beyond the column center lines in the long and in the short panel lengths, respectively, and 3 in. to 7 in. beyond the column center-line in the diagonal directions. These bars thus serve as reinforcement in compression in the region of the column and in tension at the middle portion of the panel. No drop or depressed head at the column capital was used. The average measured depths to the center of gravity of the reinforcement were 7.04 in. for positions of negative moment and 8.55 in. for positions of positive moment.

The reinforcement for the wall panels is slightly in excess of that for interior panels. Spandrel beams 12 by 22 in. in cross section extend between exterior columns.

The position of the reinforcing bars is shown in Fig. 76.

51. *The Test.*—It was desired to make the amount of load handled as small as possible consistent with developing reasonably high stresses. Instead of covering four entire panels with 200 lb. per sq. ft. and then covering two entire panels with 400 lb. per sq. ft., it was believed to be more satisfactory to use equivalent total areas made up of parts of six panels, as shown in Fig. 77, first with a load of 200 lb. per sq. ft. (boundary dotted in the figure), then rearranging the load to give 400 lb. per sq. ft. over the other parts (boundaries in full line), then adding enough to the latter area to make 500 lb. per sq. ft. This plan gave more complete loading over the principal band of reinforcement and produced negative moments in this region greater than would be obtained under other likely conditions with only two panels loaded. This arrangement of the 500-lb. load also gave easier access to gage lines for the measurement of stress in the vicinity of the columns.

Aside from determining the action of the type of reinforcing, it was expected to obtain information bearing on (a) lateral distribution of stress in bands midway between columns and at the columns and (b) longitudinal distribution of stress along rectangular and diagonal spans. Fig. 78 shows relative location of gage lines on the upper and under surfaces of the test floor.

When the forms were ready and before the concrete had been poured small beveled iron plugs were nailed to the forms in such position that when the forms were removed the smaller base of each plug was in the plane of the under surface of the slab and in proper location for the gage holes used for the measurement of deformation in concrete. Corks were attached to the bars at such places as gage holes in the reinforcement were desired. These steps greatly facilitated the

preparation for the test. The concrete was poured on July 30, 1913, and at that time six concrete control cylinders 7½ in. in diameter and 16 in. long were made from concrete taken from the mixer in use on the job. The cylinders were tested at the University of Illinois, November 13 and 14, 1913.

The floor test was made October 22 to 30, 1913. For loading material bags of cement were used. An area equivalent to four panels, shown

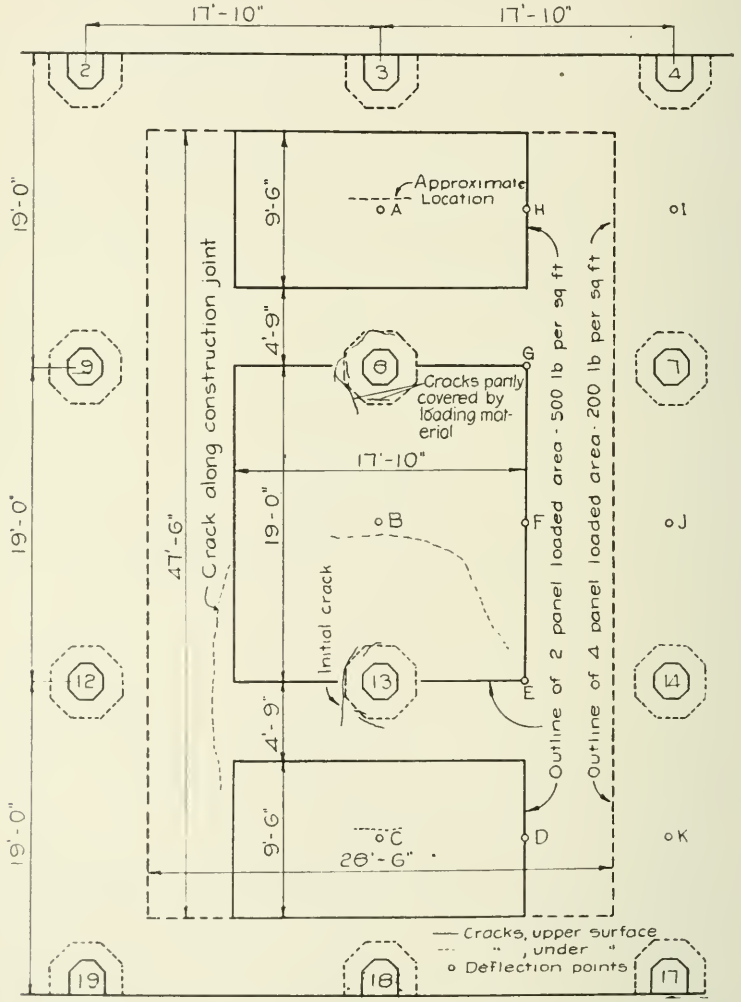


FIG. 77. PLAN INDICATING DISPOSITION OF LOAD AND LOCATION OF DEFLECTION POINTS ON TEST FLOOR OF CURTIS-LEGER COMPANY BUILDING.

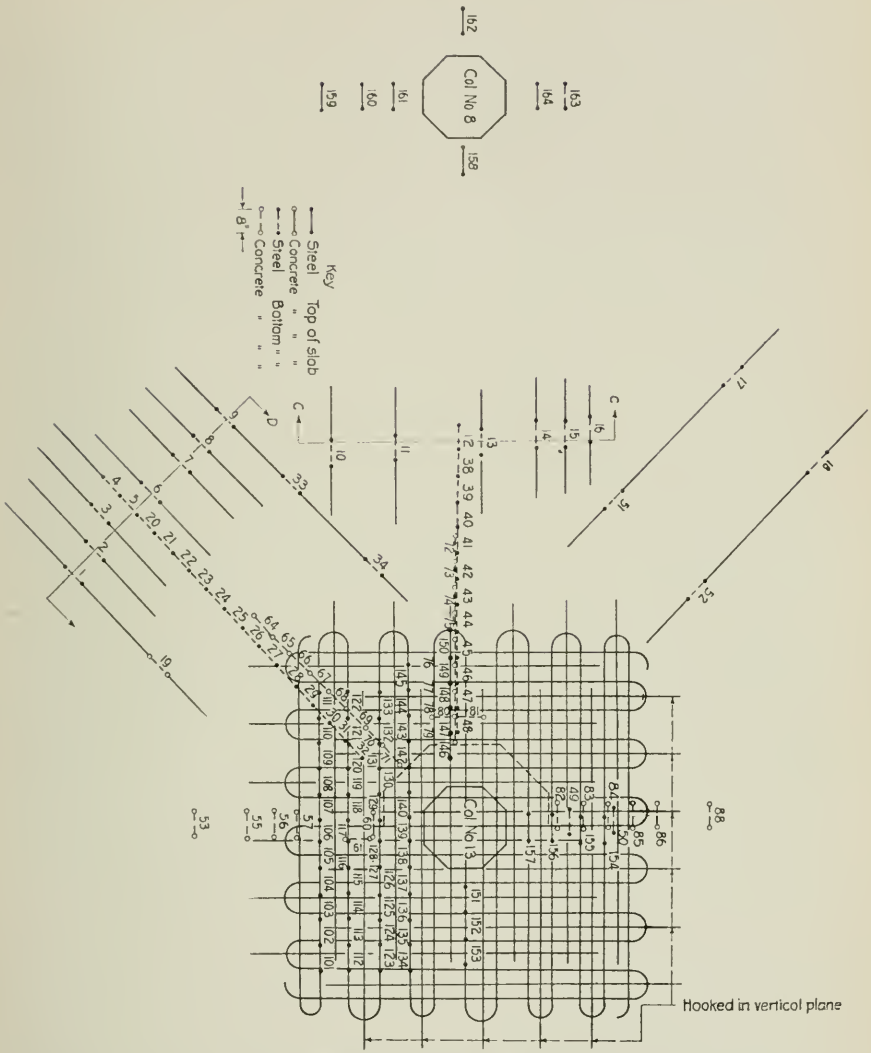


FIG. 78. LOCATION OF GAGE LINES ON UPPER AND UNDER SURFACES OF TEST FLOOR CURTIS-LEGER COMPANY BUILDING.

in Fig. 77 was loaded to 200 lb. per sq. ft., the design live load. The load was then removed from one-half of the area, and the total load was concentrated on an area equivalent to two full panels, shown in Fig. 77. Enough additional load was later applied to the two-panel area to bring the average intensity of the applied load up to 500 lb. per sq. ft. over this area of two panels.

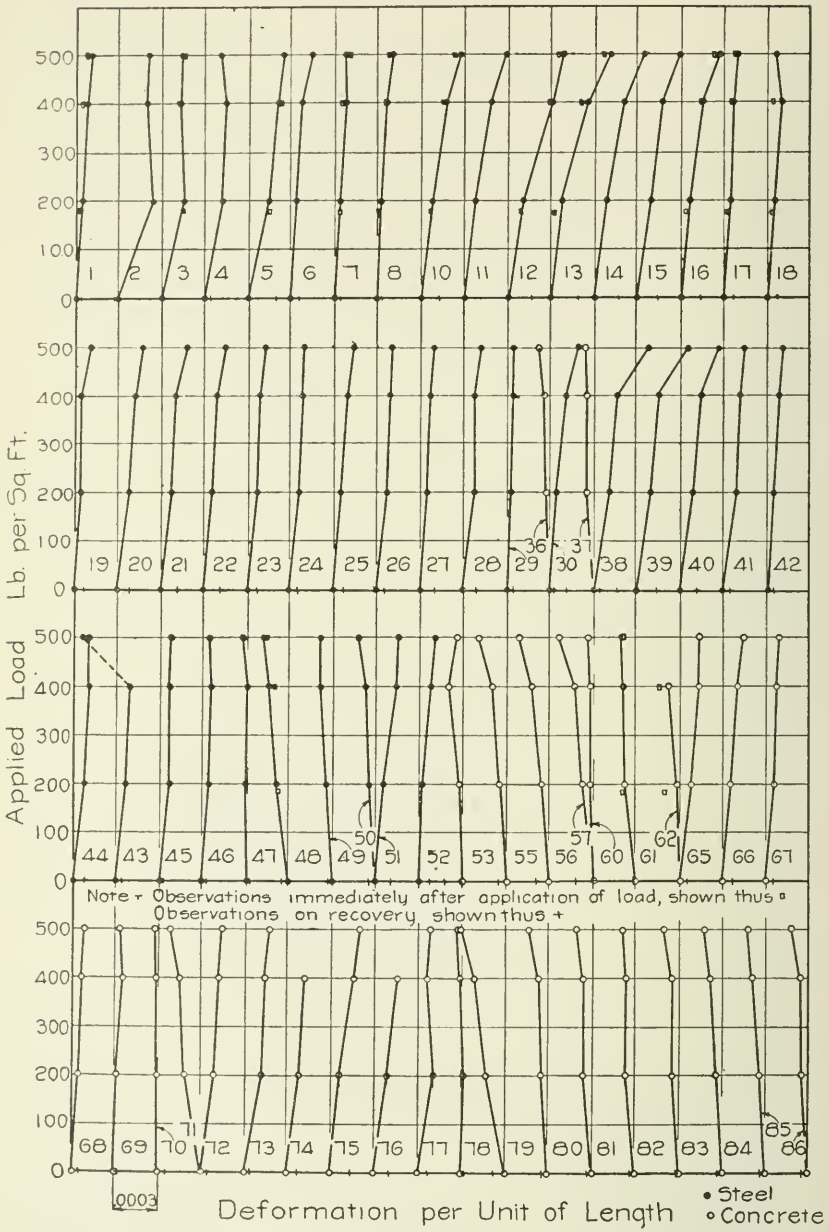


FIG. 79. LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES 1 TO 86 OF TEST FLOOR OF CURTIS-LEGER COMPANY BUILDING.

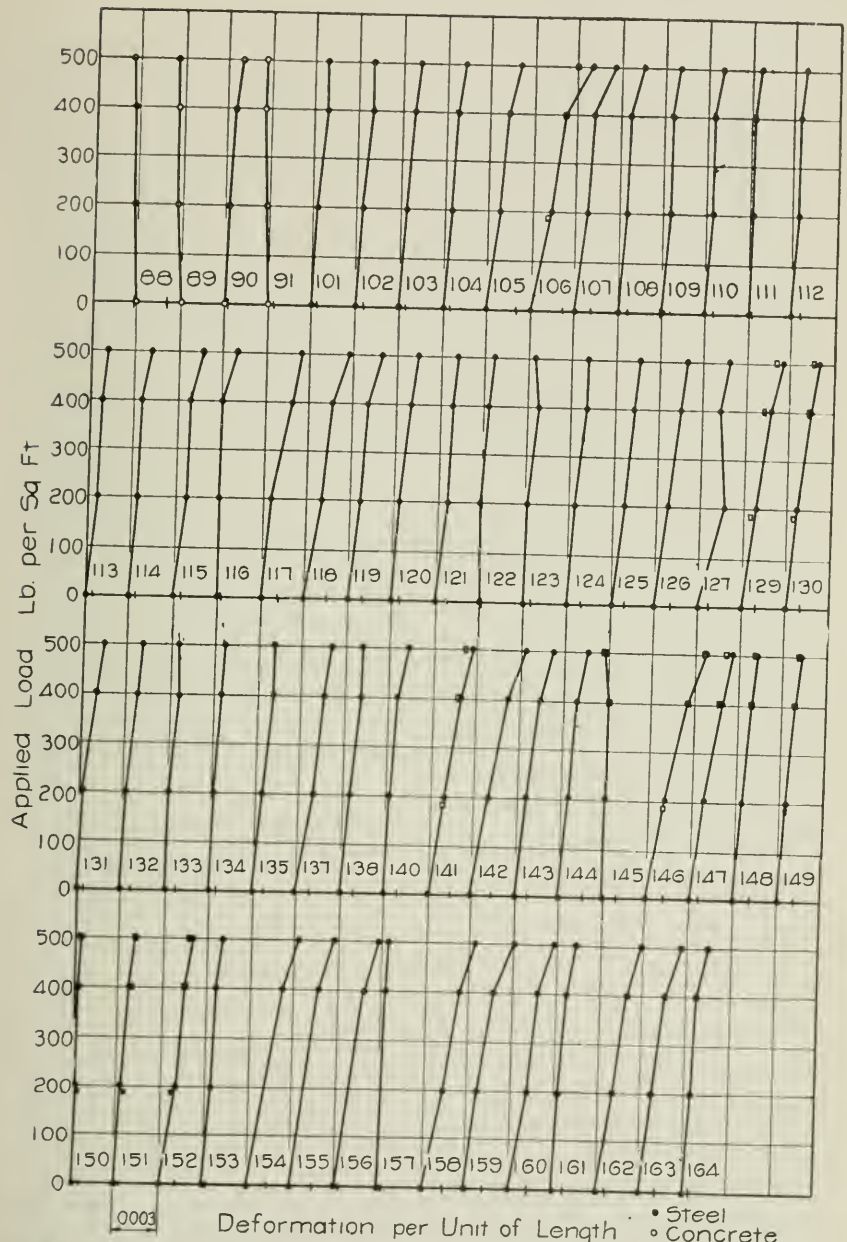


FIG. 80. LOAD-DEFORMATION DIAGRAMS FOR GAGE LINES 88 TO 164 OF TEST FLOOR OF CURTIS-LEGER COMPANY BUILDING.

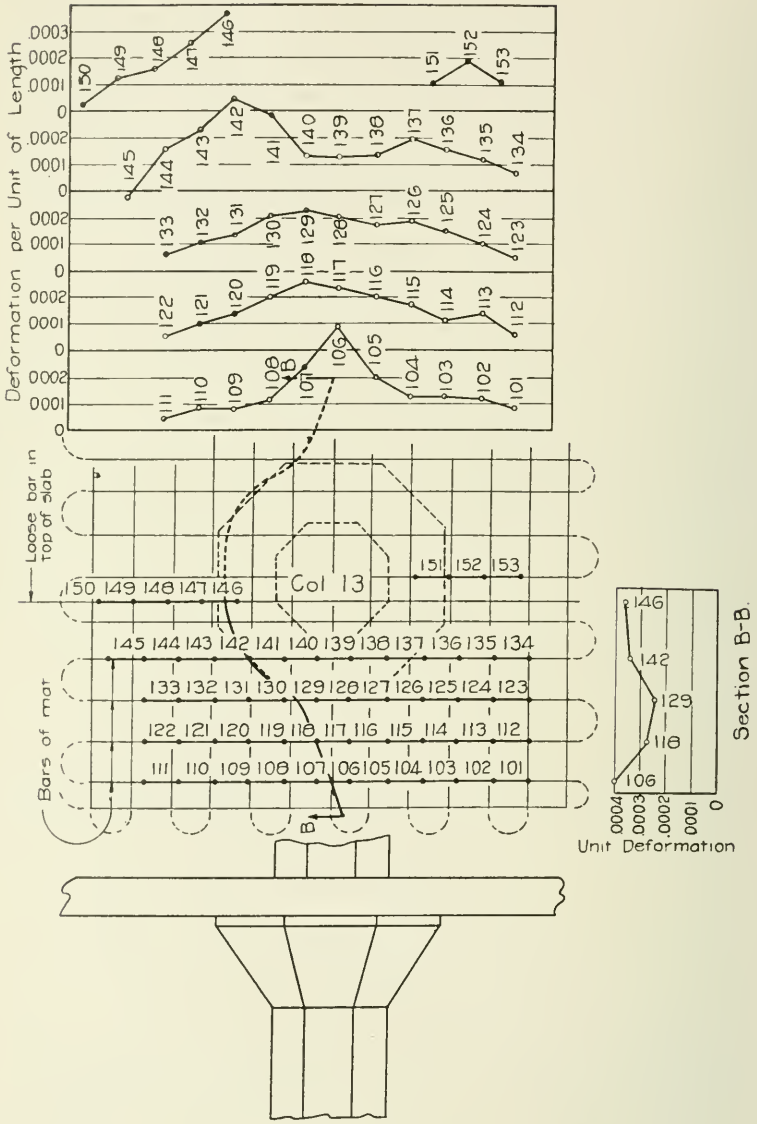


FIG. 81. LOCUS OF POINTS OF HIGHEST STRESS IN REINFORCEMENT IN TOP OF SLAB NEAR COLUMN 13 OF TEST FLOOR OF CURTIS-LEGER COMPANY BUILDING.

Deformation readings were taken on a few gage lines in a number of the most significant positions immediately after completing the loadings of 200 lb., 400 lb., and 500 lb. per sq. ft. When each load had been in place 12 hours or more a complete series of readings was taken. For the load of 500 lb. per sq. ft. the complete series of readings was not begun until the load had been in place 46 hours. When the recovery readings were begun, the greater portion of the load had been off the floor for more than 12 hours, but a small portion had been removed less than three hours before.

52. *Control Cylinders.*—The control cylinders gave strengths ranging from 1230 to 1650 lb. per sq. in. and initial moduli of elasticity ranging from about 2,300,000 to 4,700,000 lb. per sq. in. As a fair working value and one which facilitates comparison with the results of other tests, 3,000,000 lb. per sq. in. has been used as the modulus of elasticity in cases in which it is desirable to interpret unit-deformation into stress.

53. *Tension at Capital.*—Load-deformation diagrams for all gage lines are shown in Fig. 79 and 80. Unit-deformations in the reinforcement which extends in the direction of the 19 ft. side of the panel, found at a floor load of 500 lb. per sq. ft. are plotted in Fig. 81 also. From this the locus of the highest stress has been determined and is shown in this figure as section B-B which, for lack of complete information, assumes that gage lines 154, 155, and 156 (see Fig. 78) lie on this locus. The deformations on this section are shown in the right-hand portion of the figure. The highest deformation on this section corresponds to a stress of 11,000, the lowest to 7,000, and the average to 9,300 lb. per sq. in.

54. *Tension Midway Between Columns.*—Fig. 82 shows the distribution of deformation among bars of the rectangular and diagonal bands of reinforcement midway between columns at the applied load of 500 lb. per sq. ft.

Section C-C for the rectangular band lies midway between columns and is cut at right angles by the long side of the panel. The highest deformation was found in one of the central bars of this band, and corresponds to a stress of 11,500 lb. per sq. in. The average deformation corresponds to 9,100 lb. per sq. in.

Section D-D of Fig. 82 lies normal to the diagonal band at the center of the panel. This section lies near the edge of the area covered by the 500 lb. load, and the deformations here probably were smaller than would have been found if the entire area of this panel had been loaded. This is indicated by the fact that the deformations were

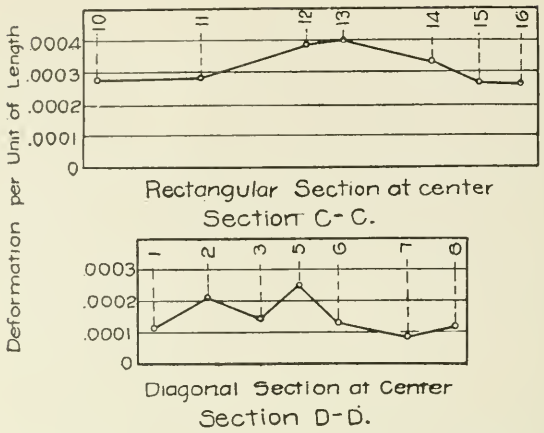


Fig. 82. Tensile Unit-deformation, Midway between Columns, in Reinforcement of Rectangular and of Diagonal Bands in Test Floor of Curtis-Leger Company Building.

changed only slightly by the shifting of the load from the four-panel area to the two-panel area.

A comparison of the stresses at the gage lines crossed by cracks at construction joints with those at similarly situated gage lines not crossed by cracks shows that considerably larger tensile stress in the reinforcement may be expected at points where the tensile strength of the concrete is impaired than at points where the concrete is intact.

On the short side of the panel no measurements of deformation in the rectangular bands were taken midway between columns because the load was not so placed as to develop representative stresses at that point.

55. *Compression at Column.*—The maximum measured compressive unit-deformation in the concrete was 0.00031. It was found at gage line 79, close to the capital of column 13 on the line of column centers in the direction of the 19-ft. span. The compressive unit-deformation at the corresponding position (gage line 71) on the diagonal line through column centers was 0.00021. If a modulus of elasticity of 3,000,000 lb. per sq. in. be assumed, these unit-deformations correspond to stresses of 930 and 630 lb. per sq. in., respectively. Compressive unit-deformations in these gage lines are shown in Fig. 79, 83, and 84.

To investigate the lateral distribution of compressive stress at the column, measurements of deformation in concrete were taken parallel to the direction of the long side of the panel across the center-line of

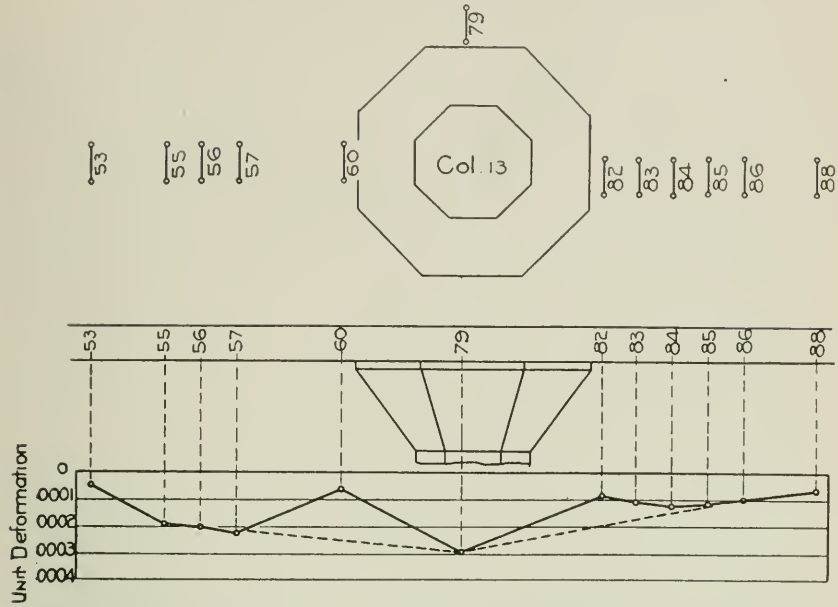


FIG. 83. LATERAL DISTRIBUTION OF COMPRESSIVE DEFORMATION ON BOTTOM OF TEST FLOOR OF CURTIS-LEGER COMPANY BUILDING.

columns, at intervals in the direction of the short side. The extent of area covered by these measurements is shown by the position of gage lines indicated (see Fig. 83). Gage lines 88 and 53 were 84 in. and 76 in. from the center of column 13. The results plotted in Fig. 83 indicate that as far out on either side of column 13 as measurements were taken some compression was developed in a direction normal to the section. If the stress had been uniform and equal to that at gage line 79, a width of about 110 in. would have been required to develop the same total stress as that which was found in the width of about 180 in. It should be noted that gage lines 60 and 82 are close to the column head. By reason of the stiffness of the column head the deformations at these gage lines may not be expected to be as large as may be found farther away from the column head or at points in front or in rear of these positions.

56. *Points of Zero Tension and Zero Compression.*—The manner and amount of variation in deformation along elements of the slab passing over the center of the column in the rectangular and diagonal directions are shown in Fig. 84. The location of the gage lines shown may be found in Fig. 78. An examination of Fig. 84 shows that on the rectangular band the point of zero compression was closer to the

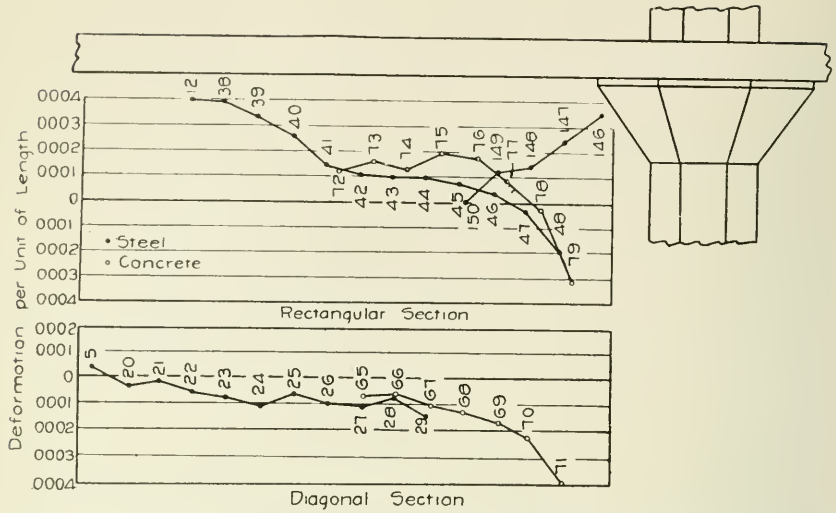


FIG. 84. LOCATION OF POINTS OF ZERO UNIT-DEFORMATION ON UPPER AND UNDER SURFACES OF TEST FLOOR IN CURTIS-LEGER COMPANY BUILDING.

edge of the column capital than was the point of zero tension. No explanation of this is offered. For the diagonal direction, section F-F, the point of zero deformation was found on only the under surface. Table 17 gives the positions of the various points of zero stress.

TABLE 17.

POSITION OF POINT OF ZERO DEFORMATION.

Two panels loaded. Applied load 500 lb. per sq. ft.

Point of Zero Unit-Deformation	Distance from center of column	
	Inches	Ratio to panel length
Bottom; Direct band	42	.184
Bottom; Diagonal band	45	.144
Top; Direct Band, Central Bar	57	.25
Top; Direct Band, other Bars	45	.197
Top; Average, Weighting other bands 3	48	.21

57. *Deflection.*—Load-deflection diagrams for the eleven points shown in Fig. 77 and marked A, B, C, etc., are given in Fig. 85. The maximum deflection at B under the full panel load of 500 lb. per sq. ft. live load was 0.16 in. immediately after the application of this load and 0.17 in. after the load had been in place for 48 hours. These deflections are 1/1425 and 1/1340 of the 19-ft. span.

58. *Cracks.*—In Fig. 77 the locations of cracks found at full load

have been shown by a solid line for the upper surface and by a dotted line for the under surface. These cracks were very fine, the largest being at a construction joint which extended along the direction of the 19-ft. span half way between columns 12 and 13.

59. *Recovery.*—Table 18 gives data on recovery at some of the more important gage lines. This table shows that the recovery was more complete in compression regions than in tension regions whether the measurements were of steel or of concrete deformations. It seems probable that this phenomenon may have been due to the formation of

TABLE 18.

PRINCIPAL FULL-LOAD UNIT-DEFORMATIONS AND AMOUNTS OF RECOVERY.

Load of 500 lb. per sq. ft. applied over two panels.
Plus indicates extension and minus indicates shortening.

Location of Gage Lines	Gage Line	Full-load Deformation	Residual Deformation	Ratio of Residual to Full-load Deformation
On tension reinforcement midway between columns	10	+.00028	+.00012	.62
	11	+.00028	+.00000	.00
	12	+.00039	+.00016	.43
	13	+.00041	+.00020	.50
	14	+.00033	+.00017	.50
	15	+.00027	+.00014	.53
	16	+.00027	+.00013	.50
	Average			.45
On tension reinforcement at capital	106	—.00040	—.00021	.52
	107	—.00025	—.00012	.47
	117	—.00025	—.00012	.47
	129	—.00024	—.00009	.37
	142	—.00035	—.00014	.40
	146	—.00039	—.00012	.31
	Average			.42
On compression reinment at capital	36	—.00008	+.00001	*.13
	37	—.00005	—.00002	*.40
	48	+.00019	+.00005	.27
	49	—.00010	+.00002	.22
	50	+.00012	+.00004	.31
	Average			.27
On concrete at capital	55	+.00020	+.00004	.20
	56	+.00020	+.00009	.45
	57	+.00023	+.00011	.49
	71	+.00021	+.00004	.20
	79	+.00031	+.00008	.25
	80	+.00014	+.00003	.20
	81	+.00012	+.00006	.50
	83	+.00010	+.00001	.10
	84	+.00012	+.00003	.24
	Average			.29

*Not included in average.

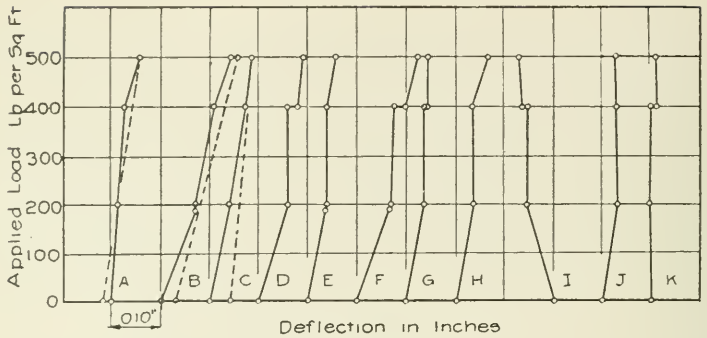


FIG. 85. LOAD-DEFLECTION DIAGRAMS FOR TEST FLOOR OF CURTIS-LEGER COMPANY BUILDING.

cracks, the fractured surfaces of which could not come together again perfectly after the tensile stress was removed.

60. *Summary.*—The following summary is intended to give the main features of the results of the test:

1 With the load of 500 lb. per sq. ft. distributed over an area equal to that of two panels with a view of making its effect in producing stress as severe as possible, the maximum stress in the reinforcement at the column and midway between columns was about 11,000 lb. per sq. in. Calculations made on the basis of design most commonly used in Chicago give a stress of about 25,000 lb. per sq. in. for this load. However, it seems probable that if a larger area had been loaded to the same intensity the stress developed would have been somewhat larger.

2 The highest unit-deformation observed was near the column on the under side of the slab in the concrete and was measured in the direction of the longer span. Based on a modulus of elasticity for the concrete of 3,000,000 lb. per sq. in. this unit-deformation corresponds to a stress of 930 lb. per sq. in.

3 The point of zero unit-deformation on the under surface of the slab was closer to the column than that on the upper surface. For this reason the location of the point of inflection is not known with certainty, but the indications are that it was at a distance of about two-tenths of the panel length from the center of the column.

4 The deflection under twice the design live load plus the dead load was about 1/1400 of the span.

5 The cracks were very small, the largest being along a construction joint. The stresses in gage lines crossing this crack were enough larger than the tension in similar gage lines not crossing a

crack to indicate that the tensile strength of the concrete adds considerably to the resistance of the slab at this load.

6 The recovery was more complete in regions of compression than it was in regions of tension regardless of whether the measurements were taken on concrete or on steel.

VII. General Comments.

61. *General Comments.*—As was remarked at the beginning, the circumstances surrounding the floor test of a building are unfavorable to securing definite and uniform quantitative results. The distribution of the resistance of the structure to parts beyond the portion which is loaded and the effect of the tensile strength of the concrete, greatly modify the action of the structure. The physical conditions connected with the tests are unfavorable to securing exactness. Conclusions drawn from such tests must be of a general nature and must be confined to the general behavior of the structure. The following comments are given:

1 The stresses measured in the reinforcing steel were relatively low. It is felt that the values of these stresses should not be taken as representative of the stresses which may be developed in the structure when it is loaded over a large area for a considerable time. That this view is not inconsistent with the general practice in designing reinforced concrete may be seen by examining laboratory tests of reinforced concrete beams which have percentages of reinforcement comparable with those in the flat slabs tested. In such beams measured stresses of 5,000 to 20,000 lb. per sq. in. in the steel account for only one-fourth to one-half of the external bending moment. In the tests of flat slabs there is no indication that the tensile resistance of the concrete contributes less to the apparent strength of the structure than is the case with beam construction. It is evident that attention must be given to the mechanics of the structure in determining the requirements for making designs.

There is difficulty in evaluating the compressive deformations of the concrete in terms of stress, since the modulus of elasticity of the concrete in the slab may not agree with the values determined from test specimens. The observations on compression are useful in finding the distribution of compressive stresses.

2 For negative moment the locus of maximum stress in a direction perpendicular to a panel edge was a line which followed the column capital for nearly 180° and merged into the panel edge a little distance away from the column capital. In the Schulze Baking Company

Building the measurements were of compression on the under side of the depressed head. In the Worcester test structure and in the Curtis-Leger Building the measurements were made on the tension reinforcement.

3 In the Shredded Wheat Factory the tensile stresses resisting negative moment across the panel edge would average as high at locations intermediate between columns as exist at points close to the column. It is apparent that the actual distribution along a section of negative bending moment would be affected by the size and spacing of the bars crossing the section.

In the Schulze Baking Company Building also it appears that bars across a panel line at a location midway between columns developed resistance to negative bending moment.

Information having a bearing on the distribution of tensile stresses across panel lines was not obtained in any other test discussed in this bulletin.

4 In the building in which there were depressed heads around the column capitals and in which information was obtained on the distribution of the compressive stresses over the section of maximum negative moment, the Schulze Baking Company Building, there were indications that the compressive stresses of the negative moment were taken almost entirely in the portion of the section within the width of the depressed head and that there was very little compression in the thin portion of the section between depressed heads. In the Worcester slab, which had no depressed heads, at the load of 215 lb. per sq. ft. the compressive stresses in the section of maximum negative moment were distributed along the section for the full width of the panel, although the stress midway between columns was less than that closer to the column. In the Curtis-Leger Building compressive stresses were found in the section of maximum negative moment as far away from the column capital as measurements were taken.

5 In the building having a relatively large thickness of depressed head, the Schulze Baking Company Building, the compressive stresses on the under side of the thin portion of the slab close to the depressed head and perpendicular to its edge were nearly as large as those in the same direction on the depressed head close to the column capital.

6 An increase in the deformations in the section of maximum positive moment was found when the loaded area was changed from a group of panels to a row of panels. This change of loading was made in the Shredded Wheat Factory and in the Soo Terminal. How much of the increase may have been due to a proportionally smaller contri-

bution by the tensile resistance of the concrete is not known, but it is evident that the positive moment must have been increased considerably by this change in loading.

7 High bending deformations, due to eccentric loading, were found in columns located at edges of loaded areas. In the Shredded Wheat Factory, a severe bending moment in a column of the basement story was observed when panels of the first floor on one side of this column were loaded. Even with nine panels loaded bending deformations were found in interior columns, evidently due to difference in the slab moments on the two sides of the column. In this case the bending was in a direction opposite to that found when the column was at the edge of the loaded area. In the Soo Terminal, a one-story structure, marked bending phenomena were observed in columns at the edge of the loaded area, and tensile deformations were found of such amount that even considering the compression due to dead load the tensile resistance of the concrete must have been exceeded. The position of the point of inflection of the elastic curve of flexure was in fair agreement with the usual analysis. In the Schulze Baking Company Building, the bending of columns at edges of the loaded area was an important feature of the action of the structure in the test, the largest bending apparently occurring in a column at a corner of the loaded area. In the Worcester Slab Test, the bending of certain wall columns and corner columns was apparent to the eye, and large tensile deformations were observed in the column reinforcement. Although the bending action was not different from that which may be obtained by analysis, it seems well to call attention to the phenomena observed, since provision for resisting the bending moment produced by the eccentric loading of columns (both wall columns and interior columns) may be overlooked by some designers.

8 In the one building in which load was applied to a wall panel having a lintel beam, the Shredded Wheat Factory, diagonal cracks were found on the interior side of the lintel beam near its ends. None was found on the outside of the beam. The cracks extended upward and away from the ends of the beam. The phenomenon was probably the result of the twisting action produced by bending moment developed in the slab at its edge by the load on the wall panel and transmitted to the lintel beam through the monolithic connection between the slab and the beam.

9 In the two one-story structures tested, the Soo Terminal and the Worcester test structure, the unevenness of settlement of the footings was sufficient to interfere with interpretation of the results. In

a building of several stories the rigidity of the structure may be expected to cause it to settle more as a unit. It is evident that in a one-story structure unusual precautions should be taken to guard against uneven settlement.

10 The tests which have given most definite results and results most useful for comparison with analytical treatment have been made on slabs whose thickness was small in relation to the span.

11 Progress in obtaining experimental knowledge of flat slab structures may best be made through a series of tests on structures designed solely for test purposes and planned systematically to bring out the fundamental differences between different types of design and the effect of varying certain elements of design. Occasional tests of floors may give interesting information, but the differences in design and construction among the different structures may be so unsystematic as to make the results not comparable, rendering them useful mainly for judging of workmanship and the sufficiency of the design.

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THE STRENGTH AND STIFFNESS OF STEEL UNDER BIAXIAL LOADING

BY

ALBERT J. BECKER

ONTARIO RESEARCH FOUNDATION
47 Queen's Park
NOV 16 1938
TORONTO

BULLETIN No. 85

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 85 APRIL, 1916

THE STRENGTH AND STIFFNESS OF STEEL UNDER BIAXIAL LOADING.*

BY ALBERT J. BECKER,

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*This bulletin embodies the principal data of the thesis of Albert John Becker, presented in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering in the Graduate School of the University of Illinois, June, 1915, together with further experimental data taken by the author to extend the investigation.

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THE STRENGTH AND STIFFNESS OF STEEL UNDER BIAXIAL LOADING.

I. Introduction.

1. *Scope of Investigation.*—The purpose of this investigation was to determine the laws governing the strength and stiffness of mild steel when subjected to combined stress produced by two tensions at right angles to each other or by a compression combined with a tension at right angles. In order to give a satisfactory basis for comparison of results, the plan of investigation provided that the ratio between the two stresses be kept constant throughout the test of a specimen, and J. B. Johnson's tangent method of determining the "yield point" or "apparent elastic limit" was selected.

The specimens tested were drawn steel tubes of uniform size and practically of uniform thickness. These tubes were subjected to an axial load and to internal pressure. The only variable was the ratio of the circumferential stress to the axial stress. Comparison has been made only in the test results from sets of tubes cut from a single length of seamless drawn tubing. By means of strain gage readings a knowledge of the distribution of stress on the cross section was obtained; no assumptions were made except that of uniform distribution of the circumferential tensile stress throughout the thickness of the tube wall.

The investigations of strength and of stiffness were carried on simultaneously, but the results are discussed separately. The points investigated are:

(a) The change of yield-point stress of the material with increasing ratios of circumferential tensile stress to axial tensile or compressive stress.

(b) Stiffness of the material (strains accompanying stress) for increasing ratios of circumferential tensile stress to axial tensile or compressive stress.

No discussion has been given of the engineering applications, for it is realized that while these applications are important, more work is needed to establish the conclusions reached. When this has been done and all the work has been correlated, it will be a simple matter to make an application of these principles to engineering design.

2. *Acknowledgment.*—All the tests were made in the Laboratory of Applied Mechanics of the University of Illinois, under the supervision of Professors A. N. Talbot and H. F. Moore, to whom acknowledgment is made for their suggestions and criticisms and for the interest they

have shown in the progress of the investigation. Acknowledgment is also made to Mr. J. O. Draffin, research fellow in the Engineering Experiment Station, for his assistance in the conduct of the various tests. It is also desired to make an acknowledgment to the Joint Committee on Stresses in Railroad Track for the use of the new model 4-in. Berry Strain Gage.

3. *General Statement.*—When a steel bar is tested in tension or compression, certain phenomena are observed which have been incorporated as fundamental facts in the theories of the elastic behavior of bodies under stress. In such a test, both the strength and the stiffness of the material are observed, the former by noting the yield point and ultimate strength, the latter by observing the unit-strains corresponding to successive loads and computing the modulus of elasticity. Repeated experiments have shown that for material of the same composition and

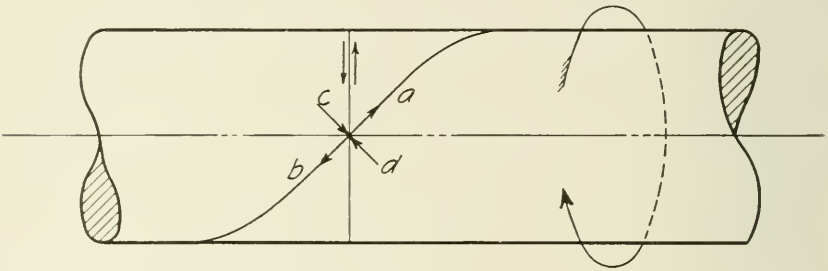


FIG. 1. ILLUSTRATION OF STRESSES PRODUCED ON OBLIQUE PLANES IN A BAR SUBJECTED TO TORSION.

treatment, these results are practically constant and can be used as a basis of design. The strength of any material of construction cannot be determined by mathematical analysis, neither can its stiffness. Poisson's ratio, modulus of elasticity, yield point, and Hooke's law are experimental results.

When an investigation of combined stress is attempted, there arises the question of the extent to which the calculations may be based upon the values obtained in the experiments in simple tension, compression, and shear. Constants determined by uni-directional loading cannot be indiscriminately applied to bi-directional loading. Theories have been evolved in which these constants are used by taking account of the interaction of the applied stresses. The analyses for these are correct from the mathematical standpoint, but the soundness of the basic assumptions can be demonstrated only by experiment.

Different combinations of simple stresses are possible, and it may be expected that the same analysis will not apply to all combinations. The presence of shearing stress in a bar subjected to simple tension and the tensile and compressive stresses accompanying the shearing stress due to a torsional load indicate that the governing conditions depend upon the relative strength of the material in shear, tension, and compression. A cast iron bar tested in torsion fails in tension on an oblique plane, because the tensile strength is less than the shearing strength. It is, therefore, logical to suppose that different stress combinations will produce failures differing in character for different materials.

4. *Combined Stress.*—Three types of stress applications are possible, uni-directional or simple stress, bi-directional or biaxial, and tri-directional. The first is illustrated by a specimen subjected to tension or compression in an ordinary testing machine. Bi-directional or biaxial stress is the application of two stresses in the same plane acting in directions at right angles to each other. Tri-directional stress is the application of three stresses at right angles to each other. The condition of biaxial stress is more important, from the point of view of the engineering applications, than that of three stresses at right angles to each other.

The possible combinations of biaxial stress are as follows:

Tension with tension.
Tension with compression.
Compression with compression.
Compression with tension.
Shear (torsion) with tension.
Shear (torsion) with compression.

These may be divided into three classes, tension with tension and compression with compression forming the first, tension with compression and compression with tension the second, and the combination of either tension or compression with torsion forming the third. The third class includes also two special cases of the second class; for a simple torque is equivalent to two equal principal stresses, one compression and the other tension, so that a torque combined with tension or compression can be reduced to the case of tension combined with compression or vice versa. This equivalence will readily be seen by considering a bar of circular cross-section subjected to torsion alone, Fig. 1. The stress on a plane at right angles to the bar is a pure shearing stress, depending in intensity upon the diameter of the bar and upon the torque. But this is not the only plane of stress. As in a bar in simple tension, so in this case

there are planes on which both tensile and shearing stresses occur; there are also planes upon which no shearing stresses occur. Referring to Fig. 1, with the torque as shown by the arrow, the stress on the 45° plane CD is tension, and on plane AB at right angles to this plane, the stress is compression. This is equivalent to a biaxial loading which develops a tensile and a compressive stress at right angles to each other and each equal to the shearing stress. It should be noted that there are stresses on oblique planes which may control the strength of the material.

Applications of combined stress are to be found in the familiar examples of the steam boiler for tension combined with tension, and of the crank shaft for tension or compression combined with torque. Biaxial stresses occur in flat plates and in flat concrete slabs or girderless floors.

II. Theories of the Strength of Materials Under Combined Stress.

5. *The Six Theories.*—The mathematical discussion of stresses and strains in a thin tube under axial load and internal pressure is given in Appendix II, page 58. It follows closely the method used by Love* in his work on the theory of elasticity, to which those who wish to investigate the subject further are referred.

Six theories have been advanced to cover the problems of the strength of material under combined stress. Two of them are empirical, one is developed from a molecular hypothesis, one from the mathematical theory of elasticity, and two from static relations of stresses. Three of these theories have found considerable favor and are given first.

6. *The Maximum Strain Theory.*—In the mathematical theory of elasticity, after the relations between stress and strain are established for simple stress, three equations of the following types are derived:

$$E\epsilon_1 = \sigma_1 - \frac{1}{m}(\sigma_2 + \sigma_3),$$

where σ_1, σ_2, and σ_3 are the three stresses at right angles to each other, E is the modulus of elasticity assumed constant in all directions, ϵ_1 is the unit-strain in the direction of σ_1, and $\frac{1}{m}$ is Poisson's ratio.† Stresses

*The Mathematical Theory of Elasticity, A. E. H. Love.

†A stress in any direction produces strain in that direction and also strain at right angles to that direction. The numerical ratio between the unit-strain at right angles to the direction of the force and the unit-strain in the direction of the force is called Poisson's ratio.

are considered positive if tension, and negative if compression. $E\epsilon_1$ is called by various writers the reduced stress, the true stress, or the ideal stress, but as the term stress is generally used by engineers to mean an internal resisting force which holds external forces in equilibrium it seems best to refer to it merely as $E\epsilon$. Writing two equations similar to the above for $E\epsilon_2$ and $E\epsilon_3$, the three equations for the reduced stress are obtained. The maximum strain theory takes these three equations and assumes that whatever the combination of stresses, the material will fail when the maximum strain (which will be in the direction of the greatest stress) reaches a value equal in magnitude to that at the yield-point stress in simple tension or compression. $E\epsilon$ at the yield-point stress for any combination of stresses must be the same, provided the yield-point stress is the same for tension as for compression. For ductile materials, E is usually assumed to be constant and it follows that ϵ must be the same when the yield-point stress of the material is reached, no matter what combination of stresses is used. But for a brittle material, where E varies, the strain ϵ must vary in an inverse ratio; that is, the product remains constant.

The maximum strain theory, or St. Venant's theory as it is sometimes called, holds that when a material is subjected to two or three stresses at right angles to each other, its strength is increased if the stresses are of like sign and that its strength is diminished if the stresses are opposite in sign. Thus two tensions or two compressions will produce an increase in the elastic strength of the material, whereas a tension combined with a compression produces a reduction in strength. For a stress ratio of one to one, both stresses tension, the material will be increased in strength 43 per cent if Poisson's ratio is 0.3, while if one stress is tension and the other compression, it will be weakened 23 per cent for the same stress ratio.

If in the equation for reduced stress given above, σ_2 and σ_3 are zero, the case is that of a bar in simple tension (compression is expressed as negative tension) and dividing both sides of the equation by ϵ_1, the result is the equation of the modulus of elasticity.

For combined stress according to this theory, then, the strain accompanying a given stress is changed by the addition of another stress at right angles to the first. It is increased if the stresses have unlike signs and diminished if they have like signs. Also, the strain ϵ is the measure of $E\epsilon$ (the reduced stress) and the material will not reach the yield point until the strain ϵ reaches the value corresponding to the strain obtained in simple tension at the yield point. It should be

emphasized that all elastic theory holds only within the elastic limit, or more correctly within the limit of proportionality, where E remains constant for an individual stress-strain diagram. But the slight variation up to the yield point, even though the value of E does change slightly, does not invalidate the theory, and the yield point is commonly taken as the limit of the discussion.

The maximum strain theory is based upon the mathematical theory of elasticity. Temperature effect is neglected and Hooke's law is assumed to hold rigidly. Herein lies its weakness, for the maximum strain theory, like the mathematical theory of elasticity, is dependent upon the accuracy of the relation assumed between stresses and strains. It has been shown* that there is a cooling of a bar of metal as the stress is increased up to the yield-point stress and it is also well known that Hooke's law is only an approximation.† A very good approximation it is, to be sure, for engineering purposes, but lack of isotropy in the materials, cold working and similar causes tend to change conditions, so that a slight deviation from Hooke's law may be observed considerably before the yield-point stress is reached. While the maximum strain theory has a good foundation, it must not be expected that the measured strains upon a body known to be not wholly isotropic, will conform exactly to this theory of stiffness.

The question of strength is quite different, for there is no assurance that the strains are the true measures of strength. Reasonable as the assumption may be, it is an assumption whose correctness must be demonstrated by experiment.

7. *The Maximum Stress Theory.*—The maximum stress theory, or Rankine's theory as it is sometimes called, virtually assumes that whatever the ratio of the stresses in the two directions and whether they are of like or opposite sign, the material will reach the yield point when, and only when, one of the stresses reaches the value corresponding to the yield-point stress in simple tension or in compression, as the case may be. It takes no account of Poisson's ratio as affecting strength and assumes that a material is neither weakened nor strengthened by the addition of a second stress at right angles to the first. If, then, this theory holds, the material should reach its yield point when the greater stress reaches the yield point stress for uni-directional loading.

8. *The Maximum Shear Theory.*—In the preceding theories failure

*C. A. P. Turner, Trans. Am. Soc. C. E., 1902. Lawson and Capp, Inter. Assn. Test. Mat., 1912. Ew. Rasch, Inter. Assn. Test. Mat., 1909.

†Hedrick, Engineering News, Sept. 16, 1915.

by yielding is considered to take place in tension or compression, whereas the maximum shear theory, or Guest's law as it is sometimes called, holds that all failures are failures by yielding due to shear when the shearing unit-stress reaches the shearing yield-point stress. Therefore, if loads are gradually applied to two specimens developing simple stress in one and combined stress in the other but so as to keep the shearing stresses the same in each specimen, the yielding failure in the two cases will be identical.

The basic principle of the maximum shear theory, that the failure in combined stress is the result of the shearing stress reaching the shearing yield-point stress, when carried to its logical conclusion demands that when two of the principal stresses are zero the failure is still due to shear. A steel bar subjected to axial tension only must therefore fail in shear. The maximum shear in this case occurs on a 45° plane and its intensity is one-half the tensile unit-stress. If the yielding due to shearing stress occurs at the same time as yielding due to tensile stress the yield point unit-stress of the material in shear must be just one-half that in tension, but if the shearing yield-point stress is reached first—as this theory maintains—then the ratio is somewhat less than one-half.

If the stresses which are combined are a compression and a tension, the resulting maximum shearing unit-stress is one-half the sum of the tensile and compressive unit-stresses. When the tensile and compressive stresses are equal, the intensity of the shearing stress is equal to the intensity of the tensile or compressive stresses and failure will take place by shear unless the shearing yield-point stress is equal to or greater than that of either tension or compression. It seems entirely possible, then, that failure may be caused under certain conditions by shear and that in other cases its intensity may be insufficient to cause yielding, the tensile or the compressive yield-point stress being reached first.

Considering compression as negative tension, there are two kinds of elementary stress treated in mechanics—tension and shear. They are quite distinct and have different accompanying phenomena. While a definite relationship may be established between the shearing and tensile stresses, the material may fail either in tension or in shear. This is suggested by the fact that mild steel in torsion gives a square break, a shearing failure, but cast iron tested in torsion breaks along a helicoid, failing in tension because the material is weaker in tension than in shear.

This duality of conditions while not entirely overlooked, has been advanced heretofore solely to form two distinct theories of failure, but these have not been connected. The possibility that both shear and

tension may govern, each within certain limits, has apparently not been mentioned in the publications and discussions on this subject. Mallock* has stated a dual law which is quite different from that discussed above. He proposes a volume extension limit and a shear limit, each dependent upon the other, and assumes that the material will fail when the limit of either is reached. This is quite distinct from the simple stresses as controlling factors in the failure of the material, but it recognizes the possibility of dual control.

The usual stress derivation for combined stress given in textbook is based upon the static equilibrium of forces and an application is made to a circular shaft in combined bending and torsion. A solution is given for the maximum normal stress and shearing stress on oblique planes, and safe working stresses are assigned. The assignment of working stresses in shear and tension fixes an arbitrary ratio of shear to tension, and the larger of the two shaft diameters determined by the two formulas is to be taken.

9. *The Internal Friction Theory.*—A short cylinder of brittle material when tested in compression fractures by shearing along a diagonal plane which, if failure be due to shear, should make an angle of 45° with the axis, since this is the plane of greatest shearing intensity. But the angles observed in experiments differ from 45°. In the attempt to explain this variation the theory of internal friction has resulted. When two particles under stress tend to slide over each other, a condition is set up similar to that of ordinary sliding friction. On the supposition that this resistance is similar to sliding friction, one of the laws governing the latter is applied; namely, that the coefficient of internal friction is independent of the load or stress. Therefore, slipping will occur along the surface of the plane inclined at an angle β with the axis of the specimen such that $\beta = 45° - \frac{\phi}{2}$ for compression and $\beta = 45° + \frac{\phi}{2}$ for tension. ϕ is the angle of friction and $\tan \phi = \mu$, the coefficient of friction. If the limiting friction per unit of surface is the same for tension and for compression, then the normal stress on the surface of slipping, at the instant when yielding begins, must be the same in each case, since this is $\frac{1}{\mu}$ times the limiting friction.

It has been said that the chief difference between the internal friction theory and the maximum shear theory is that the former is based

*Proc. Royal Society of London, 1909.

upon a maximum resistance to sliding, while the latter is based upon a maximum shearing stress. If the angle of friction is zero, the internal friction theory becomes the maximum shear theory.

10. *Mohr's Theory.**

Let k_1 = the shearing yield-point stress.

Let k_3 = the stress in compression and in tension (equal) which together produce a shearing stress equal to the shearing yield-point stress, k_4.

Let k_1 = the tensile yield-point stress.

Let k_2 = the compressive yield-point stress.

Mohr derives the formulas:

$$k_3 = \frac{k_1 k_2}{k_1 + k_2} \text{ and } k_4 = \frac{1}{2}\sqrt{k_1 k_2}$$

The usual theory developed from the static relation of stresses gives for two equal stresses of unlike sign the following relation for the stress intensities:

Shearing stress = $\frac{1}{2}$ (tensile stress + compressive stress) which is the same as Mohr's theory when the tensile and compressive yield-point stresses are equal. Mohr's theory is an attempt to modify the shearing yield-point stress according to the tensile and compressive yield-point stresses. When these are equal this theory presents nothing new, for it then coincides with the maximum shear theory. If the yield-point stresses are different, Mohr's theory brings in a new relation regarding the shear failure in combined stress. It is virtually an acceptance of the maximum shear theory with the definition of the value of that shear at the yield-point.

11. *Wehage's Theory.*†—This theory is based upon a few experiments on cross-shaped pieces of paper submitted to tension in two directions at right angles to each other. If the material has a different yield-point stress in the two directions, the following elliptic relation is given as an empirical deduction:

$$\left(\frac{t_1}{T_1}\right)^2 + \left(\frac{t_2}{T_2}\right)^2 = 1$$

T_1 and T_2 are the yield-point or the ultimate stresses in the two directions (as, for instance, with and across the direction of rolling), and t_1 and t_2 are the applied stresses in the corresponding directions. When $T_1 = T_2$, this elliptic relation becomes a circular one.

This theory assumes that the material is *weakened* by the applica-

*Zeitschrift des Vereines Deutcher Ingenieure, 1900.
†Zeitschrift des Vereines Deutcher Ingenieure, 1905.

tion of two tensions for the reason that such stresses tend to lessen the cohesion between the fibers. The assertion is also made that a compression combined with a tension should strengthen the material by increasing this cohesion, although no formula is proposed.

12. *Graphical Presentation of Three Theories.*—A graphical presentation frequently serves to give a better idea of the working of a theory or formula and for this reason the three most important theories

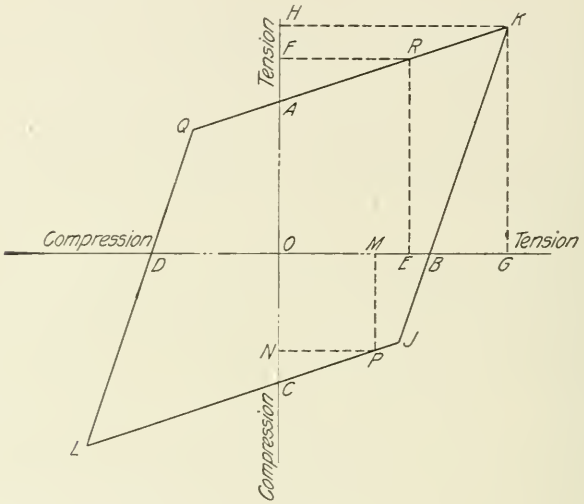


Fig. 2. Graphical Representation of Stresses According to the Maximum Strain Theory.

are represented in Fig. 2, 3, and 4, for the four combinations of simple tension and compression. To make the presentation more general, different yield-point stresses in compression and in tension have been assumed where this is possible.

Maximum Strain Theory. Let OA (Fig. 2) and OB represent the yield-point stress in simple tension and OC and OD that in compression. A tensile stress equal to OE would require a tensile stress equal to OF at right angles to cause yielding. For two equal tensile stresses the condition of yielding would not be reached until each stress attained the value OG, equal to OH. The increase in strength is OG — OB.

For a compression combined with an equal tension, yielding would occur when each stress attained the value O N, equal to O M. The other two quadrants are similar, two compressions producing the same relative

effect as two tensions, and a tension and compression producing a corresponding effect to a compression and a tension.

Maximum Stress Theory. Yielding takes place in tension or in compression and since the stress in one direction is not affected by a second stress at right angles to the first the diagram will be a square. The center of the square, however, will not be the origin of co-ordinates since the tensile and compressive yield-points will in general be different. If a tensile stress OB or a compressive stress OD, Fig. 3, equal to the yield-point stress, is applied in one direction, any stress, OE less than the yield-point stress in tension, may be applied at right angles without causing further yielding. In other words a second stress acting at right

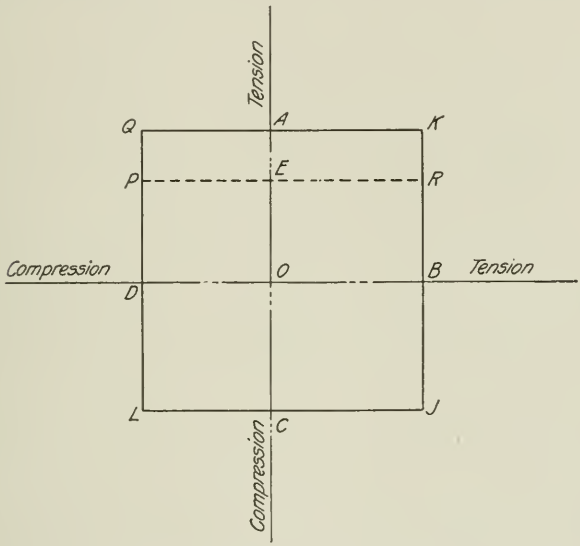


Fig. 3. Graphical Representation of Stresses According to the Maximum Stress Theory.

angles to the first yield-point stress does not change the yield-point stress of the material.

Maximum Shear Theory. The first and third quadrants (Fig. 4) correspond to the maximum stress theory. This follows from the fact that the shearing stress equals one-half the difference between the greatest and the least of the three principal stresses. For biaxial loading one of the three principal stresses is zero and in the first and third quadrants the other two are of like sign, hence the shearing stress will be one-half the greatest stress. But the limiting shearing stress must be constant,

therefore the greatest limiting principal stress must be constant and for like stresses (first and third quadrants) the diagram corresponds to the maximum stress theory. For a combination of tension and compression (second and fourth quadrants) the lines CB and AD are inclined at an angle of 45°, because the tensile stress plus the compressive stress is a constant and is equal to twice the shearing stress.

$$t + c = \text{constant}.$$

By setting t and c each equal to zero in turn, it is seen that t must equal c, and this theory demands an equal yield-point stress for tension

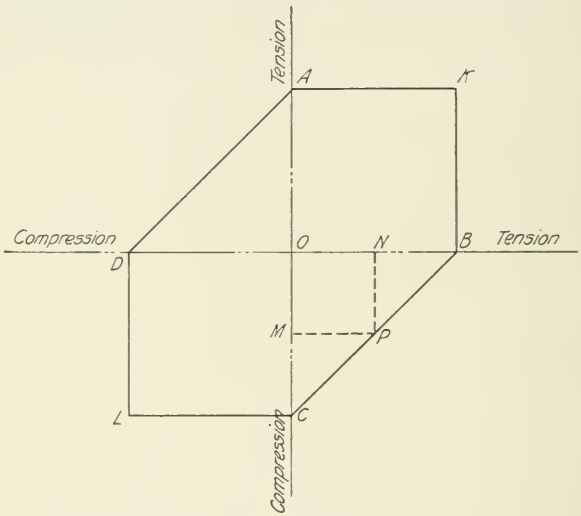


Fig. 4. Graphical Representation of Stresses According to the Maximum Shear Theory.

and compression. Two equal stresses of unlike sign will then cause yielding of the material when each stress equals ON or OM.

III. Experimental Work.

13. *Form of Specimen.*—The selection of the type of specimen to be used in the experimental work was a problem of considerable difficulty. Specimens subjected to direct tension or compression in two directions were not considered because of complications produced by the method of application of the load. A cube subjected to compression in two directions could easily have been set up, but the friction between the bearing blocks and surfaces of the cube introduces inequalities and resistance to the change in cross section which could easily vitiate the results.*

*See Zeitschrift des Vereins Deutscher Ingenieure, 1900, p. 1530.

A large number of short square steel bars, closely spaced to form in effect a bearing block, were considered not to obviate this difficulty sufficiently. Similarly, a tension specimen held at the four edges would not be practicable. Direct stress application seemed out of the question, and recourse was first had to bending to produce stresses in two directions at right angles to each other.

The first biaxial stress experiments in this series of tests were made upon flat cross-shaped specimens subjected to cross bending to produce two compressions or two tensions at right angles to each other. The stress distribution was so far from regular that no safe comparisons could be made. Such difficulties were encountered that this form of test specimen was discarded.

After a preliminary test, thin tubes were adopted as the form of test specimen. They proved satisfactory on account of the certainty with which biaxial stress of known magnitude could be applied by means of an axial load in a testing machine and internal hydrostatic pressure producing a circumferential tension. This method gives two well defined principal stresses at right angles to each other, the stress in the third direction being small since it varies from the intensity of the hydrostatic pressure on the inside to zero on the outside. It is much easier to cover the total range of stress ratios by the use of hydrostatic pressure and axial tension or compression in the tubes, than to use torque and axial load on solid bars. The latter method is inferior to the tube tests since only a small portion of the material is carried to the yield-point stress. The experiments are more successful when as much of the specimen as possible is uniformly stressed, and the best condition is that wherein the entire specimen is uniformly stressed. This is true both on account of the pronounced yield-point effect and on account of the smallness of the strains to be measured. The thinness of the wall and the relatively large tube diameter made the stresses practically uniform throughout the tube. It may be expected that the stress-strain diagrams will show a much sharper break than for solid bar specimens and the yield point is more positively determined. There are no greater eccentricities of application of load when using the tube than when working with a solid bar, and on account of the greater diameter of the tube, this eccentricity is relatively less important.

Strains were measured by means of a Berry strain gage, using a 2-in. gage length in the cross bending tests and a 4-in. gage length in the tube tests. The accuracy and reliability of an instrument of

this type has been demonstrated repeatedly and reference is made to the tests by A. N. Talbot and W. A. Slater on reinforced concrete buildings, as given in Bulletin No. 64 of the Engineering Experiment Station of the University of Illinois, to show what results may be

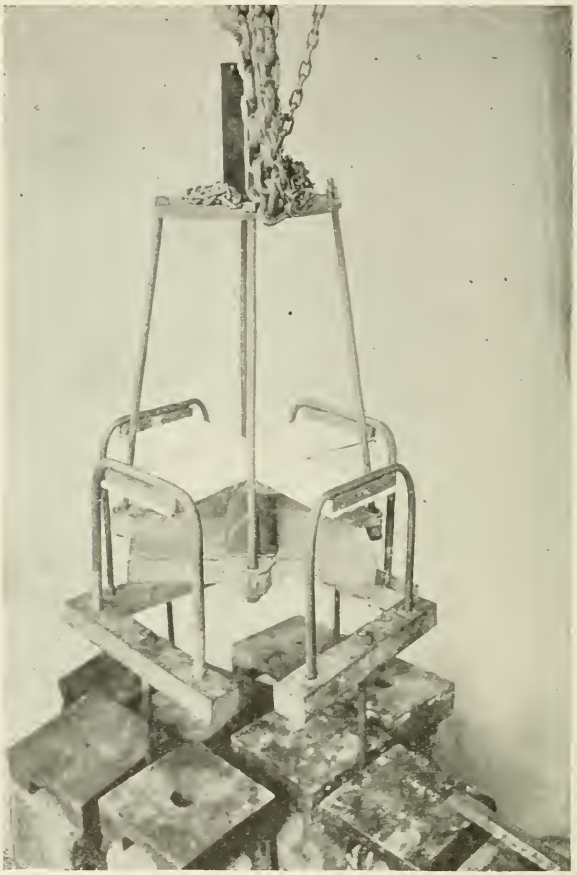


FIG. 5. VIEW SHOWING CROSS-BENDING TEST SPECIMEN UNDER LOAD.

achieved with such an instrument. A discussion of the strain gage and its use is given in a paper by Slater and Moore in Vol. XIII of the Proceedings of the American Society for Testing Materials.

The use of the strain gage marks a decided advance in the measurement of strains. With this instrument it was possible in these tests to take twenty-eight readings on as many gage lines for each increment

of load, whereas other investigators have been able to take four at the most and often only two. The advantage of a portable instrument over an attached one is very great and the rapidity of operation and freedom from danger of jarring the instrument as well as the ability to read overlapping gage lines, as was done in these tests, marks a decided step in advance.

14. *Cross-bending Tests.*—The set-up for the bending tests is shown in Fig. 5. Two specimens were prepared from ¼-in. soft steel plate of the shape shown in the figure. Tension specimens were prepared from the portions cut away. In order to have the upper surface unobstructed for the use of the strain gage, the beam was loaded as an overhung beam with four equal loads placed symmetrically one on each projection of the cross-shaped specimen. The center part of the cross was thus subjected on the top to two tensions at right angles to each other.

Load was applied by placing known weights on the yokes at the ends of the arms of the specimen, thus giving a definite bending moment. The strains were measured over 2-in. gage lines with a Berry strain gage. Instead of a uniform stress over the center portion of the test piece, the readings showed a considerable variation. The effect of the sharp reentrant angles at the corners in changing the lines of stress must have been considerable, for the yield point was reached first at the corners. The lines of yielding spread inward along a line making an angle of approximately 45° with the center lines. As the load was increased these lines divided, curving toward the adjacent corners, gradually changing direction and becoming parallel to the lines of symmetry of the specimen shortly before the lines from adjacent corners joined. New lines formed beside the first ones and others appeared outside the center of the cross. The latter were straight and parallel to the support. The lines are clearly shown in Fig. 6, which is from a photograph of the compression side of the first specimen tested. The lines marking the square from corner to corner and the center lines were used to lay out the specimen and must not be confused with the lines of yielding. The specimen, considered as a beam, widens abruptly for the center four inches, but the effect of this increased width in carrying stress was slight. The places of greatest stress were near each corner and to measure the maximum strain would have required a very short gage line. This stress condition is due to the form of the specimen rather than to combined stress.

15. *Tube Specimens.*—Specimens made from 6-in. tubes with

¼-in. walls were used. Four lengths of seamless drawn tubing were bought in the open market and made into test specimens. A series number was given to the specimens cut from a length of tubing and each specimen was numbered individually. The number of the test specimens

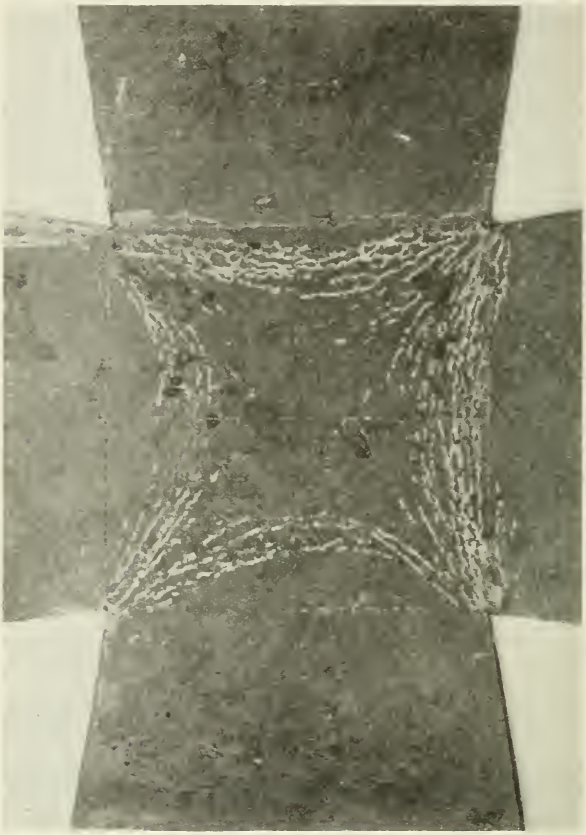


FIG. 6. VIEW SHOWING COMPRESSION SIDE OF CROSS-BENDING TEST SPECIMEN AFTER TEST.

in each series cut from each length of tubing and the character of the stresses applied are as follows:

Series	Number of Specimens	Specimen Number	Character of Combined Stress
1	5	1-2-3-4-5	Tension with tension
2	4	6-7-8-9	Tension with tension
3	6	1-2-3-4-5	Compression with tension
4	5	8-9-10	Tension with tension

Tube No. 6 of Series 3 and tube No. 7 of Series 4 were tested in torsion only. There was a marked difference in the physical properties of the material of the four lengths of tubing. This is shown by the stress-strain diagrams of the tensile tests made on specimens cut from the tubes. The yield point stress varied from 21,500 lb. per sq. in. to

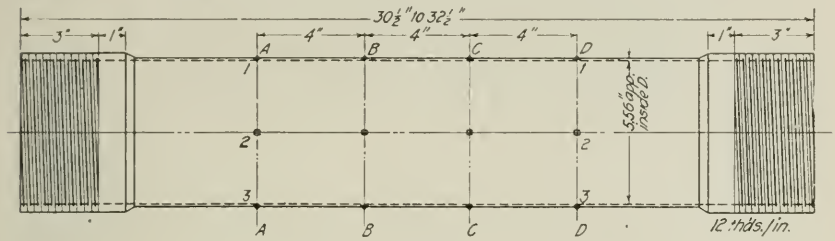


Fig. 7. Dimensions of Tube Test Specimen.

50,000 lb. per sq. in., Series 1, 2, 3, and 4 having yield-point stresses of 42,500, 21,500, 24,000, and 50,000 lb. per sq. in. respectively. The tubes were not annealed, but the first three series gave very uniform results for all gage lines, and showed a decided change at the yield point. The specimens of Series 4 showed a much greater variation. The behavior was that of hard, brittle steel of quite irregular composition.

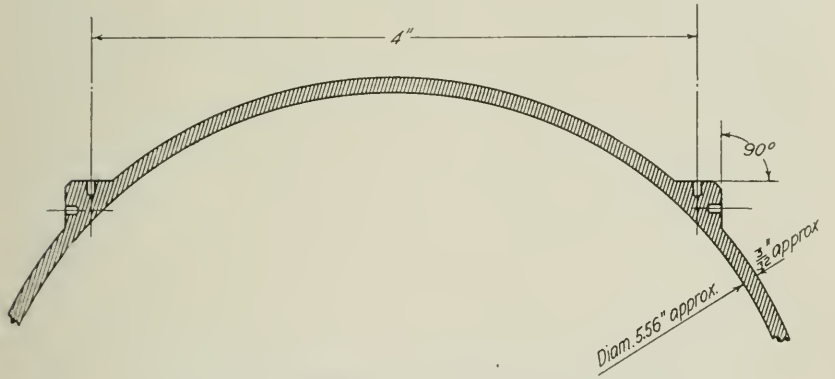


Fig. 8. Arrangement of Gage Holes on Tubes.

There was little reduction of area and the rupture was sharp and sudden, both in the tension specimens and in the one tube that broke during testing. The stress-strain diagram for the Series 4 show only qualitative results. These tubes were not suited for a test of this character, the inner and outer circumferences of the tube before machining were not concentric circles, and some gage lines gave diagrams that

curved throughout, similar to the diagrams of drawn wire. There was no well-defined yield point. As the stress-strain diagrams did not give positive results, no use will be made of this series.

16. *Preparation of the Tubes.*—The test specimens were first

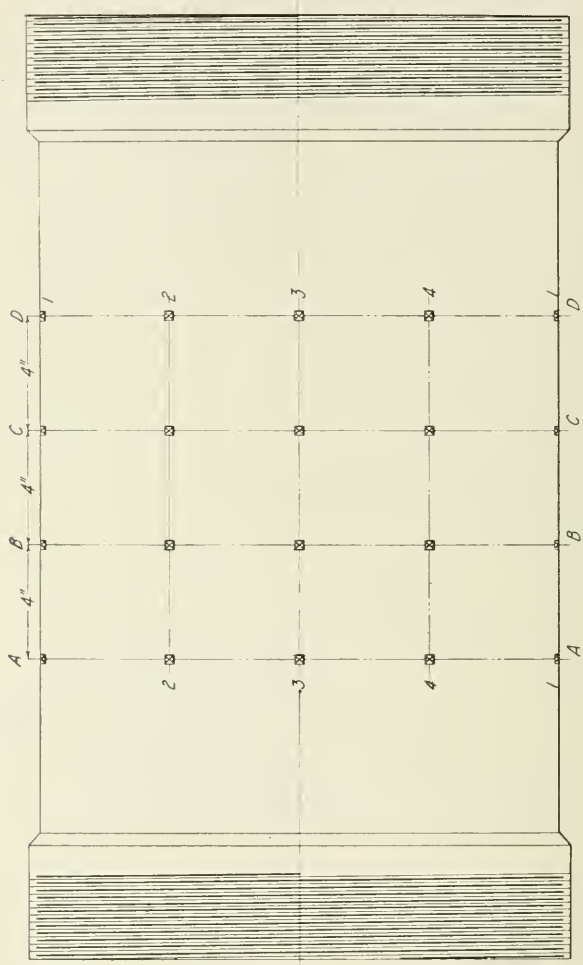


Fig. 9. Location of Gage Lines on Developed Outer Surface of a Tube.

bored out for the entire length on a horizontal boring mill and then turned to the dimensions shown in Fig 7. Each tube was threaded on the two ends with a taper thread of twelve threads per inch over a length

of three inches. The tube was left full thickness for about an inch beyond the threads to furnish a bearing for packing. The remainder of the tube was turned to an approximate thickness of 3/32 in. except for four bands of 1/4-in. width spaced four inches apart along the tube. The greater part of these bands were afterwards milled off leaving four projections on each band for the gage holes. The tube was thus spanned with four circumferential gage lines each four inches long. The axial gage lines used one of the two holes so that the projections could be reduced to the smallest possible size. This gave four rows of three axial gage lines each, twelve in all, and four bands of four circumferential gage lines, sixteen in all, making it necessary to take twenty-eight readings, exclusive of the standard bar and check readings for each increment of load. The standard bar readings are necessary in tests with the strain gage to detect variations in the instrument due to temperature or jarring of the points.

Fig. 9 shows the position of the gage lines on a developed surface of a tube specimen. The circumferential bands were lettered A, B, C, and D; the axial lines were numbered 1, 2, 3, and 4. Thus an axial gage line would take two holes in the same axial line, but in two consecutive circumferential bands. It would, consequently, be called by the letters of the bands, in order, and by the number of the axial line. Thus AB 3 would be an axial gage line spanning the distance between the circumferential bands A and B and lying along the axial line 3. As soon as the tube was machined the numbering was fixed and the projections on the A band marked with small prick punch marks to identify the axial lines. In this way the readings for the thickness of the tube walls could be correlated with the strain gage readings. The gage holes were drilled by hand using a No. 54 drill. They were not reamed.

The boring of the tube caused a slight change of shape of the cross section due to the removal of the inner skin of metal, and after the outside was turned the thickness was uniformly varying, usually having two points of maximum thickness diametrically opposite, and at 90° from these, two points of minimum thickness. This renders the tube slightly elliptical (but not over 0.02 in. in 5.50 in.) and of varying thickness. While the variation in thickness was as high as 15 per cent in some cases, it apparently did not affect the averages of the readings, although the individual circumferential curves show the effect of this variation and the effect of the water pressure in making the tube more nearly cylindrical.

17. *Determination of the Thickness of Tube Walls.*—The principle of the apparatus adopted for measuring the thickness of the tube walls is that a micrometer caliper with a very deep throat. Fig. 10 shows the apparatus with the tube in position for a zero reading. A 4½ by 2½ by 7/16-in. T-bar was clamped at one end to a support and a stiff wooden bar was bolted to it. At one end of the wooden bar an Ames Dial read-

Fig. 10. View of Apparatus Used in Measuring Thickness of Tube Walls.

ing to thousandths of an inch was fastened so that the plunger rested on a steel ball (*a* Fig. 10) embedded in the stem of the T-bar. To determine the thickness of the tube wall the plunger of the dial was raised, the tube was slipped over the T-bar and rested on the steel ball. Two other steel balls (*b* and *c* Fig. 10) were embedded in the stem of the T-bar, one on each side of the ball under the plunger at such a distance from it that the tube always swung free on the center ball and one of the others. The ball under the plunger was slightly higher than either of the others to insure a bearing on it at all times. When the plunger of the dial was in contact with the tube, the thickness of the tube was the difference between the reading then taken and the zero reading. Zero readings were obtained by suspending the tube in two fine wire slings in such a manner

that its weight came on the T-bar in the same way as when the tube swung on the steel balls. With the plunger of the dial in contact with the steel ball, the initial or zero reading was taken for every position of the tube along an axial line. As the T-bar was a cantilever with two point loading, this gave slightly different zero readings for the various positions of the tube, but any error arising on account of the deflection of the apparatus was removed. With the tube in a given position and with the plunger on the ball, a reading was taken after a traverse of two axial lines. These readings were taken to detect any possible change in the apparatus and are not zero readings. They correspond to the standard bar readings when using a strain gage. A set of check readings was taken and the average of the two readings was used. Readings were taken to tenths of a division (ten-thousandths of an inch) and tube thicknesses are given in thousandths of an inch.

It is thought that this method of measurement is accurate and the check results obtained with a micrometer after the tube had been cut, have borne out this conclusion. The tube must be of relatively large diameter to apply this method, but with 6-in. tubes no difficulty was experienced.

18. *Method of Testing.*—Two steel castings were designed to fit over the ends of a tube. The stresses carried by these heads were comparatively low, for the maximum load was but 167,000 lb., and the material was about ¾ in. thick at the thinnest part. The castings were machined all over and threaded internally, at one end to receive the tube and at the other to receive a 4-in. bar which served to apply the tension. The two threaded portions were separated by about an inch of metal which served to retain the water under pressure in the tube. These castings are shown in Fig. 11.

To withstand the water pressure, two layers of ⅜-in. hydraulic packing were used in an ordinary four-screw stuffing box. The heads were recessed to receive the packing and the gland, while the tube walls were left nearly full thickness for about an inch beyond the threads to furnish a firm bearing for the packing. After the packing was adjusted to position there were no perceptible leaks although pressures up to 1,800 lb. per sq. in. were used. Fig. 11 shows the general arrangement of the apparatus for the tension tests.

All the tests except the torsion tests were made in the 600,000-lb. Riehle machine of the Laboratory of Applied Mechanics of the University of Illinois. By using spherical seats with careful centering of the specimens in the machine, the eccentricity of loading was reduced

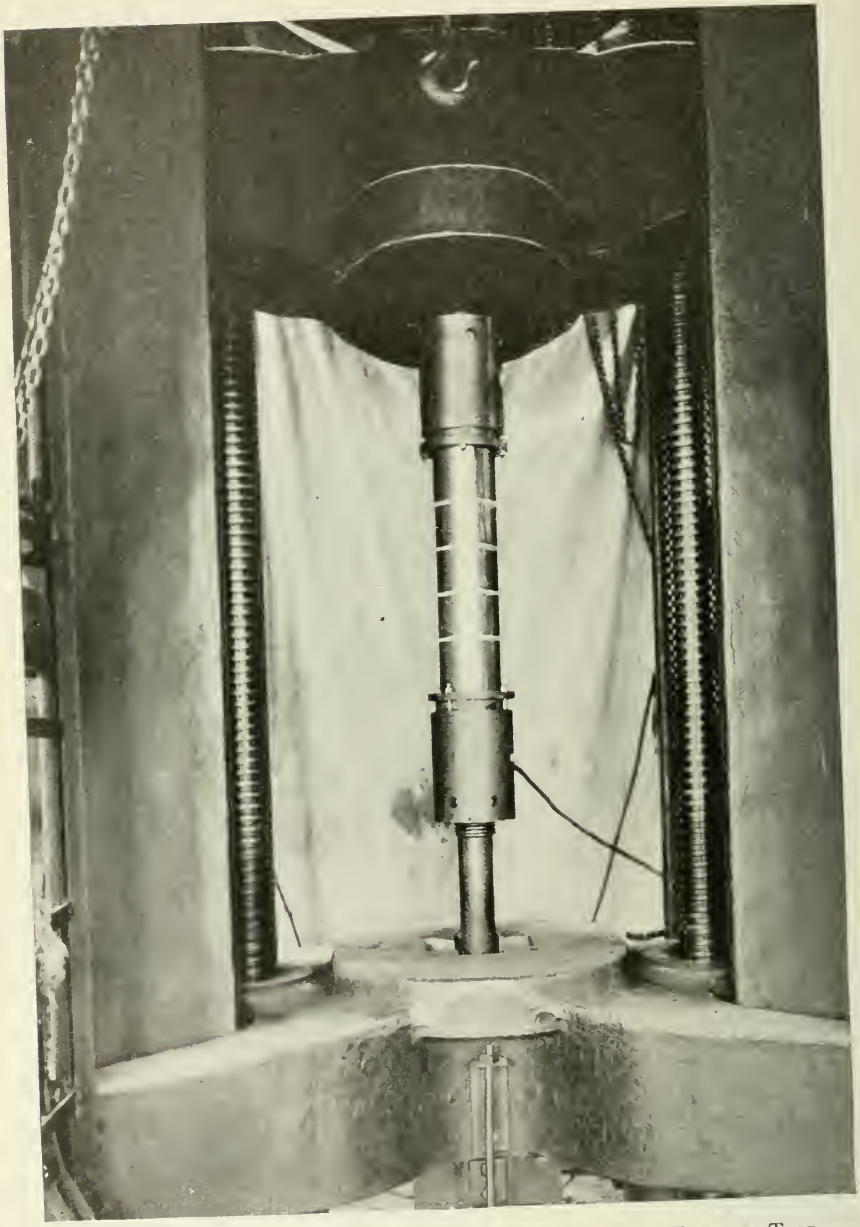


FIG. 11. VIEW SHOWING ARRANGEMENT OF APPARATUS FOR TENSION TEST.

to a minimum and the bending stresses were low as shown by the uniformity of the individual stress-strain diagrams. In the tension tests the nuts of the 4-in. bars bore directly against the spherical seats, the lower one being inverted. For the compression tests the bars were removed and the tube heads bore directly against the spherical seats and the upper spherical seat was inverted. The length of thread on the specimens tended to give a good distribution of load and the distance of the first gage hole from the end of the thin part of the wall (6½ in.) together with the thinness of the wall itself, were sufficient to insure a high degree of uniformity of stress. Holes were drilled into each head to connect into the interior of the tube; the hole in the lower head was for connection to the pump and the hole in the upper head was for the purpose of filling the tube with water. Each hole was tapped with a ½-in. pipe tap.

The torsion tests were made in a 230,000-lb. in. Olsen Torsion Machine. The heads were screwed on the specimens as in the other tests, and short steel bars threaded on one end transmitted the torque from the machine to the steel heads. Two fixed wooden clamps with 40-in. arms, one of which carried a pointer and the other a scale, were used to measure the angle of torsion over a known gage length.

19. *Character and Sequence of the Tests.*—Three tests in each of Series 1 and 2 were first made. These were the tests in direct tension, the tests with the ratio of tensile stresses equal to 0.475 and with this ratio equal to 0.92. The results of these tests were worked up before the remainder of the tests in the series were made, so that the other ratios could be chosen to the best advantage. As the difference in the strength of the steel in the direction of drawing and across it would complicate the problem, and as it was not intended to raise the question of the variation in strength in different directions throughout the specimen, the highest ratio of circumferential stress to axial stress used was made less than 1.0, being 0.92.

The average area of the inner cross section of the tubes was about 24.50 sq. in., and the axial load due to the water pressure was 2,450 lb. per 100 lb. per sq. in. water pressure. To produce a ratio of circumferential tension to axial tension equal to 0.50 required a machine load of $4 \times 2{,}450 - 1 \times 2{,}450 = 7{,}350$ lb. per 100 lb. per sq. in. water pressure, since the water pressure acts with the machine load. For axial compression combined with circumferential tension, the two quantities would be added instead of subtracted, since the water pressure tends to reduce the machine load. Dividing the net axial load (9,800 lb. per

100 lb. per sq. in. water pressure) by the cross-sectional area of the tube gives the unit axial stress. A slight error is introduced by using the inside diameter of the tube rather than the mean diameter, for in order that the circumferential tension shall be exactly twice the axial tension when the water pressure alone is acting, the mean diameter must be used to compute the axial tension. This error is about 1½ per cent, which represents the variation of the circumferential tensions from the mean. The stress ratios for Series 3 and 4 were planned complete and carried out as planned. The stress ratios used in the four series are given in Table 1.

The strain gage used was a 4-in. Berry strain gage made for the Joint Committee on Stresses in Railroad Track and loaned by that Committee. It has invar steel sides and shows a negligible correction for temperature. Two standard bars were used to detect any variation of the instrument due to jarring or striking the fixed point. All data have been corrected for variation in the standard bar readings. To avoid variation due to change of temperature of the tubes, they were usually

TABLE 1.

OUTLINE OF TEST SPECIMENS AND TESTS.

Series No.	Tube No.	Ratio of Circumferential to Axial Stress	Stress Combination
1	5	0.00	Axial tension only
	1	0.24	Tension with tension
	2	0.475	" " "
	4	0.69	" " "
	3	0.92	" " "
2	9	0.00	Axial tension only
	7	0.475	Tension with tension
	8	0.92	" " "
	6	0.92	" " "
3	4	0.00	Axial compression only
	2	0.20	Compression with tension
	5	0.30	" " "
	3	0.60	" " "
	1	0.90	" " "
	6	1.00	Torsion only
4	9	0.00	Axial tension only
	10	0.30	Tension with tension
	8	0.50	" " "
	11	0.80	" " "
	7	1.00	Torsion only

filled with water in the evening and by the time the test began the next day the tube and water were at a temperature that scarcely changed during the entire test.

20. *Test Operations.*—The initial load in all cases was small, producing an average axial unit-stress of approximately 4,000 lb. per sq. in. This load was applied after the specimen had been carefully centered and the spherical seats tried. A load sheet was prepared for each test which gave the required machine loads, the approximate yield point, the water pressure, and unit-stress. When the load was increased the water pressure was increased first and then the machine load.

The record of a test was a combination of the ordinary record and a graphical one. Co-ordinate paper was used and was divided into a series of rectangles, one for each standard bar and gage line. Along one side of this rectangle the instrument reading was noted and this reading was then plotted against the machine load. In this way the progress of the test was very evident and any doubtful reading was checked. When the nature of the curve is well known, it is advisable to see that the results for any gage line that do not show some systematic sequence of plotted points are checked to insure their accuracy. If this is not done false breaks may sometimes be obtained in the curve. If the error is experimental, the check reading will correct it, and if the stress suddenly departs from the straight line law, the check reading will be a repetition of the first reading and will give greater confidence in the result. Though but few errors were discovered and corrected, the result justifies the method employed. Whenever there are variations from the straight line in the stress-strain diagram, these are indications of a change in the rate of taking stress. As the load changes, the distribution of stress over a given cross section often changes, so that at one point there may be a rapid increase in the elongations for one increment of load, while in an adjoining gage line the change is slight. The next load increment may bring about a complete reversal of the conditions shown by the previous instrument readings.

Whatever variation occurs in one gage line, usually it is reflected in one or more of the others, so that the average takes out all these peculiarities. This is especially true of the circumferential readings.

It will be seen that the circumferential gage line readings give the correct unit-strain, the chord length being used and not the arc length. Circumferential readings are subject to the tendency of the tube to become truly cylindrical under water pressure. For low water pressures

TABLE 2.

DATA OF TUBES.

Tube No.	Inside Diameter Inches	Location	Tube Walls. Average Thickness, Inches	Sectional Area, Sq. In.
		Series 1.		
		AB	0.089	1.590
1	5.564	BC	0.088	1.570
		CD	0.086	1.527
		AB	0.087	1.543
2	5.558	BC	0.087	1.543
		CD	0.088	1.570
		AB	0.087	1.543
3	5.561	BC	0.087	1.543
		CD	0.087	1.543
		AB	0.085	1.508
4	5.563	BC	0.084	1.481
		CD	0.083	1.472
		AB	0.091	1.623
5	5.554	BC	0.092	1.641
		CD	0.092	1.641
		Series 2.		
		AB	0.083	1.467
6	5.579	BC	0.083	1.467
		CD	0.080	1.422
		AB	0.091	1.623
7	5.588	BC	0.091	1.623
		CD	0.091	1.623
		AB	0.094	1.678
8	5.560	BC	0.094	1.678
		CD	0.094	1.678
		Series 3.		
		AB	0.106	1.891
1	5.573	BC	0.106	1.891
		CD	0.112	2.001
		AB	0.114	2.040
2	5.561	BC	0.111	1.981
		CD	0.111	1.981
		AB	0.108	1.934
3	5.566	BC	0.107	1.912
		CD	0.106	1.889
		AB	0.116	2.076
4	5.581	BC	0.114	2.041
		CD	0.112	2.004
		AB	0.094	1.688
5	5.622	BC	0.094	1.688
		CD	0.096	1.724
		AB	0.084	1.509
6	5.634	BC	0.084	1.509
		CD	0.083	1.490

this was sufficient in some cases to change the stress from a tension to a compression or vice versa.

21. *Diagrams and Tables.*—Stress-strain diagrams representing the general average results of the axial and circumferential gage lines are given in Fig. 17, 18, and 19, while sample diagram showing the average results at different sections of the tube for both the tension-tension and the compression-tension experiments are given in Fig. 13 to 16. Diagrams of the experimental results of Series 1 and 2 are to be found in Fig. 22, and those of Series 3 in Fig. 23. A comparison of the theories of the strength of materials under combined stress is made in Fig. 24, while Fig. 26 and 27 illustrate some of the work of other investigators. An outline of the test specimens and tests and the principal data of the tubes are given in Tables 1 and 2. Table 3 is given as a sample of the data for a single tube, tube No. 4 of Series 1. These data have been reduced and corrected for standard bar readings. All the original and reduced data as well as the stress-strain diagrams are on file at the Laboratory of Applied Mechanics of the University of Illinois.

IV. Discussion of Results.

22. *The Criterion of Strength.*—There are three possible stress limits any one of which may be the criterion of the strength of material —limit of proportionality, yield point, and rupture or ultimate strength. It is recognized that there may be a sharp distinction between the laws governing ductile materials and the laws governing brittle materials, for such a distinction is observed in the stress-strain diagrams and in compression and torsion failures. Since this discussion is limited to ductile materials, conditions will be treated only as they apply to such materials.

It would appear at first thought that the limit of proportionality would be the proper basis upon which to determine the relative strength of material. The mathematical theory of elasticity is based upon Hooke's law generalized, engineering practice bases its computations largely upon this same law, and several investigators have used the limit of proportionality (which they called the elastic limit) as their criterion, notably Hancock* and Turner.†

The limit of proportionality, or p-limit, is defined as the stress at which the constancy of the ratio of stress to strain ceases; that is, the modulus of elasticity is a constant up to this stress. It is often stated

*American Society for Testing Materials, 1905, '06, '07, '08.
†Engineering, London, February 5, 1909.

Table 3.

Test Data of Axial Gage Lines Tube No. 4, Series 1.

Ratio of Circumferential Tension to Axial Tension 0.69.
Inside Diameter of Tube 5.563.

AB Gage Lines
Average Thickness of Tube .085 in. Area of Section 1.508 sq. in

Water Pressure lb. per sq. in.		Axial Load due to W. Pressure	Machine Load pounds	Total Axial Load pounds	Av. Axial Unit Stress lb. per sq. in.	Reading on Gage Line				Differences				Av. Diff.	Av. Unit Elongation
Gage Rdg.	Corrected					AB1	AB2	AB3	AB4	AB1	AB2	AB3	AB4		
100	100	2430	5100	7500	4950	80.1	109	280	625	0	0	0	0	0.	0
300	300	7290	14100	21400	14200	77.0	5.9	2.9	581	3.1	50	6.1	44	47	.00024
500	500	12150	23050	35200	23300	72.5	0.0	17.1	527	7.6	109	109	98	9.8	.00049
700	700	17000	32100	49100	32500	66.1	94.0	11.9	478	14.0	169	16.1	147	15.4	.00077
900	900	21900	41100	63000	41700	61.0	86.9	5.0	419	19.1	240	23.0	206	21.7	.00109
950	950	23100	43400	66500	44000	60.3	84.1	3.5	403	19.8	268	24.5	216	23.2	.00116
1000	1000	24300	45600	69900	46200	59.0	82.0	1.8	397	21.1	289	26.2	228	24.8	.00124
1050	1050	25500	47900	73400	48500	57.3	79.0	99.0	379	22.1	319	29.0	246	26.9	.00135
1100	1090	26500	50100	76600	50900	55.2	76.4	97.7	361	24.9	345	30.3	264	29.0	.00145
1150	1140	27700	52400	80100	53100	53.1	73.3	95.3	340	27.0	376	32.7	285	31.5	.00158
1200	1190	29200	54500	83500	55300	50.9	69.0	93.0	321	29.2	419	35.0	304	34.1	.00171
1250	1240	30100	56800	86900	57600	47.0	65.3	88.0	282	33.1	456	40.0	343	38.3	.00192
1300	1290	31300	59100	90400	60000	30.9	57.9	74.7	161	49.2	530	53.3	464	50.5	.00253

BC Gage Lines
Average Thickness of Tube .084 in. Area of Section 1.481 sq. in.

Water Pressure lb. per sq. in.		Axial Load due to W. Pressure	Machine Load pounds	Total Axial Load pounds	Av. Axial Unit Stress lb. per sq. in.	Reading on Gage Line				Differences				Av. Diff.	Av. Unit Elongation
Gage Rdg.	Corrected					BC1	BC2	BC3	BC4	BC1	BC2	BC3	BC4		
100	100	2430	5100	7500	5060	92.1	79.5	35.0	63.5	0	0	0	0	0	0
300	300	7290	14100	21400	14400	87.0	74.8	29.9	58.4	5.1	4.7	5.1	5.1	5.0	.00025
500	500	12150	23050	35200	23800	81.9	69.0	24.7	53.6	10.2	10.5	10.3	9.9	10.2	.00051
700	700	17000	32100	49100	33100	76.0	63.9	19.1	49.0	16.1	15.6	15.9	15.5	15.8	.00079
900	900	21900	41100	63000	42500	68.9	57.2	13.0	42.9	23.2	22.3	22.0	20.6	22.0	.00110
950	950	23100	43400	66500	44800	67.2	55.0	11.0	41.2	24.9	24.5	24.0	22.3	23.9	.00120
1000	1000	24300	45600	69900	47200	65.0	53.5	10.3	40.0	27.1	26.0	24.7	23.5	25.4	.00127
1050	1050	25500	47900	73400	49400	62.5	51.5	8.4	38.5	29.6	28.0	26.6	25.0	27.3	.00137
1100	1090	26500	50100	76600	51900	59.9	49.2	6.9	36.6	32.2	30.3	28.1	26.9	29.4	.00147
1150	1140	27700	52400	80100	54100	56.3	47.4	4.8	34.3	35.8	32.1	30.2	29.2	31.8	.00159
1200	1190	29200	54500	83500	56400	52.3	44.8	1.5	30.3	39.8	34.7	33.5	33.2	35.3	.00177
1250	1240	30100	56800	86900	58600	47.8	41.5	97.3	23.1	44.3	38.0	37.7	40.4	40.1	.00201
1300	1290	31300	59100	90400	61100	30.0	36.0	73.8	91.0	62.1	49.2	61.2	72.5	61.3	.00307

CD Gage Lines
Average Thickness of Tube .083 in. Area of Section 1.472 sq. in

Water Pressure lb. per sq. in.		Axial Load due to W. Pressure	Machine Load pounds	Total Axial Load pounds	Av. Axial Unit Stress lb. per sq. in.	Reading on Gage Line				Differences				Av. Diff.	Av. Unit Elongation
Gage Rdg.	Corrected					CD1	CD2	CD3	CD4	CD1	CD2	CD3	CD4		
100	100	2430	5100	7500	5100	68.2	44.9	95.0	40.9	0	0	0	0	0	0
300	300	7290	14100	21400	14600	64.1	39.5	89.8	34.9	4.1	5.4	5.2	6.0	5.2	.00026
500	500	12150	23050	35200	23900	59.0	34.7	84.0	30.9	9.2	10.2	11.0	10.0	10.1	.00051
700	700	17000	32100	49100	33300	53.0	29.0	78.9	25.0	15.2	15.9	16.1	15.9	15.8	.00079
900	900	21900	41100	63000	42900	47.3	23.2	74.0	19.4	20.9	21.7	21.0	21.5	21.3	.00107
950	950	23100	43400	66500	45100	46.1	22.0	73.0	18.3	22.1	22.9	22.0	22.6	22.4	.00112
1000	1000	24300	45600	69900	47500	43.9	20.0	71.3	17.0	24.3	24.9	23.7	23.9	24.2	.00121
1050	1050	25500	47900	73400	49700	41.0	17.1	68.3	15.2	27.2	27.8	25.7	25.7	26.6	.00133
1100	1090	26500	50100	76600	52200	38.8	14.1	67.0	13.9	29.4	30.8	28.0	27.0	29.0	.00145
1150	1140	27700	52400	80100	54400	35.4	11.1	65.0	11.8	32.8	33.7	30.0	29.1	31.4	.00157
1200	1190	29200	54500	83500	56800	32.0	7.4	62.5	9.0	36.2	37.5	32.5	31.8	34.5	.00173
1250	1240	30100	56800	86900	59000	26.9	4.0	55.0	4.5	41.3	40.7	40.0	36.4	39.6	.00198
1300	1290	31300	59100	90400	61500	8.0	86.0	47.2	90.0	60.2	58.9	47.8	50.9	54.5	.00273

that the distinction between yield point and p-limit is very slight and that it really makes no material difference which is used. But a glance at the stress-strain diagrams in Fig. 13 to 16, will show that in some cases the modulus of elasticity changes and that the diagram consists of a broken line instead of a straight line nearly up to the yield point. This fact, due to the lack of isotropy in the material and to the mechanical work done upon it, makes it difficult to get consistent results by using the p-limit as a criterion. When the material has been cold worked, the stress-strain diagram often curves away from a straight line slowly and the exact point of departure is not easily located. Special treatment of the material usually affects the yield point in the same way in different specimens, but not the p-limit.

The use of rupture or ultimate strength as a criterion of the strength of ductile materials still persists in the case of simple stresses, and specifications ordinarily require that the ultimate strength of the material shall have a certain value. But this is an indirect measure of the toughness rather than of the strength, and in the best specifications the yield point (or elastic limit as it is frequently but incorrectly called) is specified as well. Conditions at rupture give no indication of those existing at the yield point and whatever value a knowledge of the conditions attending rupture in a ductile material may have, no conclusions can be drawn from them which may safely be applied to the period preceding the yield point. As engineering design deals principally with stresses within the yield-point stress, rupture cannot be considered as the criterion, even though Bridgman* in his tests on thick cylinders uses it and decries the use of the yield point. When the distribution of stress is unknown and no extensometers are used to measure strains, rupture is the only criterion available.

For ductile material that has not been worked cold, the stress-strain diagram shows a very decided change in character when the material passes the yield point. When the material has been cold-rolled or cold-drawn, the yielding is more gradual and the curve, instead of breaking sharply, departs more gradually from a straight line. If the specimen of the cold-rolled or cold-drawn material is tested in simple tension with an extensometer, and the load is slowly but steadily applied, the roll of the curve is apparent a short time before the yield point is registered by the drop of the beam.

As all the investigations hereinafter described were made with instruments to measure the strains, some criterion must be adopted that

*Phil. Mag., July, 1912.

is applicable to a stress-strain diagram. The first deviation from a straight line (p-limit) is an indefinite point to locate, and, after considering everything that has been noted above, the method proposed by the late J. B. Johnson was adopted. This is called by him the "apparent elastic limit," although it is here taken as the yield point. This method empirically locates a point at which there is evidently some plastic action and furnishes a very convenient method for comparison of results. It is defined as the unit-stress at which "the rate of deformation is 50 per cent greater than it is at zero stress." Fig. 12 shows the application to a stress-strain diagram. Let O B E be a stress-strain diagram drawn in the usual manner. Then A O B is the angle determining the slope

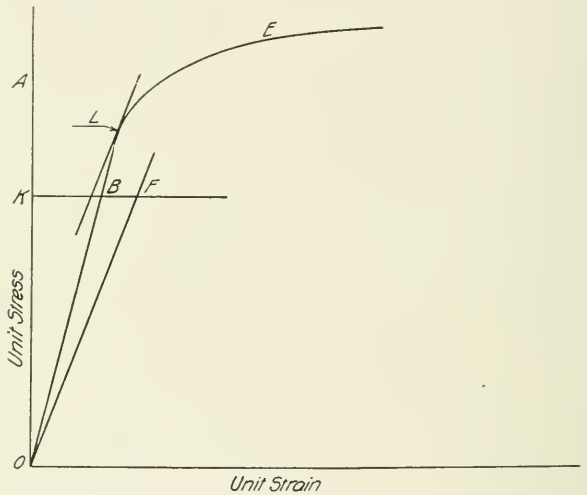
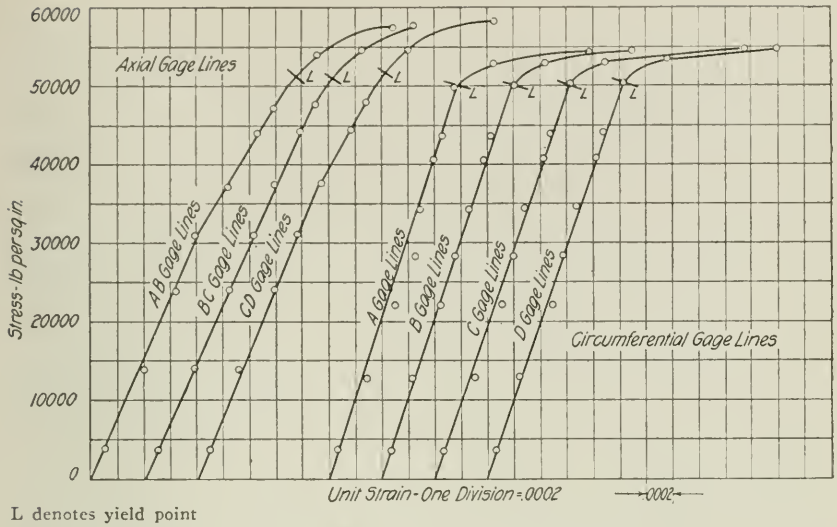


FIG. 12. STRESS-STRAIN DIAGRAM SHOWING JOHNSON'S APPARENT ELASTIC LIMIT.

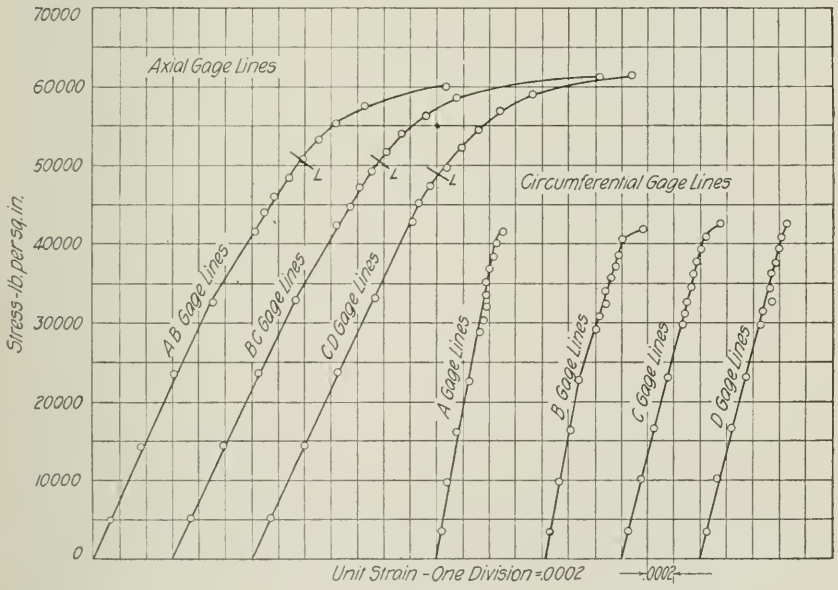
at zero stress. At any point K lay off horizontally a distance K F equal to 1.50 times K B. Then O F is the slope 50 per cent greater than the slope at zero stress. A parallel to O F drawn tangent to the curve B E, locates the point of tangency L and the corresponding stress is the yield-point stress.

23. *Strength.*—In the tabulation of the results of the tests of tubes under biaxial stress, the average of the strains measured on the four gage lines intersected by any cross section was taken as the strain at that section of the tube. Thus, the strains for the axial gage lines, A B1, A B2, A B3, and A B4 of a tube (for notation see page 23 and



L denotes yield point

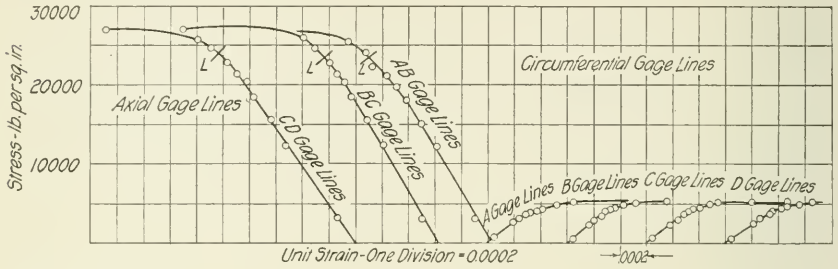
FIG. 13. STRESS-STRAIN DIAGRAMS FOR TUBE NO. 3, SERIES 1. RATIO OF CIRCUMFERENTIAL TO AXIAL TENSION, 0.94.



L denotes yield point

FIG. 14. STRESS-STRAIN DIAGRAMS FOR TUBE NO. 4, SERIES 1. RATIO OF CIRCUMFERENTIAL TO AXIAL TENSION, 0.69.

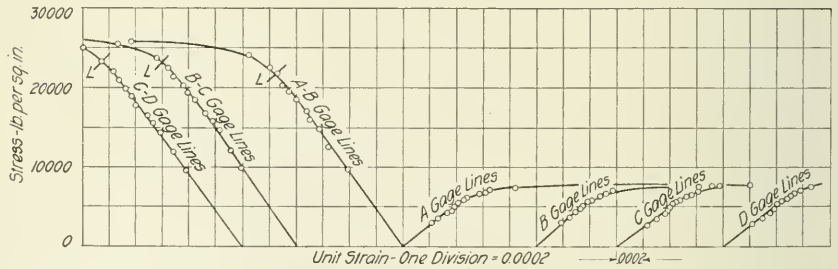
Fig. 9) were averaged, and this average is taken as the strain of the A B gage lines of that tube. Likewise for the B C and C D gage lines. For the circumferential gage lines, 1–2–A, 2–3–A, 3–4–A, and 4–1–A were averaged; that is, the four gage lines made a complete traverse of the circumference. There are then three sets of average results for the



L denotes yield point

Fig. 15. Stress-Strain Diagrams for Tube No. 2, Series 3. Ratio of Circumferential Tension to Axial Compression, 0.20.

axial gage lines and four for the circumferential gage lines of each tube. The curves formed from these average results (see Fig. 13 to 16 for samples) were then used to obtain the general average results for each



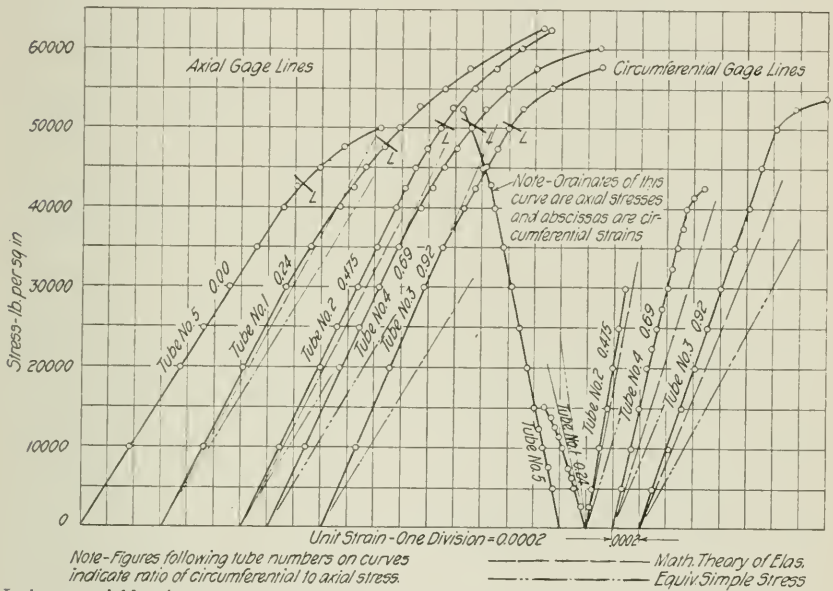
L denotes yield point

Fig. 16. Stress-Strain Diagrams for Tube No. 5, Series 3. Ratio of Circumferential to Axial Tension, 0.30.

of the tubes. That is, the general average results for the circumferential strains represent the average obtained from all the circumferential gage lines in any one tube, and the general average results for the axial strains the average obtained from all the axial gage lines. The only exception is in the case of tube No. 1, Series 1, where the averages of the A B gage lines are omitted in the general average. Each general average curve represents the average results of twelve axial gage lines or sixteen circum-

ferential gage lines. These general average stress-strain diagrams are given in Fig. 17 to 19.

The yield-point stresses are quite uniform for the different sets of gage lines and in close agreement with those of the general average curves. Because of this uniformity, the use of the general average curves as a basis of comparison seems justified. The circumferential strains are plotted with the apparent circumferential tensile stresses as ordinates, except in the case of the tubes where no internal pressure was applied. In these cases the ordinates are the axial stresses, so that it



L denotes yield point

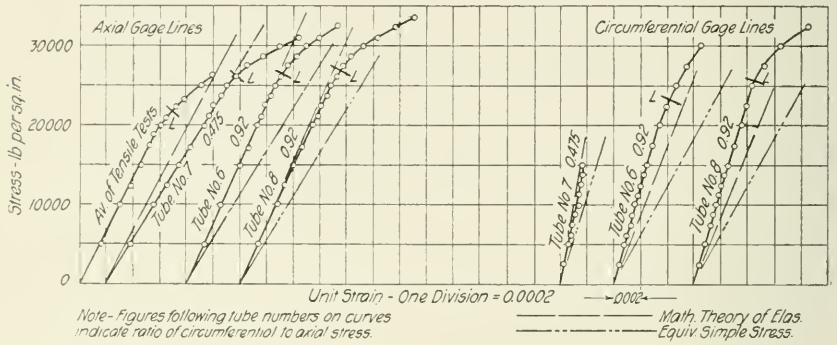
FIG. 17. STRESS-STRAIN DIAGRAMS SHOWING GENERAL AVERAGES FOR SERIES 1. TENSION WITH TENSION.

is easy to determine Poisson's ratio, which is given in Fig. 19 by the ratio of abscissas, corresponding to the same stress, on the two curves of tube 4, such as r to r'.

If diagrams are drawn having the yield-point unit-stresses as ordinates and the ratio of the circumferential tension to axial tension or axial compression as abscissas, a comparison can be made with the results reached by the different theories. For the combination of tension with tension, the maximum stress theory and the maximum shear theory demand that the yield-point stress shall be constant for all ratios.

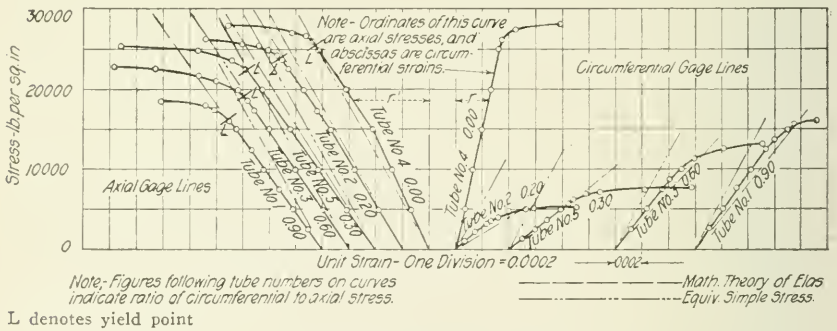
Mohr's theory and the internal friction theory have the same requirements; Wehage's theory demands a reduction in the yield-point stress and the maximum strain theory demands an increase in proportion to the increase of stress ratio.

For the combination of compression with tension the maximum stress theory demands that the yield-point stress shall be constant for all



L denotes yield point

FIG. 18. STRESS-STRAIN DIAGRAMS SHOWING GENERAL AVERAGES FOR SERIES 2. TENSION WITH TENSION.



L denotes yield point

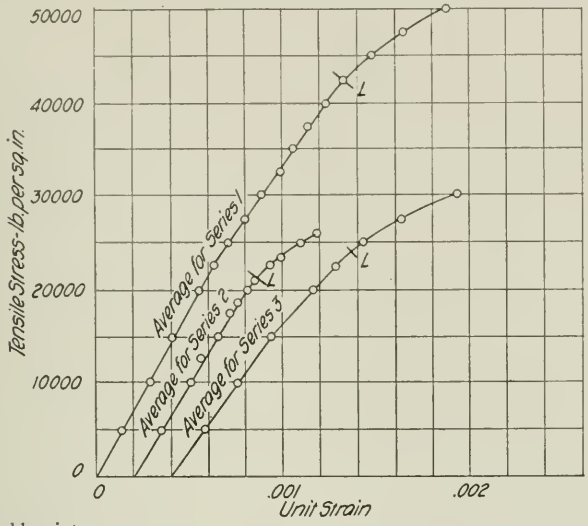
FIG. 19. STRESS-STRAIN DIAGRAMS SHOWING GENERAL AVERAGES FOR SERIES 3. COMPRESSION WITH TENSION.

stresses, while the maximum strain theory, the internal friction theory, the maximum shear theory, and Mohr's theory demand a decrease in the yield-point stress as the stress ratio increases.

What is the law that governs? Referring to the stress-strain curves of the general averages of the axial gage lines, Fig. 17, 18, and 19, it will be seen that for Series 1 and 2 as the stress ratio increases the yield-point stress rises unmistakably until the value of the stress ratio of

0.50 is reached. Beyond this the yield-point stress remains constant, no matter what the stress ratio. For Series 3 the yield-point stress steadily diminishes as the stress ratio increases.

Since for the case of compression combined with tension all the



L denotes yield point

FIG. 20. TENSION TESTS OF SMALL SPECIMENS FROM TUBES OF SERIES 1, 2 AND 3. RESULTS OF TEN TESTS FOR EACH SERIES.

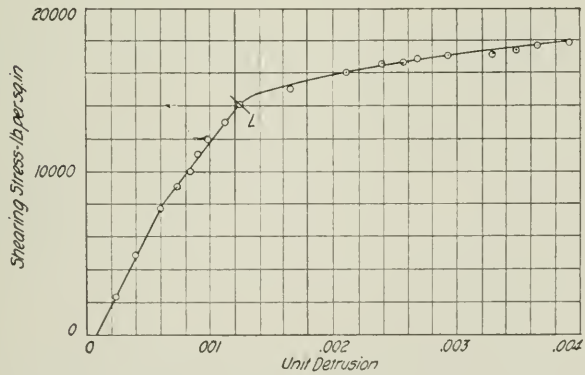


FIG. 21. TORSION TEST OF TUBE NO. 6, SERIES 3.

theories except one demand a decrease of the yield-point stress as the stress ratio increases, while for tension combined with tension the maximum strain theory is the only one which calls for the increase that

has been observed. Series 1 and 2 will be discussed first and the results of Series 3 compared with the conclusions drawn from the results of Series 1 and 2.

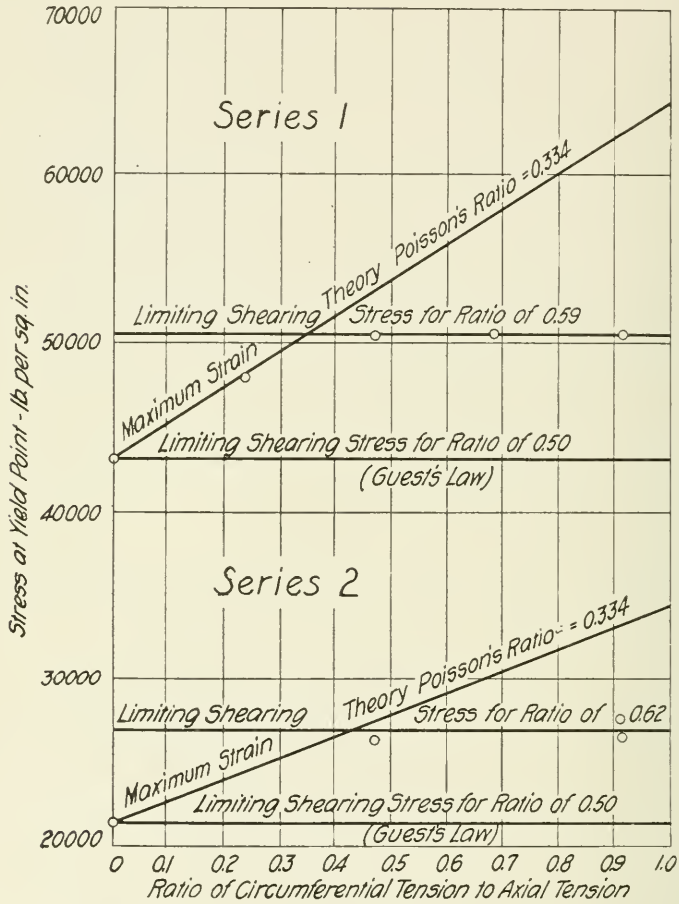


Fig. 22. Diagram Giving Yield Point Stresses and Stress Ratios for Series 1 and 2.

In Fig. 22 the yield-point stresses of Series 1 and 2 are plotted against the ratio of circumferential tension to axial tension. The line of the maximum strain theory is then drawn through the yield-point stress determined in simple tension (stress ratio zero), the inclination being determined by Poisson's ratio (0.334). For Series 1 the yield-point stress was taken from the test of tube No. 5 and for Series 2 the

average of the tension tests of twenty small specimens cut from tubes of Series 2 was taken.*

The determination of Poisson's ratio is discussed on page 46. A line of constant yield-point stress is drawn which best fits the experimental points for stress ratios of 0.50 or above. It is seen that the line of the maximum strain theory fits the experimental points up to its intersection with the line of constant yield-point stress, and that thereafter the line of constant yield-point stress well fits the points. This line of constant yield-point stress may also be a line of constant shearing stress. If, as Fig. 22 seems to indicate, tension ceases to be a governing factor and the shearing stress becomes dominant, two things must be true for the line of constant yield-point stress:

(a) The shearing unit-stress must actually reach the shearing yield-point stress as determined by tests in pure shear, and

(b) Since the shear is one-half the maximum principal stress, this maximum principal stress must remain a constant.

The first condition is important only in so far as showing that the shearing yield-point stress must be greater than one-half the tensile yield-point stress; otherwise the shear would be dominant at all times. The latter is the contention of the maximum shear theory. Counting compression a negative tension and with the principal unit-stresses numbered in the order of their magnitude, p_1, p_2, p_3, the criterion for shearing stress is:

$$\text{Shearing unit-stress} = \tfrac{1}{2}\,(p_1 - p_3),$$

but as the third principal stress is zero, this reduces to $\tfrac{1}{2}\,p_1$. The water pressure inside the tube does not constitute a third principal stress (compressive), for all the readings of the strains were taken on the outside of the specimen where the third principal stress was undoubtedly zero, if the atmospheric pressure is neglected.

The maximum shear theory carried to its logical conclusion requires that the yield-point stress of the material subjected to two stresses of like sign at right angles shall not vary from that reached in simple tension, for the shear is the determining factor at all times. If the theory holds in this form, a horizontal line drawn through the

*The tension test of tube No. 9, Series 2, the first test made, did not furnish the necessary data on account of an unexpectedly low yield point. It is thought that the use of the yield-point stress obtained from the average curve for the specimens from the tubes of Series 2 (see Fig. 20) is justified because the break of the curve of the small specimens from the tubes of Series 1 agrees closely with the break in the curve obtained from tube No. 5 of that series (axial load only), 42,500 lb. per sq. in. and 43,000 lb. per sq. in. respectively. The yield-point stress obtained from the average curve of the specimens from the tubes of Series 2 (21,500 lb. per sq. in.) has been taken as the yield-point stress in simple tension of Series 2 and the value of Poisson's ratio obtained from Series 1 has been used for Series 2.

yield-point stress in simple tension should pass through all the points. Instead, it touches only the initial point. Experiments* have shown that for ductile material the ratio of the shearing yield-point stress, obtained by torsion tests, to the tensile yield-point stress varies with the material, but usually lies between 0.55 and 0.65, in the majority of

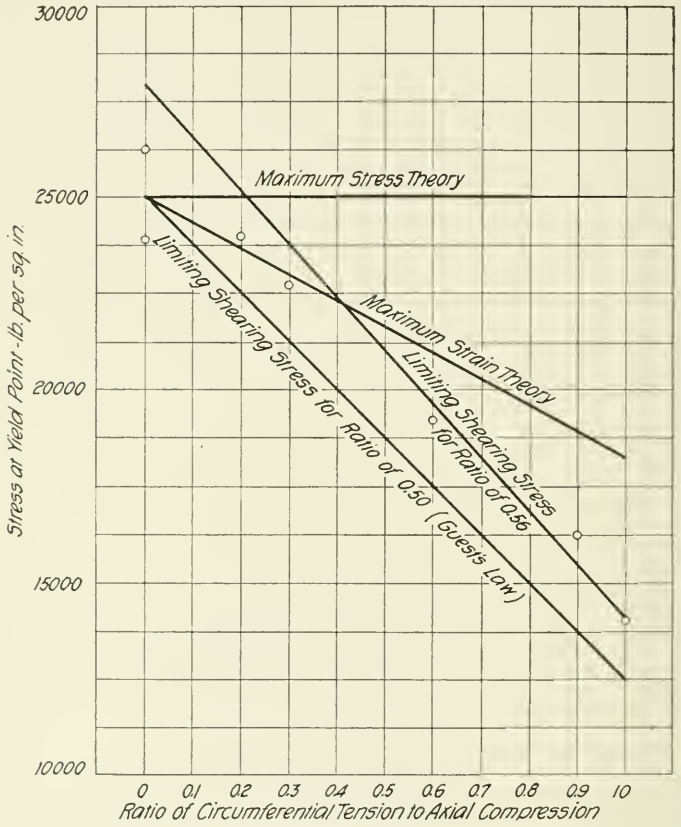


FIG. 23. DIAGRAM GIVING YIELD POINT STRESSES AND STRESS RATIOS FOR SERIES 3.

tests ranging near 0.60, which is the commonly accepted value. A few tests show a ratio less than 0.50, but they are relatively small in number. With a ratio of 0.60, the shearing yield-point stress line would lie above the line through the yield-point stress in simple tension by an amount equal to 0.20 of the latter stress. The exact location of the line will

*L. B. Turner, Engineering, London, February 5, 1909.

vary with the material, but as long as the ratio of the yield-point stresses is above 0.50, there is the hiatus between this condition and that demanded by the above form of the maximum shear theory.

The horizontal line through the experimental points in Fig. 22 is evidently the limit of the shearing strength. It corresponds to a ratio of shearing yield-point stress to tensile yield-point stress of 0.59 for Series 1 and 0.62 for Series 2, which values agree well with the majority of experiments.

These tests indicate that there are two laws covering the case of combined stress when the stresses are both tension and act in two directions at right angles. Apparently the point at which the change in law occurs depends upon the ratio of the yield-point stress in shear to that in tension and the change from one law to the other may occur at different ratios of the principal stresses for different materials. It is important to establish this ratio of yield-point stresses, for if it is not approximately constant the use of combined stress formulas will require a knowledge of such a ratio for all materials.

Before discussing Series 3, the two laws just referred to will be applied to the other combinations of stress and a comparison made with the maximum stress theory, the maximum strain theory, and the maximum shear theory. Assuming the ratio of the shearing and tensile yield-point stresses to be 0.60 and the tensile and compressive yield-point stresses equal, the co-ordinates of the rectangle A B C D (Fig. 24) represent the maximum stress theory, the rhombus Q K J L the maximum strain theory, and the figure A K_1, B C L_1, D A the maximum shear theory. The line A M K_2, N B represents the two laws in the tension-tension quadrant, while B R S C represents them in the tension-compression quadrant. The lines M K_2 and K_2N are parallel to the axes and at such a distance from them that the ordinate of M K_2 and the abscissa of K_2N are each 1.20 times O A or O B, the tensile yield-point stress. R S is parallel to B C and at such a distance from it that one-half the sum of the ordinate and abscissa of any point between R and S is equal to 0.60 of O B or O C. The construction of the other two quadrants is such that the figure is symmetrical about the bisectors of the quadrants. The diagrams, Fig. 22, showing the comparison of theory and experiment for Series 1 and 2 correspond to A M K_2 in the tension-tension quadrant.

In Fig. 23 the yield-point stresses of Series 3 are plotted as ordinates and the stress ratios of circumferential tensile stress to axial compressive

stress as abscissas. Before discussing the various theories in connection with the experimental results, the starting points of the theoretical lines must be fixed. The maximum shear theory demands the same yield-point stress in tension and in compression; the maximum strain theory and the maximum stress theory do not. From the average curve of ten specimens cut from a ten-inch remnant of the original tubing from

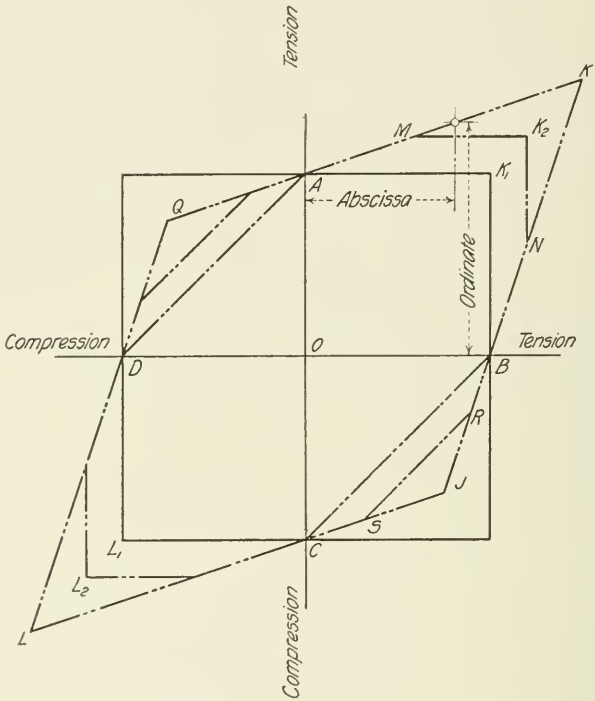


Fig. 24. Representation of Yield Point Strengths for Combined Stresses According to the Maximum Stress Theory, the Maximum Strain Theory, and the Maximum Shear Theory.

which the tube specimens of Series 3 were cut, (Fig. 20) the tensile yield-point stress was found to be 24,000 lb. per sq. in. The compressive yield-point stress obtained from tube No. 4 (no internal water pressure) was 26,250 lb. per sq. in. With the demand of the maximum shear theory for equal yield-point stress in tension and in compression it seems correct to take as the initial point of the line of that theory the average of these values, or 25,100 lb. per sq. in. The lines of the maximum strain theory and of the maximum stress theory were also drawn through

this average value of the yield-point stresses and compressive yield-point stresses. The use of this average value is believed to be justified by the observed fact that the yield-point stress of low-carbon steel in tension is found in nearly all cases to have the same numerical value as the yield-point stress in compression. In determining the line for the maximum strain theory a value of Poisson's ratio of 0.395 was used. This value was obtained from the test of tube No. 4, Series 3 (Fig. 19). A line of constant shearing stress has been drawn through the yield-point stress (14,000 lb. per sq. in.) obtained from the torsion test, since simple torsion produces tensile and compressive stresses of equal intensities and hence corresponds to a stress ratio of unity (see Fig. 21). The lines of internal friction theory and of Mohr's theory practically coincide with the maximum shear theory.

An inspection of Fig. 23 shows that for Series 3 as well as for Series 1 and 2, the experimental results follow the maximum strain theory up to a certain stress ratio and then follow a line of constant shear which is the maximum shear developed. The ratio of the shearing yield-point stress from the torsion test to the average of the tensile and compressive yield-point stresses is 0.56. The question of the neglect of water pressure as a third stress does not enter in this series, for taking the stresses in the order of their magnitude the compression due to the water pressure becomes intermediate between the circumferential tension and the axial compression, so that the maximum shearing stress is equal to one-half the sum of the axial compressive stress and the circumferential stress. This series leads to the same conclusions as the other two, although the ratio of the shearing and tensile yield-point stresses is somewhat lower.

The net result of this investigation as it affects the strength of steel under combined stress in two directions at right angles to each other—biaxial loading—is that instead of a single law, whatever its nature, as has heretofore been assumed, there are two distinct laws governing the strength of the material, each law dominant within its limits. These two laws are the maximum strain theory and the maximum shear theory; the first governs until the shearing yield-point stress of the material is reached, after which the shear theory holds. The exact point of the change from one law to the other depends upon the ratio of the shearing yield-point stress to the yield-point stress in simple tension and compression.

24. *Stiffness.*—Although strains have been measured in many tests

heretofore made, no attempt seems to have been made to determine the law of stiffness. It has been taken for granted that the deductions of the mathematical theory of elasticity, as embodied in St. Venant's theory, hold, or else no attention has been paid to strains except as related to the strength of the material in the determination of the yield point or so-called elastic limit. The weakness of the mathematical theory of elasticity lies in its generalization of Hooke's law and the neglect of the temperature changes, so that the strains obtained in tests of isotropic materials will only closely approximate the computed values. The effect of shear in producing strain has been neglected and is small before the yield-point stress is reached, but the variation of shearing strength in different directions throughout the specimen, the possibility of a change in Poisson's ratio with increasing stress, the possibility of a different Poisson's ratio and modulus of elasticity with and across the direction of rolling or drawing, enter to complicate the problem. The material experimented upon is not the isotropic substance assumed in the theory. Lines have been drawn on the stress-strain curves of the general averages of the axial gage lines, Fig. 17, 18, and 19, giving the strains as computed by the mathematical theory of elasticity using the values of Poisson's ratio* and modulus of elasticity obtained from tests in simple tension and in compression. These lines agree quite closely with the observed values except in the case of tube No. 1 of Series 1, and tube No. 7 of Series 2, the former showing lower strains and the latter greater strains than the computed values. Apparently within the range of application of the mathematical theory of elasticity, where E is constant, the strains follow the theory with sufficient exactness to say that the theory holds. Lines have also been drawn to represent the strains corresponding to a simple tensile or compressive stress equal to the greater principal stress.

For the circumferential lines there have been drawn on the stress-

*The values of Poisson's ratio for the tubes tested in simple compression and in simple tension were obtained by dividing the circumferential unit-strain taken from the general average curves of these tubes (which is the same as the diametral unit-strain) by the corresponding axial unit-strain. For Series 1 this ratio for tube No. 5 is 0.334; for Series 3, obtained from tube No. 4, it is 0.395. The modulus of elasticity of Series 1 is 27,200,000 lb. per sq. in., and for Series 3 it is 29,500,000 lb. per sq. in. An examination of the axial and circumferential stress-strain diagrams of tube No. 5, Series 1, Fig. 17, and of tube No. 4, Series 3, Fig. 19, shows that in the first case (tension) Poisson's ratio remains practically constant, diminishing about 6 per cent after the yield-point stress has been passed, but that in the second case (compression) this ratio increases to almost 0.50 after the yield-point stress has been passed. There is no reason why Poisson's ratio should be constant for all kinds of steel, and it may well be that tension and compression tests on the same material will show different results. It is not known what the effect of the hollow specimen is in changing this ratio for tension or compression tests, but it is thought that the method used to obtain Poisson's ratio is accurate and reliable. It is to be noted that for the compression tests the value of both Poisson's ratio and the modulus of elasticity are higher than for the tension tests.

strain curves of the general averages, Fig. 17, 18, and 19, lines giving the strains computed by the mathematical theory of elasticity and also the strains accompanying simple tensile stresses equal to the circumferential stresses. The values of Poisson's ratio and the modulus of elasticity are taken the same as for the axial lines. The lines of the mathematical theory of elasticity do not fit as well as in the case of the axial strains. It can be seen that to fit the experimental points of Series 1 and 2, it is necessary to use a higher value for both the modulus of elasticity and Poisson's ratio, the latter requiring the greater change. An increase in Poisson's ratio will increase the strains of tube No. 1 and lower those of the other tubes of these two series. An increase of the modulus of elasticity will diminish all the strains proportionally. It will be recalled that the value of Poisson's ratio obtained in compression tests was high, 0.395. For Series 3, where Poisson's ratio is higher than for Series 1 and 2, the modulus of elasticity alone need be increased. With a higher modulus the computed circumferential curves fit the experimental curves quite closely except for tube No. 5. There is a strong probability that both Poisson's ratio and the modulus of elasticity vary in the two directions with and across the direction of drawing. It is scarcely probable that the law changes, and the close agreement between the computed and observed values for the axial strains gives strong support to the belief that all the strains follow the requirements of the mathematical theory of elasticity. The indications are that the modulus of elasticity and Poisson's ratio may be different in different directions throughout the steel, in much the same way that Bauschinger has shown that the shearing strength of rolled steel varies in different directions.

In Series 1 and 2, Fig. 17 and 18, for the tubes tested with a stress ratio of 0.92, the yield-point stress in the circumferential direction was practically the same as the yield-point stress in the axial direction, but the circumferential curves show a more sudden yielding of the material. In Series 3, Fig. 19, for the tube tested with a stress ratio of 0.90, the circumferential yield-point stress was lower than that in an axial direction. All the circumferential stress-strain curves of Series 3 show a sharp, sudden break when the yield-point stress in the axial direction is reached, no matter what the circumferential stress was. Granting that for Series 3 the value of Poisson's ratio increases to 0.50 above the yield-point stress, this is not sufficient to account for the great increase in the strains. It must be that the shearing stresses, which have passed the shearing yield-point stress, produce shearing strains of sufficient

magnitude to account for this increase in the circumferential strains. This explanation is more strongly suggested by the stress-strain curves of Series 1 and 2, where Poisson's ratio remains nearly constant. The circumferential stress-strain diagrams that continue to show an increase in strain after the axial yield-point stress has been passed are those from the tubes whose axial yield-point stresses lie on the line of constant shear of Fig. 22. Those that do not show an increase at this time are from the tubes whose axial yield-point stresses lie on the line of the maximum strain theory.

That the shearing strains accompanying the axial stress can affect the circumferential strains is shown by the stress-strain diagrams for the circumferential lines of tube No. 4, Series 1, Fig. 14. The circumferential stress-strain diagram continues straight for a short distance after the yield-point stress has been passed in the axial direction, the circumferential stress corresponding to the axial yield-point stress being 34,500 lb. per sq. in., approximately. This is during the stage intermediate between the elastic and plastic conditions. When the axial curve breaks sharply, the circumferential curve changes direction also. If Poisson's ratio were the only factor, all the diagrams, with the possible exception of those of tubes No. 3, 6, and 8, where a high stress ratio was used, would show diminishing strains with increasing stress after the yield-point stress in the axial direction had been passed. This means that the tendency to reduce the diameter of the tube, due to the rapidly increasing axial strains, would be greater than the tendency to increase the diameter produced by the increase of the water pressure. But the curves of tubes No. 2 and 4 show an increasing strain (increasing tube diameter) even though the circumferential stresses were well below the yield-point stress of the material. These two tubes are the ones whose yield-point stresses lie on the line of constant shear, Fig. 22, and without the assistance of the shearing strains in producing circumferential strains, the curves of these two tubes would show a diminishing circumferential strain as the circumferential stress increased after the axial yield-point stress had been passed. The shear which causes yielding in an axial direction is on a different plane from that causing yielding in a circumferential direction. The former shear acts along a plane which passes through the direction line of the circumferential tension and cuts the axis of the tube at an angle of 45°. The latter shear acts on a plane which passes through the direction line of the axial tension and is parallel to the axis of the tube making an angle of 45° with the direc-

tion line of the circumferential stress. These shearing stresses are of different magnitudes, according to the ratio of the stresses, and each is equal to one-half the principal stress cut by its plane at an angle of 45°.

Apparently the strains follow the requirements of the mathematical theory of elasticity for all stress ratios, but there may be different values of Poisson's ratio and the modulus of elasticity for the axial and circumferential directions. After the yield-point stress in one direction has been passed the shearing strains have a considerable influence upon the deformations in the second direction.

25. *Comparison With the Methods and Results of Other Investigations.*—Attention is called to several points of difference between the method of investigation here recorded and the methods used by others. The greatest difference lies in the use of a portable extensometer to measure strains, the strain gage, whereby a large number of measurements were taken, both along the specimen and around it. Previous investigations used a fixed extensometer which measured strains along one or two gage lines, or, in some cases, used no strain measurements. No assumptions of uniform stress distribution were made, in the present series, for the strain gage records the variations and the gage length can be varied to suit the needs. With readings taken on a large number of gage lines for every load increment, a certain positiveness of result is attained which is impossible with attached instruments and few gage lines. Local effects are thus minimized. Another difference lies in the larger size of the specimens tested and in the smaller ratio of thickness of tube wall to diameter. Because of the form of specimen and the method of applying load, the stress was nearly uniform throughout the specimen; there was no "helping" effect by understressed material, no point of maximum stress to be located. The use of Johnson's apparent elastic limit method for determining yield-point stress gives a definite point for comparison.

An attempt was made to keep a definite ratio between circumferential and axial stresses throughout the test of each tube, so that comparison might be made later for these ratios. As far as possible, it was intended with a set of specimens cut from a given length of tubing to cover the entire range of stress ratio within tension-tension or compression-tension quadrants. The experiments reported by others and referred to in this section show generally a haphazard ratio of stresses, and the loads used were such that a definite stress was produced in one direction and then the other stress was increased until yielding took place.

The earliest important investigation of this subject was that reported by J. J. Guest* in 1900. The tests were made upon small steel, copper, and brass tubes about 1¼ in. outside diameter and varying in thickness from 0.025 in. to 0.034 in. Tests were made in combined torsion and

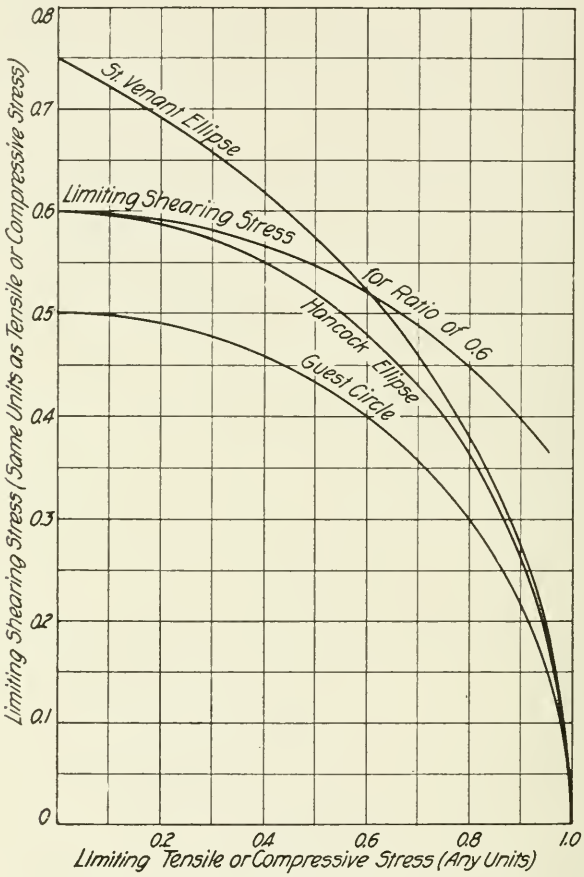


Fig. 25. Relation Between Shearing Stresses Due to Torsion and the Tensile or Compressive Stresses Due to Axial Load or Bending.

axial tension, in torsion and circumferential tension, and in axial and circumferential tension. The strains were measured by a two-point extensometer, and although it was attached to the outside of the tube, the full hydrostatic pressure was counted as a third principal stress (compressive). Other than the tests on tube No. 1 of Guest's investi-

*Phil. Mag., 1900.

gation, there are but two tests where the stress ratio of circumferential tensile stress to axial tensile stress is 0.50 or less, and test No. 1 and one of the others follow the maximum strain theory closely. The yield-point stress was used as the basis of comparison, and each test was carried just beyond the yield point. Criticism may be made of the repeated use of the same specimen, since the yield-point stress is raised by repeated loading beyond the yield-point stress of the first test. It is not stated whether the tubes were annealed between tests. The results are taken to justify the maximum shear theory, and in the main they do within the field investigated, since the majority of the tests had a stress ratio between 0.50 and 1.00 within which limits the shear theory undoubtedly holds. The tests also show that the maximum shear developed is greater than one-half the yield-point stress in simple tension.

Following Guest comes the work of C. A. M. Smith,* W. A. Scoble,† E. L. Hancock,‡ and Wm. Mason* on bars and tubes in torsion and tension or compression and on small tubes in compression and internal pressure. All these results are used to justify the maximum shear theory which demands that the shearing yield-point stress is equal to one-half the yield-point stress in simple tension. With one exception, however, that of Scoble's tests reported in 1906, the maximum shear developed is greater than one-half the yield-point strength in tension, which, as noted above, was also found in Guest's tests. The majority of these tests—like Guest's—are in the region where the stress ratio is greater than 0.50. These tests cover the entire four quadrants of combined stress.

The tests of Professors Smith and Hancock will be shown on diagrams similar to Fig. 25 (Fig. 26 and 27), in which the ordinates represent the shearing stress due to torque and the abscissas represent the tensile or compressive stress due to axial load or bending. The diagram of Fig. 25 will be discussed before the tests are taken up. The shearing stress is plotted to twice the scale of the tensile or compressive stress. If a circle with a radius equal to the tensile yield-point stress is drawn with O as a center, it will represent the relation between the shearing and the direct stress which produces a combined stress causing yielding required by the maximum shear theory. It will be observed that the shearing yield-point stress must therefore equal exactly one-half the tensile yield-point stress. A circle with radius equal to the shearing yield-point stress obtained from tests in simple torsion (0.6 the tensile yield-

*Inst. Mech. Engrs., 1909.
†Phil. Mag., 1906
‡Am. Soc. for Testing Materials, 1905, 6, 7, 8.

point stress) is shown; also, the ellipse representing the St. Venant or maximum strain theory, beginning at the tensile yield-point stress. The two laws as advanced in this bulletin require that the maximum strain theory hold to the intersection of the St. Venant ellipse and the circle for limiting shearing stress for ratio of 0.6, and that then the shear

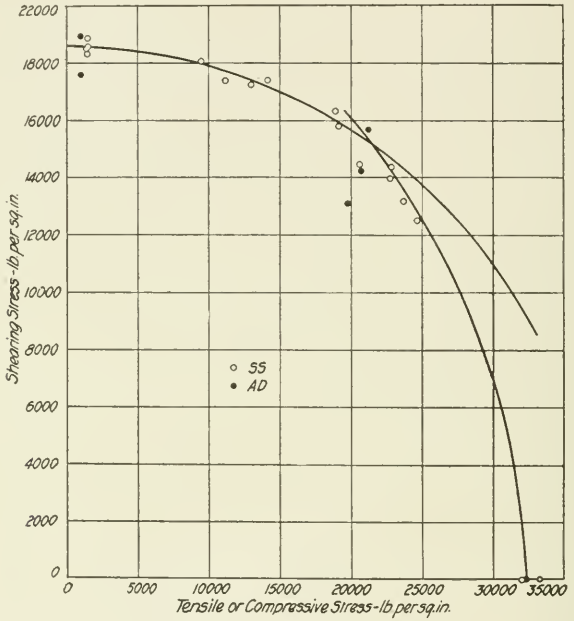


Fig. 26. Results of Tests, by C. A. M. Smith.

shall govern. Hancock's ellipse has been added to show how closely he came to the results here advanced.

The results of Smith's and Hancock's tests have been plotted in Fig. 26 and 27. Both compression and tension have been plotted on the same side of the diagrams (symmetry permitting this), and the results of the different tests have been changed proportionally in order to compare them with a single set of theoretical curves. A comparison of the two laws herein proposed with the experimental results of these investigations show that the experimental results fit these laws better than the maximum shear theory which the tests were taken to prove.

Fig. 26 shows the results of C. A. M. Smith's tests on S. S. and

A. D. steel. Professor Smith maintains that the shearing yield-point stress of steel is one-half the tensile yield-point stress within small limits and quotes Turner's tests* to prove his point. His own tests do not bear out his contention and Turner's tests show considerable variation, averaging about 0.54 for this ratio. Professor Smith's tests are examples of careful work, but the interpretation of the tests as an unqualified endorsement of the maximum shear law cannot be accepted.

Mr. Scoble's tests seem to indicate that the shearing yield-point stress is lower than half the tensile yield-point stress. This result may possibly be accounted for by the way the shearing yield-point stress was located. This stress was taken at the intersection of the straight line of the elastic portion of the stress-strain diagram with a line drawn through the diagram beyond the yield point. Since a stress-strain curve for torsion breaks more quickly than a tension curve, it may be that the determination of the shearing yield-point stresses are affected by this. Scoble's method of measuring the bending moment by means of the deflection of the beam may be in error, for the law of deflection under the combined stress would be influenced by the very law he was seeking to determine.

The results of Professor Hancock's tests† are shown in Fig. 27. The curves of the maximum shear theory and the maximum strain theory have been drawn as well as his ellipse. Hancock used the p-limit as his criterion. He alone of these investigators realized the shortcomings of the maximum shear theory and endeavored to remedy them by fitting an ellipse to the experimental results. The ellipse fits quite closely, but while it is a close approximation, it does not fit the results as closely as do the curves representing the two laws herein proposed. His ellipse is empirical, while the combination of the maximum strain theory with the maximum shear theory has a foundation in the theory of the strength of materials.

Since torsion combined with compression or tension can be resolved into a case of tension combined with compression, Smith's and Hancock's tests fall in the fourth quadrant and show the applicability of the two laws there.

Mason's tests on tubes in compression and internal pressure show that the maximum shearing stress developed is greater than the shearing stress developed in simple compression. The average of all his

*Engineering, London, February 5, 1909.

†Proceedings of the American Society for Testing Materials, 1908.

tests in which a constant stress ratio of one to one was used, gives this maximum shearing stress as 0.60 of the compressive yield-point stress. As all the tests had the one stress ratio, it is not possible to make a comparison with theories, but the point thus located falls on the line of the two laws.

Minor inconsistencies are to be expected in experimental work of this nature, both on account of the variation in the material tested and

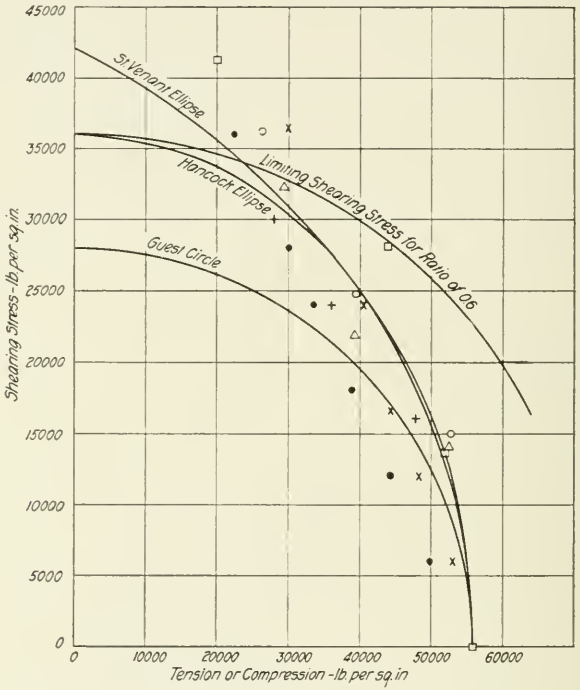


Fig. 27. Results of Tests, by E. L. Hancock.

on account of the apparatus used. The number of tests made by these investigators is insufficient to establish completely any theory, but a careful study of the published data will lead to the conclusion that the two laws, the theory here advanced, conform more closely to the experimental results than any single law.

26. *Summary and Conclusions.*—The following summary deals with the method of investigation and with the deductions which have been made from the data. As this is the first investigation of combined stress wherein a portable strain measuring instrument—such as the strain

gage—has been used, it is felt that considerable emphasis may be laid upon this fact. The size of the specimen is much larger than any heretofore used. These conditions tend to give more trustworthy results.

The experimental conclusions are:

1. The use of a portable strain measuring instrument is a decided advantage since it makes it possible to take measurements on a large number of gage lines for each increment of load, obviating to a large extent the effect of local variations in the test specimen.
2. The use of large tubes with thin walls gives quite uniform stress distribution, the yield-point stress is more positively determined, and the effect of eccentricity of loading is less than with solid bars on account of the larger diameter of the tube.
3. With large tubes the thickness of the tube walls can be accurately determined.
4. Flat plates in cross bending give uneven distribution of stress and are not satisfactory for biaxial loading tests.

The deductions which have been made from the experimental data are:

5. With increasing values of the ratio of the biaxial stresses the yield-point strength follows the maximum strain theory until the value of the shearing stress reaches the shearing yield point, then the shearing stress controls according to a maximum shear theory. There are thus two independent laws each dominant within proper limits instead of some single law as has heretofore been assumed.
6. Because these two laws govern the strength of ductile materials under biaxial loading, the ratio for simple stresses of the shearing yield-point stress to the tensile yield-point stress is important.
7. The stiffness follows the requirements of the mathematical theory of elasticity for all stress ratios, but the values of Poisson's ratio and the modulus of elasticity may be different in the two directions, with and across the rolling and drawing of the steel.
8. The results of the tests reported by previous investigators conform better to the two laws of strength than to any single law.

APPENDIX I.

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APPENDIX II. MATHEMATICAL TREATMENT.

1. *Stresses and Strains.*—The analysis of stress and strain in elastic materials known as the mathematical theory of elasticity embodies the most complete and elaborate theory of the action of elastic bodies under stress. The following brief presentation of the mathematical theory of elasticity as it applies to the problem of the investigation follows largely the treatment of Love.* It will be desirable to outline briefly the work leading up to the derivation of the general equations of the mathematical theory of elasticity connecting stress and strain before taking up the derivation of the equations of stress and strain in a cylinder under internal pressure and an axial load.

In the theory of elasticity the relations between three sets of magnitudes must be considered.

1. The displacements of the points of the strained body. If the ordinary rectangular system of coordinates is used for reference, the displacement s of a point due to the strain is resolved into components u, v, w parallel respectively to the X, Y and Z axes.

2. The strain components. Let ϵ_1, ϵ_2, ϵ_3 denote the strains in the directions of the X, Y, Z axes, respectively; then

$$\epsilon_1 = \frac{\partial u}{\partial x}, \epsilon_2 = \frac{\partial v}{\partial y}, \epsilon_3 = \frac{\partial w}{\partial z} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The components of shearing strain are defined as follows:

$$\epsilon_{23} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, \epsilon_{31} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \epsilon_{12} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \quad \ldots\ldots\ldots\ldots (2)$$

Here ϵ_{23} denotes the shearing strain in the plane YZ, etc. Along with the strain components may be included the components of the rotation of an element of the body. If the displacement involves a rotation ω of the element as a whole and this rotation be resolved into X, Y and Z components, then these components are given by the relations

$$\omega_1 = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \omega_2 = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \omega_3 = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad \ldots\ldots\ldots\ldots (3)$$

3. The stress components. The six stress components may be denoted by σ_1, σ_2, σ_3; σ_{23}, σ_{31},σ_{12}. σ_1 is the stress in the direction of the X-axis on a plane perpendicular to the X-axis; similarly for σ_2 and σ_3. σ_{23} is the stress in the direction of the Y-axis over a plane perpendicular to the Z-axis; therefore it is a shearing stress.

*The Mathematical Theory of Elasticity, A. E. H. Love, 1904.

The stress components must satisfy certain conditions of equilibrium, which are expressed by three equations of the following type (assuming that the body forces, such as gravity, may be neglected, and that the body is at rest).

$$\frac{\partial \sigma_1}{\partial x} + \frac{\partial \sigma_{12}}{\partial y} + \frac{\partial \sigma_{13}}{\partial z} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The six strain components ϵ_1, ϵ_2, etc., and the six stress components are connected by certain relations. Hooke's law, the linear relation between stress and strain, is the basis of these relations. Each stress component is taken as a linear function of the six strain components; thus

$$\left.\begin{aligned} \sigma_1 &= a_1\epsilon_1 + a_2\epsilon_2 + a_3\epsilon_3 + a_4\epsilon_{23} + a_5\epsilon_{31} + a_6\epsilon_{12} \\ \sigma_2 &= b_1\epsilon_1 + b_2\epsilon_2 + b_3\epsilon_3 + b_4\epsilon_{23} + b_5\epsilon_{31} + b_6\epsilon_{12} \end{aligned}\right\} \quad \ldots\ldots\ldots (5)$$

etc.

A consideration of the work done in deforming a body leads to the conclusion that there must exist a so-called strain-energy function V, such that

$$\sigma_1 = \frac{\partial V}{\partial \epsilon_1}, \quad \sigma_2 = \frac{\partial V}{\partial \epsilon_2}, \text{ etc.}$$

It follows that the function V must be a homogeneous quadratic function of the six strain components and must have therefore 21 terms. The number of coefficients apparently 36 in eq. (5) is thereby reduced to 21 by relations of the form $a_2 = b_1$, $a_3 = c_1$, $a_4 = d_1$, etc.; that is, V is the symmetric determinant of the quadric $\sum_1^6 c_{ij}\,\epsilon_{ij}$.

If the body is isotropic, these 21 coefficients can be reduced to two. Denoting by λ one of these remaining coefficients and by μ one-half the difference between the two coefficients, the following relations between stresses and strains are established:

$$\left.\begin{aligned} \sigma_1 &= \lambda\Delta + 2\mu\epsilon_1 \\ \sigma_2 &= \lambda\Delta + 2\mu\epsilon_2 \\ \sigma_3 &= \lambda\Delta + 2\mu\epsilon_3 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

where σ_1, σ_2 and σ_3 are the stresses along the X, Y and Z axes respectively and ϵ_1, ϵ_2 and ϵ_3 are the corresponding strains. Δ is the dilatation and is equal to the sum of ϵ_1, ϵ_2 and ϵ_3.

Let $\frac{1}{m}$ = Poisson's ratio

E = the modulus of elasticity.

Applying the relations between stress and strain to a bar in simple tension, the following relation between Poisson's ratio and the modulus of elasticity is established. Both $\frac{1}{m}$ and E are to be determined from tests in simple tension or compression.

$$E = \frac{\mu(3\lambda+2\mu)}{\lambda+\mu} \quad \ldots\ldots (7)$$

$$\frac{1}{m} = \frac{\lambda}{2(\lambda+\mu)} \quad \ldots\ldots (8)$$

G = the shearing modulus of elasticity = μ.

$$G = \frac{1}{2}\left(\frac{m}{m+1}\right)E \quad \ldots\ldots (9)$$

The values of λ and μ may now be established in terms of E and $\frac{1}{m}$.

$$\lambda = \frac{Em}{(m+1)\,(m-2)} \quad \ldots\ldots (10)$$

$$\mu = \frac{E\,m}{2\,(m+1)} \quad \ldots\ldots (11)$$

Adding the second and third equations of (6)

$$\sigma_2 + \sigma_3 = 2\lambda\,\Delta + 2\mu\,(\epsilon_2 + \epsilon_3)$$
$$= 2\,\lambda\,\Delta + 2\mu\,(\Delta - \epsilon_1)$$
$$\Delta = \frac{\sigma_2 + \sigma_3 + 2\mu\epsilon_1}{2\,(\lambda+\mu)}$$
$$\lambda\,\Delta = \frac{1}{m}\,(\sigma_2 + \sigma_3 + 2\mu\epsilon_1)$$
$$\sigma_1 - 2\mu\,\epsilon_1 = \frac{1}{m}\,(\sigma_2 + \sigma_3) + \mu\epsilon_1\left(\frac{\lambda}{\lambda+\mu}\right)$$

$$\left.\begin{aligned} \sigma_1 &= \frac{1}{m}\,(\sigma_2 + \sigma_3) + E\epsilon_1 \\ \text{Similarly}\quad \sigma_2 &= \frac{1}{m}\,(\sigma_1 + \sigma_3) + E\epsilon_2 \\ \sigma_3 &= \frac{1}{m}\,(\sigma_1 + \sigma_2) + E\epsilon_3 \end{aligned}\right\} \quad \ldots\ldots (12)$$

Rearrangement of Eq. (12) gives

$$\left.\begin{aligned} E\epsilon_1 &= \sigma_1 - \frac{1}{m}(\sigma_2+\sigma_3) \\ E\epsilon_2 &= \sigma_2 - \frac{1}{m}(\sigma_1+\sigma_3) \\ E\epsilon_3 &= \sigma_3 - \frac{1}{m}(\sigma_1+\sigma_2) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

These are the three fundamental equations connecting stress and strain. $E\epsilon_1$, $E\epsilon_2$ and $E\epsilon_3$ are called by various writers the reduced stresses, the true stresses, or the ideal stresses.

2. *Stresses and Strains in a Thin Tube.*—For bodies of cylindrical form it is convenient to use cylindrical coordinates r, θ and z instead of the rectangular system x, y, z. The z coordinate is measured parallel to the axis of the tube, r denotes the radial distance from the axis, and θ the angle of an axial plane from some chosen initial plane.

Denoting by u, v and w the displacement components, as before (u radial, w axial and v perpendicular to a radius r) the three strain components ϵ_r, ϵ_θ, ϵ_z are given by the relations

$$\epsilon_r = \frac{\partial u}{\partial r}, \epsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}, \epsilon_z = \frac{\partial w}{\partial z} \quad \ldots\ldots\ldots\ldots (14)$$

and the corresponding stress components are given by the equations

$$\left.\begin{aligned} \sigma_r &= \lambda\Delta + 2\mu\epsilon_r \\ \sigma_\theta &= \lambda\Delta + 2\mu\epsilon_\theta \\ \sigma_z &= \lambda\Delta + 2\mu\epsilon_z \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Expressions for the shearing strain and stress components may be deduced, but they are not needed in the present investigation.

In the case of a hollow cylinder under internal pressure, conditions of symmetry require that the displacement v shall be zero; hence the expression for ϵ_θ in (14) reduces to $\epsilon_\theta = \frac{u}{r}$. Furthermore, it is permissible in the case under consideration to assume a condition of plane strain, in which all points in a cross section of the cylinder experience the same displacement w in the z direction. With this assumption, $w = az$, where a is a constant, and therefore $\epsilon_z = a$. With these simplifying assumptions, the expression for the dilatation takes the form

$$\Delta = \frac{\partial \mu}{\partial r} + \frac{u}{r} + a$$

If now general expressions for the displacements u and w are found, eq. (14) will give the strain components and eq. (15) the stress com-

ponents. The conditions of equilibrium must, however, be satisfied, that is relations analogous to (4) must be introduced. It is possible, however, to eliminate the stress components by the aid of eq. (6) or eq. (15) and thus to express the equilibrium conditions in terms of displacements only. Thus from (4) and (3) may be derived the relation

$$(\lambda+2\mu)\frac{\partial\Delta}{\partial x}-2\mu\left(\frac{\partial\omega_3}{\partial y}-\frac{\partial\omega_2}{\partial z}\right)=0 \quad \ldots\ldots\ldots\ldots (16)$$

with two similar; and in cylindrical coordinates a similar process leads to the relation

$$(\lambda+2\mu)\frac{\partial\Delta}{\partial r}-2\mu\left(\frac{1}{r}\frac{\partial\omega_3}{\partial\theta}-\frac{\partial\omega_2}{\partial z}\right)=0 \quad \ldots\ldots\ldots\ldots (17)$$

In the case under consideration the rotation components ω_2 and ω_3 must be zero (ω_2 may have a small finite value at the extreme ends of the tube), hence (17) reduces to

$$(\lambda+2\mu)\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}+a\right)=0 \quad \ldots\ldots\ldots\ldots (18)$$

Integration of this equation leads to the following relation for the displacement:

$$u=Cr+\frac{D}{r}$$

in which C and D are constant.

Introducing this expression for u in the expression for ϵ_r, ϵ_θ and Δ, the result is

$$\epsilon_r=\frac{\partial u}{\partial r}=C-\frac{D}{r^2}$$

$$\epsilon_\theta=\frac{u}{r}=C+\frac{D}{r^2}$$

$$\Delta=2C+a$$

Hence the relations (15) become

$$\sigma_r=\lambda(2C+a)+2\mu\left(C-\frac{D}{r^2}\right)$$

$$=2C(\lambda+\mu)-2\mu\frac{D}{r^2}+\lambda a \quad \ldots\ldots\ldots\ldots (20)$$

$$\sigma_\theta\lambda=(2C+a)+2\mu\left(C+\frac{D}{r^2}\right)$$

$$=2C(\lambda+\mu)+2\mu\frac{D}{r^2}+\lambda a \quad \ldots\ldots\ldots\ldots (21)$$

$$\sigma_z=\lambda(2C+a)+2\mu a$$

$$=2C\lambda+(\lambda+2\mu)a \quad \ldots\ldots\ldots\ldots (22)$$

To determine the constants C and D, we have the conditions
$\sigma_r = -p_1$ the internal pressure, when $r = r_1$, the internal radius
$\sigma_r = -p_0$ the external pressure, when $r = r_0$, the external radius
From (20)

$$-p_1 = 2C(\lambda+\mu) - 2\mu\frac{D}{r^2} + \lambda a$$

$$-p_0 = 2C(\lambda+\mu) - 2\mu\frac{D}{r^2} + \lambda a$$

whence

$$2\mu D = (p_1 - p_0)\frac{r_1^2 r_0^2}{r_0^2 - r_1^2} \quad \ldots\ldots (23)$$

$$2C(\lambda+\mu) = \frac{p_1 r_1^2 - p_0 r_0^2}{r_0^2 - r_1^2} - \lambda a \quad \ldots\ldots (24)$$

Equations (23) and (24) may be written

$$2\mu D = T \quad \ldots\ldots (23a)$$

$$2C(\lambda+\mu) = S - \lambda a \quad \ldots\ldots (24a)$$

It will be observed that $S = \frac{p_1 r_1^2 - p_0 r_0^2}{r_0^2 - r_1^2}$ gives the mean intensity of tensile stress in a cross-section of a closed tube due to the internal fluid pressure p_1, with external pressure p_0. In the test the axial stress was in part applied by the testing machine; hence its value may be denoted by kS, where k is a constant that becomes equal to 1 when the axial stress is one-half of the hoop tension. In the test an axial stress was applied by the testing machine and this must be added to the axial stress due to internal pressure. Hence the *total* axial stress may be taken as kS. Putting kS for σ_z in (22) and combining with (24a), we have two equations for the determination of a and C, namely:

$$kS = 2\lambda C + (\lambda + 2\mu)a$$

$$2C(\lambda+\mu) = S - \lambda a$$

From these the following results are readily obtained:

$$a = \frac{S}{E}\left(k - \frac{2}{m}\right) \quad \ldots\ldots (25)$$

$$C = \frac{S}{E}\left[1 - \frac{1}{m}(1+k)\right] \quad \ldots\ldots (26)$$

Also from (23a)

$$D = \frac{T}{2\mu} = \frac{T}{E}\frac{1+m}{m} \quad \ldots\ldots (27)$$

The strain component ϵ_θ is now found.

$$\epsilon_\theta = C + \frac{D}{r^2} = \frac{1}{E}\left\{ S\left[1 - \frac{1}{m}(1+k) \right] + \frac{T}{r^2}\frac{1+m}{m} \right\} \quad \ldots\ldots (28)$$

The value of ϵ_θ at the outer surface of the cylinder, where the strain was measured, is found by taking $r = r_0$. Substituting now the proper expressions for S and T, and putting $r = r_0$, (28) becomes

$$(\epsilon_\theta)_0 = \frac{1}{E}\left\{ \frac{p_1 r_1^2 - p_0 r_0^2}{r_0^2 - r_1^2}\left[1 - \frac{1}{m}(1+k) \right] + (p_1 - p_0)\frac{r_1^2}{r_0^2 - r_1^2}\frac{1+m}{m} \right\} \quad \ldots (29)$$

Since p_0 is small compared with p_1, the terms $p_1 r_1^2 - p_0 r_0^2$ and $(p_1 - p_0)\, r_2$ may be considered equal. With this approximation (29) becomes

$$(\epsilon_\theta)_0 = \frac{1}{E}\left\{ \frac{p_1 r_1^2 - p_0 r_0^2}{r_0^2 - r_1^2}\left(2 - \frac{k}{m} \right) \right\} \quad \ldots\ldots (30)$$

If we consider a closed cylindrical tube with internal hydrostatic pressure p_1 and external pressure p_0, the net load producing axial tension is

$$\pi(p_1 r_1^2 - p_0 r_0^2)$$

and the area of the cross section of the tube is

$$\pi(r_0^2 - r_1^2)$$

Denoting the load by P and the area by A, we have

$$S = \frac{p_1 r_1^2 - p_0 r_0^2}{r_0^2 - r_1^2} = \frac{P}{A}$$

$$(\epsilon_\theta)_0 = \frac{P}{EA}\left(2 - \frac{k}{m} \right) \quad \ldots\ldots (31)$$

$$\epsilon_z = a = \frac{P}{EA}\left(k - \frac{2}{m} \right) \quad \ldots\ldots (32)$$

In the test the axial load applied was $kP = L$; hence

$$(\epsilon_\theta)_0 = \frac{L}{EA}\left(\frac{2m - k}{k\,m} \right) \quad \ldots\ldots (33)$$

$$\epsilon_z = \frac{L}{EA}\left(\frac{k\,m - 2}{k\,m} \right) \quad \ldots\ldots (34)$$

The corresponding values of $E\epsilon$ (reduced stresses) are

$$E\,(\epsilon_\theta)_0 = \frac{L}{A}\frac{2m - k}{k\,m} \quad \ldots\ldots (35)$$

$$E\epsilon_z = \frac{L}{A}\frac{km - 2}{k\,m} \quad \ldots\ldots (36)$$

The actual stresses are

$$\sigma_z = k\,S = \frac{k\,P}{A} = \frac{L}{A}$$

$$\sigma_\theta = S + \frac{T}{r^2}$$

$$\sigma_r = S - \frac{T}{r^2}$$

In the preceding discussion the results have been obtained in terms of p_1 and p_0 the absolute internal and external fluid pressures. Evidently p_0 is the pressure of the atmosphere. In the test the internal pressure p_1 was measured by the gage, and no account was taken of the external pressure p_0. This procedure is justified by the following results:

Let $p' = p_1 - p_0$ = internal gage pressure.

Then
$$S = \frac{p_1 r_1^2 - p_0 r_0^2}{r_0^2 - r_1^2} = \frac{p_1 r_1^2 - p_0 r_0^2 - p_0 r_1^2 + p_0 r_1^2}{r_0^2 - r_1^2}$$

and
$$= (p_1 - p_0)\,\frac{r_1^2}{r_0^2 - r_1^2} - p_0\,\frac{r_0^2 - r_1^2}{r_0^2 - r_1^2} = \frac{p' r_1^2}{r_0^2 - r_1^2} - p_0$$

$$T = (p_1 - p_0)\,\frac{r_0^2 r_1^2}{r_0^2 - r_1^2} = p'\,\frac{r_0^2 r_1^2}{r_0^2 - r_1^2}$$

From (38) the hoop tension at the outer surface is

$$(\sigma_\theta)_0 = S + \frac{T}{r_0^2} = \frac{p' r_1^2}{r_0^2 - r_1^2} - p_0 + \frac{p' r_1^2}{r_0^2 - r_1^2} = 2\,\frac{p' r_1^2}{r_0^2 - r_1^2} - p_0$$

Since p_0 is entirely negligible in comparison with S, we may take

$$(\sigma_\theta)_0 = 2S = \frac{2L}{k\,A} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

For the corresponding stress at the inner surface, we have

$$(\sigma_\theta)_1 = S + \frac{T}{r_1^2} = p'\,\frac{(r_1^2 + r_0^2)}{r_0^2 - r_1^2} = 2S + p' = \frac{2L}{k\,A} + p' \ldots\ldots (41)$$

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BY

HERBERT F. MOORE

And

W. M. WILSON

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THE STRENGTH OF WEBS OF I-BEAMS AND GIRDERS

BY HERBERT F. MOORE,
ASSISTANT PROFESSOR IN THEORETICAL AND APPLIED MECHANICS

AND

W. M. WILSON,
ASSISTANT PROFESSOR OF STRUCTURAL ENGINEERING

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THE STRENGTH OF THE WEBS OF I-BEAMS AND GIRDERS

I. Introduction.

1. *Preliminary.*—In designing beams and girders it is usual to consider that the bending action is resisted by the flanges, and the shearing stress by the web. There is a tendency, however, for webs of deep girders to fail by buckling; and there are complex stresses set up at the junction of the web and flange. There are also crushing stresses in the web over the supports of the girder or under the points of application of concentrated loads.

When a girder is subjected to flexure the material on one side of the neutral axis is subjected to longitudinal tensile stresses and the material on the other side is subjected to longitudinal compressive stresses. The material at any point in the girder is also subjected to longitudinal and to transverse shearing stresses of equal intensity. The longitudinal tensile and compressive stresses are equal to zero at the neutral axis and increase to a maximum at the outer edges of the flanges. The shearing stresses are equal to zero at the outer edges of the flanges and increase to a maximum at the neutral axis. Since the shearing stress is equal to zero at the outer edge of the girder the flange carries little and the web practically all of the shear on a transverse section.

In the design of a girder the longitudinal stress in the outer edge of the flange is limited to a safe value for the material in tension or compression, and the average shearing stress in the web (obtained by dividing the maximum total shear upon a transverse section by the cross-sectional area of the web) is kept within safe limits for the material in shear. Although, according to the elastic theory of beams, points intermediate between the neutral axis and the outer edge of the flange are subjected to both longitudinal tension or compression and to transverse and longitudinal shear, and although it is known that these combined stresses result in diagonal tensile (or compressive) and shearing stresses which are greater than the component stresses producing them, these diagonal stresses are not considered in the design of the girder. The view has been held that the diagonal tensile or compressive stresses do not materially exceed the simple longitudinal stress at the outer edge of the flange, and that the diagonal shearing stress does not materially exceed the shearing stress at the neutral axis.

It has been shown by tests that a tensile (or compressive) stress produces a strain in a direction at right angles to the the line of action of the stress, the term strain being here used to designate deformation and not stress which is used to designate an internal resisting force. Recent tests, notably those of Dr. Becker,* indicate that the tendency of a material to fail depends, within certain limits, upon the *strain,* and that a stress in one direction, while it does not set up lateral *stress,* does set up lateral *strain,* and affects the strength of the material. An analysis of diagonal stresses in a girder shows that, in general, a load which sets up a stress in a diagonal direction sets up a second stress at right angles to that direction. Hence, in considering the strength of a girder to resist a diagonal stress any stress at right angles to that diagonal stress must be taken into account.

The tests reported in this bulletin were made to study the web strains in I-beams and girders so designed that the primary failure would be a web failure. The test data obtained were used in conjunction with a mathematical analysis made to determine the importance of the diagonal strains and the methods of failure of girders.

2. *Previous Tests of the Web Strength of I-Beams and Girders.*—Not many tests of I-beams and girders in which the primary failure was web failure have been made. Table 1 (reprinted from bulletin 68 of the Engineering Experiment Station of the University of Illinois) gives the results of a few such tests.

3. *Acknowledgment.*—This Investigation was a part of the research work of the Department of Theoretical and Applied Mechanics and of the Department of Civil Engineering, and was conducted under the general supervision of Professor A. N. Talbot, of the Department of Theoretical and Applied Mechanics, and Professor I. O. Baker, of the Department of Civil Engineering. Wherever reference has been made to the work of other investigators, credit has been given in the text.

II. MATHEMATICAL ANALYSIS OF STRESS AND STRAINS IN WEB MEMBERS.

4. *Notation and Units.*—The following notations are used:

S = tensile or compressive stress (lb. per sq. in.); (various subscripts are used to denote stresses in various parts of the web).

S_s = shearing stress (lb. per sq. in.); (shearing stresses in various parts of the web are denoted by S'_s, S''_s, etc.).

*Bulletin No. 85, Engineering Experiment Station, University of Illinois.

P = force acting upon a girder or any small element (pounds).

ϕ = angle between the diagonal plane and the plane of one side of a element of a girder.

E = modulus of elasticity in tension or compression (lb. per sq. in.).

F = modulus of elasticity in shear (lb. per sq. in.).

ϵ = unit-strain in inches per inch (an abstract number).

M = bending moment at any section of a girder (pound-inches).

I = moment of inertia of the cross-section of a girder (inches)^4.

c = distance from the neutral axis of a girder to the outer fibers (inches).

V = vertical shear at any section of a girder (pounds).

t = thickness of the web of a girder (inches).

$(a_1 c_1)$ = "static moment" of any portion of the cross-sectional area of a girder about the neutral axis (inches)^3.

$\left(\frac{l}{r}\right)$ = slenderness ratio for the web of a girder acting as a long column under compression (an abstract number).

h = clear depth of web of girder between flanges (inches).

b = length of a bearing block over a support or under a concentrated load (inches).

Δ_f = deflection of a girder due to direct flexure (inches).

Δ_s = deflection of a girder due to shear (inches).

Δ = deflection of a girder due to both flexure and shear (inches).

a = area of the cross-section of a girder (square inches).

l = length of span of a girder (inches).

l_1 = distance from support of a girder to nearest concentrated load (inches).

λ = Poisson's ratio, the factor of lateral strain (an abstract number).

Throughout this bulletin the term *strain* is used to denote the deformation caused by stress; it is not used as a synonym for stress.

5. *Common Formulas of Girder Design.*—The tensile or com-

Table 1.

Web Failure of I-Beams

Reprint from Bulletin 68 of the Engineering Experiment Station of the University of Illinois

1.	Beam	12-in., 31.5-lb. I-Beam	12-in., 31.5-lb. I-Beam. Web Planed Thin	12-in., 31.5-lb. I-Beam. Web Planed Thin	12-in., 31.5-lb. I-Beam. Web Planed Thin	Beth'm Girder Beam 30-in., 175-lb.	20-in. Special* Built-up Girder
2.	Span, ft.	2.92	3.00	3.00	3.00	20	15
3.	Loading	Two Points 4⅞ in. Each Side of Center	Two Points 4⅞ Each Side of Center	Two Points 4⅞ Each Side of Center	Two Points 4⅞ Each Side of Center	Quarter Points	One Load at Quarter Point
4.	Material	Steel	Steel	Steel	Steel	Steel	Steel
5.	Number tested	2	1	1	1	3	1
6.	Tested by	Univ. of Illinois	Univ. of Illinois	Univ. of Illinois	Univ. of Illinois	Marburg (Univ. of Pa.)	Turneaure (Univ. of Wis.)
7.	Fiber stress due to direct flexure, lb. per sq. in.	33,500	28,200	19,300	12,700	31,000	Sidewise buckling prevented by bracing
8.	Vertical distance between flanges, inches (h)	10.52	10.52	10.52	10.52	26.8	14
9.	Thickness of web, inches (t)	0.35	0.28	0.19	0.16	0.69	0.14
10.	Slenderness ratio for web $\frac{l}{r} = l\sqrt{12}\,\frac{h \text{ sec. } 45°}{t}$	148	184	272	324	190	490
11.	Load at failure, pounds	190,100	160,500	109,600	72,100	538,400	106,000 (Approx.)
12.	Computed fiber stress (shear and also compression) at middle of web, lb. per sq. in.	25,800	27,200	27,400	21,400	14,800	26,500
13.	$\frac{4\pi^2 E}{(\frac{l}{r})^2}$	53,900	34,900	16,000	11,300	32,700	4,900
14.	Length of block under support, inches	6	6	6	6	12	.....
15.	Computed fiber stress (compression) in web adjacent to support, lb. per sq. in.	45,300	47,800	48,200	37,600	32,500	Stiffeners used at ends and under load
16.	Yield-point strength of material at root of flange, lb. per sq. in.	31,700	33,100	34,000	32,200	28,200	37,700

*Test reported in full in the Journal of the Western Society of Engineers for 1907, p. 788. Failure by sidewise buckling was prevented by bracing. The load at failure is given by Dean Turneaure as that at which very great distortion had taken place and noticeable buckling in the web occurred. Excessive compressive stress in the web adjacent to reactions and concentrated loads was prevented by using stiffeners, well fitted to the flanges.

pressive stress in the outer fibers of the flanges of a girder is denoted by the equation

$$S=\frac{M\,c}{I} \quad \ldots\ldots (1)$$

in which S is the stress in the outer fiber, M the bending moment, I the moment of inertia of the cross sectional area of the girder and c the distance from the neutral axis to the outer fiber. The longitudinal or the transverse shearing stress at any point in a girder is given by the formula

$$S_s=\frac{V}{It}(a_1\ c_1) \quad \ldots\ldots (2)$$

in which S_s is the shearing stress, V the total shear on a vertical section through the point, I the moment of inertia of the cross-sectional area of the girder, t the thickness of the beam at the point considered, a_1 the area of that part of the cross-section between the point and the extreme fiber, and c_1 the distance from the center of gravity of the area a_1 to the neutral axis of the beam.

For I-beams and built-up girders it is customary to use an approximate method for obtaining the shearing stress at any transverse section of the web; the total shear V is divided by the cross-sectional area of the web, and the quotient is taken as the shearing stress.

For the derivation of the above formulas the reader is referred to any standard author on the mechanics of materials, such as Merriman, Boyd, Murdock, Slocum, or Morley.

6. *Discussion of the Theory of Web Stress.*—The stresses on a transverse section of a beam in flexure consist of shearing stresses parallel to the section and tensile and compressive stresses normal to the section. The stress on a longitudinal section normal to the force plane consists of a shearing stress parallel to the section. Fig. 1 (a) represents a small element of a beam in flexure. Before the beam was subjected to flexure the element was rectangular. The plane *ab,* normal to the paper, is a transverse plane, and *cb* is a longitudinal plane normal to the force plane. The forces acting upon the element will produce a stress upon the oblique plane *ac* which makes an angle ϕ with *bc.* This stress may be resolved into two components, one parallel and the other normal to *ac.* The relation between the stresses on the diagonal plane and the stresses on the faces of the element depends upon the value of ϕ. Let Fig. 1 (a) represent any element having a length *dx,* a width *dy,* and a thickness normal to the paper equal to

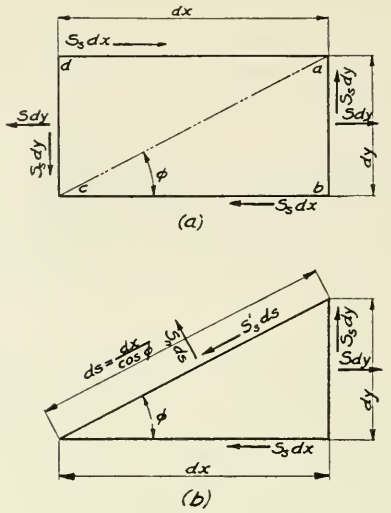

FIG. 1. STRESSES ACTING ON AN ELEMENT OF A GIRDER

unity. The normal stress per unit area on the ends of the element is represented by S and the shearing stress per unit area on the sides of the element, which are normal to the paper, is represented by S_s. The normal stress per unit area on the oblique plane (see Fig. 1 (b)) is represented by S_n and the tangential stress is represented by S_s'. It can be proven that*

$$S_n = \tfrac{1}{2}S\ (1 - \cos 2\phi) + S_s \sin 2\phi \quad \ldots\ldots (3)$$

$$S_s' = \tfrac{1}{2}S \sin 2\phi + S_s \cos 2\phi \quad \ldots\ldots (4)$$

The stresses S_n and S_s' are functions of ϕ. Their maximum values are given by the equations*

$$\text{Max } S_n = \tfrac{1}{2}S \pm \sqrt{S_s^2 + (\tfrac{1}{2}S)^2} \quad \ldots\ldots (5)$$

$$\text{Max } S_s' = \sqrt{S_s^2 + (\tfrac{1}{2}S)^2} \quad \ldots\ldots (6)$$

It should be noted that Max S_n and Max S_s' are not in the same direction.

The stress S may be either tension or compression. When S is tension the plus sign before the radical is used to find the maximum tensile unit stress S_n, and the minus sign before the radical, to find the maximum compressive unit-stress S_n. The latter is always normal to the former.

If stresses S_1 and S_2 at right angles to each other are acting on small particles, S_1 causes strain at right angles to its direction and

*Merriman, "Mechanics of Materials," 10th ed., p. 264.

thus influences the total strain of the particle in the direction of S_2. If S_1 and S_2 are both tensions or both compressions, the strain in the direction of either stress is *diminished* by the lateral strain due to the other stress; if S_1 is a tension and S_2 a compression (or vice versa) the strain in the direction of either stress is *increased* by the lateral strain due to the other stress. Following what they believe to be the best practice, the writers use the term *stress* to mean an internal resisting force set up by the action of external forces. A stress in any direction causes a lateral *strain* at right angles to that direction, but, accepting the above definition of stress, does not cause a lateral stress.

Whether or not the strain at right angles to a stress influences the *strength* of a material has for a long time been a subject of discussion among students of the mechanics of materials. Three theories of the strength of materials under combined stresses have been advanced:

(a) The strength of a material depends only on the normal *stress.* If this theory is accepted, the transverse strain produced by a stress does not affect the strength, and a particle under the action of two stresses at right angles to each other is just as near failure (and no nearer) as it would be under the action of the greater of the two stresses acting alone. This is called the "maximum stress theory."

(b) The strength of the material is dependent upon the *strain.* If this theory is accepted, a particle under the action of stress in one direction is in greater danger of failure than it would be under the action of two stresses of no greater magnitude, alike in sign, and acting in two directions at right angles to each other. This is known as the "maximum strain theory."

(c) The strength of the material depends on the maximum *shearing* stress set up by the action of the various stresses. This is known as the "maximum shear theory."

The latest, and in some respects the most conclusive, tests which bear on this subject are those of Becker (University of Illinois Engineering Experiment Station, Bulletin No. 85). Becker studied the behavior of thin tubes in which axial tension or compression was produced by means of a testing machine and transverse tension was set up by means of internal water pressure. He found that for steel the second theory, the maximum strain theory, held unless the maximum shearing stress developed was greater than about 0.60 of the tensile or compressive stress. Within certain limits the strength of the material seems to depend on the strain, and beyond those limits it seems to depend on the shearing stress.

For expressing the value of a strain in this bulletin, the numerical value of $E\epsilon$ (the product of the unit strain and the modulus of elasticity, 30,000,000 lb. per sq. in. for steel) is given instead of the numerical value of the strain. Thus, in speaking of the safe working strain for a bar in tension, instead of saying that the safe unit strain is 0.000533 inches per inch length, the writers have used the expression, the strain corresponding to a value of $E\epsilon$ of 16,000 lb. per sq. in. or, more briefly, the $E\epsilon$ value of 16,000 lb. per sq. in. It is thought that this method of expressing the strain enables the engineer, who is not in the habit of thinking in terms of strain, to compare more easily the combined strains in girders with the strains resulting from the simple stresses with which he is familiar. $E\epsilon$ is not a stress, but a simple stress in one direction only with a magnitude equal to $E\epsilon$ would produce the same structural damage to the material as is produced when the strain ϵ occurs.

In the following equation S_1 and S_2 are two stresses at right angles to each other* and λ is Poisson's ratio

$$E\epsilon = S_1 - \lambda S_2 \quad \ldots\ldots (7)$$

$$E\epsilon_2 = S_2 - \lambda S_1 \quad \ldots\ldots (8)$$

If a stress is compression it is to be taken as negative; if the strain is positive it is an elongation; if negative, a contraction. The stresses S_1 and S_2 produce shearing stresses along planes oblique to their lines of action, and it can be proven that the maximum shearing stress S_s' set up by the two stresses is given by the equation.†

$$S_s'' = \tfrac{1}{2}(S_1 - S_2) \quad \ldots\ldots (9)$$

III. Tests

7. *Specimens.*—The specimens of the series of tests reported in this bulletin (1914 series) comprised six 12-inch I-beams and two 24-inch built-up girders. The webs of the I-beams were planed thin and the webs of the girders were made of thin plates. The webs in all of these test specimens were thinner than those used in standard practice. Had standard practice been followed, the webs would have been so strong in comparison with the flanges that the primary failure would probably not have been a web failure, but a failure due to some other cause, such as direct flexure or sidewise buckling of the compression flange. To investigate the web strength of beams it was

*Lanza, "Applied Mechanics," pp. 868-869.
†Merriman, "Mechanics of Materials," 10th ed., p. 363.

necessary to use specimens in which web failure would be the primary failure. Hence, thin-webbed specimens were used.

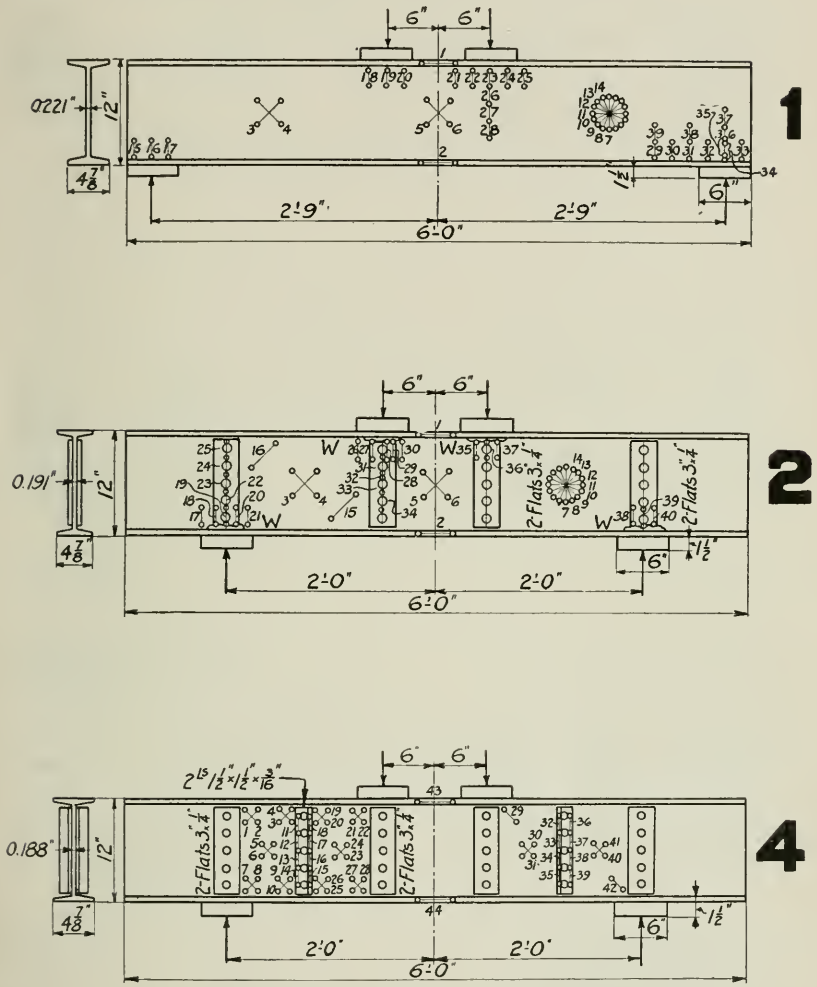

Fig. 2. I-Beams 1, 2, and 4 Showing Location of Gage Lines (w Denotes an Oxy-acetylene weld)

Fig. 2 and 3 show the shape and size of the I-beam specimens. Beams 1 and 5 had no stiffeners; beams 2 and 6 had flat stiffeners on the web adjacent to the bearing blocks. Flats were used instead of angles so that there would be very little resistance to the buckling of

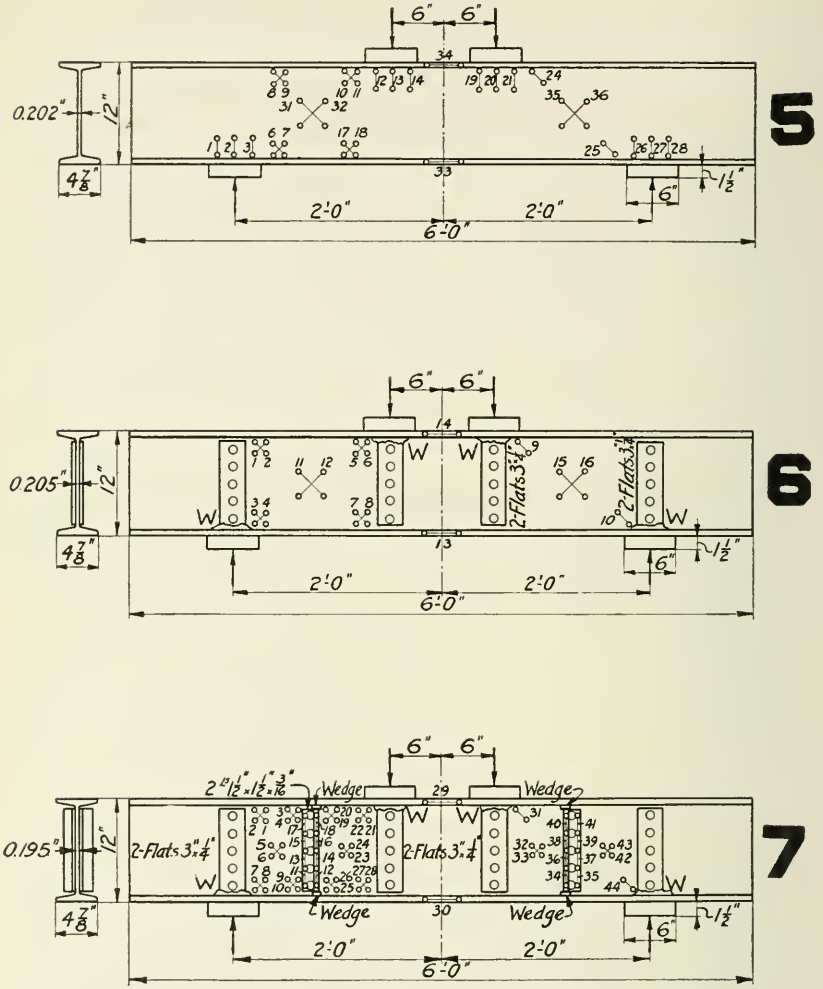

Fig. 3. I-Beams 5, 6, and 7 Showing Location of Gage Lines (w Denotes an Oxy-acetylene weld)

the web. The flats were welded to the flanges to insure a good bearing. For beams 4 and 7 there were flat-stiffeners at the bearing blocks and angle stiffeners between bearing blocks. Fig. 4 and 5 show the built-up girders. Girder 8 had angle-stiffeners at the bearing blocks and at intermediate points; girder 3 had angle-stiffeners at the bearing blocks only.

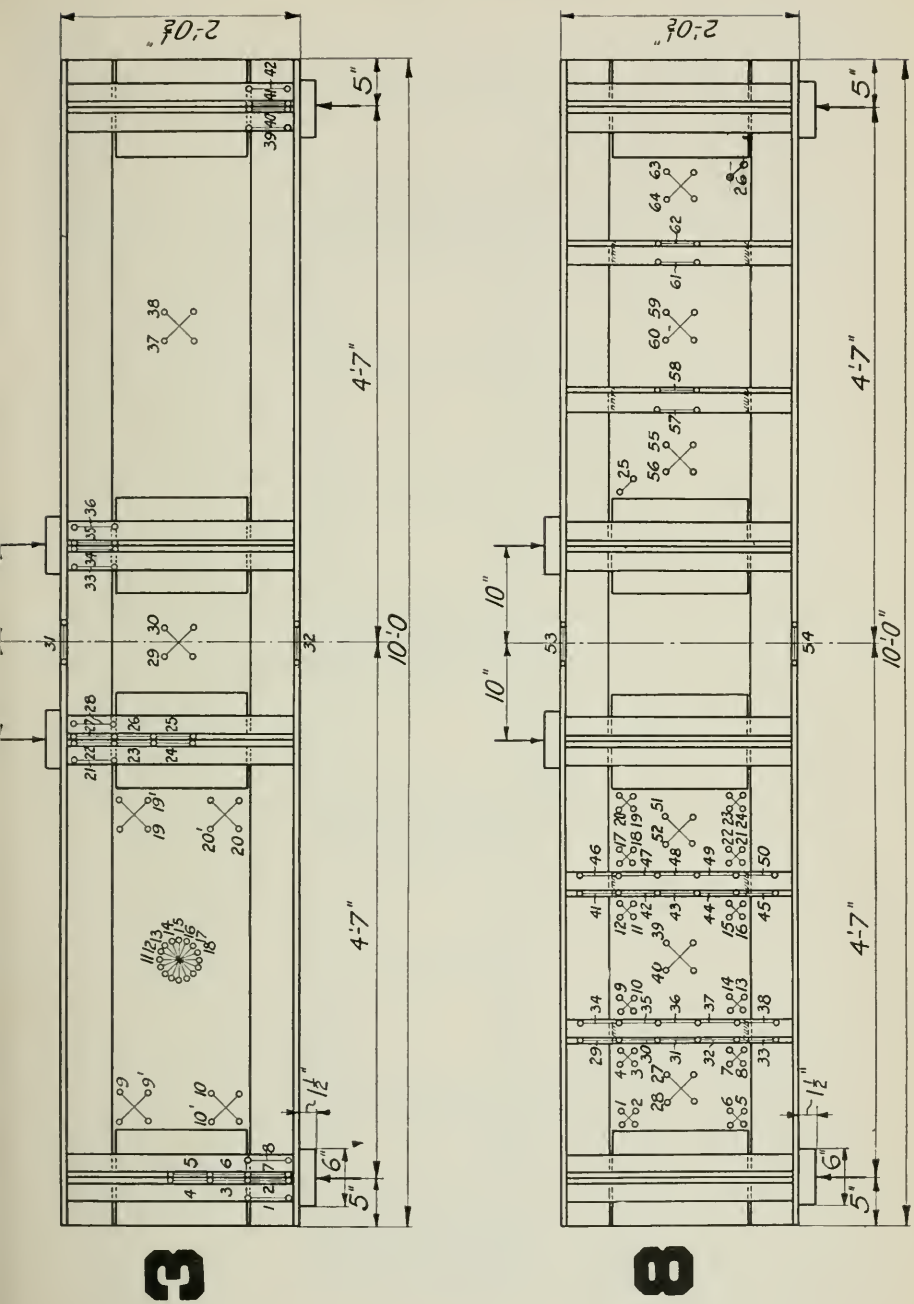

FIG. 4. BUILT-UP GIRDERS 3 AND 8 SHOWING LOCATION OF GAGE LINES

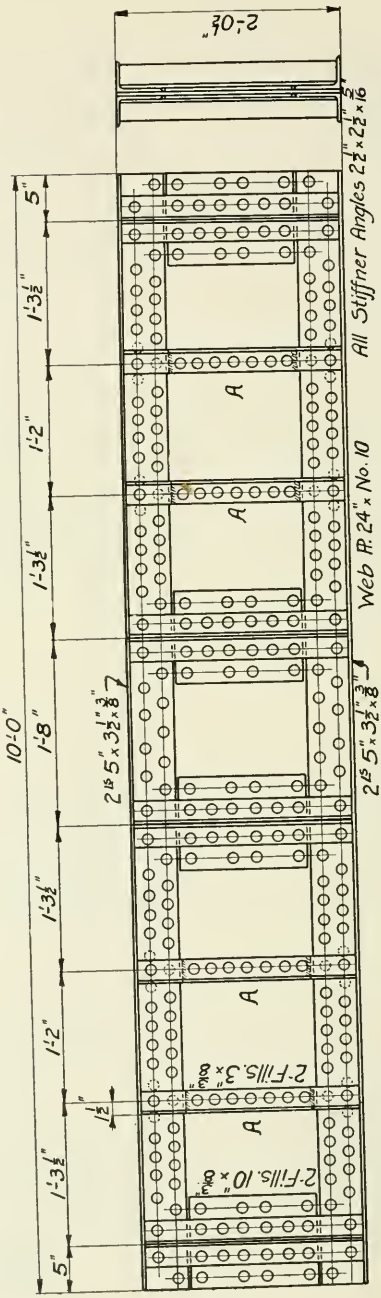

FIG. 5. WORKING DRAWING OF BUILT-UP GIRDERS 3 AND 8 (STIFFENERS MARKED A OMITTED FOR GIRDER 3)

For the purpose of determining the quality of the material in the beams, test pieces were cut from some portion of each beam which had not been under heavy stress during the test. For the I-beams the specimens were cut from the portion overhanging the end bearing. Tension specimens were cut from the webs, from the flanges, and from the points at which the webs join the flanges. Shear specimens were cut from the webs. For the girders these specimens were cut from the central portion of the webs where the shear was zero. A typical tension specimen is shown in Fig. 6 and a typical shear specimen in Fig. 7.

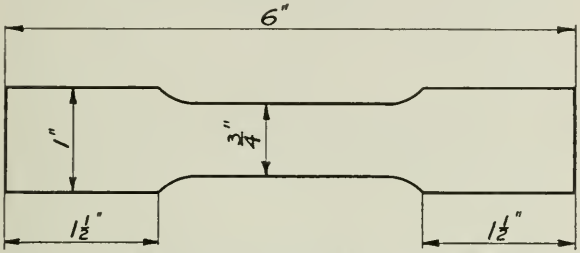

FIG. 6. FORM OF TENSION SPECIMEN CUT FROM GIRDER

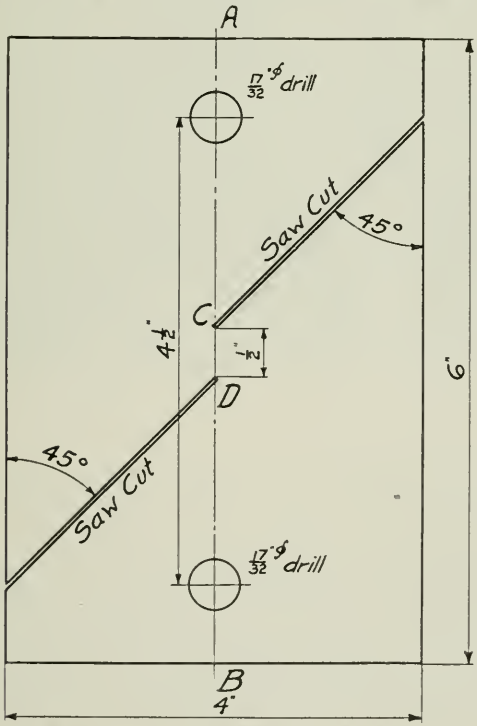

FIG. 7. FORM OF SHEAR SPECIMEN CUT FROM WEB OF GIRDER

The form of the shear specimen was suggested by Mr. Malcolm Westergaard. When axial pull is applied to the shear specimen along the line AB (Fig. 7) shear occurs along the line CD.

8. *Apparatus.*—Beam 1 was tested in a 200,000-lb. Olsen vertical-screw testing machine. The other tests were made in a 600,000-

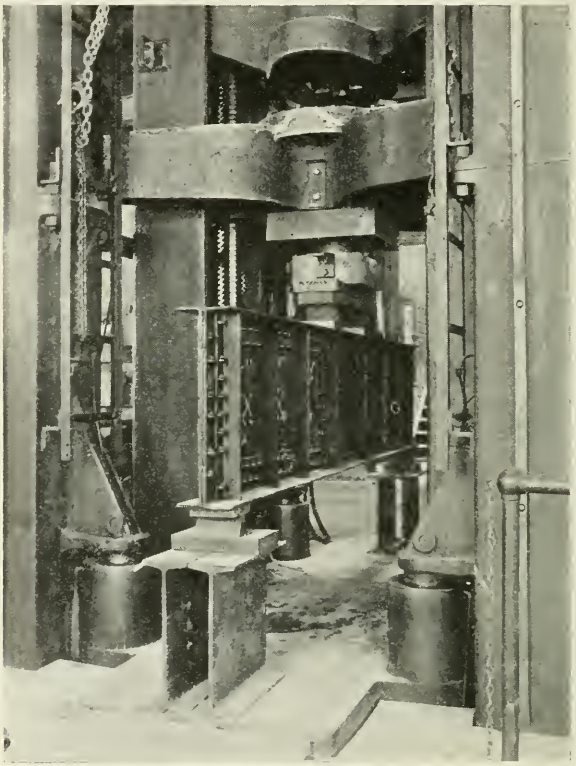

FIG. 8. GIRDER IN TESTING MACHINE IN POSITION FOR TESTING

lb. Riehle vertical-screw testing machine. In all tests, loads were applied at two points equidistant from the center of the span. Fig. 8 shows a specimen in the testing machine.

Strains were measured by means of Berry strain gages. The locations of the gage lines for the different strain measurements are shown in Fig. 2, 3, and 4. Each gage line is denoted by two small circles, one for each end of the line, joined by a straight line. The gage lengths used were two inches and four inches, and, as Fig. 2, 3, and 4,

are reproduced to scale, the length of any gage line is apparent. Each gage line was given an identifying number as shown in the figures. For each gage line on one side of the specimen there was a mating line directly opposite on the specimen. The lines for the opposite sides of the specimen were distinguished by the letters *E* or *W*, *N* or *S*, following the line number.

The deflections of beams 3, 4, 5, 6, 7, and 8, were measured at the middle of the span. Several forms of deflectometer were used, the most convenient being the level bar shown in Fig. 9. The point *A* of

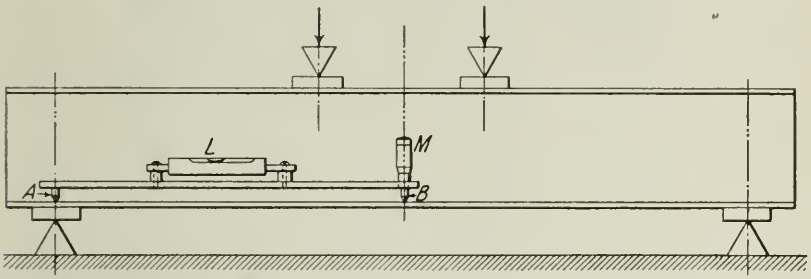

FIG. 9. LEVEL BAR FOR MEASURING DEFLECTION OF GIRDER. (THE POINT *A* CAN BE SCREWED INTO VARIOUS HOLES ALONG THE BAR FOR DIFFERENT LENGTHS OF SPAN)

the instrument is set on the beam over the end bearing, and the point *B*, which is at the end of a micrometer screw *M*, is set at mid-span. With zero load on the beam the micrometer screw is adjusted, raising or lowering the point *B*, until the level bubble *L* is in mid-position. With any load on the beam the same process is repeated, and the difference between this micrometer reading and the reading for zero load gives the deflection at mid-span. The point *A* is placed in a prick-punch hole, and the point *B* in a cold-chisel mark made along the flange of the beam. The sensitiveness of the level bubble was such that twenty seconds change of angle from the horizontal caused the bubble to move one division of its scale.*

9. *Data and Results.*—Readings of the strain gages and readings of the deflectometer were taken for various loads up to the ultimate. The strain gage readings were corrected for variation of temperature by means of an unstressed standard bar.† The value of $E\epsilon$ along

*Since these tests were made a later form of level bar has been constructed in the shops of the Laboratory of Applied Mechanics. In this new level bar, designed by Mr. H. R. Thomas, the screw micrometer shown in Fig. 9 has been replaced by a leveling screw which actuates the plunger of a direct reading dial gage micrometer. This later form of level bar is much quicker in operation than the earlier form.

†For detailed discussion of the use of the strain gage see: Proceedings of the American Society for Testing Materials, 1913, Slater and Moore on "The Use of the Strain Gage in Testing Materials"; also, Bulletin No. 64 of the Engineering Experiment Station of the University of Illinois, "Tests of Reinforced Concrete Buildings under Load," by Talbot and Slater.

each gage line was determined from the strain gage readings assuming a modulus of elasticity of 30,000,000 lb. per sq. in. Curves were plotted for each beam with values of applied loads as ordinates and values of $E\epsilon$ along the gage lines, determined from the strain page readings, as abscissas. These curves are given in Fig. 14-22. The load at failure for each beam is given in Table 2. Fig. 10, 11, and 12 are reproduced from photographs of the beams after failure.

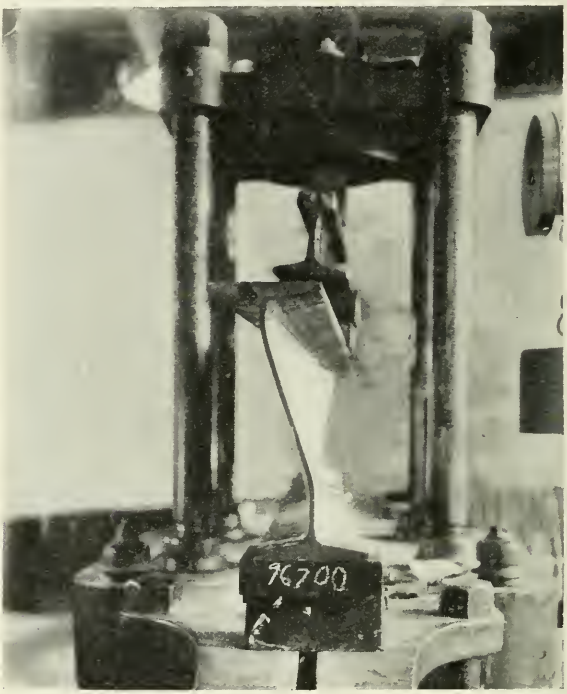

Fig. 10. I-Beam 1 after Failure

Fig. 11. Girder 3 after Failure

FIG. 12. GIRDER 8 AFTER FAILURE

The local yielding of the built-up girders is shown in Fig. 11 and 12. During the progress of the tests local yielding was made evident by the flaking of the mill scale and the paint on the specimen.

During a test three stages of structural failure were noted: (1) Local overstress, shown by flaking of paint and high local strain gage readings. (2) Signs of general yielding of the girder as a whole. (3) Final collapse under the ultimate load.

10. *Determination of the Yield Point of Girders.*—The writers believe that the second stage of failure, as given in the preceding paragraph, gives the best indication of the limit of load-carrying capacity of girders for static loads. The term "yield point of the girder" is used to designate this stage of general yielding. The general yielding of an I-beam or girder is shown by the "knee" of the load-deflection curve, and an examination of the load-deflection curves shown in Fig. 16-22 shows that the departure of a curve from a straight line is fairly well marked. In determining the yield point of a girder from the load-deflection curve the method of J. B. Johnson was used.* The yield

TABLE 2.

FAILURE OF GIRDERS (1915 TESTS)

Girder Number	Load at Yield Point of Girder, pounds.	Load at Ultimate, pounds.	Manner of Failure.
1	89,500	96,700	Local compression over bearing block followed by twist of web.
2	85,500	138,000	Wrinkling of web due to diagonal compression.
3	101,500	188,000	Wrinkling of web due to diagonal compression.
4	73,500	141,500	Wrinkling of web due to diagonal compression.
5	82,000	104,500	Local compression over bearing block followed by twist of web.
6	86,500	120,500	Wrinkling of web due to diagonal compression.
7	82,000	144,000	Wrinkling of web due to diagonal compression.
8	112,500	220,800	Wrinkling of web due to diagonal compression.

*J. B. Johnson's method consists in finding, by means of drawing a tangent line, the point on any stress-strain curve (or load-deflection curve) at which the strain is increasing 50 per cent more rapidly than its initial rate of increase. See J. B. Johnson, "The Materials of Construction," pp. 18-20.

point obtained from the load-deflection curve does not differ greatly from a yield point obtained from a curve showing the average web-strains at mid-web. The yield point for those girders for which no deflection was measured was determined from the average web-strain (along a 45-degree gage line) at mid-web. The yield point of the girder occurs under loads greater than those causing the first evidence of local structural distress (shown by scaling of paint or excessive strain along individual gage lines) and at loads less than the ultimate. The yield point seems to be a fairly reliable criterion of the beginning of serious structural damage to the girder as a whole. The loads at the yield points, the ultimate loads, and the manner of failure are given in Table 2.

IV. Discussion of Results.

11. *Relation between the Actual and the Theoretical Strains.*— Equation (7) gives the theoretical value of $E\epsilon$ for any gage line in a girder. In this equation S_1 is the computed unit stress along a gage line, and S_2 the computed unit stress normal to the gage line and in the plane of the web. The values of S_1 and S_2 can be determined from equation (3).

To illustrate the use of these equations consider gage line 19 of girder 3 (see Fig. 4 and 16). A total load of 1,000 lb. on the girder will produce on the transverse section at the middle of gage line 19, a compressive stress S of 70.7 lb. per sq. in. as computed by equation (1), and a shearing stress S_s of 153.4 lb. per sq. in. as computed by equation (2). $\phi = -45$ degrees. Substituting these qualities in equation (3) gives

$$S_1 = \tfrac{1}{2}(-70.7)\,[1-\cos(-90^\circ)]+153.4\sin(-90^\circ) = -188.75 \text{ lb. per sq. in.}$$

This is compression along gage line 19. S_2, the stress at right angles to S_1, is also given by equation (3). S and S_s are the same as for S_1, and $\phi = +45^\circ$. Substituting these values in equation (3) gives $S_2 = \tfrac{1}{2}(-70.7)\,(1-\cos 90^\circ) + 153.4 \sin 90^\circ = 118.05$ lb. per sq. in. This is tension normal to gage line 19.

λ (Poisson's ratio) for steel is about $\tfrac{1}{3}$. Substituting values of S_1, S_2, and λ in equation (7) gives

$E\epsilon = -188.75 - \tfrac{1}{3}(118.05) = -228.10$ lb. per sq, in. for 1,000 lb. load on the girder.

This is slightly less than the value given by the average of the strain gage readings on the two sides of the web at gage line 19, as shown in Fig. 16.

A comparison of the strains as computed by the above method with the strains measured by the use of the strain gage may be made by means of the diagrams shown in Fig. 14-22. In these diagrams the full lines represent values of $E\epsilon$ corresponding to the strain as determined from the readings of the strain gage on the two sides of the girder and the dot and dash lines give the values of $E\epsilon$ as calculated by the formulas given in the preceding pages.

Comparison of measured strain with computed strain was made on 163 gage lines for the eight test pieces. Of these 163 gage lines, 40 were near bearing blocks where local compression materially modified the strains. For 107 of the remaining 123 gage lines the measured strain and the computed strain agreed closely, and for most of the other gage lines there were evidences of local bending action which might explain the discrepancy. The good general agreement between measured strain and computed strain furnishes an experimental confirmation of the theory on which the computation is based. This theory is therefore used as the basis of the computation of the maximum strains, and the shearing stresses in the girder webs.

12. *Maximum Shearing Stress in the Web.*—The longitudinal and transverse shearing stresses in the web of a girder are a maximum at the neutral axis at which the tensile and compressive stresses are zero. Equation (6) shows that it may be possible to have a diagonal shearing stress greater than the longitudinal and transverse shearing stress at the neutral axis. The longitudinal and transverse shearing stresses vary inversely as the thickness of the girder; hence they are much smaller in the flange than in the web adjacent to the flange. The tensile and compressive stresses vary directly as the distance from the neutral axis. Therefore, in the case of the girders under consideration, the maximum diagonal shearing stress is either at the neutral axis or at the inner edge of the flange and under a load. The maximum diagonal shearing stress can be determined by the use of equation (6). The shearing stresses in girders 1 to 8, calculated by the above methods, are given in Table 3.

In all I-beams, except No. 1 and No. 5, there are stiffeners under the loads. The stresses given in Table 3 are at the outer edges of these stiffeners. The moment is slightly greater immediately under the load, but the stiffener helps the web to resist the shear at this point.

The quantities used in the calculation of the maximum shearing stresses are also given in Table 3; the longitudinal tensile or compressive stress in the web at the inner edge of the flange is given in Column 2; the longitudinal and transverse shearing stresses at the same

Table 3.

Shearing Stresses in Webs of Girders

All stresses are given in pounds per square inch

No. of Test	Stress for 1000 Pounds Load on the Girder				Maximum Shearing Stress in Web at Yield Point of Girder.	Maximum Shearing Stress in Web at Ultimate of Girder.	Yield Point in Shear of Material in Web (from Tests of Specimens).
	Longitudinal Stress at Inner Edge of Flange (Tension or Compression)	Longitudinal and Transverse Shearing Stress at Edge of Flange.	Maximum Diagonal Shearing Stress in Web.	Longitudinal and Transverse Shearing Stress at Neutral Axis‡			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	353	172	246*	206	22,000	23,800	24,400†
2	219	203	230	236*	20,200	32,600	26,500†
3	108¶	151	161*	158	16,300	30,300	25,700
4	220	206	234	240*	17,600	34,000	25,400†
5	236	190	224*	224*	18,300	23,400	25,100†
6	217	190	219	224*	19,400	27,000	26,100†
7	219	198	226	232*	19,000	33,400	23,700†
8	108¶	151	161*	158	18,100	35,500	26,100

*Maximum shearing stress in the web of the girder.
†Taken as 0.60 of the yield point in tension. The yield point in shear was not well defined for these particular specimens.
‡Computed by Equation (2).
¶Stress on center line of inner row of rivets.

point are given in Column 3; the maximum shearing stresses in the web at the inner edge of the flange in Column 4; and the shearing stresses at the neutral axis are given in Column 5. The maximum shearing stress in the web, the greater of the quantities in Columns 4 and 5, is indicated by an asterisk. All of the above stresses are given in lb. per sq. in. per 1000-lb. load on the girders. The maximum shearing stresses in the web when the beam is loaded to the yield point are given in Column 6.

A comparison of Columns 3 and 4 of Table 3 shows that, except for I-beam 1, the diagonal shearing stress in the web at its junction with the flange is not materially greater than the longitudinal and transverse shearing stress at the neutral axis. In the case of I-beam 1 the high diagonal shearing stress is due to the fact that the longitudinal tensile (or compressive) stress at the point at which the flange joins the web is much greater than the transverse and longitudinal shearing stress at the same point.

To illustrate the possible importance of diagonal shearing stresses consider the following numerical example. A girder is supported upon end supports 70 ft. apart. It carries two concentrated loads of 450,000 lb. each, one located 9 ft. 8 in. from each end support. The girder is made up of one web plate 90 in. x ½ in., and (for each flange) two 6 in. x 6 in. x ¾ in. angles, and two cover plates 14 in. x ¾ in.

Neglecting the weight of the girder, the shear is constant between a load and the adjacent support, and is equal to 450,000 lb. The maximum bending moment occurs under a load and is equal to 450,000 x 116=52,200,000 pound-inches.

The web area equals 90×0.5 or 45 sq. in. and the average shearing stress as usually figured is 10,000 lb. per sq. in. The moment of inertia of the net section is 153,100 $(\text{in.})^4$ and the distance from the neutral axis to the outer edge of the flange is 46.75 in. The longitudinal stress in the outer fiber, as given by equation (1), is, therefore, 15,930 lb. sq. in.

The maximum diagonal stress occurs at the point at which the flange is connected to the web. For shear consider this point to be on the center line of the inner row of rivets, a distance of $45.25 - 4.75 = 40.50$ in. from the neutral axis. The longitudinal stress varies directly as the distance from the neutral axis and at the point in question is equal to 13,800 lb. per sq. in. Consider the shear at the neutral axis on a transverse section under a load. The unit shearing stress at any point is given by equation (2). For the girder consider, $V=450{,}000$ lb., $I=183{,}200$ $(\text{in.})^4$, $t=\frac{1}{2}$ in., and $a_1c_1=2{,}204$ $(\text{in.})^3$ for

the gross area. Then from equation (2) S_s equals 10,800 lb. per sq. in. Consider next the shear on the same transverse section on the gage line of the angles, a distance of 40.50 in. from the neutral axis. For this point the value of a_1c_1 is 1,794 $(\text{in.})^3$, and V, I, and t have the same values as in the previous case. S_s equals 8,800 lb. per sq. in.

The maximum diagonal shearing stress at the edge of the angles, S_s^1, given by equation (6), is 11,200 lb. per sq. in. This value is 400 lb. per sq. in. or 3.7 per cent greater than the value of the shearing stress at the neutral axis (10,800 lb. per sq. in.).

In this girder the longitudinal tensile and compressive stresses are as high at a section at which maximum shear occurs as good practice will permit, and therefore the excess of the diagonal shearing stress at the inner edge of the flange over the transverse and longitudinal shearing stress at the neutral axis should, if ever, be of importance. Since this difference is only 3.7 per cent it would seem that the excess of diagonal shearing stress over shearing stress at the neutral axis is not, in general, of much importance. It is true that if the shearing stresses had been low at the inner edge of the flange while the longitudinal tensile and compressive stresses had been high, the diagonal shearing stress might have been materially greater than the shearing stress at the neutral axis. But the very supposition on which this condition is based; namely, that the shearing stresses should be low, makes it certain that they will not be the criterion of strength of the girder, and that the difference between the two low shearing stresses will be of no importance.

The shearing stress at the neutral axis, as computed by equation (2) is 10,800 lb. per sq. in.; whereas the value obtained by dividing the total shear by the gross area of the web (the method usually used in girder design) is 10,000 lb. per sq. in. The excess of the former value over the latter is about 8 per cent. For the I-beams and built-up girders tested the difference between the values obtained by the two methods of calculation is also about 8 per cent. The relation between the transverse and longitudinal shearing stress at the neutral axis, as obtained by equation (2), and the approximate value obtained by dividing the total shear upon a section by the area of the cross-section of the web depends upon the relation between the area of the cross-section of the web and the area of the cross-section of the flange. In the example given above, in which the area of cross-section of the web is not large compared with the area of cross-section of the flange, equation (2) gives values for the shearing stress about 8 per cent higher than those obtained by approximate method. Calculations for

girders, in which the area of the web is large compared with the flange area, show that the difference between the sharing stresses as computed by the two methods may be as great as 20 per cent. Therefore, in the design of a girder, if the value obtained by dividing the total shear by the area of cross-section of web is more than 80 per cent of the allowable stress for the material in shear, a check computation of the shearing stress should be made, using the more exact formula, equation (2). The computing of the diagonal shearing stresses at the inner edge of the flange seems unnecessary.

13. *Maximum Tensile and Compressive Strains in the Web.*—The longitudinal tensile and compressive stresses increase with the longitudinal distance of a point from the support and vary as the transverse distance from the neutral axis of the girder. The longitudinal and transverse shearing stresses decrease as the distance from the neutral axis increases, but they are nearly as great at the point at which the flange joins the web as at the neutral axis. However, after this point is passed the decrease is very rapid. The maximum diagonal tensile and compressive stresses depend upon the shearing stress and upon the longitudinal tensile (or compressive) stress. In the case of the beams under consideration the maximum tensile and compressive diagonal stresses occur at the inner edges of the flanges, under the load point for beams which have no stiffeners, and at the outer edge of the middle stiffeners for beams which have stiffeners.

The maximum diagonal tensile or compressive stress is given by equation (5). The values of S and S_s to be used in this equation for a load of 1,000 lb. on the girder are given in Columns 2 and 3 of Table 4. A diagonal tensile or compressive stress at a given point in the web in one direction is accompanied by a second diagonal stress at the same point normal to the first. If the first is the maximum diagonal stress, the second is the minimum. If the longitudinal stress is tension, the maximum diagonal stress is given by equation (5) if the upper or plus sign is used, and the minimum diagonal stress is given when the lower or minus sign is used. In considering the strength of the web it is necessary to get the value of $E\epsilon$ corresponding to the maximum strain. The value of $E\epsilon$ is given in equation (7). Substituting in equation (7) the values of the maximum and the minimum diagonal stress for S_1 and S_2, as given by equation (5) there is obtained

$$E\epsilon=\frac{S}{2}(1-\lambda)+(1+\lambda)\sqrt{S_s^2+\left(\frac{S}{2}\right)^2} \qquad (10)$$

Table 4.

Diagonal Tensile and Compressive Stresses in Webs of Girders

All stresses are given in pounds per square inch

No. of Test	Longitudinal Stress in Web Adjacent to Flange for 1000 lb. Load on Girder.	Shearing Stress In Web Adjacent to Flange for 1000 lb. Load on Girder.	Maximum Value of $E\epsilon$ along a Diagonal for 1000 lb. Load on Girder.	Maximum Value of $E\epsilon$ along a Diagonal at Yield Point of Girder.	Compressive Stress along 45-degree Line at Neutral Axis at Yield Point of Girder. (S_y)	Ratio of Clear Vertical Depth to Thickness of Web.	Ultimate Buckling Strength of 45-Degree Strip by Euler's Formula for Fixed-Ended Columns (S_b)	$S_y : S_b$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	353	172	447	40,000	18,700	48	21,500	0.87
2	219	203	381	32,500	19,500	55	16,200	1.20
3	104*	151	250	25,400	20,000	101	4,800	4.16
4	220	206	385	28,300	17,300	56	15,800	1.09
5	236	190	377	30,800	17,100	53	17,400	0.98
6	217	190	364	31,400	18,300	52	18,200	1.00
7	219	198	375	30,700	18,800	54	16,900	1.11
8	104*	151	250	27,900	22,200	101	4,800	4.63

*Stress on center line of inner row of rivets.

The values of $E\epsilon$ corresponding to a load of 1,000 lb. on the girder are given in Column 4 of Table 4. The values of $E\epsilon$ at the yield point of the girders are given in Column 5 of Table 4, and are repeated in Column 3 of Table 5. The longitudinal stresses at the

TABLE 5.

COMPARISON OF LONGITUDINAL STRESSES IN EXTREME FIBER WITH MAXIMUM VALUES OF $E\epsilon$ FOR GIRDERS

No. of Test	Longitudinal Stress in outer Fiber of Girder at Yield Point of Girder* lb. per sq. in.	Maximum Value of $E\epsilon$ along a Diagonal at Yield Point of Girder lb. per sq. in.	Ratio of $E\epsilon$ along a Diagonal to Longitudinal Stress
1	36,100	40,000	1.11
2	21,400	32,500	1.52
3	15,800	24,600	1.56
4	18,400	28,300	1.53
5	22,200	30,800	1.39
6	21,400	31,400	1.46
7	20,600	30,700	1.49
8	17,500	27,000	1.54

*Computed by the commonly used flexure formula, Equation (1).

outer edges of the flanges are given in Column 2 of Table 5. Since the transverse stress at the outer edge of the flange is zero, the values in Column 2 of Table 5 represent also values of $E\epsilon$ for the outer fibers of the flanges, and since the values in Column 2 are for the same transverse section of the girder as those in Column 3, a direct comparison of the values of the two columns can be made. The ratios of the values in Column 3 to those in Column 2 are given in Column 4 of Table 5. An examination of these ratios shows that the maximum value of $E\epsilon$ along a diagonal in girder 1 is only slightly greater than the maximum longitudinal stress at the outer edge of the flange, but that for the other girders the values of $E\epsilon$ are much greater than the longitudinal stress at the outer edge of the flange. An examination of equation (5) shows that the relation between the two quantities depends on the relation between the transverse shearing stress and the longitudinal stress. This fact is apparent from Table 5 also, which shows that the values of $E\epsilon$ along a diagonal are not much greater numerically than the longitudinal stresses for cases in which the shearing stress is low compared to the longitudinal stress, but that for cases in which the shearing stress is high compared with the longitudinal stress the values of $E\epsilon$ along a diagonal are decidedly higher than the longitudinal stress at the outer edge of the flange. However, for the girders tested the ratio of the shearing stress to the longitudinal stress in the outer fibers is higher than ordinarily occurs in practice, and this fact tends to exaggerate the excess of the values of $E\epsilon$ over those of the longitudinal stress.

A consideration of the girder described in section 12 will illustrate the possible importance of the diagonal strain in a girder designed in accordance with current practice. The maximum diagonal strain occurs at a point at which the flange is connected to the web. For tensile and compressive strains this point is taken on the center line of the inner row of rivets, a distance of 4.75 in. from the backs of the angles, and 45.25–4.75=40.50 in. from the neutral axis. The longitudinal stress varies directly as the distance from the neutral axis, and at the point in question is equal to $15{,}930 \times \frac{40.50}{46.75} = 13{,}800$ lb. per sq. in. In computing the shearing stress at this point the numerical values to be used in equation (2) are the same as those used on p. 26, except the value of a_1c_1 which in this case is 1,794 (in.)^3. Using equation (2), S_s=8,800 lb. per sq. in.

The maximum value of $E\epsilon$ obtained from equation (10), is 19,530 lb. per sq. in. This is 3,600 lb. per sq. in. or 22.6 per cent greater than 15,930 lb. per sq. in., the longitudinal stress at the outer edge of the flange.

In this example the excess of the value of $E\epsilon$ along a diagonal over the longitudinal stress at the outer edge of the flange is so great (22.6 per cent) that it should have been considered in the design of the girder. This indicates that in the design of structural steel girders in which maximum shear and maximum moment occur *at the same transverse section of the web*, it is not safe to allow an average transverse shearing stress of 10,000 lb. per sq. in. and at the same time to allow a longitudinal stress of 16,000 lb. per sq. in. in the outer fibers of the flanges. It is evident that the diagonal strains in such girders should be given special attention.

14. *Buckling of Web.*—The tendency of webs of girders to fail by buckling is well illustrated by the action of the webs of girders 3 and 8 as shown in Fig. 11 and 12. The exact analysis of buckling action in the web of a girder would be extremely complicated, and the following approximate analysis is in common use: A narrow strip of web making an angle of 45 degrees with the longitudinal axis of the girder is regarded as a column carrying an average stress over its cross-section equal to the shearing stress at the neutral axis (which is equal to the compressive stress on a 45 degree line at the neutral axis—the maximum compressive stress at that point). The length of this column is taken as $h\sqrt{2}$, in which h is the clear depth of web. The column is regarded as fixed-ended. Since the web is thin

the slenderness ratio l/r is large for the strip, and Euler's column formula, which gives good results for very slender columns, may be used. Applying Euler's formula for fixed-ended columns, the computed web stress at failure by buckling S_c, becomes

$$S_c = \frac{1.64\,E}{\left(\frac{h}{t}\right)^2} \qquad (11)$$

The compressive stress at the neutral axis at the yield point of the girders tested is given in Column 6 of Table 4; the ultimate strength of a 45-degree strip as obtained by Euler's formula in Column 8; and the relation between these two quantities is given in Column 9.

It will be seen for all girders except 3 and 8 that the load at the yield point of the beam corresponds quite closely to the load for the failure of the web-column as given by Euler's formula. In the case of girder 3 the stiffeners at the ends apparently helped stiffen the whole web, though the unsupported length of web was much greater than is allowable under standard specifications. In the case of girder 8 the web was supported against buckling by stiffeners at intermediate points. It would seem that the webs of girders are capable of developing a shearing stress at the neutral axis equal to the ultimate stress on a 45-degree strip considered as a column, figured by Euler's formula for columns with fixed ends, unless the stress given by that formula is higher than the yield-point strength of the material in shear. An examination of Tables 1 and 3 shows that before the *ultimate* of any test girder had been reached the yield-point strength in shear of the web material was developed (unless failure was due to local compression of the web over bearing blocks), but that the *yield point* of the girder was reached before the web material was stressed to its yield point in shear. It should be kept in mind that the yield point of a girder is the practical limit of its load-carrying capacity.

15. *Local Web Compression Adjacent to Bearing Blocks.*—The exact determination of the stresses in the web of an I-beam or girder adjacent to a bearing block would involve a consideration of the combined effect of the shearing stresses and the longitudinal stresses in the web adjacent to a bearing block. It would also involve a knowledge of the law of variation of the transverse stress from the flange towards the neutral axis and of the distribution of the pressure along

the bearing block. Since exact knowledge of these conditions is lacking, it is thought that a simple, approximate treatment of the local stress adjacent to bearing blocks will serve the structural engineer as well as a more elaborate analysis, especially since the simple analysis gives results in fair agreement with tests.

The determination of the local compressive stress in the web of an I-beam or girder, adjacent to a bearing block is discussed in Bulletin No. 68 of the Engineering Experiment Station of the University of

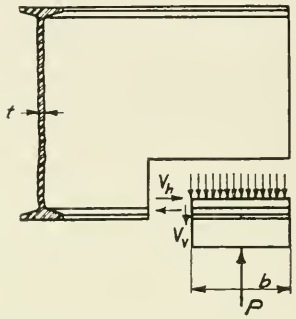

Fig. 13. Diagram of Compressive Stress in Web of Girder over a Bearing Block

Illinois. The following is quoted from that bulletin, with a few changes in notation:

"There may be an excessive compressive stress near the junction of web and flange and adjacent to a concentrated load or reaction. What has been referred to previously as failure by twisting of ends of I-beams is in most cases primarily caused by excessive local compression at the root of the flange.

"An approximate method of computing the compressive stress at the root of the flange adjacent to a concentrated load or an end reaction, has been given by C. W. Hudson as follows:* Imagine a small piece cut from the flange and web of an I-beam immediately over a bearing block (Fig. 13), and imagine this piece to be held in equilibrium by the elastic forces which act on it while it is in its place in the beam. The forces are (1) the pressure of the reaction at the bearing block P; (2) the compression in the web which equals $S_w tb$, when S_w=the average intensity of compressive stress, t= the thickness of web, and b=the length of bearing block; (3) a horizontal shearing force V_h; (4) a vertical shearing force V_v and (5) horizontal

*Engineering News, Dec. 9, 1909.

tensile or compressive stresses. Very little of the total shear would be balanced by the small internal shearing stress in the flange of an I-beam, and if the section considered be taken at the root of the flange, an equation may be written without serious error as follows:

$$V_v = V_h = 0$$

Then, the compressive stress on the web is balanced by the reaction on the bearing block. The compressive stress may be regarded as uniformly distributed, and an equation may be written as follows:

$$S_w = \frac{P}{bt} \quad \ldots\ldots\ldots\

Table 6.

Failure of I-Beams by Local Compression in the Web Adjacent to a Bearing Block

I-beam	Tested by	Thickness of Web in.	Length of Bearing Block in.	Load at Failure lb.	Local Compressive Stress Adjacent to Bearing Block by Hudson's Formula lb. per sq. in.	Yield Point of Material at Root of Flange lb. per sq. in.	Remarks
30 in. 175 lb. Beth. Girder	Marburg	0.69	12	538,400	32,200	28,200	
12 in. 31.5 lb. with planed web	U. of Ill.	0.35	6	190,100	45,300	31,700	1913 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.28	6	160,500	47,800	33,100	1913 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.19	6	109,600	48,200	34,000	1913 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.16	6	72,100	37,600	32,300	1913 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.22	6	96,700	36,500	38,000	Test 1 1914 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.20	6	104,500	41,600	44,000	Test 5 1914 series

lowance for the fact that the strain gage readings are taken over a gage length of which the center is necessarily some distance from the root of the flange. The approximate relation between the strain and the distance from the root of the flange can be obtained by comparing the strains indicated by the readings on gage lines 34, 35, 36, 37, and 23, 26, 27, and 28 of Test 1 shown in Fig. 14.

Assuming that the strain varied directly as the distance from the root of the flange, the stress at the root of the flange would be about 1.25 times as great as the stress indicated by the strain gage readings nearest the flange. Table 7 gives the compressive stress in the web

TABLE 7.

COMPRESSIVE STRESS IN WEBS OF I-BEAMS ADJACENT TO A BEARING BLOCK

I-beam	Thickness of Web in.	Length of Bearing Block in.	Gage Line (See Figs. 2 and 3)	Stress for a Load of 1000 lb. lb. per sq. in.	
				From Strain Gage Measurements	Computed by Hudson's Formula
1–1914 series	0.221	6	16 at end	426	376
			35 at end	400	376
5–1914 series	0.202	6	2 at end	500	412
			27 at end	472	412

directly over the center of a bearing block as determined from the strain gage readings (corrected for distance from root of flange), and as computed by Hudson's formula. The results show an actual fiber stress over the center of the bearing blocks slightly greater than that given by Hudson's formula.

Considering both the results of tests to failure and results of strain gage tests, Hudson's formula seems fairly reliable.

The writers wish to call especial attention to the importance of considering the local compression over a bearing block for an I-beam or a girder. If there is not sufficient bearing area in the web, stiffeners must be provided to prevent the flange from folding over towards the web (Fig. 10). Tendencies toward such folding action were observed in several tests, and the inequality of the strains on opposite sides of the web (test 1, gage lines 18, 19, 20, 21, 22, 23, 24, test 5, gage lines 12, 13, 14, 19, and 20) indicates bending action as well as compression. Angle stiffeners placed over the supports as shown in Fig. 8 check this tendency to bend. It is of the highest importance that such stiffeners fit closely against the flange. An excellent illustration of this point is furnished by the behavior of the built-up gir-

ders during the tests. The stiffeners at points at which concentrated forces are applied, for girders 3 and 8 are angles having their outstanding legs ground to fit the outstanding legs of the flange angles, whereas the corresponding stiffeners for the I-beams are flats. Girders 3 and 8 maintained a vertical position much better than did the I-beams, indicating that it is an essential feature at points at which concentrated forces are applied, to have the stiffeners support the outstanding legs of the flange as much as possible.

16. *Functions of Stiffeners.*—Stiffeners, as commonly used on plate girders, perform two functions. Those placed at frequent intervals along the girders prevent the web from buckling as a column because of diagonal compression. Stiffeners placed under concentrated loads or over supports distribute the concentrated force, which would otherwise be delivered directly to the flange, over a considerable portion of the depth of the web. These latter stiffeners assist also in preventing the buckling of the web.

The spacing of stiffeners is usually determined more or less arbitrarily. The General Specifications of the American Railway Engineering Association for Steel Railway Bridges, 1910 edition, state, "There shall be web stiffeners generally in pairs, over bearings, at points of concentrated loading, and at other points where the thickness of the web is less than 1/60 of the unsupported distance between flange angles. The distance between stiffeners shall not exceed that given by the following formula, with a limit of six feet (and not greater than the clear depth of the web):

$$d = \frac{t}{40}\,(12{,}000 - s).$$

in which d=clear distance between stiffeners of flange angles, t=thickness of web, s=shear per sq. in."

According to these specifications the spacing of the intermediate stiffeners is in a general way a function of the shearing stress; but stiffeners are required, no matter how low the stress may be, if the ratio of the unsupported width to the thickness of the web exceeds 60.

The ratio of the unsupported width to the thickness of the web for the I-beams tested was less than 60 in all cases. For girders 3 and 8 this ratio was 100, and therefore, according to the specifications quoted, intermediate stiffeners were required. Girder 8 was equipped with intermediate stiffeners in accordance with the specifications; whereas girder 3 was provided with stiffeners only at points at which concentrated forces were applied. The load at the yield point for girder 3

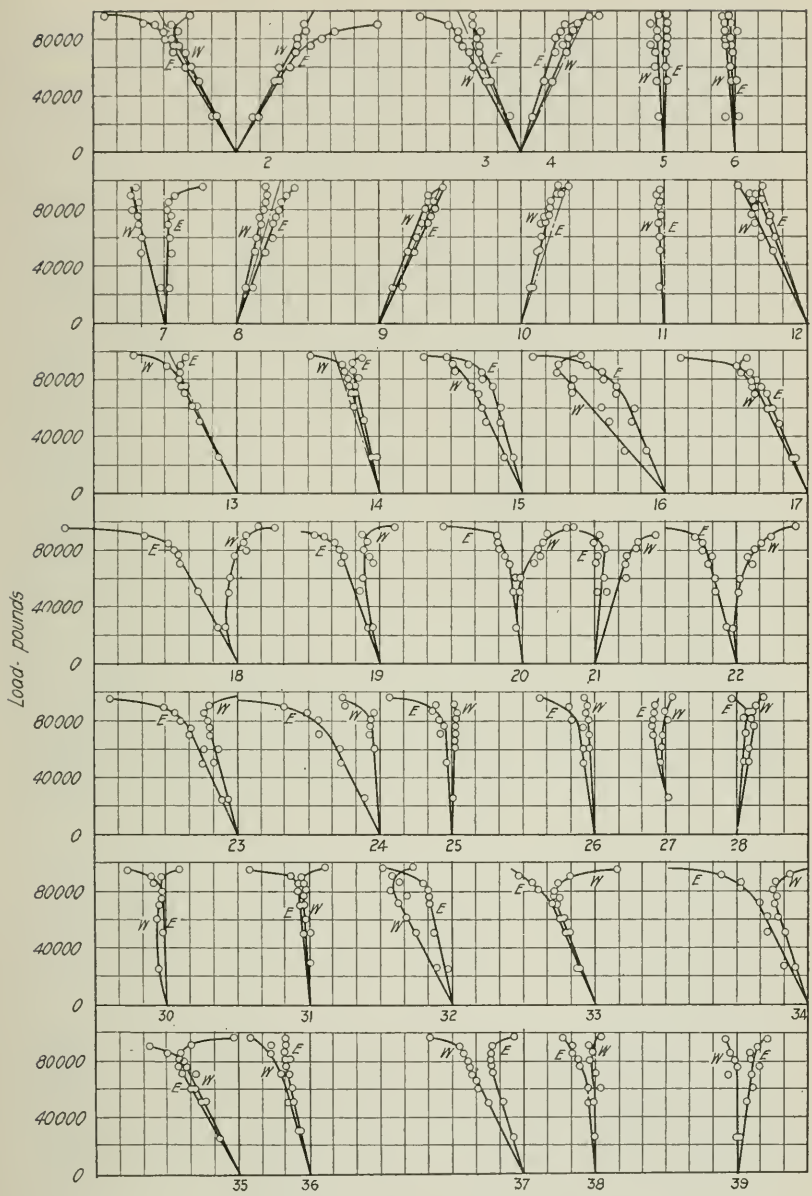

FIG. 14. STRAIN DIAGRAMS FOR I-BEAM 1

Note: 1 horizontal division = 10,000 lb. per sq. in. as indicated by the strain gage. Numbers refer to gage lines as given in Fig. 2. Tension plotted to right; compression to left. Measured quantities ————; computed quantities —— — —— — ——.

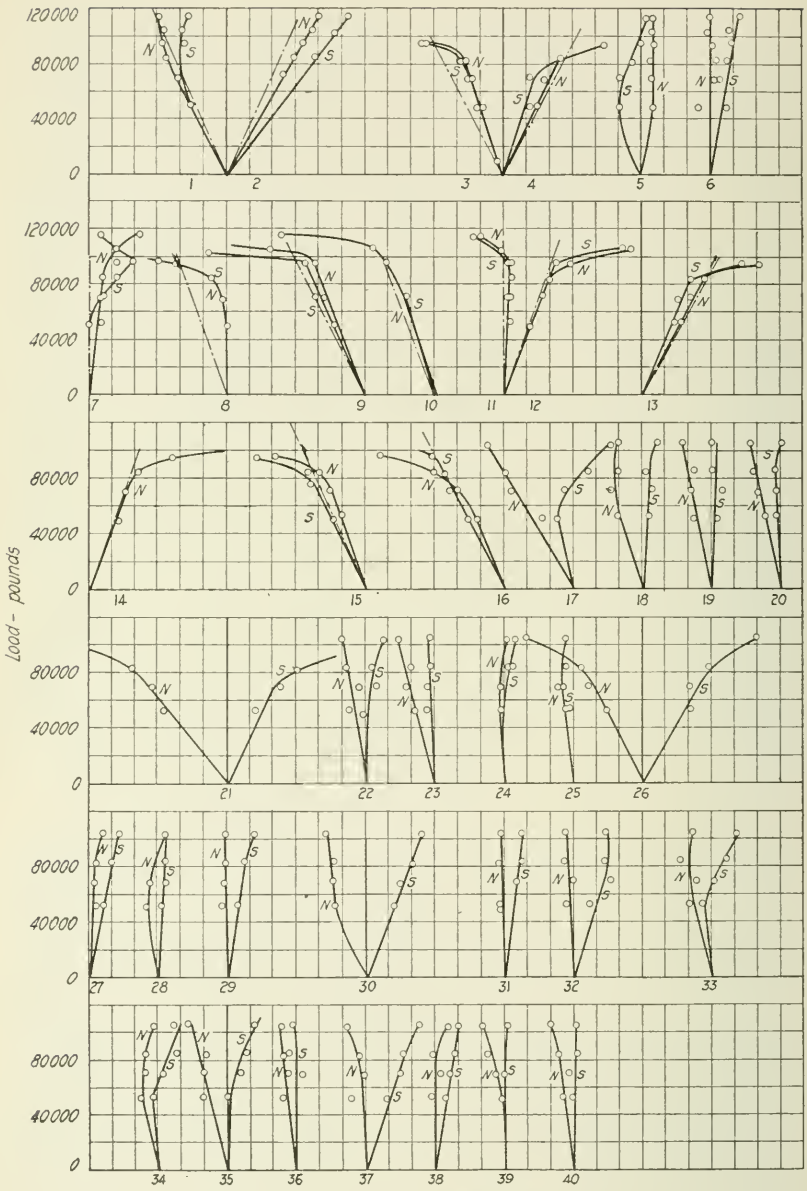

FIG. 15. STRAIN DIAGRAMS FOR I-BEAM 2

Note: See Fig. 14.

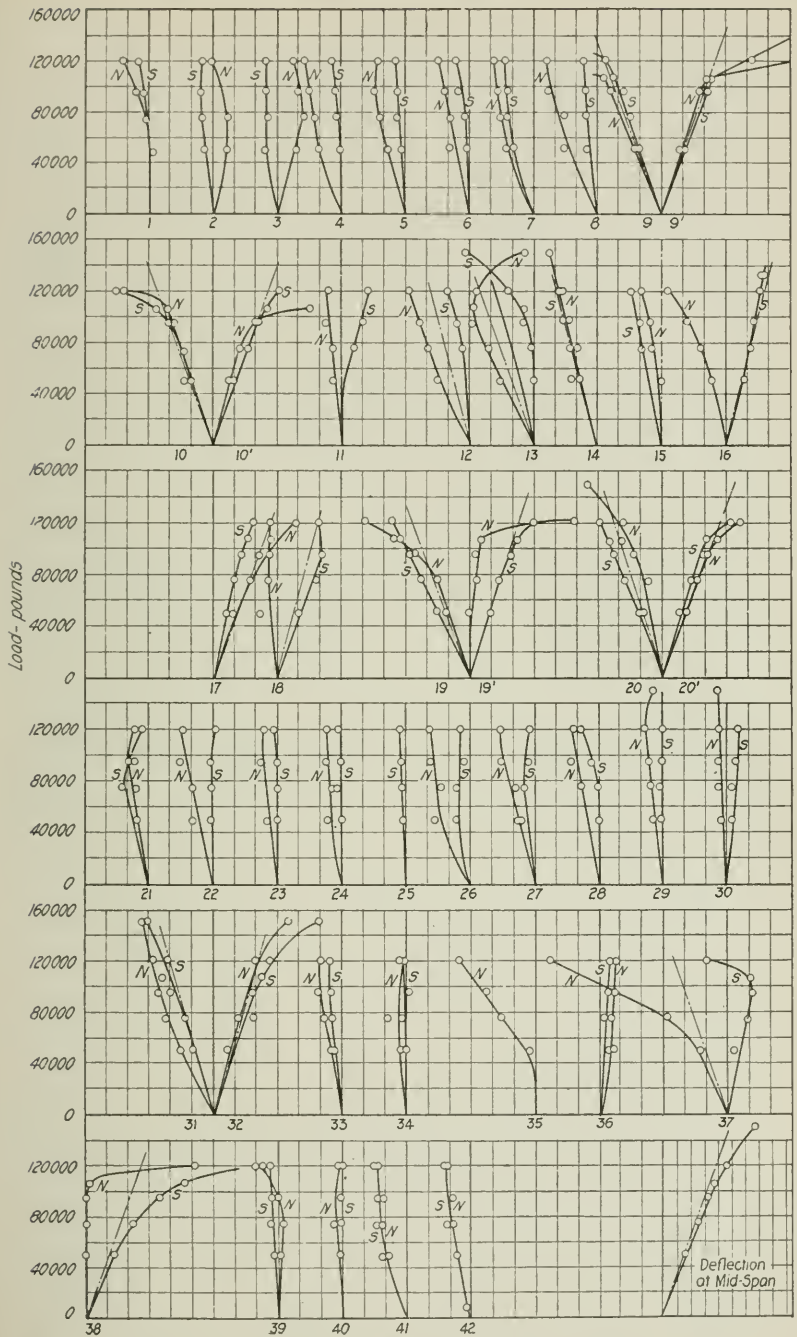

FIG. 16. STRAIN DIAGRAMS FOR BUILT-UP GIRDER 3

Note: See Fig. 22.

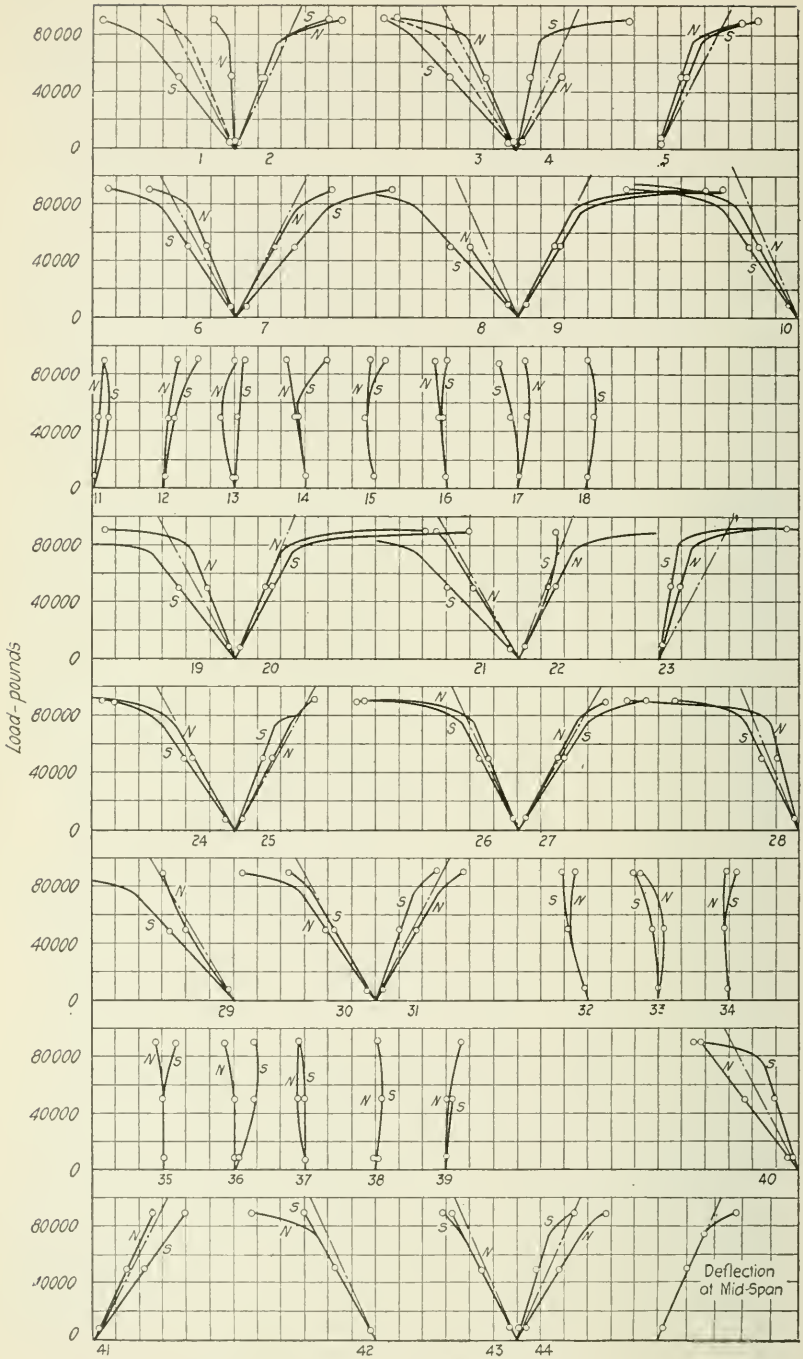

FIG. 17. STRAIN DIAGRAMS FOR I-BEAM 4

Note: See Fig. 14.
1 horizontal division=0.02 inch (for deflection curve only)

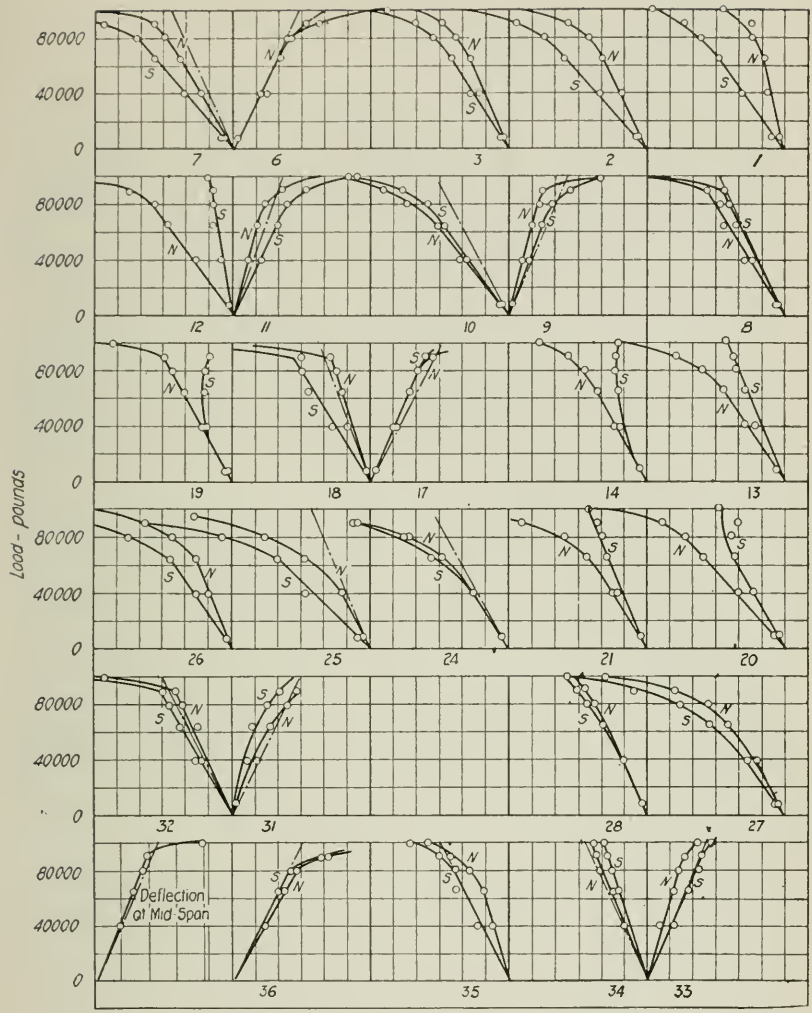

FIG. 18. STRAIN DIAGRAMS FOR I-BEAM 5

Note: 1 horizontal division = 10,000 lb. per sq. in, as indicated by the strain gage.
1 horizontal division = 0.02 inch (for deflection curve only)
Numbers refer to gage lines as given in Fig. 3.
Tension plotted to right; compression to left.
Measured quantities ———; computed quantities —— — —·— — ——.

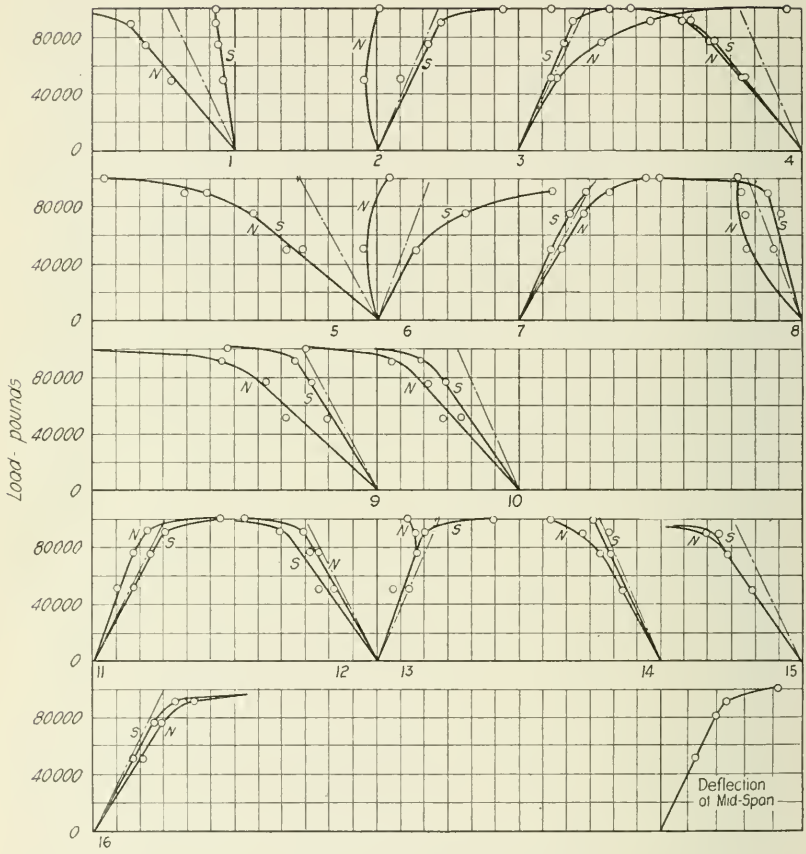

FIG. 19. STRAIN DIAGRAMS FOR I-BEAM 6

Note: See Fig. 18.

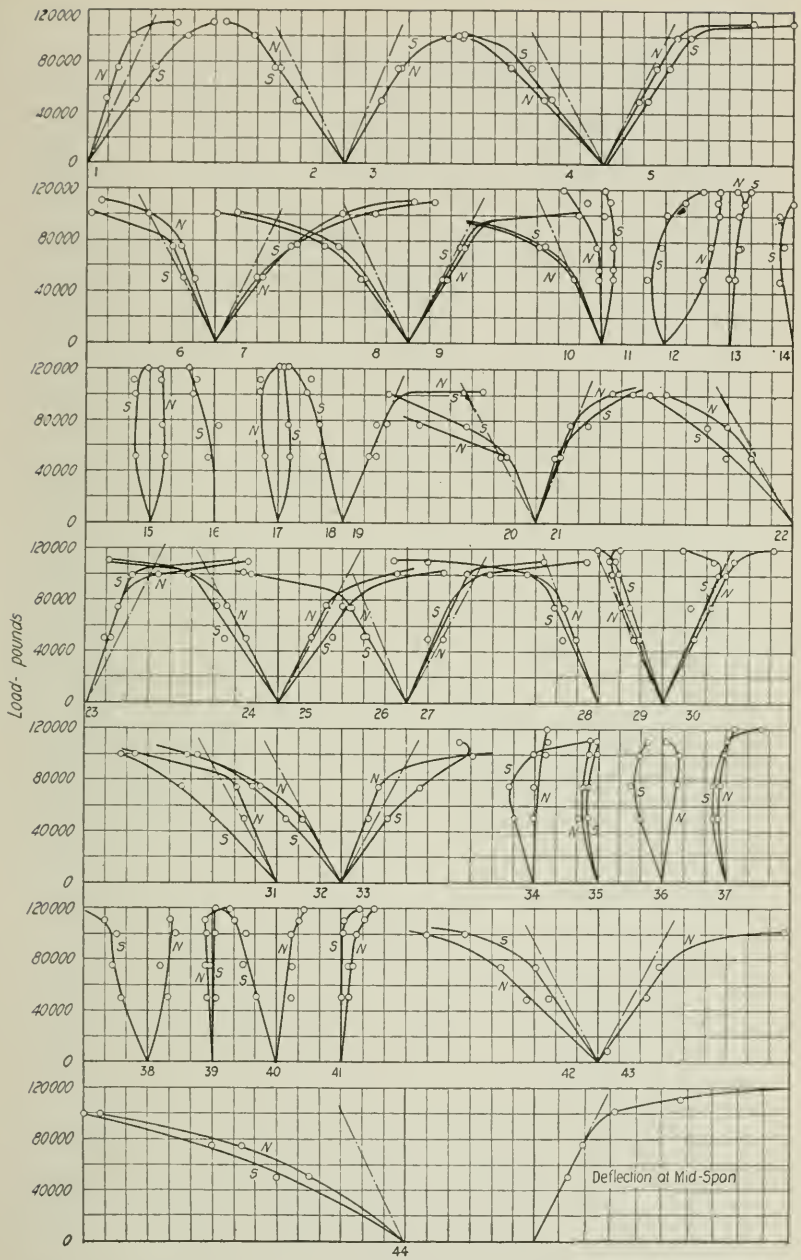

FIG. 20. STRAIN DIAGRAMS FOR I-BEAM 7

Note: See Fig. 18.

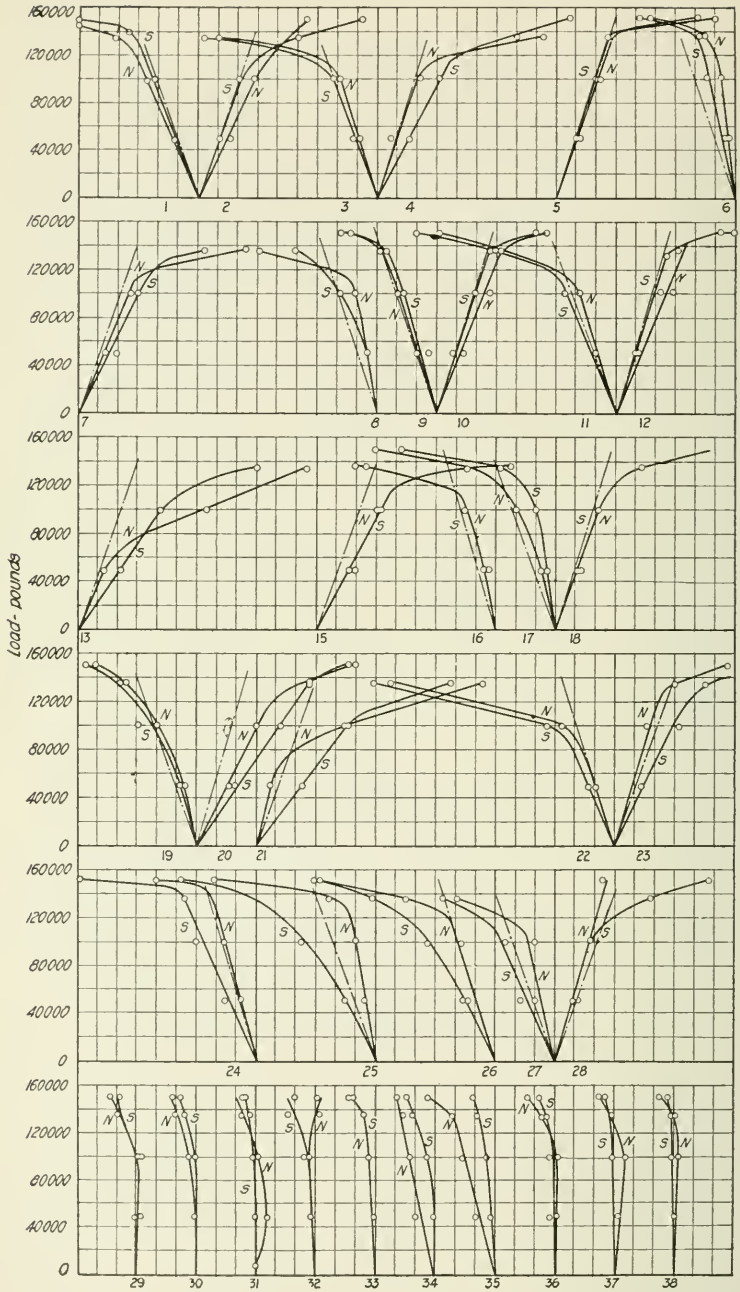

FIG. 21. STRAIN DIAGRAMS FOR BUILT-UP GIRDER 8,—I

Note: See Fig. 22.

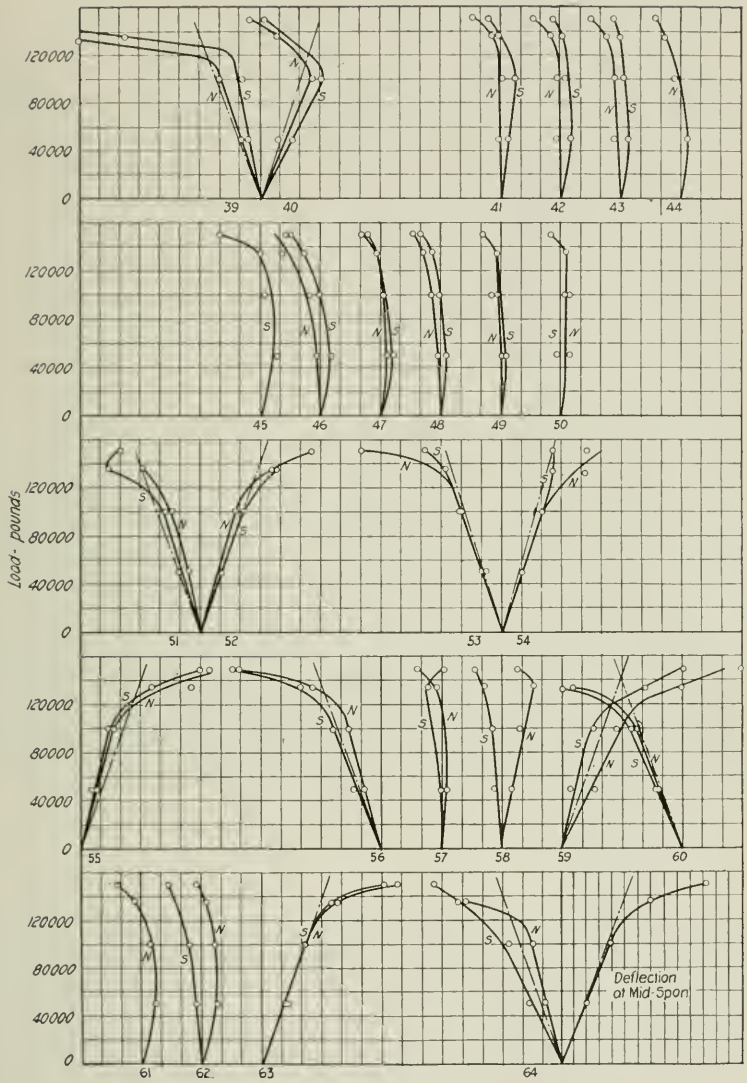

FIG. 22. STRAIN DIAGRAMS FOR BUILT-UP GIRDER 8,—II

Note: 1 horizontal division = 10,000 lb. per sq. in. as indicated by the strain gage.
1 horizontal division = 0.02 in. (for deflection curve only)
Numbers refer to gage lines as given in Fig. 4.
Tension plotted to right; compression to left.
Measured quantities ———; computed quantities —— — —— — ——

was 101,500 pounds, and for girder 8, 112,500 pounds. That is, the addition of intermediate stiffeners to a girder for which the ratio of the unsupported width to the thickness of the web was 100, increased the yield point of the girder only about 10 per cent. This is remarkable in view of the fact that the proportions of the girders were such that the web received a relatively higher stress than the flanges, a condition which is usually reversed in practice. While the girder without stiffeners does not have a yield point quite as high as the girder with stiffeners, the fact that the deflection curve is practically straight up to the yield point indicates that it may be safe to build girders without intermediate stiffeners if the ratio of the unsupported width to the thickness of the web exceeds 60. However, it is necessary to decrease the working stress allowable in the web as this ratio becomes greater.

For the determination of the proper working stress for webs without intermediate stiffeners three calculations should be made: (1) The maximum shearing stress (which may be taken equal to the shearing stress at the neutral axis) should not be greater than the safe working stress in shear for the web material. (The strength in shear of web material is discussed in the next section "Strength of Materials in I-beams and Girders".) (2) The value of $E\epsilon$ corresponding to the maximum diagonal strain should not exceed the value of the safe tensile or compressive working stress for the material. (3) The maximum diagonal compressive stress at the neutral axis should not exceed the safe stress as given by Euler's formula for fixed-ended columns, which when applied to webs of I-beams and girders becomes

$$S_w = \frac{1.64\ E}{f\left(\frac{h}{t}\right)^2} \qquad (11a)$$

in which S_w equals the average working stress in the web, f=the factor of safety, E=the modulus of elasticity (30,000,000 lb. per sq. in. for steel), h=the unsupported depth of the web between flanges, and t=the thickness of the web.

It may be necessary to provide stiffeners for girders having relatively thin webs, irrespective of the stress, in order that the girder may not be injured in handling.

17. *Strength of Material in the I-beams and Girders.*—Specimens for tension tests and shear tests were cut from the beams and girders as described in section 7 "Specimens." The results of the tension tests are given in Table 8, and the results of the shear tests are given in Table 9. In all tension tests the yield point was determined by the

Table 8.

Tension Tests of Material in I-Beams and Girders

Each value is the average of two or more tests

Girder (or I-beam)	Specimens from	Yield Point lb. per sq. in.	Ultimate lb. per sq. in.	Elongation in 2 inches per cent
1	Flange	40,200	66,800	35.0
	Web	40,600	67,500	29.5
	Root of flange	38,000	65,700	27.7
2	Flange	42,600	67,600	27.5
	Web	44,200	68,200	27.7
	Root of flange	39,200	67,700	31.5
3	Web	41,800	54,100	35.7
4	Flange	37,500	66,900	36.1
	Web	42,300	68,900	29.7
	Root of flange	45,800	70,100	27.0
5	Flange	36,200	66,900	34.2
	Web	41,800	68,900	28.2
	Root of flange	46,000	69,200	27.0
6	Flange	39,300	67,400	33.5
	Web	43,500	66,500	27.0
	Root of flange	49,200	69,800	28.7
7	Flange	41,400	67,700	35.5
	Web	39,500	65,300	28.7
	Root of flange	43,800	68,200	30.2
8	Web	36,900	53,700	36.0

Table 9.

Shearing Tests of Material From Webs of I-Beams and Girders

Each result is the average of two or more tests

Girder (or I-beam)	Yield Point lb. per sq. in.	Ultimate lb. per sq. in.
1	Not well defined	53,300
2	do.	53,300
3	25,700	49,000
4	Not well defined	52,300
5	do.	54,400
6	do.	50,500
7	do.	53,100
8	26,100	47,900

drop of the beam of the testing machine, and in shear tests by the first noticeable stretch as shown by a pair of dividers. The results of the tension tests indicate that the material at the root of the flanges of the I-beams was well rolled, since the specimens cut from this part of the I-beams showed fully as high strength as did the specimens from the webs and from the flanges. The yield-point strength of the shear specimens cut from the webs of the I-beams was not clearly defined, but definite yield points in shear were obtained for the specimens cut from the webs of the two built-up girders. Several specimens from each built-up girder were tested. These shear tests

indicate a ratio of yield-point strength in shear to yield-point strength in tension of about 0.65, a ratio not widely different from the value 0.60 which is commonly used. If a fiber stress of 16,000 lb. sq. in. is allowable for structural steel in tension, a stress of about 10,000 lb. per sq. in. would be allowable for steel in shear. This agrees with the usual practice.

18. *Deflection of Test Girders.*—As a matter of interest, though secondary in importance to the determination of strength properties, the observed deflection at mid span of the girders under load has been compared with the deflection computed by means of the formulas commonly given in texts on the mechanics of materials. The girders tested had such short spans that the deflection due to shear, about 20 per cent of the total, is important. This deflection is calculated from the formula

$$\Delta_s = \frac{P}{2} \frac{l_1}{aF} \qquad (13)$$

in which Δ_s is the deflection at mid span due to shear, P is the total applied load, l_1 is the distance from support to the near of the two symmetrically spaced loads, F the modulus of elasticity in shear (taken as 12,000,000 lb. per sq. in. for steel) and a is the total area of cross-section of the beam. This formula is readily derived from the discussion given in Merriman's "Mechanics of Materials," 10th ed. p. 320.

The deflection due directly to flexure is *

$$\Delta_s = \frac{Pl_1}{EI}\left(\frac{l^2}{16} - \frac{l_1^2}{12}\right) \qquad (14)$$

in which l is the total length of span of the beam, E is the modulus of elasticity in tension and compression (taken as 30,000,000 lb. per sq. in. for steel), I the moment of inertia of the cross-section of the beam, and other symbols are the same as given for equation 14.

The total deflection Δ is then $\Delta_s + \Delta_f$. In the deflection curves of Fig. 16-22, the deflection computed by the preceding formulas is shown by the dot and dash line, and the observed deflection by solid lines. The computed and observed values agree very closely.

The theoretical curve of deflection due to shear alone for a girder loaded at two symmetrical points is made up of two inclined straight lines for the end portions, joined by a horizontal straight line for the

**Boyd, "Strength of Materials," p. 115.

middle portion. The general tendency of the girders to assume such a shape under excessive shearing strain is well shown in Fig. 10 and 11.

19. *Summary.*—The following summary is given:

(1) The measured strains in various parts of the six I-beams and two built-up girders agree closely with the strains as computed by the ordinary elastic theory if due allowance is made for the lateral strain (Poisson's ratio effect).

(2) The maximum shearing stress in an I-beam or a built-up girder is in some cases the shearing stress at the neutral axis, and is in other cases the diagonal shearing stress caused by the combined stresses in the web at its junction with the flange. However, the two are usually nearly equal, and, in general the shearing stress at the neutral axis may be used in designing girders.

(3) A common approximate method of computing the shearing stress in the web of a girder is to divide the total shear upon a transverse section by the area of the cross-section of the web. If the value given by this method is more than 80 per cent of the allowable stress in shear for the material, a check computation for shearing stress should be made, using the more precise formula, equation (2).

(4) The yield point (not the ultimate strength) of the material in shear should be regarded as the ultimate shearing stress which can be developed in the webs of girders. The ratio of the yield point in shear to the yield point in tension for structural steel is about 0.6, and the ratio of the allowable shearing stress to the allowable tensile or compressive stress may be taken at the same value.

(5) The maximum tensile or compressive strain is an I-beam or built-up girder is in some cases the longitudinal strain in the extreme fibers of the flange, and is in other cases the diagonal strain in the web adjacent to the flange. The diagonal strain may be enough greater than the longitudinal strain to make it desirable to consider the former in the design of a girder. The value of $E\epsilon$ corresponding to the maximum strain should not in any case exceed in magnitude the safe working stress of the material in tension. (The safe stress in tension for structural steel is usually taken at 16,000 lb. per sq. in.)

(6) In the case of girders having no stiffeners except at points at which concentrated forces are applied, the web is capable of developing the lowest of the following critical values: (1) the yield-point strength of the material of the web in shear; or (2) the compressive strength of the web as computed by Euler's formula, considering a 45-degree strip as a fixed-ended column subjected to a compressive stress equal to the transverse shearing stress at the neutral axis; or

(3) a diagonal strain equal to the strain at the yield point of the material in tension.

(7) It would seem that the ability to resist buckling of thin webs without intermediate stiffeners had been underestimated.

(8) Stiffeners at supports and under concentrated loads are very necessary. These should be well fitted to the flanges.

(9) The local compressive stress in the web of a girder when no stiffeners are used at points at which concentrated forces are applied may be computed with a fair degree of accuracy by the use of Hudson's formula (see p. 32 for detailed discussion). Even if this stress is low, the use of stiffeners at points at which concentrated forces are applied diminishes the danger of lateral bending of the beam at the junction of the web and the flange.

(10) The deflection of the girders as measured and as computed by the ordinary elastic theory agrees closely when the deflection due to shear is considered (see p. 48 for discussion of formulas). For short-span beams the deflection due to shear may be as much as 20 per cent of the total.

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

Circular No. 3. Fuel Tests with Illinois Coal (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907), by L. P. Breckenridge and Paul Diserens. 1909. *None available.*

Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

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Bulletin No. 11. The Effect of Scale on the Transmission of Heat Through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

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Bulletin No. 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

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Bulletin No. 16. A Study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*

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BULLETIN No. 87

CORRECTION OF ECHOES AND REVERBERATION IN THE AUDITORIUM, UNIVERSITY OF ILLINOIS

BY

F. R. WATSON

And

JAMES M. WHITE

ONTARIO RESEARCH FOUNDATION
47 Queen's Park
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TORONTO

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 87 MAY, 1916

THE CORRECTION OF ECHOES AND REVERBERATION IN THE AUDITORIUM AT THE UNIVERSITY OF ILLINOIS

BY

F. R. WATSON, Associate Professor of Experimental Physics, at the University of Illinois, and

PROFESSOR J. M. WHITE, Supervising Architect

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THE CORRECTION OF ECHOES AND REVERBERATION IN THE AUDITORIUM AT THE UNIVERSITY OF ILLINOIS

I. Introduction.

1. Preliminary.—The work described in this bulletin may be considered as a continuation of an earlier investigation on "Acoustics of Auditoriums."*

Figure 1 shows the floor plans of the auditorium under investigation. The interior approximates a sphere cut off on the lower surface by the sloping floor of the room. There is a balcony, but no gallery. The balcony projects 12 feet over the main floor at the sides and 34 feet in the rear. The stage is built out into the room instead of being set back behind a proscenium arch as originally designed, the stage house having been omitted to reduce the cost of the building.

The domed ceiling is supported on four equal arches, and the side walls above the gallery are double curved surfaces. The limited appropriation for the building made it impossible to embellish the surfaces of the walls and ceiling, and therefore, they were left practically plain, which increased their power to reflect sound and cause echoes. There are no windows in the room, the daylight lighting being exclusively through a ceiling light 30 feet in diameter in the center of the dome.

The results set forth in the previous bulletin are briefly as follows. A systematic investigation of the acoustical properties of the Auditorium at the University of Illinois was carried on for several years. "Cut and try" methods of cure were avoided. It was shown by theory and experiment that the usual acoustical faults in a room are due first, to a reverberation, or undue prolongation of sound, and second, to echoes; both of these defects being caused by the reflection of sound from the walls. Various methods of cure were considered,—the effect of padding and paneling the walls, the possible advantage of installing wires† and sounding boards,‡ and finally, the action of

*Bulletin No. 73 of the Engineering Experiment Station, University of Illinois.

†"Inefficiency of Wires as a Means of Curing Defective Acoustics of Auditoriums." Science, Vol. 35, p. 833. 1912.

‡"The Use of Sounding Boards in an Auditorium." Physical Review, Vol. 1 (2), p. 241, 1913. Also The Brick Builder, June, 1913.

the ventilating system.* The conclusion was drawn that the most effective cure lay in padding the walls with materials which absorb sound.

An experimental diagnosis of the acoustical properties of the Auditorium was made. This was done by tracing the path pursued by a small bundle of sound when it was sent in a definite direction and noting what became of it after reflection. Several methods of tracing sound were tried before a suitable one was found. A ticking watch backed by a reflector, or a metronome enclosed in a box having a directed horn gave definite data. However, a hissing arc light with a parabolic reflector was much more satisfactory and gave conclusive results. Enough data were secured in this way to show the general behavior of the sound in the room and also to indicate how the chief echoes were set up.† Attempts were then made to secure satisfactory acoustics by hanging curtains and draperies at critical points suggested by the diagnosis. This result was finally secured by suspending four large pieces of canvas in the dome.

From the acoustical standpoint, the Auditorium was then in a much improved condition. The canvas, however, was very unsightly and did not accord with the architectural features of the room. It was therefore proposed that the materials used to correct the acoustics be installed in such manner as to remedy this fault. It was also proposed at this time to install a pipe organ, to decorate the interior of the room, and to change the lighting system.

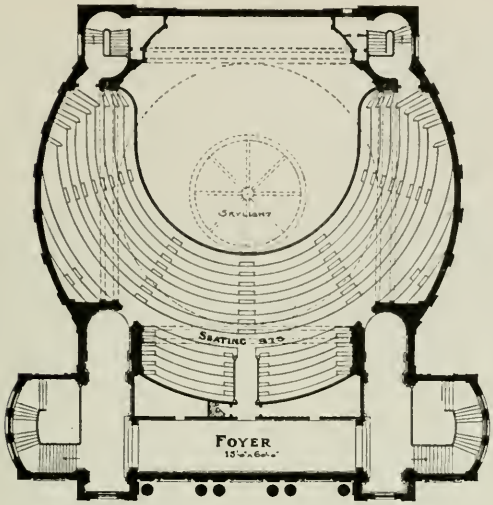
2. *Object of the Bulletin.*—The object of this bulletin is to describe the changes that were made in the Auditorium to carry out the proposals just mentioned, and especially to show how the acoustical properties were modified.

II. Preliminary Acoustical Investigation.

It was desired that the materials used to correct the acoustics be installed in such manner as to conform with the architectural features of the Auditorium. This introduced a new problem since in the provisional cure the canvas sheets in the dome hung with very little conformity to the curvature of the walls. A further complication appeared when it was found by calculation that the amount of material necessary to correct the reverberation was insufficient to pad all the

*"Air Currents and Acoustics of Auditoriums." Engineering Record. Vol. 67, p. 265, 1913.

†"Echoes in an Auditorium." Physical Review, Vol. 32, p. 231, 1911.



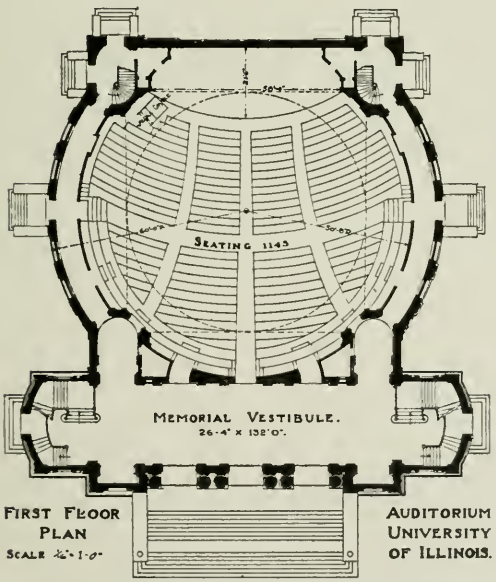


FIG. 1. FLOOR PLANS SHOWING INTERIOR OF THE AUDITORIUM AT THE UNIVERSITY OF ILLINOIS WHICH WAS CORRECTED FOR ECHOES AND REVERBERATION.

walls that produced echoes. It was desirable to eliminate the echoes, but it was regarded as risky to install too much sound absorbing material, owing to the danger of making the Auditorium too dead for sound.

Because of these difficulties it was decided to carry on further experiments and to secure more data before deciding on the final cure. Accordingly, one large curved wall was covered with strips of one-inch hair felt, 30 inches wide, placed vertically and 30 inches apart so as to leave bare spaces between them. This arrangement was satisfactory for several reasons; it did not change the curvature of the wall; it used only half the amount of material necessary to cover the entire wall; and because of diffraction and interference effects, it was theoretically more efficient in breaking up the reflected sound than if the same material were spread continuously over the whole surface. Although encouraging, the results were not so marked as expected in diminishing the echoes.

On the basis of this experiment, plans were made for covering other walls in a similar way, except that the hair felt was to be mounted on wooden ribs built out from the wall surface. Such an installation seemed more likely to break up the incident sound than the first plan of mounting the hair felt snugly against the wall. The sound wave on striking these outer felt strips would suffer partial reflection and change of phase, while the remaining portion of the sound would pass through the open spaces and be spread out by diffraction and reflection from the walls. The hair felt strips would oppose the incident and reflected waves, thus breaking up the original sound and diminishing its intensity and possibility of producing echoes.

Because the scaffolding erected for the use of the workmen interfered with the passage of sound waves, the efficiency of this method of placing the felt could not be tested step by step as the material was mounted. The test was deferred, therefore, until the installation was completed. In the meantime the pipe organ was installed, the interior was redecorated, and the lighting system changed, so that only the combined effect of all these factors on the acoustics could be investigated.

III. Modifications of the Interior of the Auditorium.

3. *Installation of the Pipe Organ.*—The organ was mounted in a unique way by dividing it into two parts and placing them in lofts 24 feet above the ends of the stage with a distance of 75 feet between centers. This arrangement placed the organ at a considerable distance

above the audience. The absence of any vertical surface between the lofts and the audience room prevented any visible arrangement of the organ pipes, but the necessary free exit of the sound was provided for by the construction of ornamental plaster grills covering the pendentives on either side of the stage. (See Fig. 2.)

FIG. 2. VIEW TOWARD THE STAGE SHOWING THE GRILL WORK FOR FREE PASSAGE OF SOUND FROM THE CONCEALED ORGAN. THE ORGAN CONSUL IS SHOWN TO THE LEFT. CARPET IS REMOVED FROM STAGE IN PREPARATION FOR AN ORCHESTRA CONCERT

4. *Method of Mounting Hair Felt.*—The hair felt was mounted on thin furring strips which were bent to fit the curvature of the surfaces. The dome above the arches and the double curved side walls and single curved rear wall above the balcony were padded in this way. The felt was mounted in vertical strips on the west side wall as shown in Fig. 3. Fig. 4 shows the wall after the material was installed and decorated.

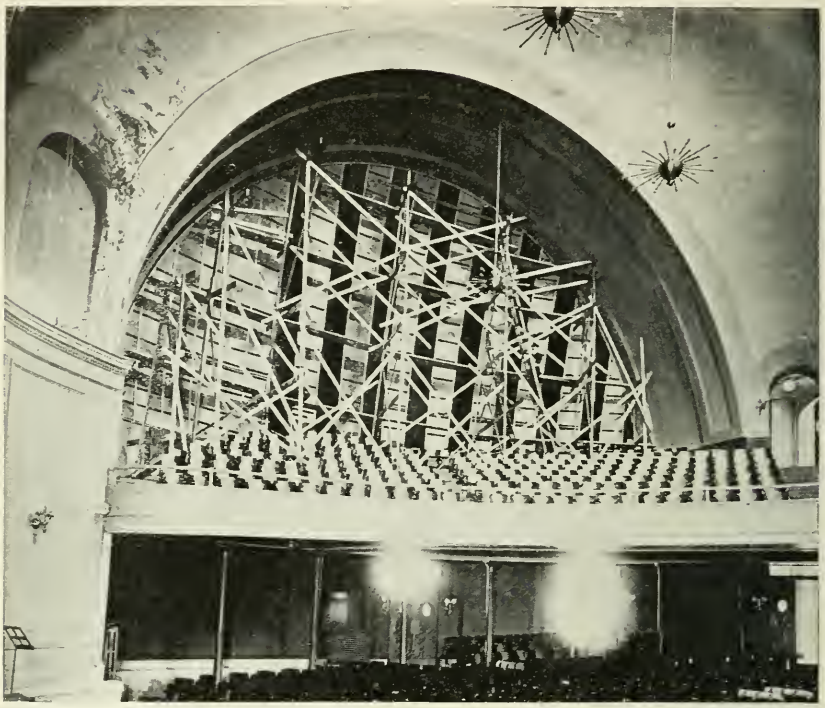


FIG. 3. PHOTOGRAPH SHOWING THE MOUNTING OF THE HAIR FELT IN VERTICAL STRIPS. THE MATERIAL WAS FASTENED TO THIN FURRING STRIPS WHICH COULD BE BENT TO CONFORM TO THE CURVATURE OF THE SURFACE

On the east balcony wall the felt was mounted on wooden ribs so that it stood concentric with the plaster surface at a distance of one foot. Eighteen inches below the edge of the skylight in the dome radial strips of felt which approached the wall until they touched at the crown of the arches, were mounted on wooden ribs. (Fig. 5.) The hair felt used was the Akustikos Felt developed especially for correction of acoustical faults by the H. W. Johns-Manville Company under the direction of Professor Sabine.

Before the changes were made in the Auditorium, Professor Sabine visited the building at the invitation of President James. After this visit, he wrote to President James as follows: "If such confirmation of the results of Professor Watson's investigation is necessary, please permit me to assure you that you will obtain an excellent effect from following out his suggestions in all detail." The final installation was

FIG. 4. PHOTOGRAPH SHOWING THE SIDE WALL OF FIG. 2 WHEN COMPLETED. A REP OF SUITABLE COLOR WAS STRETCHED OVER THE ENTIRE SURFACE AND DECORATED. IT WAS NECESSARY FOR THE FREE PASSAGE OF THE SOUND THAT THE MATERIAL USED IN DECORATING SHOULD NOT CLOSE THE PORES OF THE REP

modified somewhat from the original plans, but the general features were maintained.

5. *The Decoration and the Lighting System.*—The modification of the lighting system involved the elimination of the suspended fixtures. The wall brackets were retained, but the main lighting was changed to a semi-indirect system with reflectors above the arches and around the skylight. An ivory tone was selected for the basic color in the redecoration. Ornamentation was stenciled and painted on the various walls and surfaces to give a unified effect. With the exception of the ornamental borders the rep covering the padded surfaces was left its natural color. The difference between the old and new interiors is shown in Figs. 6 and 7. The modifications relieved the auditorium of its cheerless, barn-like interior.

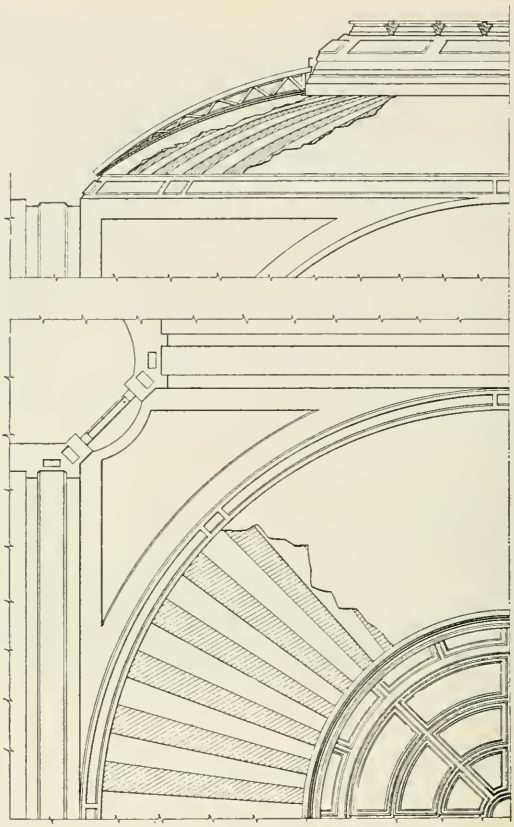


FIG. 5. DIAGRAM SHOWING THE DOME SURFACE WITH WOODEN RIBS SEPARATING THE HAIR FELT IN RADIAL STRIPS. THIS FALSE CEILING ARRANGMENT WAS THOUGHT MORE EFFECTIVE FOR ABSORPTION OF SOUND THAN IF THE MATERIAL WERE MOUNTED SNUGLY AGAINST THE SURFACE

IV. FINAL ACOUSTICAL INVESTIGATION.

The remodeled Auditorium has been tested under varied conditions for music and speaking, and popular opinion has pronounced the acoustics satisfactory. A speaker with a moderate voice can be heard distinctly by auditors in the most distant seats. The music of the new pipe organ, according to experts, is satisfactorily rendered. The room is also suited for orchestra music, though for this case, it has been found advantageous to follow the usual custom of leaving the wooden floor of the stage bare of carpet so as to reenforce the sound from the instruments.

While the Auditorium has proved to be generally satisfactory, a detailed investigation of the acoustical effects secured by the modification of the room was thought desirable. A request was made, accordingly, that auditors report any echoes or acoustical disturbances however slight they might be. About a dozen replies were received, and on the basis of these and other considerations, a systematic investigation was undertaken.

FIG. 6. PHOTOGRAPH SHOWING AUDITORIUM BEFORE CHANGES WERE MADE

The acoustical results, beneficial and otherwise, may be anticipated by considering the changes made. According to Sabine, the hair felt installed would reduce the reverberation. This would also eliminate echoes if installed on certain surfaces in accordance with the analysis; but, since the amount of material used to correct the reverberation was insufficient to cover all the walls, acoustical defects might still be set up by the unpadded surfaces, especially by the pendentives. The pipe organ, by generating musical sounds that emerged through the pendentives in the dome, might introduce new acoustical disturbances. The openings made in the surfaces of two of the pendentives for the passage of the organ music would reduce the general

FIG. 7. PHOTOGRAPH SHOWING NEW INTERIOR OF AUDITORIUM. THE SUSPENDED LIGHTING FIXTURES WERE REMOVED, THE INTERIOR REDECORATED, AND THE REAR WALL IN THE ALCOVE PADDED

reverberation and would also diminish echoes. The changes in the decoration and in the lighting system would produce little effect.

6. *Investigation of Echoes.*—Tests were made in several ways to determine the presence of echoes. The opinion offered by auditors that the echoes had generally disappeared was, of course, the most satisfactory evidence. One test was made by talking through a megaphone toward different walls (Fig. 8). The sound was generated inside a small house and its direction of propagation controlled by two megaphones, one being pointed toward an observer and the other toward a wall which previously gave echoes. No distinct echo could be obtained by speaking simultaneously into the two megaphones. The ticks of a metronome produced very little additional effect, but when a sharp intense metallic sound was tried, echoes were obtained from the unpadded walls but only faint responses from the padded walls. The intense hissing sound of an arc light backed by a parabolic reflector gave more pronounced results. It showed that the padded walls produced a marked effect in reducing the intensity of the sound.

FIG. 8. PHOTOGRAPH OF STAGE SETTING SHOWING EXPERIMENTAL HOUSE WITH MEGAPHONES USED FOR TESTING ECHOES

The effect of the unpadded pendentives in the rear dome surface is shown in Fig. 9. The cone of incident sound received by each pendentive is small and, after reflection, spreads over a large area. It was therefore anticipated that little disturbance would result. This prediction was not entirely correct since the echoes reported by auditors, so far as could be ascertained, came from these two walls. An echo was perceptible when the speaker faced directly toward one of these pendentives so that the profile of his face was seen by an auditor seated at one side of the auditorium. The direct sound coming to the auditor was then diminished while the reflected sound was augmented, thus producing an echo.

Other unpadded walls, notably the side walls under the balcony, still set up concentrations of sound. Thus, an observer at A, Fig. 10, can hear not only the direct sound from the speaker, but also the portion that is concentrated by reflection from B. He does not hear an echo because the time interval between the direct and reflected sounds is too short to enable his ear to detect them separately. The result is

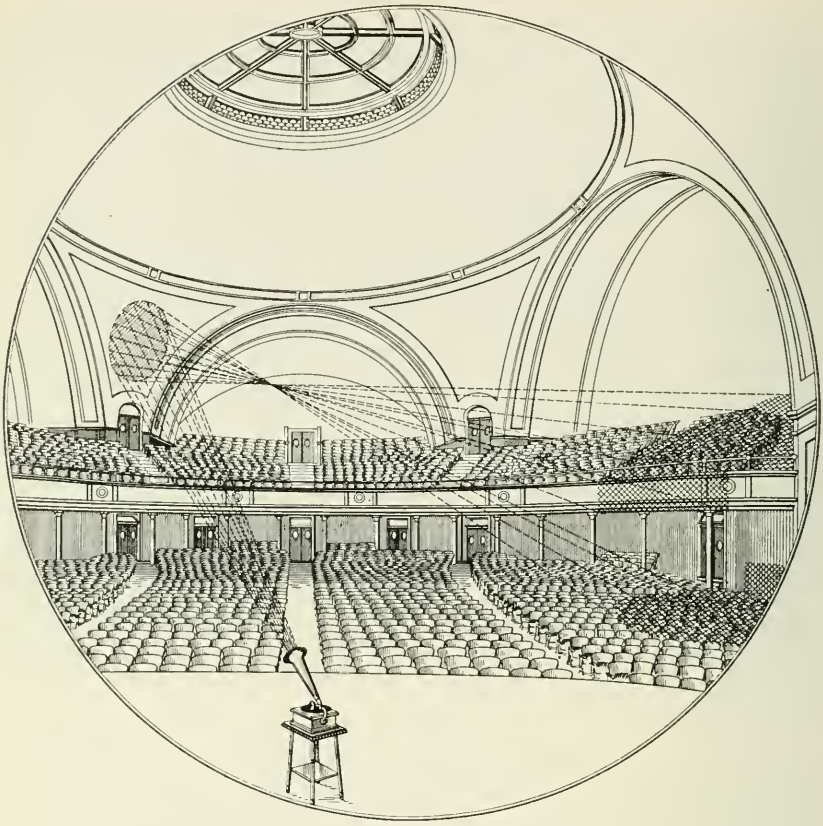


FIG. 9. DIAGRAM SHOWING THE REFLECTION OF SOUND FROM THE UNPADDED PENDENTIVE IN THE REAR WALL. ECHOES SET UP BY THIS WALL CAN OCCASIONALLY BE NOTED

much the same as if his neighbor on the side toward the wall were to say the words of the speaker in his ear at the same time that he received them from the speaker. The auditor realizes that something is peculiar about the sound but usually does not understand the cause of the trouble. An auditor at C, however, may get an echo when the speaker faces the point D.

7. *Investigation of the Reverberation.*—By means of Sabine's formula and coefficients of absorption* the time of reverberation of the Auditorium was found and a calculation was made to determine the amount of sound absorbing material necessary to correct the fault. The following tabulation shows the method employed:

*American Architect, 1900.

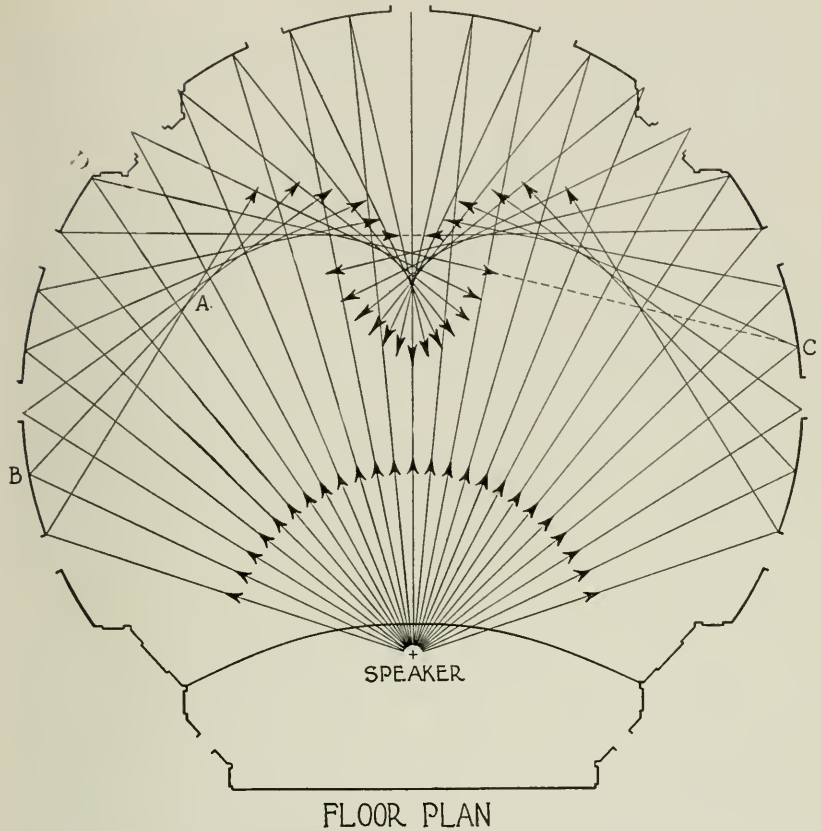


FIG. 10. PLAN OF AUDITORIUM SHOWING CONCENTRATION OF SOUND BY THE WALLS UNDER THE BALCONY

Material	Exposed Area in sq. meters	Coeff. of Absorption	Total Absorption
Plaster on lath	2000	0.0330	66.0
Plaster on tile	510	0.0250	13.0
Wood work	1630	0.0610	99.0
Glass	83	0.0270	2.3
Cocoa matting	145	0.0200	2.9
Wood seats	2150 seats	0.0082	17.7
			201
Average audience	1200 people	0.44	527
		Total	728

Volume of room.....12000 cubic meters.

Substituting these data in the formula $t=0.164\ V \div a$, in which t is the time of reverberation, V the volume of the room and a the total absorbing power, the following equation for the empty room is obtained:

$t=0.164\times 12000 \div 201=9.8$ seconds.

When an audience of 1200 people is present,

$t=0.164\times 12000 \div 728=2.7$ seconds.

This value is too great for good acoustics and a reverberation results. To correct the fault, absorbing material should be added until the time of reverberation is reduced to about 1.8 seconds; this value having been found satisfactory for halls as large as the Auditorium when used for both music and speaking.

The amount of Akustikos Felt needed to carry out the plans already described was 3315 square feet. This was less than the area necessary for felt mounted snug against the wall since the coefficient of absorption is greater when the felt is mounted out from the wall.* Calculations, which allowed for the sound absorbing power of the felt and the other alterations in the Auditorium indicated that the time of reverberation would be reduced to about 1.90 seconds with 1200 people present.

V. Discussion and conclusions.

The Auditorium fulfilled the theory held many years ago by Lord Rayleigh* that a large room with hard, non-porous walls and with few windows has a prolonged resonance, and that the best chance of improvement lies in padding the walls and ceiling with sound absorbing materials. Thus, the installation of hair felt in the Auditorium reduced the reverberation; the amount of reduction being calculated in advance by Sabine's† formula and constants of absorption.

The amount of hairfelt necessary to correct the reverberation was insufficient to cover all the walls, and it was found that some of these unpadded surfaces still produced echoes. This action was anticipated in part from the general considerations discussed by Rayleigh‡ in which the possibility of reflection of sound was shown to depend on the positions of the source and receiver of sound, and also upon the size and form of the wall compared with the wave length of the incident sound.

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*Sabine. Architectural Quarterly of Harvard University, p. 22, March, 1912.

*Theory of Sound, Vol. 2, pp. 287 and 351.

†American Architect, 1900.

‡Theory of Sound, Vol. 2, p. 287.

The installation in an auditorium of considerable sound absorbing material eliminates the objectional condition of satisfactory reverberation being wholly dependent on the sound absorbing power furnished by an audience. This means that rehearsals without an audience can be conducted satisfactorily and that a speaker addressing a small audience is not obliged to contend with a distressing reverberation.

The theoretical advantages in absorbing and breaking up sound waves when hair felt is mounted out from a wall instead of placed snugly against the surface do not appear to be so great as expected. Observers listened to sounds reflected from both types of surface and concluded that a surface having the hair felt mounted out from the wall was more efficient. The conclusions, however, should be checked by quantitative, instrumental measurements since the ear is inaccurate in its estimation of the comparative intensities of different sounds.* It appears that the felt is more effective when mounted out from the wall, but there is some question whether or not the advantages secured justify the additional expense of installation and the greater risk of fire.

The music of the pipe organ emerging in large volume from the pendentives in the dome introduced concentrations of sound different from those set up when the source of sound was on the stage. This made it desirable to pad other walls in addition to those requiring padding for the single source of sound.

The effect of the organ music confirmed one conclusion set forth by Jägert; namely, that the strength of the source of sound for good acoustics should be in correct proportion to the volume of the room. It appears that the Auditorium is too small for loud organ music since the sound in this case becomes unpleasantly intense. On the other hand, it appears that the volume is fairly well suited for softer organ music and for a weak source of sound, such as a speaker with a moderate voice. In this connection Jäger contends that an auditorium is limited in its acoustical possibilities; that if a room is too large, it is impossible to make it satisfactory for weak sources of sound. He points out also that the problem of correcting faulty acoustics must include a consideration of intensity of sound as well as of reverberation; that is, the variable factors at command, the volume and absorbing power of the room and the source of sound, must be so propor-

*Rayleigh, Scientific Papers, Vol. II, p. 132.

†"Zur Theorie des Nachhals," Sitzungsberichten der Kaiserl. Akademie der Wissenschaften in Wien. Matem-naturw. Klasse; Bd. CXX, Abt. IIa, Mai, 1911.

tioned as to give not only a suitable reverberation but also an acceptable intensity of sound. He discusses the limitations in obtaining this desired result.

Another deduction made by Jäger which applies rather directly to the Auditorium is that the ratio S/W should be large for good acoustics, in which S is the total surface of walls, furniture, and fixtures struck by the sound and W is the volume of the interior. Theoretically, this ratio is smallest for a sphere, and, since the Auditorium approximates a hemisphere, the excessive reverberation might have been predicted.

Reverberations and echoes were corrected simultaneously by installing a suitable amount of hair felt on the walls which produced echoes. To locate these walls, a new method was developed in which the source of sound was an arc light as explained earlier in this bulletin.

The investigation showed that curved walls are worse acoustically than plane walls since they produce undesirable concentrations of sound and echoes. It also appears that the openings in the pendentives for the organ music and the ventilation openings act similarly to open windows and thus reduce reverberation and diminish echoes.

One acoustical disturbance which was not corrected was that due to talking and walking in the foyer and on the stairs immediately outside the Auditorium. The sounds of footsteps and the reverberation caused by loud talking and accidental noises in the foyer could be reduced by covering the stairs and foyer with a yielding material, such as cork and by padding some of the walls.

It is apparent from this discussion that the means employed to correct the acoustics, as exemplified by this complex problem, were based upon established scientific principles and this investigation and others of like nature have served, to a large extent, to dispel the mystery surrounding the action of sound in auditoriums.

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DRY PREPARATION OF BITUMINOUS COAL AT ILLINOIS MINES

BY

E. A. HOLBROOK

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 88 JUNE, 1916

DRY PREPARATION OF BITUMINOUS COAL AT ILLINOIS MINES

BY E. A. HOLBROOK

ASSISTANT PROFESSOR OF MINING ENGINEERING

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(Photograph by E. C. Lee.)

Fig. 1. Horse Whim at a Local Mine.

Fig. 2. A Modern Steel Tipple with Concrete Rescreener.

DRY PREPARATION OF BITUMINOUS COAL AT ILLINOIS MINES.

INTRODUCTION.

Coal, as it comes from the mine, is not usually in condition for direct delivery to the consumer, but must be first subjected to treatment or preparation in order to remove impurities and to secure the sizes suitable for the different markets.

"Preparation" of coal is understood generally to include that set of operations which begins after the coal is delivered to the mine tipple, and ends when the loaded railroad cars at the mine are weighed and ready for shipment. It is, however, often difficult to state the exact points at which preparation of coal begins and ends. On the one hand, preparation encroaches on the domain of the miner because much of the impurity may be separated from the coal at the face underground and because of the natural production there of various sizes of coal. On the other hand, coal is subjected to breakage and inspection after being shipped to market, and therefore repreparation may be necessary after delivery and before consumption. Thus, preparation may enter the province of the mechanical or fuel engineer.

In 1909 a committee of the International Railway Fuel Association, appointed to inquire into the difficulties encountered in producing clean coal, reported the causes for poor coal as follows:*

(1) The physical conditions of the seam, mine, and mine equipment.

(2) The class of labor that produces and handles the coal.

(3) The conditions surrounding the sale of the coal, including the prices obtained.

Any or all of these conditions may seriously affect the preparation required by Illinois coal.

In this bulletin the mining and marketing of the coal are discussed from the standpoints of contained impurities and breakage only as these affect preparation in the tipple and auxiliary buildings. Preparation of coal may be divided into two separate and distinct processes: (1) Wet preparation, called coal washing or jigging. (2) Dry preparation, including weighing, screening, dry cleaning, and loading. The first division of the subject has been covered by F. C. Lincoln in Bulletin No. 69 of the Engineering Experiment Station of the University of Illinois, under the title "Coal Washing in Illinois," and will not be considered here.

In the early days of coal preparation in the anthracite regions of Pennsylvania coal was frequently washed or rinsed with a spray of water in order to make easier the detection of intermixed refuse and to give it a better market appearance. Afterwards, cleaning of

*Proceedings of the First Annual Convention, 1909, p. 13.

coal by jigging was introduced and called coal washing, and thus the same term was applied to two different processes of coal preparation, giving rise to some confusion. Since in this state, in at least one instance, coal has been rinsed by a spray of water before hand picking, this process will be called rinsing and not washing, in order to avoid confusion with true coal washing or jigging.

The subject of dry preparation has been divided into two parts, the first of which considers the development, the standard methods, and the products of present practice and is treated in this bulletin. The second part, which will be an engineering discussion of the machinery, appliances, and costs of preparation, will be covered by future work.

The subject matter of the present bulletin is subdivided as follows:

Chapter I. Evolution of present preparation practice.

Chapter II. Standard types of Illinois coal mine tipples or preparation plants.

Chapter III. Impurities and breakage of the coal, making preparation necessary.

Chapter IV. Sizing and sizes of Illinois coal.

This bulletin is not intended as a reference for all the machinery and appliances used in dry coal preparation in this state, but merely outlines the standard practice; therefore the inclusion and description of certain appliances do not imply preferences, but rather indicate that data were available concerning them, that they illustrate the principle of a process, or that they are in common use.

Data for this bulletin were gathered by an inspection of about fifty mine tipples in Illinois during the summer of 1914, followed by visits or letters to the offices of many of the producing coal companies in this field. Access was also had to the data of the Illinois Co-operative Coal Mining Investigation.

The author wishes to thank Prof. H. H. Stoek, in charge of the Department of Mining Engineering, of the University of Illinois, without whose cooperation the bulletin could not have been written. Various engineering firms, including the Allen and Garcia Company, The Link Belt Company, Roberts and Schaefer Company, and the Wisconsin Bridge and Iron Company, all of Chicago, aided freely by suggestions and by drawings. Acknowledgment is due the various mining companies visited for their interest and the uniformly courteous treatment received from their staffs.

CHAPTER I.

Evolution of Present Preparation Practice.

development of coal preparation.

During the year ended June 30, 1915,* the mines of Illinois produced 57,601,694 short tons of coal, valued at the mines at about $1.14 per ton or at a total value of $65,665,931.16. Of this total about 4,000,000 tons were treated by washing, while 53,600,000 tons were prepared to a greater or less degree in the dry state, principally by screening or sizing, and by picking. The run of mine coal produced amounted to more than 10,000,000 tons or 17.3 per cent of the total; lump coal amounted to 19,200,000 tons or 33.3 per cent; egg coal, 8,700,000 tons or 15.1 per cent; nut coal, 3,800,000 tons or 6.7 per cent; and pea, screenings, and slack coal, 15,900,000 tons or 27.6 per cent. The exact significance of these terms for designating sizes, is explained fully on page 103. Briefly, "run of mine" coal refers to coal shipped as it is mined, all sizes being mixed together. Lump coal refers to the largest sizes of coal, from which the fines have been taken by screening; thus, lump is made over screens ranging in size from a ¾-inch round hole up to 8-inch round hole. If the coarser sizes of screens, with 5-inch or 6-inch holes are used, the coal is passed over another set of screens having perhaps 3-inch round holes, the oversize produced being called egg coal. Nut coal refers usually to sizes made between 1-inch and 3-inch round hole screens. Pea coal is somewhat smaller, generally below 1 inch, and having only the finest sizes, say, under 5/16 inch round or square hole taken out. The smallest sizes of coal remaining after these processes are performed are usually called slack. The term screenings usually refers to sizes of coal passing 2-inch or 1¼-inch round hole screens from which no smaller sizes have been taken. The washing preparation in Illinois is confined entirely to coal under three inches in size.

The growth of the coal mining industry in Illinois, and the increasing importance of coal preparation are shown in Fig. 3, which is based on production figures compiled from the annual Illinois coal reports from 1882 to 1915. It should be remembered in considering these figures that during the earlier years a larger percentage than at present of the total tonnage came from longwall mines, which make less fine coal than the pillar and room mines.

In most bituminous coal fields the first seams mined were thick and contained the best quality of coal with a minimum of impurities. At first only lump coal was considered of value. Later it became necessary to mine relatively impure seams or thinner seams which, owing to the admixture of roof and floor impurities with the smaller amount of clean coal, might produce a coal which is relatively impure and which in either case must be cleaned and otherwise prepared for the market. Consumers also become more exacting as to size and

*Illinois Coal Report, 1915.

quality of the fuel and the general increasing value per ton permits more careful preparation of the impure coal and of the smaller sizes.

Germany, forced to use her lower grades of coal and to mine the thinner seams, led the way by introducing bituminous coal preparation

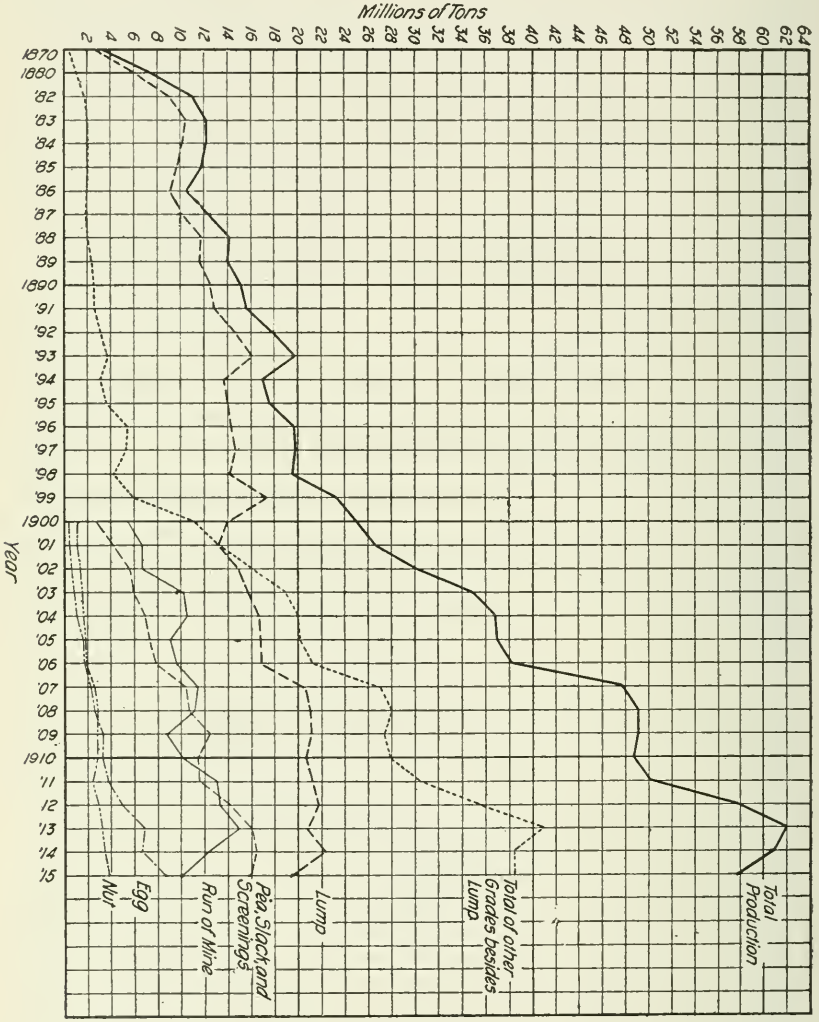

FIG. 3. YEARLY PRODUCTION OF DIFFERENT SIZES OF COAL IN ILLINOIS (1880-1915).

in the decade, 1870-1880. France and England followed as like pressure was felt, until by 1895 preparation by sizing and cleaning

was highly developed in these countries. One German colliery at the present time prepares twenty sizes and grades of bituminous coal. Belgium divides her small production into five degrees of quality and into twelve standard sizes. In general, European technique in bituminous coal preparation is more advanced than in this country.

In America the close sizing and extensive preparation of Pennsylvania anthracite have been notable for nearly half a century. Probably the first bituminous region in the United States in which close attention was given to preparation was the Fairmount region in West Virginia, in which sorting and cleaning had become general by 1900.

In Illinois close preparation has been developed although scarcely ½ per cent of the coal resources have been extracted. The attention given to careful preparation has been increasing since the early nineties, the progress during the past seven years having been especially marked. The causes for this development are:

(1) The introduction of state laws regulating the weighing of coal and the basis of payment to the miner for his coal.

(2) The various struggles and consequent agreements between operators and miners dealing chiefly with payment for mining and with the cleaning of coal.

(3) The demand of the consumers, who, having become educated by the publicity given during the past few years to the purchasing of coal on specifications, are no longer content with the grades of coal they received a decade ago.

(4) The campaigns waged by the cities to abate the smoke nuisance.

(5) The excessive competition among producers, caused not only by the operation of too many mines and the consequent desire to keep these in constant operation; but also by the maintenance of highly organized selling departments, which have a tendency to introduce new sizes and trade names for coal.

(6) The general introduction of improved machinery used in coal mine tipples to prevent breakage and facilitate preparation.

HISTORICAL DEVELOPMENT OF ILLINOIS PREPARATION PRACTICE.

Early History and Methods.—Before the general advent of railroads in the middle of the 19th century coal from Illinois mines was transported largely by boat on the Illinois and Mississippi rivers. Gordon Buchanan* states that the early railroads hauled coal into Chicago with engines that burned wood. A large proportion of the tonnage in these early days came from the northern or longwall districts in which the proportion of large clean lump or chunk coal is high. This coal requires little preparation. The earlier U. S. Census Reports, particularly those of 1870 and 1880, give statistical information concerning Illinois production, but, since little weighing was done at

*Black Diamond, May 17, 1913, p. 16.

the mines at that time, such information probably is not so accurate as that contained in later statistical records.

Little information concerning the preparation of Illinois coals prior to 1882 is available. The second biennial report of the Illinois Bureau of Labor Statistics, published in 1882, contains the first annual report on the coal industry of the state, although the first biennial report, published in 1880, contains the reports of coal mine inspectors to the governor of the state for the years 1879 and 1880. From 1882 until 1911 the Bureau issued each year an annual report or compilation of statistics on the coal mining industry of the state. After the first

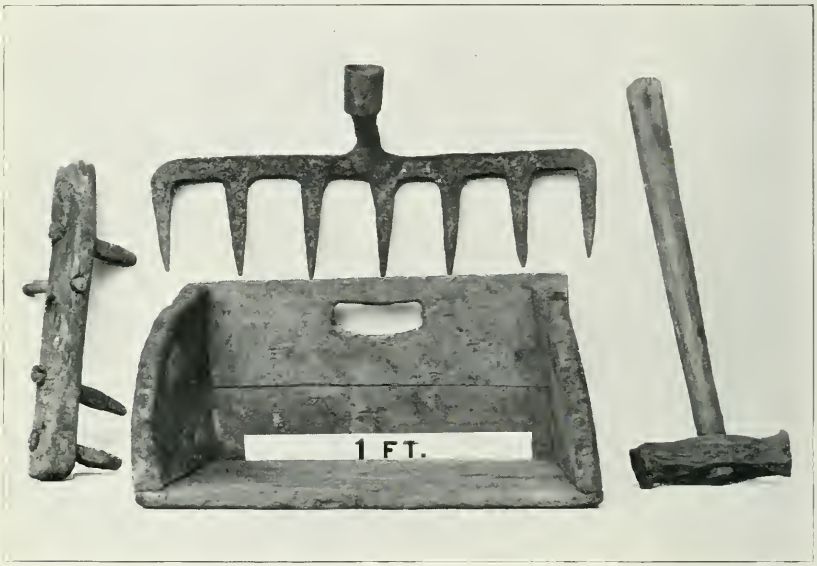

Fig. 4. The Miner's Rake (Wooden and Iron). Pan and Sledge. Museum, Department of Mining Engineering, University of Illinois.

two years it became the duty of the various mine inspectors to gather data, which was in turn compiled by the secretary of the Bureau. Since 1911 these annual coal reports have been issued by the State Mining Board.

The expression "coal is coal" might appropriately be used in speaking of the early periods of mining in Illinois, since the practice of selling coal as brought to the surface with no attempt at preparation before shipment was generally followed. A rough separation of the fines from the lump coal took place underground at the face and during the period of loading the coal into the mine cars. Instead of the customary miner's shovel of today the coal was loaded with a tined fork, having spaces between the tines of from ¾ inch to 1½

inches. This allowed the finer coal to pass through the fork, after which it was thrown back into the gob and left in the mine. While such a fork is still used at coke ovens to free the coarse coke from the "breeze" during the loading process and is also used by retail coal dealers for the purpose of delivering clean coarse coal when necessary, it has passed out of use in the preparation of Illinois coals, excepting in some of the small local and coöperative mines.

The common round hand screen, or coal miner's riddle, with square mesh wire cloth, having perforations varying from ½ inch to 2 inches in size, was also used by the loaders underground for screening out the smaller coal which was to be left in the mine and for carrying the coarser coal to the mine car.

The rake and pan method of loading coal was formerly in common use in Illinois mines.* After the largest lumps of coal had been loaded into the mine car by hand, the remainder was raked onto an iron or wooden plate or pan and transferred to the mine car, while the fine coal and dirt not gathered by the rake were left in the mine. "The pan was in fact a hand scoop made of sheet wrought iron or steel, the front end being flat and open, the sides being usually curved outward and upward with handles affixed for lifting and carrying; they were also made with straight sides. The back of the pan was straight up. The conditions under which the pan was used varied its dimensions. The purpose of the pan was to have only clean lump coal carried to the pit box or car; and whether this coal was loaded on the pan by hand or scraped on by a rake or fork, the end attained was the same. In some parts a penalty was attached to any person found loading coal with a shovel."†

Fig. 4, is a photograph of an old pan, part of a wooden rake, an iron rake, and a sledge recently found in abandoned workings in the Belleville district, and presented to the Mining Museum of the University of Illinois. These implements were practically out of use by 1884. They were, however, used in a few mines until the strike of 1897, after which their use was discontinued.

Another common method of preparation underground was by the use of "grills" or wooden bar screens. These were wooden bars, spaced ½ inch to 2 inches apart, nailed into a wooden framework, and set up at an angle of about 45 degrees. The coal, before being loaded was shoveled against these bars. The finer coal which passed through the bars was left in the mines and the oversize was rehandled and loaded. Similar practice may be seen with the common gravel screen of today.

The forms of underground preparation described made unnecessary the loading and hoisting of a product then considered useless; namely, coal equivalent in size to the screenings of today. Although a limited amount was utilized in the eighties, screenings were not even

*Report of Ill. Bureau of Labor Statistics, 1897, p. xxxv.
†W. L. Morgan, Ex-State Mine Inspector. Personal communication.

considered as a coal in the tonnage reports of the State Bureau of Labor Statistics until 1891, and in some districts it was not until 1900 that they became a commercial product. The use of screenings began to assume important proportions about 1895. The total estimated production of lump coal for the period from the beginning of the industry (about 1830) to 1895 is 273,000,000 tons. The first report including screenings in the tonnage (1891) showed that, considering the state as a whole, 17.24 per cent of the coal produced was screenings. If this average be accepted for the preceding years there was probably a total of more than 57,000,000 tons of this small coal, which was either mined and left underground to be buried beneath the falling mine roof and consumed in gob fires or, if separated on the surface, was hauled to dump piles and there destroyed by spontaneous combustion.

When the mines grew, when steam hoists and improved systems of underground haulage were generally introduced, and when labor cost underground and the value of fine coal or screenings increased, the coal was hoisted as mined and prepared in the mine surface buildings or tipple. The introduction and use of the automatic stoker with fine hole or chain grates and other special grates which made possible the generation of power from the finer sizes of coal were important factors in the increase in use and value of these small sizes of coal. Such devices were introduced into Illinois about 1890 and during the succeeding ten years their use in large power plants became general. Then the consumer with the average steam plant realized the possible saving through their use in connection with cheap coal screenings, and within a few years these stokers became common. Now, there is a constant demand for the once despised waste product, coal mine screenings.

Weighing Practice.—Formerly payment to the miner and operator alike was made almost wholly on the basis of the bushel, there being usually 25 to 28 bushels to the ton. This standard is still used at some of the country "banks" or local mines. In some places an arbitrary standard was based on the volume of the box or contents of the mine car. An early report* records prices paid to the miner based not only on the short ton of 2,000 lb., but also on tons of 2,050 lb., 2,100 lb., 2,200 lb., and 2,250 lb.

By 1880 some of the larger mines had adopted weight by scales as a basis of payment, such weights being taken by company men, while others still adhered to the volume basis. This naturally led to considerable trouble between operator and miner, which, together with the difficulty experienced by the state in gathering adequate statistics concerning tonnage, led to the passage, in 1883, of a law governing the weighing of coal at the mines. (Ill. L. 1883, p. 113.) The act required that all coal companies in the state, shipping coal by rail or by water, should provide standard track scales at the

*Report of Ill. Bureau of Labor Statistics, 1885, p. xxvii.

mines and should weigh all coal hoisted before or at the time of loading for shipment. The weights so determined formed the basis upon which the wages of the miners were computed. The miners could employ their own check weighman who should be an employee at the mine. The act seems to have referred to lump coal only, since screenings were not considered salable coal. The act also declared that all contracts for mining coal not based on the stipulated requirements should be null and void. This was the first attempt in Illinois to regulate preparation practice.

Although most of the shipping mines complied with the act, it was immediately attacked in the courts, and though held constitutional, it was declared to apply only to mines at which weight was accepted as a basis of payment (Reinecke v. People, 15A, 241). In the case of Jones v. People (110 Ill. Rep. p. 590) the law was declared to have no application if the wages of miners were computed on a basis other than that of weighing; namely, on the basis of volume; and it was not held to require miners' wages to be based on the weight of coal mined.

The law was amended (L. 1885, p. 221) by the addition of a provision requiring shipping mines to keep their weights on record for inspection by miners, inspectors, and other interested persons. The law as amended did not require the check weighman of the miners to be an employee at the mine in question, but stipulated that both the company's weighman and miners' check weighman must make affidavit faithfully to weigh and record the coal. In the test case of Millet v. People (Ill. Rep. 117, p. 294) the court decided that if an operator bought and sold coal by weight, the law compelled him to keep reliable scales for that purpose, but it did not oblige him to make contracts for coal on a basis of weight. Moreover, it was declared that the requirement that operators should keep a record of weights for public information was the taking of private property for public use without a provision of just compensation and therefore was unconstitutional.

A bill passed in the state (L. 1887, p. 235) repealed the former laws of 1883 and 1885 and substituted a new law of different wording but having about the same practical effect. It provided, "that at all mines, where miners are paid by weight, a standard scale shall be provided for the weighing of all coal hoisted or delivered." According to this law the check weighman should be an employee of the operators of the mine, and all coal delivered by the miner should be weighed and the records kept open for the inspection of the miner and other interested persons. This act was held unconstitutional (Harding v. People, Ill. Rep. 160, p. 459) because it singled out an especial class of mines, and interfered with the right of the operators and miners to contract among themselves.

A new law, designed to correct the weaknesses of the old one, was passed by the legislature in 1899 (L. 1899, p. 301), and revised in 1911 (Ill. Stat. Ann. Ch. 93, 7501). Its provisions are substantially

as follows: Operators at mines where miners are paid by weight of their output shall provide accurate scales for weighing such coal and the record so obtained shall be open to inspection by interested parties; a sworn weighman shall be provided by the company; and the miners may provide a sworn check weighman. This privilege is taken advantage of uniformly throughout the state, and there are today never less than two men in the weighroom of the mine tipple. As a result all contentions as to false weights, so prominent in the past, have disappeared.

At the present time, as a rule, at mines where the pit cars hold 1½ tons or more, the weight is read to the nearest 100 lb.; that is, the weighman and check weighman "give and take" on 50 lb. If the load weighs 4,135 lb., the miner is credited with 4,100 lb., and the company gains the extra 35 lb.; if the weight is 4,165 lb., the miner is credited with 4,200 lb. In mines with cars holding less than 3,000 lb., the weight is generally read to the nearest 50 lb., with "give and take" on 25 lb. The fact that at some mines three cars per minute are weighed, checked, and dumped from the tipple weigh box, makes it evident that considerable care and engineering skill have been devoted to bring tipple weighing to a degree which is nearly perfection. The beam scales formerly used have in many cases been replaced by the self-indicating dial scales, and in some of the newer tipples by automatic self-recording weighing devices. Thus, a just and equitable system of weighing, giving satisfaction to operator and miner alike, has been evolved.

Lump Coal vs. Run of Mine Payment.—During the same period in which the troubles over weighing occurred, a still sharper fight was being waged with reference to the quantity and kind of coal for which the miner should be paid. It had become customary at some of the larger mines to hoist all the coal mined, and to roughly clean and prepare it above ground before loading into railroad cars.

The common method of procedure then followed in Illinois, and still followed in Indiana, Michigan, Western Pennsylvania, and several other coal producing states, was to dump the coal from the mine car over a bar screen in the tipple. This bar screen consisted of a row of iron bars about ⅝ inch wide and 2 inches thick, set on edge with a space of about one inch between the individual bars, the whole rack being possibly 6 feet wide and from 8 to 15 feet long, and set at an angle of from 26 to 45 degrees. The oversize from the bar screen, or coal too coarse to pass between the bars, was weighed and sold as lump coal and the miner was paid, if by weight, according to the lump coal so produced. The finer coal passing through the screen bars, called slack or waste, was usually given away or hauled to a dump in the neighborhood. There was no market for this product since it could not be burned on the type of furnace grates then in use.

In the early eighties attempts were made to recover a part of these screenings. At a number of mines installations were made at the tipple by means of which the screenings were elevated and rescreened

in a revolving screen, having holes about ¾ inch in diameter. The material passing through this screen was called slack and was discarded. The oversize, varying from about ¾ inch to 1 inch more or less in diameter, was called nut coal. This coal was in considerable demand, especially as a domestic coal and was used by miners and by the public if the mines were located in the neighborhood of a city. The records show that by 1887 most of the large mines in the central part of the state were equipped to rescreen the small coal, and the value of this rescreened, or nut coal, as noted in several contracts made between miners and operators, was about one-third that of the lump. The proportion of nut coal so recovered to the lump varied with the districts, the method of mining, and the size of screen used, but was estimated as 13 per cent of the lump.* This treatment of coal, making lump as oversize on the bar screen, nut as oversize, and slack as undersize in the revolving screen, probably constituted the fullest dry preparation that coal received in 1887, excepting in one or two cases in which four sizes were made, two from the lump and two from the screenings, to meet the demand of domestic city trade. By 1891 several of the city mines were making three distinct sizes of screenings. The largest, called nut, was above 1 inch or ¾ inch in size; the medium size, called pea, below nut and above ½ inch or ¼ inch, and the slack below this size was thrown away.

T. B. Comstock,† writing in 1887 on coal mining in Illinois, said that Pennsylvania methods were followed blindly; that market rating of coals was based on crude trials in unskilled hands; and that one of the subjects just beginning to attract attention was the sizing of coal for market. He noted that assorted products from one or two mining plants in the state threatened to revolutionize the trade. At these plants small portable crushers and screens were placed at the car door to prepare the coal before loading, but probably the extra cost was not covered by the increased profit. Commenting on the fact that washing and other methods of preparation received little attention, chiefly because consumers did not recognize the enhanced values of prepared coal, he prophesied, "The time will come when these advantages will appear as necessities."

At a few mines in the state miners were paid for gross weight of coal hoisted, an allowance agreed upon being made for slack. For instance, at Oglesby, in the longwall field, where conditions favored a minimum of slack, 36,000 lb. gross of coal were required to obtain payment on 30,000 lb. of lump, an allowance of 16⅔ per cent being made for slack. At Mt. Olive, in the central field, 20 lb. of coal were deducted for slack from each 100 lb. mined, an allowance of 20 per cent. However, these were isolated instances, as four-fifths of the product of the state was screened, the oversize only being weighed and paid for. The thought that they did not share in the revenue from this

*Report of Ill. Bureau of Labor Statistics, 1888, p. 331.
†Eng. and Min. Jour., Vol. 44, p. 24.

merchantable nut coal was a source of considerable grievance among the miners, but it was overshadowed by a greater one; namely, the lack of uniformity in width of opening and area among the bar screens at the various mines.

In 1885 the secretary of the Illinois Miners' Protective Association issued a report, part of which dealt with the miners' grievances against the lack of uniformity of weighing, screening, and preparing coal for the market, and in 1886, the report of the Illinois Bureau of Labor Statistics (p. 549) dealt at length with the question of screens. The report, of which the following is a condensed table, covered screening practice at 218 mines in the state.

TABLE 1.

SCREENING PRACTICE IN ILLINOIS IN 1886.

Number of Mines	Tons Screened Lump Coal Produced	Space Between Main Screen Bars	Tons of Nut Produced by Second Screen (Estimated)	Tons Total Product Lump and Nut	Per Cent of Nut Coal (Not Paid for by Operators)
12	175,425	½"- ¾"	20,579	196,004	10.0
98	4,352,252	⅞"	516,631	4,868,883	10.6
23	437,074	1 "	62,972	500,046	12.6
10	354,305	1⅛"	60,356	414,661	14.6
44	1,198,739	1¼"	224,417	1,423,156	15.6
25	665,533	1½"-1⅞"	167,220	832,753	20.1
6	175,409	2"-2½"	49,705	225,114	22.1
Total 218	7,358,737		1,101,880	8,460,617	13.0

This table represents 80 per cent of the production at that time, based on lump coal. By far the most common width of bar space was ⅞ inch, 62 per cent of the product being so prepared; in fact, this width of bar space was recognized as a standard in several parts of the state. The percentage of coal passing such a bar screen is about equal in amount to that passing the 1¼-inch round hole screen common today, except that the shapes of the larger particles of coal are of course different. This common screen, however, was overshadowed by those with 2-inch and even 2½-inch spaces. Such a screen must have allowed a considerable proportion of the miner's coal to pass through, probably an amount equivalent to that passing a 3-inch or even a 4-inch round hole screen. The average area of screen used in Illinois was about 60 sq. ft., usually 5 ft. wide and 12 ft. long, but certain screens were at least 16 ft. long and some had an area as great as 130 sq. ft.

This condition existed generally throughout the country. At 146 mines,* taken at random from bituminous mines in the United States, the screens used were mostly of ⅞-inch or 1¼-inch bar, though

*Mineral Industry, Vol. 1, 1892, p. 80.

some had spaces as large as 2½ inches. Forty-four of these mines produced and paid on a run of mine basis, while 102 were operated on the screened lump payment basis.

The cleanness of separation of lump and screenings was further affected by the slope of the screens. If the screens were set at a low angle the coal passed slowly over them,—in some cases having to be pulled over, and thus a maximum of coal was sent into the nut or through sizes. One or two strikes were caused because chains or logs were hung loosely close above the bar screens. According to the operators, the lump coal sliding down the bar screen with considerable force was turned over and freed from adhering dust; according to the miners, the lump coal was broken by such devices, and passed through the screen into the nut sizes for which they received no pay.

Such lack of uniformity in the preparation of coal led the miners to demand that screens be made of uniform size and opening, and that payment be made for nut coal produced. They claimed that the average realization to the operator for nut coal was five cents per ton of lump made. Their grievances on these subjects were: (a) The practice of changing screen openings from time to time. (b) Screens of largest size were found where they were least justified by market conditions. (c) The operator using a screen of only sufficient size to clean his coal could not compete with one using a screen which passed enough nut to pay his expenses. (d) The percentage of coal going through the screens was larger than that necessary to clean it. (e) The operators encouraged the use of an excessive amount of powder in order that they might receive the benefit of the large percentage of small sizes made by it, and an extra profit on the excess powder. (f) The operators tried to force the loading of the fine coal for which the miners received no pay.

The operators claimed: (a) Coal had to be screened to get a merchantable grade of lump. (b) Fair miners' wages were based on the percentage of lump and nut sold, even though the miners were nominally paid for lump only. (c) Since different degrees of friability of coals, and different methods of mining, tended to produce a variation in the percentage of finer sizes, a variation in the size of screens was necessary to make a uniform coal for a common market. For instance, coal shot off the solid as in the central part of the state made more fines than that mined longwall and wedged down as in the northern fields. (d) It was necessary to load out the fine coal to prevent gob fires.

Three remedies were proposed: (a) To use throughout the state a uniform bar screen of not more than ⅞-inch space. (b) To pay miners pro rata per ton for whatever proportion of the product was sold. (c) To weigh and credit to the miners all coal before screening.

Although the first of the above met with considerable favor, in 1883 a bill requiring the use at all mines of a screen of uniform

dimensions and width between bars failed to pass the State Legislature. In the light of the history of coal preparation revealed during the next few years, it is a question if the passage of this bill might not have prevented the difficulty caused by the increased percentages of fine coal and refuse.

Legislation was then proposed to weigh and credit to the miner, before screening, all coal hoisted. The operators protested against payment on the "mine run" basis, their arguments being: (1) Installation of new scales and tipples would be necessary. (2) Domestic coal must be lump to command a good price. (3) An unskillful or careless miner might make double the amount of slack made by a careful one. (4) A dishonest miner could load dirt and rock in the bottom of his car. (5) The excess of powder used would shatter the coal. (6) The proposal amounted to offering a premium for dishonest work. (7) They would suffer on account of time wasted by weighing in the tipple. (8) If a carpenter is paid for his day's finished work, and not for the chips he makes, why should a miner be paid for the slack he makes?

To this report a reply was issued, signed by a committee of the United Mine Workers of Illinois, as follows:

(1) The miners' organizations during the last few years had forced some reduction in the size of screens used. (2) Where no miners' organization existed, screens were large enough to let through good sized coal. (3) Some of the mines had installed breakers to crush lump coal, proving that fine coal was not bad. (4) There was no law in Illinois limiting the size of coal screens, which varied from ⅞ inch to 2½ inches between bars and had an area of from 50 to 130 sq. ft. (5) Miners were forced to sign contracts stating that screens might be widened without violating such contracts between operator and miner. (6) Wages were based on the price of lump coal, but the operator derived an enormous revenue from screenings which was clear profit to the owner. (7) It was shown that at one mine on the Illinois Central Railroad, of 1,095 tons hoisted, 509 tons or 46 per cent, passed through the screens. (8) The allegation that practical and trained miners would attempt to produce slack was denied.

The same question of run of mine vs. lump coal payment was fought out in Ohio in 1913. A coal mining commission was appointed by the governor to inquire into the merits of the matter. In its report the commission took up exactly the same arguments for and against as those advanced in Illinois. Twenty-five years have thrown very little new light on the subject, the only additional arguments presented by the miners in Ohio being along the line of conservation. They claimed that under the lump system valuable coal was left in the ground, that pillars could not be economically robbed because of the undue crushing of the coal in them, and that they were unable to make fair wages under these conditions. The only new argument

of the operators was that a trial of many years had disclosed the detrimental effects of the mine run law in Illinois.

Whichever side had the better of the argument in Illinois, on June 10, 1891, the Illinois Legislature (L. 1891, p. 170), passed a gross weight law, making it unlawful for an operator, whose men are paid on the basis of quantity of coal, to take any portion of the same by any process of screening, or by any other device, without fully crediting the same to the miner. The law required the weighing of all coal in the pit cars before being dumped, and gave the miners freedom to choose their own checkweighman. Nearly all the mines which paid their miners by weight complied with the law, which, "as a whole, has not seriously inconvenienced the business in spite of the claims of the operator that weighing before screening was impracticable without serious reconstruction of the surface plant."*

Generally in the American bituminous coal fields, as long as the coal has been paid for on a screened coal basis, little attempt at detailed preparation has been made. When paying for run of mine coal, however, the operators are forced, through a series of new conditions, such as necessity of disposing of the finer sizes, or by an increase of refuse in the coal, to make greater refinement in its preparation. Moreover, when the operator pays the miner for all his coal regardless of size, he may more easily install whatever system or make whatever sizes best meet his market conditions, since he has no agreement with the miners concerning screens.

Operators in Illinois were now free to screen and prepare the coal as they wished. Consequently, new devices for preparation were introduced among which was the shaker screen (see p. 44), which replaced the bar screen in a number of plants. The first screen of this kind in the state was probably installed in 1890 at the Gillespie Colliery of the Consolidated Coal Co. Since that time the old bar screens have been steadily replaced by shaker screens. As late, however, as 1893, a shaker coal screen exhibited at the World's Columbian Exposition attracted considerable attention. At a number of plants the bar screen was replaced by a revolving screen. It was found, however, that lump or larger sizes of coal were badly shattered during passage through this type of screen, and their use for lump coal was gradually abandoned, until at the present time the writer does not know of a single plant in the state which uses such a screen for the separation of the coarser sizes. For screening the finer sizes of coal, under 3 inches, which are not so much affected by breakage from fall, the revolving screen held its own until a very recent period.

In 1892 the screening or gross weight law of 1891 was attacked in the courts (Ramsey v. People, Ill. Rep. 142, p. 380) and declared unconstitutional, since "it required, regardless of contract, payment on the weight of coal before it was screened, and it thus so far

*Report of Ill. Bureau of Labor Statistics, 1891, p. 47.

limits the power of employers and employees to contract, as to deprive them of property and rights without due process of law." In spite of this decision, many mining plants which had been equipped for the gross payment basis continued to use it in preference to the old method, partly because consumers had begun to demand a variety of sizes, which could be produced more easily with the new equipment. In some cases as a result of a decided increase in the percentage of screenings and contained dirt, a reversion was made to the older method. As contracts between operators and miners were generally of a local or district nature, such changes were quite easily made. On the whole until 1897 the pendulum swung towards the abandonment of the gross weight system.

In 1895 new complaints were made against the enlargement of screen apertures, and it was claimed that the weighing of the coal was inaccurate. The annual reports of the State Board of Arbitration for 1895, '96, '97, and '98 cite numerous instances of coal mine strikes, the principal causes being those just mentioned and the renewed contention by the miners for payment by the gross weight system. The case was aggravated by the general industrial depression during these years and by a decreased demand for Illinois coal, due in part to natural gas displacing coal as a house fuel in the Chicago market,* to smoke ordinances passed in the cities,† and to the increased use of Ohio and West Virginia coal in Chicago and of Missouri coal in St. Louis.‡

On June 3, 1897, the Illinois Legislature passed a new gross weight bill, designed and worded to overcome the legal objections to the law of 1891 and to accomplish all that was intended by the previous law. The law (L. 1897, p. 270) provided "that every person engaged in mining coal . . . . shall be paid in lawful money . . . . for all coal mined and loaded into the mine car by him, including lump, egg, nut, pea, and slack, or such other grades as said coal may be divided into, at such price agreed upon by the respective parties."

As in previous cases the act was attacked in the courts (Whitebreast Fuel Co. v. People, Ill. Rep. 175, p. 51). Since it was held constitutional and is the basis for present laws, not only in this state but elsewhere, it is interesting to note that, "the application of the act does not extend to cases where there is a contract for compensation upon a different basis than that specified in the act, and the employer and miner are free to contract at such price as may be agreed upon by the respective parties. It does not require that the coal shall be weighed, or that the same price shall be paid for different grades."

The efforts of the miners to obtain their demands and to bring their contracts under the provisions of the act helped to bring on the great strike of 1897, which began in Illinois July 4, 1897, and

*Min. Ind., Vol. 1, 1892, p. 80.
†Min. Ind., Vol. 4, 1895, p. 164.
‡Min. Ind., Vol. 7, 1898, p. 185.

lasted about 100 days. The strike resulted in an almost complete victory for the miners, district after district acceding to their demands for higher wages and payment on the gross weight basis when the same was not already in force. The greatest change perhaps was made in the longwall field. On November 22, 1897, a joint conference of miners and operators of this field was held in Joliet, and the gross weight scale was adopted for the entire field. The system of paying for all coal mined, provided for in terms of settlement, was considered important enough to justify the miners of the northern field accepting a rate for mining a few cents less per ton than agreed upon at the previous State Convention.

The report of the Illinois Bureau of Labor Statistics for 1897 contains a full history of the strike. From the viewpoint of preparation, before the strike at 156 of 284 mines in the state, miners were paid on the gross ton basis and at 128 on the screened ton basis. After the strike 198 mines paid the miners on the gross ton basis and only 86 paid on the screened ton basis. These totals included 86.5 per cent of the total coal mine employees in the state, and probably about the same percentage of the total tonnage. The large mines were now all in the gross tonnage column. Little change resulted in the southern fields, which previously had been gradually changing to a mine run basis, based either on weight or on volume.

Operators' and Miners' Agreements.—The miners of the state had now become firmly organized as District No. 12 of the United Mine Workers of America, and following the national convention of that order at Chicago, January, 1898, District No. 12 met the Illinois coal operators associations at Springfield, February 24-26, 1898. The wage scales adopted at this joint conference by the different state districts were "understood in every case to mean that coal is to be weighed before screening and the system of paying miners by the ton of R. O. M. (run of mine) coal shall obtain in all the mines in the state of Illinois.* Thus the present system was adopted in Illinois, not so much through the strictness of the gross weight law, as by agreement between the operator and the organized miner.

The next year (1899) the first complete state agreement regarding mining prices and general conditions was adopted at a meeting between operators and miners held in Peoria. Concerning cleaning of the coal, no reference was made in the agreement regarding the mining of impurities or the increase in amount of screenings. Testimony given at this meeting, however, showed that at Streator the percentage of screenings increased 15 per cent and at La Salle from 2 to 5 percent through the same screens, as the result of a year's trial under the new gross weight system. There was, for the last time, some talk of accepting a double standard; namely, one price for lump and another for screenings, but no action was taken.†

*Minutes of Springfield Conference.

†Proceedings Joint Convention, Dist. 12, U. M. W. of A. and Operators, Peoria, 1899.

This first year under the new system has been called "an era of good feeling in the coal mining history of the state." The old grievances and troubles had disappeared and no new ones had arisen as yet. By the time, however, that the 1900-1901 agreement was made the modern refuse question had become so acute that the following clause was inserted: "Where slate, bone, etc., are sent up by the miners, it is the duty of the car trimmer or inspector, who shall be a member of the miners' union* and appointed by the operator, to call the attention of the weighman and check weighman to the same. If they agree, the offender shall be fined fifty cents for the first offence; one dollar for the second; and two dollars for the third or subsequent offence, or discharged by the company; providing that no miner shall be discharged unless he is guilty four times in one month."

This clause was evidently not sufficient to enforce the loading of clean coal since the Peoria agreement for the year ending March 31, 1903, contained more elaborate rules regarding preparation. These rules, except for minor changes, are in force today. The pertinent sections, as given in the 1915-16 ageement, are as follows:

Section 4. "The scale . . . shall be per ton of 2,000 lb. R. O. M. (run of mine) coal, practically free from slate, bone, and other impurities."

Section 5. (b) "The system of paying for coal before screening was intended to obviate the many contentions incident to the use of screens and not to encourage unworkmanlike methods of mining and blasting coal, or to decrease the proportion of screened lump, and the operators are hereby guaranteed the hearty support and cooperation of the United Mine Workers of America in disciplining any miner who from ignorance or carelessness or other cause fails to properly . . . load his coal."

Section 6. (a) "In case slate, bone, clay, sulphur, or other impurities are sent up with the coal by the miner, it shall be the duty of whomever the company shall designate as inspector to report the same, with the estimated weight thereof, and the miner or miners so offending shall have such weight deducted from the established weight of the car and for the first offence in any given calendar month shall be fined fifty cents; for the second offence in the same month he or they shall, at the option of the operator, be fined two dollars and for the third or any subsequent offence in the same calendar month, he or they may be fined two dollars or be suspended for not to exceed six days of mine operation.

(b) "For a malicious or an aggravated case, ... the operator may either indefinitely suspend or discharge."

The term "malicious or aggravated case" is then defined "as a case in which the quantity, character, or appearance of the impurities

*The 1904 and subsequent agreements provide that the company coal inspector "shall not be a member of the union."

indicate that they were loaded with intentional carelessness or wrong purpose. In case of discharge for an aggravated case as above, the inspector shall preserve the impurities for 72 hours, Sundays and legal holidays excepted. All impurities subject to being docked shall be preserved for the balance of the working day except at mines where it is impossible to do so without seriously impeding the mine. Where it is claimed by the operator that to so preserve the impurities will serously impede the output of the mine and where it is claimed by the miner that the case is not a malicious or aggravated one, the question shall be taken up jointly for determination."

(c) "The company weighman shall post in a conspicuous place at the pit head the names of all miners dealt with hereunder."

(d) "The inspector designated by the operator . . . . shall not be a member of the U. M. W. of A., and in the discharge of his duties shall not be subject to the jurisdiction of the union. Any miner . . . . seeking to embarrass the inspector . . . . shall, at the option of the operator, be suspended two days."

(f) "The proceeds from all fines shall be paid to the miners' subdistrict treasurer, and . . . . shall not be remitted."

"The foregoing is designed to secure to the operator the loading of clean coal, while protecting the miner from any abuse of the penal code."

It is evident from the various clauses noted that every possible means is being taken to compel the loading of coal free from impurities. The present demand of the consumer for clean coal has made it necessary that care be taken to load, at least as regards lump coal, only pieces that will pass inspection by the company coal inspector, or as he is often called, the "dock boss" or "rock man."

Regulations concerning fine coal and screenings have not been so thoroughly worked out. At present the only provisions in the operators-miners agreements regarding such coal are:

Section 5. (c) "That all 'bug dust'* or machine coal cuttings when practically free from impurities shall be loaded out with the snubbings† or other coal so as to produce a merchantable R. O. M. coal."

"The above does not contemplate any change in the present method of handling bug dust or machine cuttings in Franklin county, or other mines when it is necessary to load the same out before shooting the coal, as a protection against explosions or fire."

"Where the operator desires the bug dust loaded out separately, this shall be done by the miner working in his place during his regular shift at the regular tonnage price and the company shall furnish cars promptly to load the same."

Much of the roof, "draw slate," floor, and bands which comprise the bulk of the impurities mined with the coal either break into

*See p. 101.
†See p. 106.

small pieces or tend to soften quickly on exposure to the air. Any piece of such impurity smaller than 1¼ inches in diameter passes into the screenings during the screening process of preparation. Since the impurities are small and pass quickly from sight through the screens, and since they mix with the dust and multitude of small

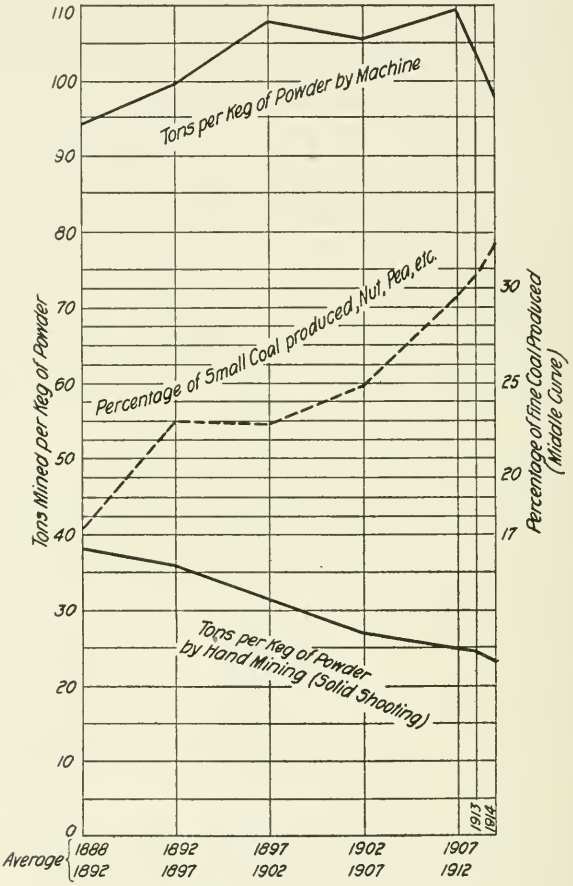

Fig. 5. Tons of Coal Mined Per Keg of Powder (1888-1914).

pieces of coal which make up the screenings, they so blend with the prevailing black that close inspection is impossible. It is sometimes easier, while loading, to break a piece of flat shale with the back of a shovel so that the pieces will enter the screenings, than it is to pick this same piece out of the coal and throw it into the gob.

From a large number of analyses the Department of Applied Chemistry at the University of Illinois determined that, on the average,

Illinois screenings contained about twice as much ash as the lump coal from the same mine. The writer has observed a like difference, varying considerably as to daily conditions at the individual mine. Samples of screenings taken on different days from the same mine often vary several per cent in ash content. The mere fact that a coal is of screenings size does not necessarily impair it for use in modern furnaces, but the value of screenings generally is lowered and sale retarded by a high and uncertain refuse content. Impurities in screenings, even though concealed to the eye, appear to their full value upon chemical analysis or in the furnace ashes. Thus, the increasing tendency of consumers to insist upon ash analysis and specifications is based on reason and is certain to continue.

The coal from some of the mines in Illinois is held in disfavor in the general market, and a differential price is made against it. Some of this coal is in every way equal in quality to that from more favored mines. Investigation reveals that carelessness in loading impurities and negligence in enforcing inspection are the main reasons for this condition.

Another clause of Section 5 in the recent joint agreements is of interest, since it relates to the effect of powder used on the sizes of coal produced. It is given as follows: "Where practicable, miners shall shoot coal with two pounds of powder or less."

In general, little or no powder is used in longwall mines, the coal being loosened by roof pressure or by wedges; in mines in which the coal is undercut only a moderate amount of powder per ton of coal is used; while in mines in which the coal is "shot off the solid," without previous preparation of the face, a much greater amount of powder is used since there is only one free face to break to.

Before the general introduction of the run of mine payment system in 1907, it was evidently to the miner's interest to shatter and break his coal as little as possible. This could only be done with a minimum of powder. The accompanying chart (Fig. 5), prepared from data in the annual coal reports, shows the number of tons of coal mined per keg of powder for each year since 1888, for both machine and hand mines (solid shooting). Since longwall mines come under the general heading of hand mines, and since they use little or no powder, they have been omitted from the compilations. The charts are based on five year averages in order to avoid accidental fluctuations.

The percentage of fine coal produced by the mines for the same period is also plotted in Fig. 5. Since the year 1900, the coal production has been reported in tons of the different sizes produced; namely, run of mine, lump, egg, pea, and slack. For general purposes of comparison the percentages since 1900 were obtained by considering run of mine as lump coal, and considering nut, pea, screenings, and slack sizes as fines. Since today these are mostly made through round hole screens, while before 1897 they were mostly made through bar screens, the first part of the curve up to 1902 shows an uncertainty

due perhaps to the change from bar to round hole during these years. There has been a regular increase in the percentage of the fine sizes produced since 1902. Thus, from 17.3 per cent in 1892 to 22.9 per cent in the period from 1897 to 1902 and to 32.5 per cent in the year 1914, is a serious increase in fine coal in the state as a whole. The rise of this curve, coincident with the increased use of powder in solid shooting mines, recorded in the lower curve, is more than a coincidence, and shows plainly the close relationship between them.

It has been stated that as the amount of powder per hole increased so increased the percentage of fatal accidents.* This fact, more than the evil of increased coal breakage, led to the passage in 1903 of a law regulating the use of powder in coal mines (L. 1903, p. 252), which limited to about 5 lb. the amount of powder used in a single charge in seams over 5½ feet thick, and to about 4 lb. if the seam were under this thickness. The sections covering the above were repealed in 1913, and substantially inserted as Section 19, Act of 1913. The coal report of 1904 (p. 3) commenting on the law of the previous year states that disregard of its provisions had been general. It did not serve to stop the increase in the percentage of fine coal.

More legislative remedies were proposed: (1) All coal should be undercut. (2) Special shot firers should be employed by the company. Had the first measure passed and been enforced, probably the percentage of coarse coal would have increased. The second measure finally became a law July 1, 1905. It provided, "that where more than 2 lb. of powder are used in one blast . . . a sufficient number of practical miners to be designated as shot firers shall be employed by the company to inspect and fire all blasts in the mine."

Evidently the miners, now freed from firing their own shots, loaded heavier than ever, taking the chance that their holes would be fired without inspection. For this reason, apparently, the law was amended in 1907 (Sections 47, 47a, and 47b) forbidding a miner to alter a hole after inspection by the shot firer and forbidding the shot firer to fire an unlawful hole.

A. Bement† comments on the increased use of powder during the last few years, and in Bulletin No. 16 of the Illinois State Geological Survey he states that its increase per ton of coal produced was over 100 per cent between 1897 and 1908.

Laws and collective agreements in Illinois as regards percentage of fine coal and the contained amount of impurities have not proved wholly successful. The writer believes further action should be taken, especially regarding the impurities and excessive dust in screenings. The close competitive market prevailing today on all sizes of coal, and especially on screenings, makes it appear imperative that in the near future this coal be placed on the market in a cleaner condition without preparation.

*"Powder Accidents in the Coal Mines of Ill." Issued by The Illinois Coal Operators' Association, 1909.

†Jour. West. Soc. Eng., June, 1909, p. 307.

The effort in Ohio, one of the mining states competing with Illinois, to avoid the difficulties in changing to the run of mine payment basis, if successful, may offer a remedy for many of the difficulties which have hindered effectiveness with the run of mine payment practice.

According to the new mining laws of Ohio, passed in 1914, payment to the miner is changed from the screened coal to the run of mine basis; but, in case the miner and his employer cannot agree, the State Industrial Commission has the power to fix, for any mine, maximum percentages of fine coal and impurities allowable in the miner's coal. Loading of excess impurity is made a misdemeanor and punishable by fine on conviction. Thus, Ohio has attempted to solve by law a matter which in other states has been regulated by collective bargaining between operator and miner.

A suggestion has been made to introduce a bonus system of payment to the miner for his lump coal, as a means of overcoming the evils of the present run of mine system in connection with breakage and impurities. Briefly stated, it is proposed to continue the present practice of paying an agreed price for all coal as mined and hoisted, but since clean lump coal is of extra value to the operator, it is suggested that he pay the miner for any excess of such product over a fixed percentage to be agreed upon for each district. This would not conflict with present state laws and would compensate both the operator and the careful and skillful miner. The possibility of such a method proving impracticable on account of difficulty in determining rapidly in the tipple the respective amounts of lump and slack in each miner's car of coal should be given consideration.

RECENT DEVELOPMENTS IN PREPARATION PRACTICE.

Fig. 3 shows that sizing and close preparation had become important enough by 1900 to justify the separation of the total production into the separate sizes produced. The public was beginning to demand sized coal. Fresh impetus to this demand was a result of the anthracite strike of 1902, when the public, deprived of closely sized anthracite, tried sized Illinois bituminous coal for the first time as a substitute. In the period from 1902-1907 many of the small companies were consolidated, and larger corporations entered the field and erected new and larger modern steel surface plants.

Beginning about 1906, considerable prominence was given to the abatement of the soft coal smoke nuisance in cities, and the burning of more closely sized coals proved to be of some advantage in overcoming this trouble. In 1908 considerable public attention was given to "The Purchase of Coal by the Government under Specifications," by Geo. S. Pope, Bulletin 428, U. S. Geological Survey, and reprinted as Bulletin 11 of the U. S. Bureau of Mines. The author (p. 5) states, "Until recent years coal consumers purchased coal merely on the statement of the dealer as to its quality,

relying on his integrity and on the reputation of the mine or district from which the coal was obtained. It is surprising that the important question of whether value was being received for the money expended was not sooner seriously considered." In speaking further (p. 8) on the size of coal as influencing combustion, and without qualifications as to kind of coal or to conditions of burning, he states that, "coal of uniform size forms the most satisfactory fuel." Attention is also called in the bulletin to the limiting values of ash and refuse allowed and to the general advantages of some specification system. Frequent articles in the general press also emphasized the advantages to the consumer of prepared coal.

The period from 1907 to 1915, especially the last five years, is notable for the remodeling of coal tipples to handle larger outputs and to meet the growing demand for cleaner and more evenly sized coal. This has necessitated improvements in mechanical details, such as weighing devices, sizing screens, picking tables, and loading booms, and has caused the introduction of special engineering features in improved rescreening plants and special dry processes for cleaning the coal. A surface plant in Illinois must be capable of handling more than 4,000 tons per eight-hour day in order to be considered one of large capacity. At a mine in Macoupin county recently a record has been made of 5,116 tons hoisted and prepared in an eight-hour day, or an average of 640 tons per hour. Upon comparing this with a record of 1,655 tons hoisted in 9½ hours, or about 174 tons per hour, made at Braidwood, Illinois in 1888, and mentioned in Colliery Engineer* of that year as being remarkable, it is evident that remarkable changes have taken place not only in underground methods, but in weighing, handling, and general preparation at the surface.

The large increase in capacity of individual mines has at times tended towards overproduction with a consequent decreased profit per ton and increased competition. The average shipping mine in Illinois is operated 172 days per year. In other words, without increasing present mine plant capacity, if the mines were worked 300 days per year, Illinois could supply over 100,000,000 tons of coal per year. This factor has led to complication regarding preparation, some plants having introduced devices on account of competition rather than through real need of such treatment for the coal.

It has been stated that Illinois has been obliged to improve preparation, in order to keep ahead of her competitors in the neighboring coal fields. The interstate commerce in Illinois coals has extended to the west, perhaps as far as Omaha, to the southwest into Texas, and to the northwest into Minnesota and the Dakotas until competition with the high grade West Virginia and other eastern coals carried over the Great Lakes has become too severe. Thus, at various places Illinois coals may come into competition with West Virginia, Ohio,

*Vol. VIII, 1888, p. 222.

Kentucky and Indiana coals from the east, with the coals of Oklahoma and Arkansas in the southwest, and with the generally lower grade coals of Missouri, Kansas, and Iowa in the west. Further west, the eastern shipment of the bituminous coals of the Rocky mountains is the determining factor in the situation.

That Illinois has kept abreast and ahead of her competitors in preparation is apparent by noting the number of new screening and preparation plants erected in these competing states within the last two or three years. All things considered, there is probably no bituminous coal district in America having refinement in preparation equal to the general Illinois field, and especially equal to that at the tipples producing coal for domestic and retail use, in which an even size and freedom of impurities visible to the eye are prerequisites.

CHAPTER II.

Standard Types of Illinois Coal Mine Tipples.

INTRODUCTION.

Tipple Units.—At Illinois mines today the coal is hoisted to the surface for screening and final cleaning, although a rough preliminary hand picking often takes place at the face underground. This necessitates for preparation a surface structure composed of one or more of the following units:

(1) A headframe or tower over the mine shaft, supporting and taking the reaction from the pulleys or sheaves over which the hoisting rope runs, and which generally must further resist the shock of the mine car being dumped.

(2) A screen structure, sometimes called, if of the common type, a shaker house or shaker, containing the particular type of screen used to separate the mine run coal into various market sizes, and also some weighing device for the coal.

(3) Picking belts, for use in cleaning the screened coal, and loading chutes, etc., and for loading it gently into railroad cars.

(4) At certain mines the mine cars are run off the mine cage before being dumped, necessitating an additional structure in which the mine cars are handled and recaged, and in which disposition is made of the waste rock brought to the surface.

In Illinois these units are grouped into one general structure at the mouth of the shaft which is called the tipple. To avoid vibration and promote stability the screen frame in the tipple is generally built with independent supports and foundation. A frequent auxiliary and separate structure is the rescreening plant or rescreener, into which the smaller coal is re-elevated, and there separated into various small sizes. In the design of these units every precaution should be taken to prevent breakage of the coal, to obtain capacity, to do clean screening, and to avoid vibration, all with a minimum of labor. These are the five requisites of good tipples.

Materials of Construction.—Until about 1900, mine tipples in Illinois were built of wood, excepting the screens, chutes, etc. Since then the use of steel in the construction of tipples has become more and more common. In June, 1911, the state legislature passed an act requiring that all structures thereafter erected on the surface within 100 feet of the mouth of any shaft, slope, or drift should be built of metal, rock, clay, cement, clay or cement products, or a combination of the same. Tipples erected since that time have been made of steel, and although concrete and other fireproof materials have been used in other parts of the country for mine tipples, they have not as yet found favor in this state.

Many of the older wooden tipples have given remarkable service, especially those well built and cared for, and in many districts are still the common form. A number of the earlier steel tipples were constructed of a comparatively large number of small steel members,

and following the custom prevalent at the wooden tipples, the sides, especially over the shaft, were tightly enclosed, usually with corrugated iron. Since the hoisting shaft is generally the upcast air shaft, the corrosive action of this moist mine air on the thin steel framing of the tipple was rapid, especially on those members closest to the mouth of the shaft. In the later steel tipples trouble from this source has been eliminated by any or a combination of the following simple precautions:

(1) Leaving the sides of the headframe open.

(2) Spreading the legs of the headframe; thus removing them from the shaft collar.

FIG. 6. CLASS I TIPPLE.

(3) Using a few large steel members for headframe construction, instead of the more quickly corroded small sections.

(4) Encasing in concrete the legs or posts close to the shaft.

(5) More frequent scraping and painting.

It is to be regretted that so many correctly designed steel headframes in this state have been allowed to deteriorate solely through lack of attention on the part of the operator. No design can withstand for many years the severe conditions imposed without being properly protected by an occasional coat of paint. If this simple

expedient is carried out, there is no reason why the steel tipple should not last almost indefinitely.

Types of Tipples.—Mine tipples differ according to (a) their structural material, (b) method of receiving coal (whether from a shaft, slope, or drift), (c) position of hoisting engines in reference to the axis of the structure, (d) type of mine car dump used, (e) method of handling mine cars of coal and waste, (f) whether or not the coal is rescreened, and (g) the type of screen used to prepare the coal.

In Illinois the structural material is uniformly wood or steel, and as shaft mines are most common, mine tipples may be grouped into three general classes or types as follows:

(1) Those equipped with self-dumping cages and with shaker screens for sizing the coal.

(2) Those equipped with self-dumping cages and with gravity bar screens for sizing the coal.

(3) Those equipped with rigid or fixed cages, with cross over dumps in the tipple to empty the mine cars, and with shaker screens for sizing the coal.

To illustrate the general practice in coal preparation, one tipple representing each of the three types just mentioned, has been chosen at random from a number of similar ones in the state. Although constructed of steel they are by no means the largest or most recent installations in each class, but each represents many features found at many of the tipples in the state. The designs illustrated are for steel construction, but they are also representative of wooden construction since there are no especial differences in principle, operation, or appliances. Any of these types may have an auxiliary rescreening plant.

Although the width or long axis of a tipple is usually perpendicular to the long axis of the shaft, in several cases, in order to accommodate a more favorable position of railroad tracks, the screen structure is built on an angle to the long axis of the shaft, and in several instances parallel to it.

Hoisting engines may be placed so that the hoisting ropes will run over the sheaves either in the same plane as the long axis of the shaft or perpendicular to it. In the first case the headframe is said to be "end pull" and the pulleys or headsheaves are tandem and in approximately the same vertical plane. Headframes in the second case are "side pull" and the pulleys are parallel. Neither type is more popular in Illinois. Both types are often used in the same district, the choice depending on the designer, the operator, the permanent position of some part of the old plant necessary to combine with the new structure, the ease of conveying coal to the boiler room, the topography, the position of railroad, etc. Accidently the three plants chosen to illustrate typical surface plants in this chapter are of the end pull type. The frontispiece (Fig. 2) shows a side pull headframe.

CLASS I. TIPPLES EQUIPPED WITH SELF-DUMPING CAGES AND WITH SHAKER SCREENS FOR SIZING THE COAL.

The type of tipple commonly used in Illinois at room and pillar mines and in which the bulk of Illinois coal is prepared for the general market belongs to Class I and is shown in the photograph (Fig. 6) and in sectional elevation (Fig. 7). The operation is best explained by following the course of the coal from mine car to railroad car.

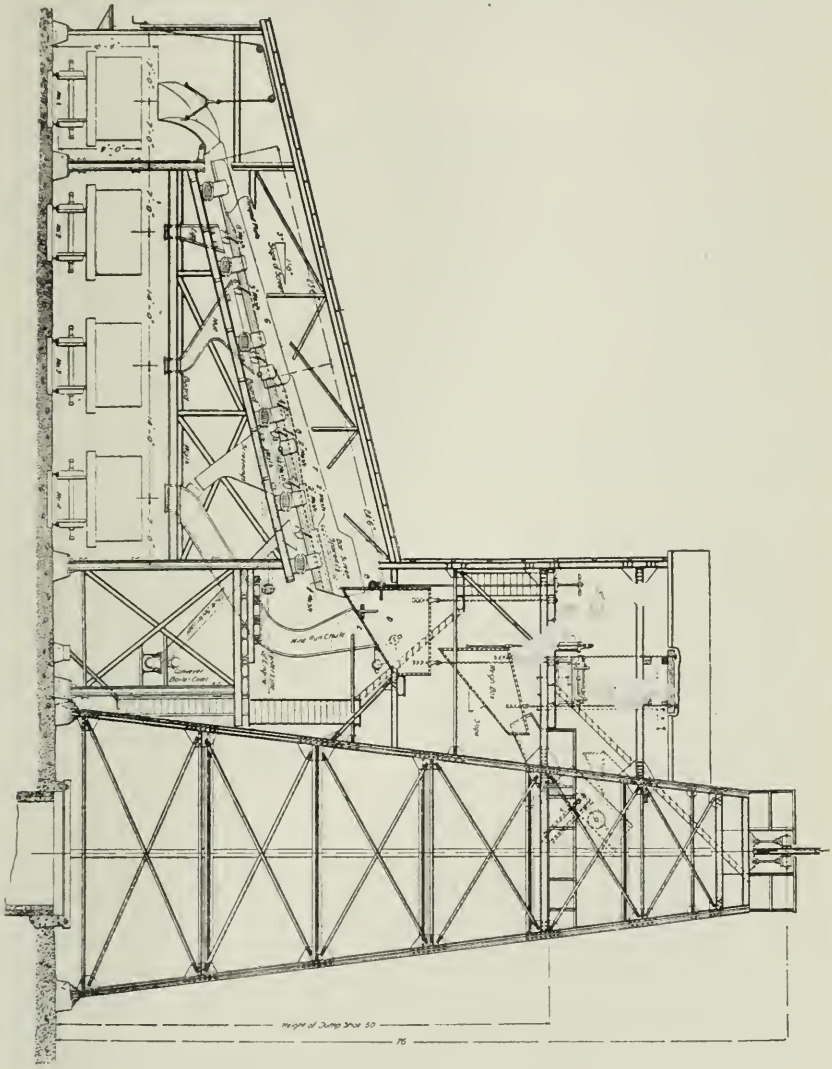

FIG. 7. ELEVATION OF CLASS I TIPPLE.

The self-dumping cage *A*, containing the end dump mine car, dumps at a height of about 50 feet above the ground, this height varying with the design of the tipple, the kind of screen used, the number of railroad tracks to be served, etc. The general scheme is to elevate the coal sufficiently before dumping to enable all necessary preparation to take place and the coal to be loaded into railroad cars without re-elevation. As the cage tips, the coal slides out of the mine car over the dump chute and into the weigh box. The weighman, or in the larger mines a check puller at the scale, takes the miner's check from the mine car in order that proper accounting for the coal contained may be rendered.

After the coal has been weighed, the gate *B*, controlled by hand or by the piston *C*, is opened, thus allowing the coal to fall into the hopper or pocket *D*. In many mines this hopper is absent, the coal being dumped directly from the weigh box on the shaking screen. At other mines an automatic feeder which supplies the coal evenly to the shaking screen is installed instead of this hopper. A valve *E* may be opened to allow rock, refuse, or run of mine coal to enter the mine run chute and thus be loaded into a railroad car on track 4. Above the screen proper is a relief bar screen 6 ft. long and with 2½-inch spaces. This prevents a rush of coarse coal on the upper fine screens and in general prevents their choking; thereby increasing capacity and the completeness of screening. The coal from *D* falls on this relief bar screen and thence to the top section of the upper shaking screen *F*, which in the case illustrated, is 10 feet wide and 24 feet 6 inches long.

The top section *F* of the shaking screen has three decks; the upper one is equipped with a screen plate with 2-inch round perforations, the middle one with a plate with ¾-inch round holes for the upper ten feet and 1¼-inch round holes for the lower end, and the bottom plate is solid, excepting for discharge gates or doors, Nos. 1, 2, 3 and 4.

The lower section *G* of the shaking screen has approximately the same area as the upper one and has two decks only. The top one is fitted with sections of plate having 3-inch round holes and 6-inch round holes as shown, and the bottom deck is solid excepting for the discharge doors.

The driving eccentrics of the shaker screens have a throw of about 6 inches and make about 100 revolutions per minute. To minimize vibration the motion of one screen opposes the other and their total weights when vibrating, including the loads of coal, are approximately the same. The screen supports are independent of the rest of the tipple. The slope of the screens is usually 3 to 4 inches per foot. The chutes leading to the cars are curved and built at a low angle in order to prevent breakage and spilling of the coal during the loading of the railroad cars. The boiler coal chute takes the finest coal, ¾-inch

screenings, to the trough conveyor leading to the boiler room of the power plant.

With such an arrangement of screens and chutes, by opening or closing variously the gates, numbered 1 to 8 inclusive, in the bottom plates of the screens, a great number of sizes of coal can be made without changing the screen plates. The possible sizes are given below in Table 2, and Table 3 gives the arrangements of the gates necessary to obtain the different combinations of sizes.

TABLE 2.

SIZES OF COAL PREPARED. CLASS I TIPPLE.

Combination Number	Size of Coal			
	On Track 1	On Track 2	On Track 3	On Track 4
1	¾" lump			¾" screenings
2	2" "		¾"x2" nut	¾" "
3	6" "	2"x6" egg	¾"x2" "	¾" "
4	1¼" "			1¼" "
5	2" "		1¼"x2" "	1¼" "
6	6" "	2"x6" "	1¼"x2" "	1¼" "
7	3" "		1¼"x3" "	1¼" "
8	6" "	3"x6" "	1¼"x3" "	1¼" "
9	6" "	1¼"x6" "		1¼" "
10	2" "			2 " "
11	3" "		2 "x3" "	2 " "
12	6" "	3"x6" "	2 "x3" "	2 " "
13	3" "	3"x6" "	3" screenings	
14	6" "		3" "	
15	6" "	6" egg run		

TABLE 3.

OPERATION OF SCREENS. CLASS I TIPPLE.

To Make Combination Number	Open Gate Number	Close Gate Number
1	2	3, 8, 4, 5, 6, 7
2	2, 4, 5	3, 8, 6, 7
3	2, 4, 5, 7	3, 8, 6
4	2, 3	8, 4, 5, 6, 7
5	2, 3, 4, 5	8, 6, 7
6	2, 3, 4, 5, 7	8, 6
7	2, 3, 4, 5, 6	8, 7
8	2, 3, 4, 5, 6, 7	8
9	2, 3, 4, 7	5, 6, 8
10	2, 3, 8	4, 5, 6, 7
11	2, 3, 8, 6	4, 5, 7
12	2, 3, 8, 6, 7	4, 5
13	4, 5, 6	2, 3, 8, 7
14	4, 5, 6, 7	2, 3, 8
15	4, 7	2, 3, 8, 5, 6

Besides these sizes mine run or unscreened coal can be loaded on track No. 1 by closing all the gates or on track No. 4 by opening the valve *E* and running the coal through the mine run chute. The usual combinations of sizes of coal prepared are 1¼-inch or 2-inch screenings, 2-inch to 3-inch nut, 3-inch to 6-inch egg, and 6-inch lump. At many tipples, having adjustable screens of this type, only the above standard sizes are prepared, since it is found that changing the screens and valves to prepare different sizes lowers the capacity, especially if close screening of the smaller sizes is attempted. If there is a demand for sized nut coal, a separate rescreening plant is used.

A plant of this size will prepare up to 4,000 tons in eight hours, and similar tipples are in operation which will handle 6,000 tons daily if required.

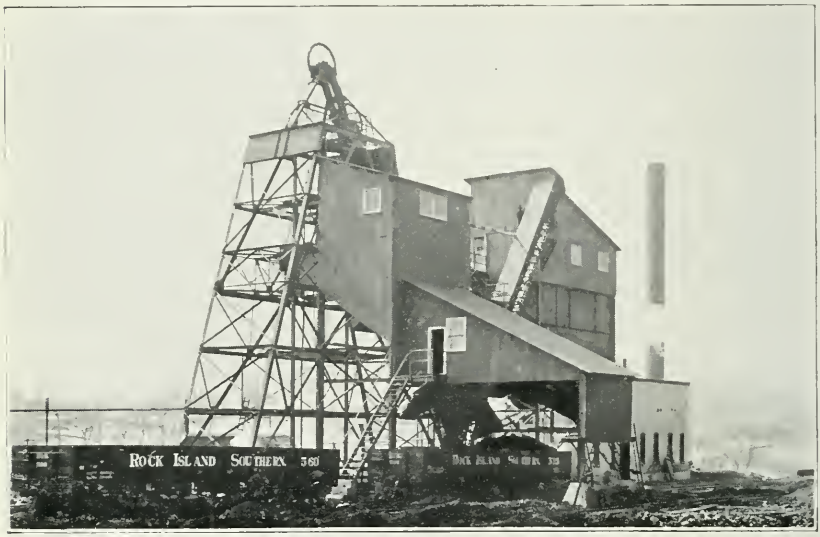

FIG. 8. CLASS II TIPPLE.

CLASS II. TIPPLES EQUIPPED WITH SELF-DUMPING CAGES AND WITH GRAVITY BAR SCREENS FOR SIZING THE COAL.

Although the gravity bar screen has been replaced generally by the shaker screen, on account of the better sizing, less breakage, and less headroom and slope required, yet at many mines, especially if the coal is prepared for a special market, the gravity bar screen is still used, because of its low cost of installation and upkeep, simplicity, and freedom of the tipple from vibration, as well as its almost unlimited capacity.

Fig. 8 shows a photograph, Fig. 9 a ground plan and end elevation, and Fig. 10 a side elevation of a bar screen tipple and accompanying

surface plant. The self-dumping cage (Fig. 10), dumps the coal from the mine car into the weigh box, and after being weighed, it flows on the top bar screen *AA'*. This bar screen is in two equal sections, 6½ feet wide and 8 feet long, and is set at an angle of 27 degrees, which is just enough to allow the coal to slide over it by gravity. The spaces between the bars are 5 inches in width. Lump coal passes over the bars *AA'* into chute *B*, set at the same angle, and from here into the railroad car on track No. 1. The coal passing the 5-inch spaces

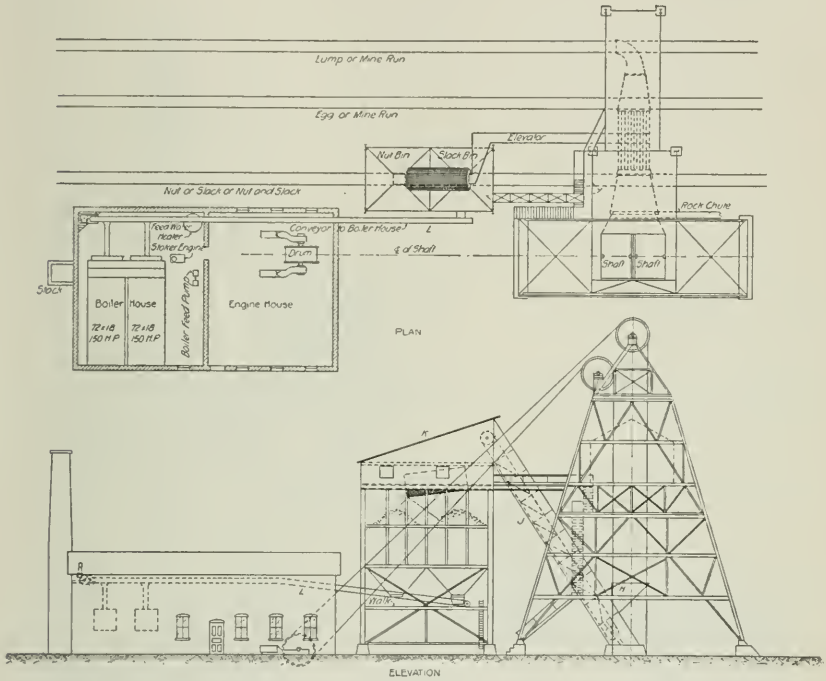

Fig. 9. Ground Plan and End Elevation of Class II Tipple.

of the screen *AA'*, falls on the lower bar screen *DD'*, which is of the same width as *AA'*, is 12 feet long, is set at an angle of 32 degrees, and has only 1-inch spaces between the bars. Thus, the coal between 5 inches and 1 inch in size passes over the lower screen into chute *E*, from which, as egg coal, it is loaded on track No. 2.

The smallest coal, having passed through screen *DD'*, is collected by chute *F* and may be loaded on track No. 3 as raw or untreated screenings. By means of a hinged gate *G* this coal may also be deflected into chute *H* and from here to the elevator *J* (Fig. 9), leading to the rescreening plant *K*. This particular rescreening plant is provided with a revolving screen 5 feet in diameter and 12 feet long,

sloping 5 degrees, and enclosed with wire screen cloth having ¾-inch diameter square meshes. The oversize falling into the steel nut bin is called rescreened nut or pea coal; the undersize ¾ inch to zero in size, called slack, falls into the steel slack bin. The capacity of each of these bins is about 60 tons. Railroad cars can be loaded underneath as desired. A conveyor *L* carries part of the slack to the boiler room.

By lowering veils or blank plates over the screen *AA'*, it is possible to load mine run coal on track No. 1. Also by opening the chute

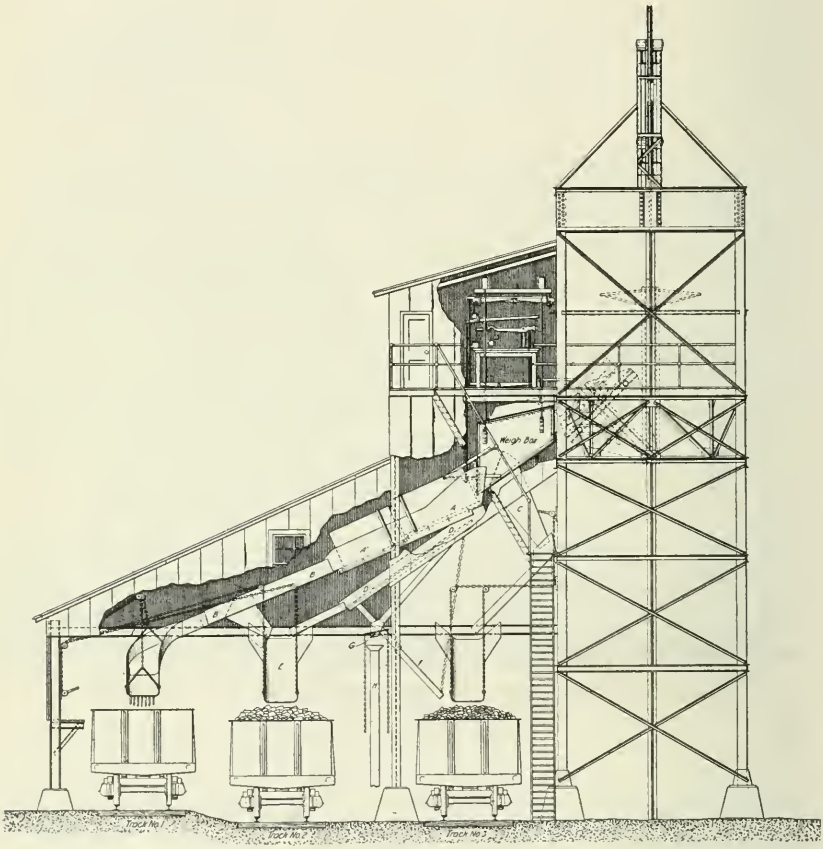

FIG. 10. SIDE ELEVATION OF CLASS II TIPPLE.

B', the oversize from the 5-inch bar screen may be loaded on track No. 2. Provision is also made for running either mine run coal or refuse from the weigh box directly into the chute *C* and from here into the railroad car on track No. 3. By using veils on the screens as required,

together with the rescreener, the combinations of sizes given in Table 4 are possible in the tipple described.

TABLE 4.

SIZES OF COAL PREPARED. CLASS II TIPPLE.

Combination Number	Size of Coal			Rescreener	
	On Track 1	On Track 2	On Track 3	Nut or Pea	Slack
1	mine run				
2	5″ bar screen lump	5″ bar egg run			
3	5″ bar screen lump	1″-5″ bar egg	raw screenings		
4		1″ bar screen lump	" "		
5			mine run		
6	Raw screenings in combinations 3 and 4 may be rescreened into			¾″ sq. to 1″ bar	¾″ sq. to zero

The average daily capacity of the tipple illustrated is 800 tons, but at individual tipples of this class in the state more than 5,000 tons daily are prepared.

CLASS III. TIPPLES EQUIPPED WITH RIGID OR FIXED CAGES, WITH CROSS-OVER DUMPS IN THE TIPPLE TO EMPTY THE MINE CARS, AND WITH SHAKER SCREENS FOR SIZING THE COAL.

The tipple illustrated by the photograph (Fig. 11), and shown in side elevation (Fig. 12), has found favor in and is confined almost wholly to the longwall field in the northern part of Illinois for the following reasons: (a) Many of the mines are from 300 to 600 feet deep and at some two seams at different levels have been worked from the same shaft. (b) The thin seams and method of working usually necessitate small mine cars holding 1 to 1½ tons, and to secure adequate hoisting capacity, two cars are often hoisted end to end on the same cage. (c) One car of waste is also often hoisted to three cars of coal, and large amounts of timber are lowered.

The loaded cars (Fig. 12), are pushed from the cage along a track having a gradient with the load of about 1½ per cent, to some form of crossover or end dump and dumped into the weigh box. The empty car runs down the sharp grade *A* (6 to 12 per cent), through a spring switch at *B* to the kickback at *C*, while returning the empty car runs by gravity down the side track *D* (1½ per cent grade), to the transfer car *E*. When two mine cars have entered the transfer car, it is moved up inclined tracks by means of an air or steam piston,

not shown, until the mine cars arrive at the level of the cage. By this time two more full cars have been hoisted and the mine cage is in the position referred to in the beginning. By means of a piston working in the long plunger F a ram G gently pushes the two empty cars into the cage, at the same time forcing the two loaded cars off.

The tracks, dumps, etc., in the tipple are in duplicate to provide for the cars coming from the cage in the other compartment of the shaft.

Mine cars containing shale or refuse are taken over the cross-over dump without dumping and switched to a track leading to the left end H of the tipple where their contents are dumped into a chute, usually by means of a horn dump, after which the cars are returned to the empty tracks D. In the same way, coal intended for local or

Fig. 11. Class III Tipple.

wagon trade is dumped into a chute at H and roughly bar screened before being loaded into the wagons.

Cars to be loaded with timber are taken through the end J of the tipple to an elevator (shown in Fig. 11), lowered to the ground, loaded, and returned to the system at E.

The screening plant in such a tipple may consist of either gravity bar screens or shaker screens. As illustrated in Fig. 12 on the shaker screen M one, two, or three sizes of coal can be prepared at once. It is of all steel construction and rests on independent foundations.

The coal dumped into the weigh box is weighed by scales in the weighroom K, the check puller at the dump having taken the miner's

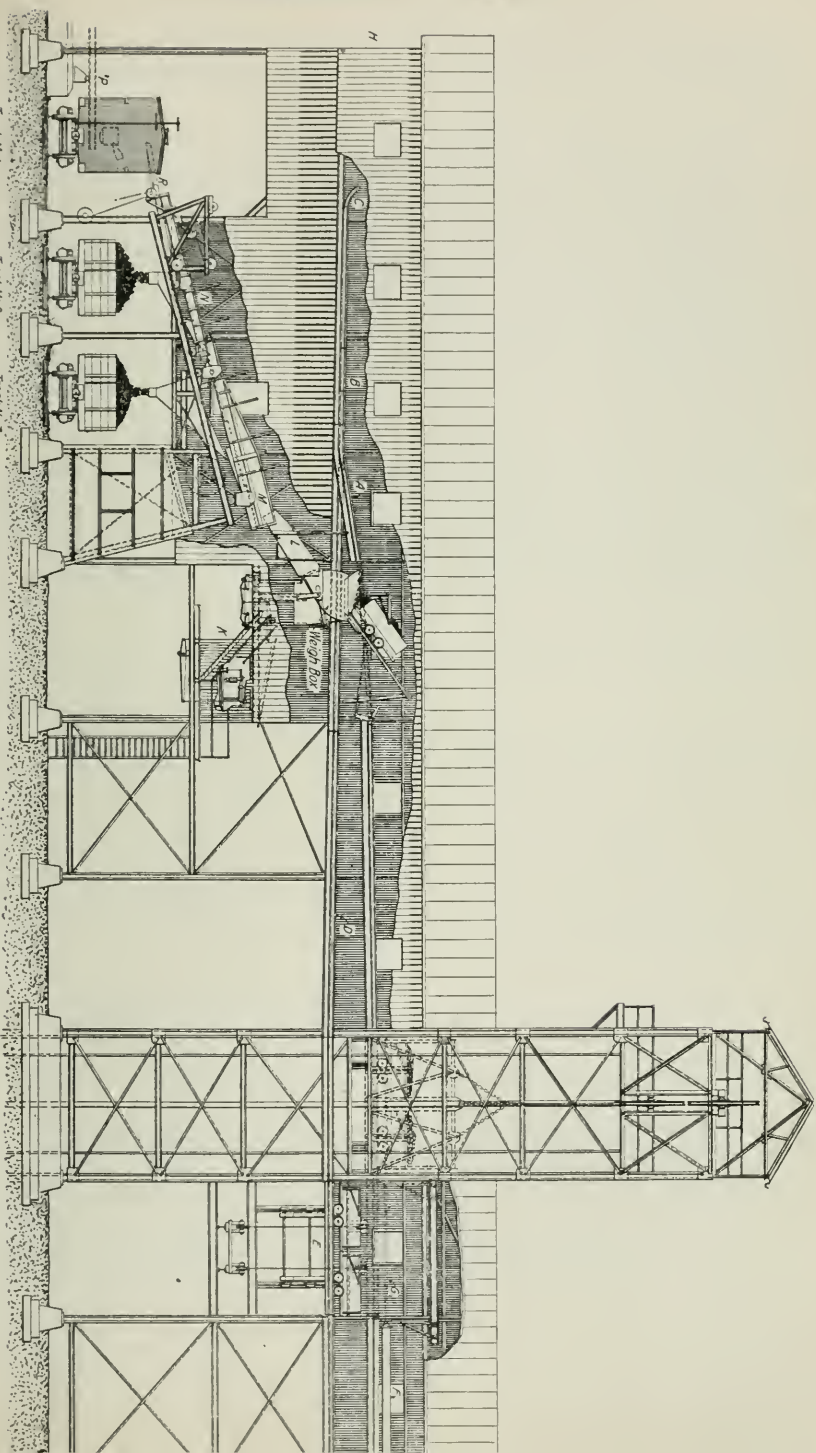

Fig. 12. Side Elevation of Class III Tipple.

check from the car and dropped it into a chute (not shown), which carries it to the weigh room, so that proper credit is given for the coal.

The coal passes over the dead plate or chute *L* to the upper shaking screen *M*, which is 8 feet wide by 16 feet long, and consists of plates with 1⅛-inch round hole perforations. The undersize from here is loaded as screenings on track No. 3. In some cases it is washed before being marketed. The oversize of the screen passes over a short dead plate to the lower shaking screen *N* of which the upper part consists of 5-inch hole plate. Thus 1¼-inch to 5-inch egg may be loaded on track No. 2. The lump or chunk coal passes over the 5-inch screen and through the adjustable chute *S* into the car on track No. 1. If it is necessary to load box cars, the box car loader is run out from *P* into the car as shown, and the chute *R* is adjusted to feed it. It is possible by veiling the screens to load mine run on either track No. 1 or No. 2, or to load 1¼-inch lump on track No. 1.

Owing to facts previously stated, the capacity of these tipples is relatively small in comparison with those at pillar and room mines, a daily production of 1,500 tons being considered large. A large proportion of the coal produced is clean lump, and consequently, most of the screening plants in tipples of this type are small and simple.

CHAPTER III.

Impurities and Breakage.

PART I. GENERAL.

Removal of impurities and avoidance of breakage are the two standards by which coal preparation is measured. The object of coal mining is not only to produce the greatest possible tonnage at the least cost, but to produce the greatest possible percentage of clean lump coal. Perfect mining would mean, not only the extraction of all the coal, but the winning of it in a perfectly clean condition with 100 per cent lump. Were this possible, subsequent mechanical breakage could easily and cheaply produce finer sizes, as needed, in as pure a condition as the original lump.

"On the whole, Illinois has pure coal compared with many sections and it is possible to clean lump by hand in the railroad car. If the market calls for a steam coal, very little preparation is necessary. If the market requires domestic, we must screen and pick and even rescreen. The biggest problem is to load without breakage."*

No seam of Illinios coal is free from at least small amounts of impurities. Since mines situated in widely separated districts over an area of approximately 36,800 square miles are worked in five different seams, great variations are possible locally in the nature and in the extent of the impurities in an individual seam and in the physical characteristics of the coal itself.

The most persistent impurity in Illinois coal is the famous "blue band" or shale band in seam No. 6, which averages 1 to 2 inches in thickness, and is situated about 20 inches above the bottom of the seam. It extends with considerable regularity over an area of at least 5,000 square miles.† Impurities such as pyrite ("sulphur") and shale bands are of frequent local occurrence in all the seams. On the whole, it cannot be said that any one district in Illinois has an advantage over others in this respect. Considerable variance occurs among mines in the same district, however, not only in the impurities in the coal, but also in the care displayed in removing the same before the coal is shipped to market.

Friability means the tendency to produce fines under like conditions. The Illinois coals are not so friable as the coking bituminous coals of the Appalachian region, and in freedom from the breakage which occurs during mining, preparation, and shipment, they are perhaps not excelled by coal from any other section of the bituminous fields of America. Taken as a whole the different seams in Illinois show considerable uniformity as regards friability, although some-

*W. R. Roberts, Black Diamond, November 30, 1912.
†T. E. Savage, Journal of Geology, Vol. 22, No. 8, 1914.

times the same seam in different districts or even in the same district will vary, depending probably on the thickness of the cover and the regularity of the bed. A seam buried under several hundred feet of cover is likely to be firm; a seam which has been tilted or folded is likely to be somewhat friable. At only two places in the state where mining is undertaken have the beds undergone any extensive regional

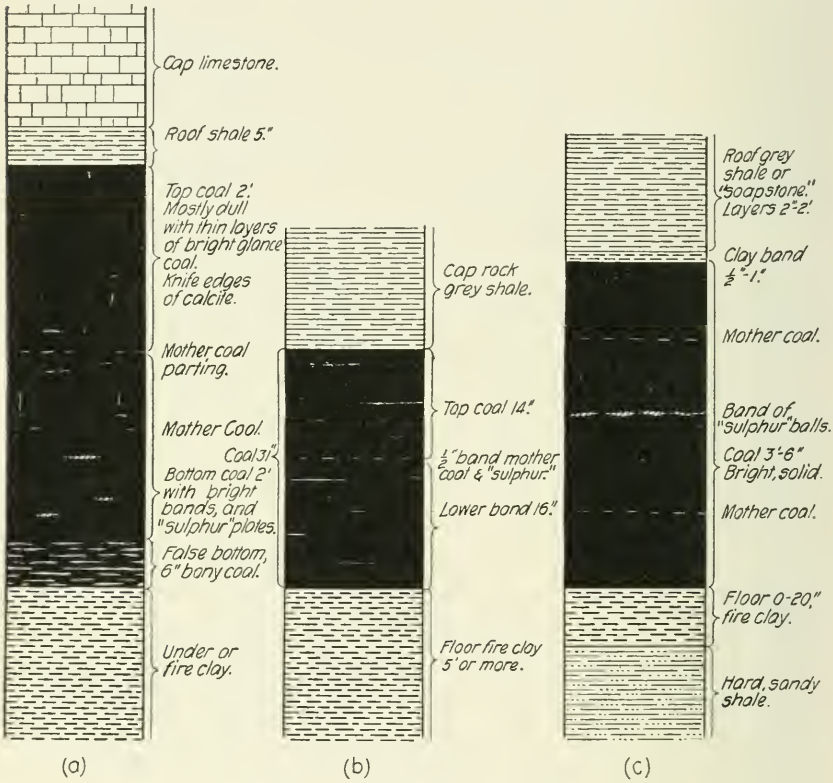

Fig. 13. Typical Sections of Seams Nos. 1 and 2.

faulting and tilting, all being practically horizontal. This has tended to keep the beds regular in general physical structure over large areas. Since Illinois coal is relatively firm, the preparation of sized coals intended for competition with the domestic sizes of anthracite has recently become a noticeable feature.

The sections shown in Figs. 13 to 16 have been prepared to illustrate the relative occurrence and distribution of impurities in the different coal seams worked in Illinois together with the nature of roof and floor. These in no way reflect on or give prominence to any one seam, since the quality of the merchantable coal is largely affected by

TABLE 5.

AVERAGE ANALYSES* OF COAL IN SEAMS DESCRIBED.

Illustration Fig. No.	Seam	Number of Samples	Proximate analysis of coal: 1st; "as rec'd," with total moisture. 2nd; "Dry," or moisture free				Sulphur	B. t. u.	Unit Coal B. t. u.
			Moisture	Volatile Matter	Fixed Carbon	Ash			
13a	1	11	15.58	39.17	35.80	9.45	4.69	10,673	.....
			Dry	46.40	42.41	11.19	5.55	12,643	14,546
13b	2	3	17.40	33.30	41.48	7.82	2.03	10,811	.....
			Dry	40.32	50.20	9.48	2.45	13,091	14,663
13c	2	33	16.18	38.83	37.89	7.08	2.89	10,981	.....
			Dry	46.34	45.21	8.45	3.45	13,101	14,528
14a, 14b	5	54	15.10	36.79	37.59	10.52	3.52	10,514	.....
			Dry	43.32	44.28	12.40	4.15	12,384	14,447
14c	5	27	6.75	35.49	48.72	9.04	2.92	12,276	.....
			Dry	38.06	52.25	9.69	3.13	13,165	14,812
15a, 15c	6	76	12.56	38.05	39.06	10.33	4.01	9,848	.....
			Dry	43.52	44.67	11.81	4.59	12,406	14,377
15b	6	58	9.21	34.00	48.08	8.71	1.53	11,825	.....
			Dry	37.45	52.96	9.59	1.68	13,025	14,585
16a	2	15	9.28	33.98	51.02	5.72	1.29	12,488	.....
			Dry	37.46	56.24	6.30	1.42	13,765	14,818
16b	6	31	14.45	35.88	40.33	9.34	2 55	10,919	.....
			Dry	41.94	47.14	10.92	2.98	12,764	14,557
16c	7	18	12.99	38.29	38.74	9.98	2.93	11,143	.....
			Dry	44.01	44.52	11.47	3.37	12,807	14,740

*S. O. Andros, "Coal Mining in Illinois," Bulletin 13, Co-operative Agreement, p. 57.

the care and skill exercised in its mining and preparation. These sections and the accompanying analyses of the coal listed in Table 5 are based mainly upon data gathered by the Illinois Coal Mining Co-operative Investigation.

The instructions given the samplers engaged in gathering the face samples for the analyses called for the exclusion of all impurities in the seam more than ⅜ inch thick, or thinner partings or impurities if, in the judgment of the sampler, such are excluded by the miner in loading the coal. The latest directions of the U. S. Bureau of Mines for taking face samples are practically the same, with the addition that "lenses or concretions of 'sulphur' or other impurities more than 2 inches in maximum diameter and one-half inch thick are excluded if, in the judgment of the sampler, they are being excluded by the miner from the coal as loaded out of the mine or as shipped."* It is evidently assumed that these are the maximum sizes the miner would allow to enter the car with the coal. The commercial sizes of prepared coal may be higher or lower in impurities and ash than these face samples, depending largely on the care used in preparation.

Seam No. 1 in Mercer county (Fig. 13a), averages 4 feet in thickness. The coal has weak or incipient cleavage, along which plates and films of pyrite or calcite may be developed, and locally, sulphur bands are interbedded with the coal. Directly over the coal is a shale band 2 to 5 inches thick, which tends to disintegrate and fall on exposure to the air, and which may, therefore, mix with the coal. Below the mineable coal there may be a thin band of bone giving way to a soft fireclay, that swells badly on exposure. This has been likened to bread rising. The output from this seam is scarcely large enough to affect the general market.

Seam No. 2 in McDonough county (Fig. 13b), averages about 30 inches in thickness and consists of a top and a bottom band which are separated as shown. The coal as a whole is built up of fine laminations of bright and dull coal. Irregular bands and lenses of pyrite occur in places. The roof is a grey shale or "soapstone," and the floor fireclay; usually soft.

Seam No. 2 in LaSalle county (Fig. 13c), averages about 3 feet, 6 inches in thickness. The coal is uniformly long grained, hard, bright and firm. It tends to split parallel to the bedding, being aided by the mother coal layers. One or two irregular bands of sulphur balls are found in some places. The roof is a brittle blocky gray shale or soapstone separated from the coal by a clay band ½ inch to 1 inch thick. The floor, in which undercutting is often done, may vary from fireclay to a sandstone. In general, this is a clean seam and little or no picking is necessary to prepare an excellent lump coal for the market. Considerable fire clay, which may be removed by washing, is mixed

*A. C. Fieldner, "Notes on the Sampling and Analysis of Coal." U. S. Bureau of Mines, Technical Paper 76, p. 8.

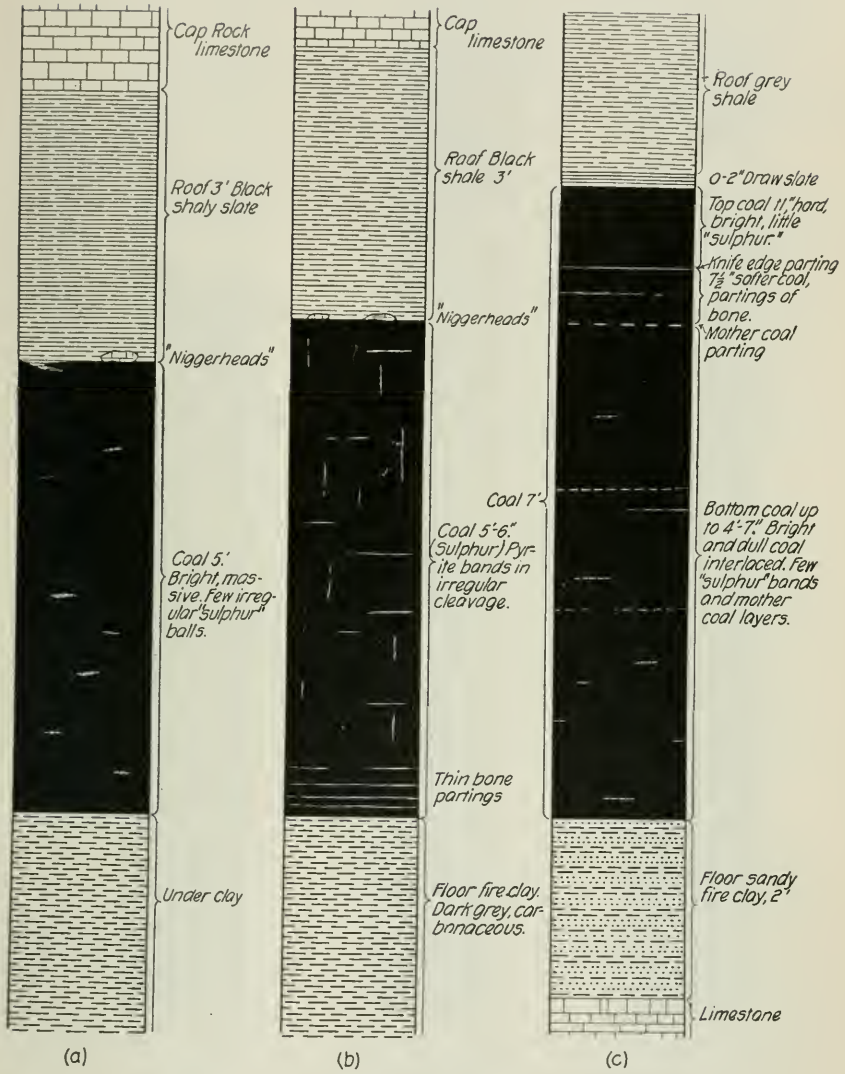

FIG. 14. TYPICAL SECTIONS OF SEAM NO. 5.

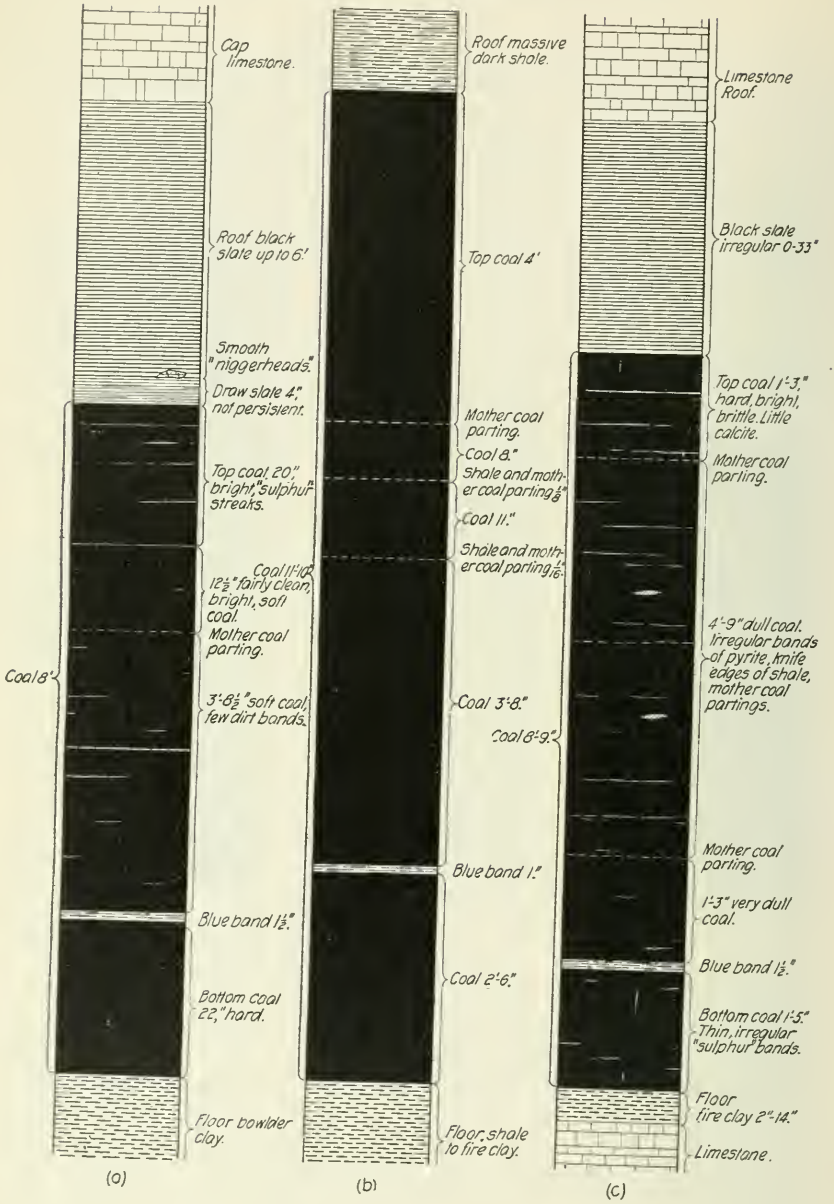

FIG. 15. TYPICAL SECTIONS OF SEAM NO. 6.

with some of the screenings.* As an illustration, raw screenings from this field containing 28.87 per cent ash, are reduced by washing to nut coal containing 8.75 per cent ash and to slack coal containing 9.96 per cent ash.† The top coal is generally harder than the bottom.

Seam No. 5 at Lincoln, Logan county (Fig. 14a), consists of about 5 feet of bright massive coal. There are some irregular balls and bands of sulphur and other impurities present. The roof contains frequent concretions or "niggerheads."

Fig. 14b shows the same seam in the Springfield district. The sulphur here often is irregularly distributed in thin plates in the cleavage planes of the coal. The seam in this district is relatively free from impurities. The chief of these is sulphur in vertical faces. Also, in places a rather high percentage of ash is noted in the lump coal, due to a slight tendency of the coal to be bony. This same tendency, however, results in a coal that is unusually tough, hard and firm, and able to withstand breakage as well as any in the state. Rolls, horsebacks, and clay veins, which are of frequent local occurrence, are often avoided by careful mining. The fact that of these clay veins the large ones are soft, and the small ones are hard, has considerable effect on the cleanness of the adjacent coal.

Fig. 14c shows the seam mined in Saline county, and correlated by the State Geological Survey with seam No. 5. In general it is a bright laminated coal, the hard top coal being succeeded by a much softer coal in the middle of the seam, and this by a harder coal on the bottom. The noticeable feature of this seam here is its low moisture content. This coal has been compared favorably with the famous Hocking Valley coal.‡ Locally, in the roof there are bone and stringers of coal up to 3 feet or more above the true coal. The roof is generally a hard calcareous shale, while the fireclay bottom in places is sandy and heaves badly when wet. Only incipient cleavage is developed, and the coal has a good reputation for hardness.

Seam No. 6 (Fig. 15a) in the Standard district east of East St. Louis is usually divided into several benches by partings, sometimes of mother coal, often by sulphur or by shale bands, which include the blue band. In many places the roof consists of a thin layer of dark drawslate, locally called clod, containing frequent niggerheads. In places in which this drawslate is absent, the cap limestone forms a firm roof. According to A. J. Moorshead,¶ the coal is harder where limestone forms the direct roof. The floor, although a clay shale containing frequent boulders, as a rule does not heave, except when wet. In the mines in which the coal is thick (8 feet) the top coal

*A. Bement, "Illinois Coal Fields." J. W. S. E., June, 1909.

†F. C. Lincoln, "Coal Washing in Illinois." Bulletin No. 69, Engineering Experiment Station, University of Illinois, p. 90.

‡A. Bement, "The Illinois Coal Field." J. W. S. E., Vol. 14, 1909, p. 319.

¶Colliery Engineer, 1914, p. 435.

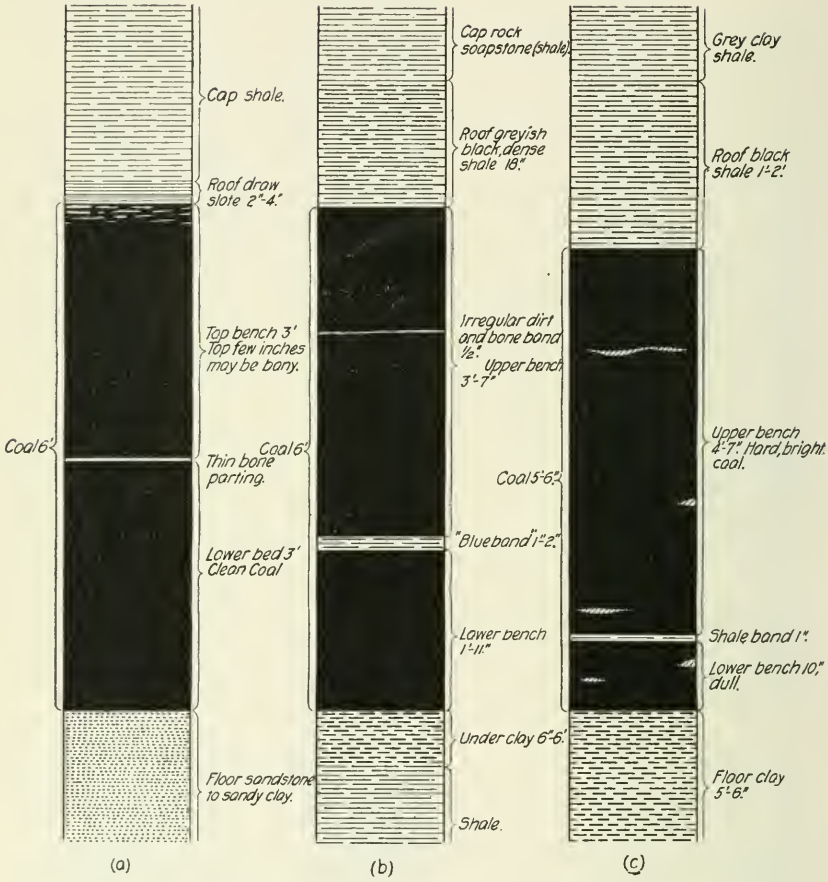

FIG. 16. TYPICAL SECTIONS OF SEAMS NOS. 2, 6, AND 7.

shown is often left in order to hold the roof in place. Since the district covered in the analyses embraces fifteen counties in whole or in part, considerable difference is evident in local physical characteristics. On the whole, the coal contains enough bands of impurities to require care and attention in removing them during its preparation.

Seam No. 6 in Williamson and Franklin counties (Fig. 15b), varies from 7½ to 14 feet in thickness. Excepting for the blue band and locally sulphur in thin, irregular vertical plates, the coal is clean. Although from the standpoint of heating value, it is not the best coal in the state, its toughness, strength and clean glossy appearance make it a favorite, especially for domestic trade and for northwest shipments often requiring several handlings enroute. Also, it withstands storage well. In places a definite east and west vertical cleavage affects to some degree the sharpness of its breakage. The top coal is frequently left in place.

The coal in seam No. 6 in Macoupin county (Fig. 15c), is often more brittle than that in the same seam farther south. This county is included in the average analysis given for seam No. 6, Fig. 15a.

Seam No. 2 in Jackson county (Fig. 16a), is often called Big Muddy or New Kentucky. Its chief characteristics are general freedom from impurities and high fixed carbon, as compared with most other coals of the state. The thin bone parting varies from zero up to 30 feet in thickness, in which case only the lower and better bed is mined. The cleavage is more pronounced than in any other district in the state, the coal breaking freely northeast and southwest, thus making blocky lumps of a bright lustre. As a rule, the floor is more stable than those in the other districts. The former use of this coal for the manufacture of blast furnace coke indicates its generally low sulphur content. A. Bement called this coal the best west of the Appalachian region,* and it is to be regretted that the district is so limited.

Seam No. 6 in the Danville district (Fig. 16b), called the Grape Creek field, consists of two benches, separated by the blue band, the upper bench being usually duller and dirtier than the lower. The roof, varying from a gray to a dark shale with little bedding, breaks away easily and falls, often in conchoidal masses, and thus is mixed more or less with the coal. The underclay swells readily. The bands and nodules of pyrite present are usually thick enough to allow hand separation. Rolls occur in both floor and roof, and frequent horsebacks tend to mix considerable fine impurity with the coal.

Seam No. 7 (Fig. 16c), as it occurs in the Danville district, is the uppermost seam exploited commercially in the state. The feature of the bed is the large lenses and bands of sulphur, which can be easily

*Black Diamond, June 27, 1914, p. 53.

removed in mining and preparation. In general, the lower bench of the seam is hard and dull, overlaid by a softer, cleaner coal near the center of the seam and again by a harder bright coal at the top. The cap rock, a gray clay shale, is soft, and where open cut mining is practiced is dug and removed by steam shovels. The bottom is soft and rolls are frequent.

PART II. IMPURITIES.

Impurities in the Coal Bed.—The diagrams or logs of the various Illinois coal seams (Figs. 13-16), show that they contain various thin bands or streaks of impurities, such as are found in practically all bituminous seams. In order to discuss these impurities, they have been divided by authorities into groups relating either to their origin, position in the seam, or ease of removal. The writer prefers a grouping by origin which divides these coal impurities into three general groups which have been variously named as follows:

Group 1. Innate, Intermixed, Inherent, Normal Ash, or Inseparable Impurities.

Group 2. Sedimentary, Interbedded, Intercalated, or Separable Impurities.

Group 3. Infiltrated, Extraneous, Interstitial, Segregated, Subsequent, or Precipitated Impurities (Sometimes Separable).

Group 1. Innate or Inseparable Impurities.—All the terms listed under this group have been used to describe the true or normal ash of the coal. Since coal is a product of plant remains, it must contain the original mineral matter or ash of such plant. Trees or plants of the present day contain about one per cent ash, but since ancient plant remains have undergone great changes and losses of weight through partial decomposition before arriving at their present stage of coal, and since the mineral matter remains without loss in the decreasing residue, the inherent ash may amount to a much larger percentage in the coal than in the original vegetable matter. Such impurity may be considered a part of the original coal substance. It is present in every piece of coal and cannot be removed or altered by any process of preparation. Certain coals have probably as low as 2½ per cent inherent ash. The author determined approximately by a number of experiments the inseparable or inherent impurities in Illinois coal. Each sample was crushed to pass a ¼-inch mesh screen and then subjected to a "float and sink" test in a solution of 1.35 specific gravity, which allowed the purest coal to float and the separable impurities to sink. The results indicated that the inseparable impurities varied from 3 to 7 per cent, with an average of about 5 per cent.

If the coal bed were formed slowly by accumulations under sheltered and shallow water conditions, and if this water were clear and without sediment, a coal would result containing only inherent ash. If, however, the waters of the marsh contained sediment, such as might arise from the influx of rivers, etc., the slow settlement of this

slimy mineral matter simultaneously with the growth of the coal forming bed would produce a coal containing a uniform mixture of this impurity and coal. Such an impurity would increase the ash content, and as it increased in amount would cause the coal to assume more and more a rocklike character, both in appearance and physical characteristics. Since such sediment is, in most cases, a clay-like slime, or clay in a more or less colloidal state, which becomes shale under heat and pressure, its presence in increasing percentages tends to lower rapidly the commercial value of the coal. These impure coals are called bony coal or simply "bone," and by English engineers "bass." Geologically, there may be a range of bone coals containing as much as 50 per cent clayey matter; beyond this point the substance is no longer coal, but a carbonaceous shale, with pure shale as a limit.

If these conditions persisted to a slight degree only during the formation of the coal bed, this admixture of impurity, while increasing slightly the ash content of the coal, would produce through its own cementing tendency a harder and firmer coal than the average. As an illustration, the clean lump from seam No. 5 in the Springfield and Peoria districts is usually slightly higher in ash than the usual clean coal from some other districts, but is of such a recognized hard nature that special provision is made for its mining in the agreement between operators and miners.

If these conditions favoring the formation of bone coal were periodic or infrequent during the growth of the seam, certain benches only of the coal seam would be bony. At many places in this state the bottom bench of the seam is of a higher ash content and is harder than the upper benches, although not bony to a degree which interferes with its commercial value. This is illustrated by seam No. 7 in the Danville district. On the whole, the coal seams of Illinois are unusually free from benches of true high ash bone coal, and trouble is caused by it locally in one or two seams only.

The moisture and oxygen in the coal are also inherent impurities, but since they are removed only by weathering, heating and similar methods, they are not impurities removable in the dry preparation of coal at the mines. S. W. Parr says, "A coal with 14 per cent moisture may reach the consumer with 10 per cent only, therefore the value per ton is greater than when the coal left the mine."* This, however, is probably due to part of the moisture in Illinois coals being held mechanically in the pores and drying out on exposure; therefore, it is not an impurity under control of the operator in the preparation of the coal.

The effect of oxygen in coal is fully discussed by David White, who shows in Bulletin 29, U. S. Bureau of Mines, that oxygen is an inert constituent in coal, and unit for unit of weight takes the place of so much combustible material.

*Bulletin No. 16, Ill. State Geol. Survey, p. 227.

FIG. 17. THE "BLUE BAND."
AA=LINE OF BLUE BAND.

Group 2. Sedimentary or Interbedded Impurities.—Bone coal may be formed in layers or bands intermixed with and separable from the better coal. Such bands may be of any percentage mixture of pure coal and mineral matter and range in thickness from a knife edge to a dimension that separates the coal into two benches. Moreover, such a band may be either flat or lenselike in shape, and either local or of even thickness over a considerable area.

During mining these bands tend to break free from the coal into characteristic flat pieces. As they approach a pure shale in composition ($Al_2O_3 2H_2O + XSiO_2$) and also containing varying percentages of lime ($CaCO_3$), the color becomes lighter, usually a stony gray, and the specific gravity increases. This makes easy, detection and removal of the coarsest bands by the miner at the face, or by the picker in the better light of the tipple above ground. If these bands are in coal below two or three inches in size and above 5 to 10 per cent in quantity, their removal is accomplished only by some mechanical means, chiefly washing. From the larger sizes of coal, approximately those above three inches, and not exceeding 5 to 10 per cent in quantity, these shale bands can be removed by hand picking, although at the present time the largest percentage of such an impurity hand picked in an Illinois tipple does not exceed 3 per cent of the total coal, or about 4.5 per cent of the sizes above screenings.

Bands of bone coal, containing more than 50 per cent of coal substance, are dark in color and not greatly different from the coal in weight. This renders their removal difficult, whether attempted by hand picking or by washing, which process depends for success upon a considerable difference of specific gravity between coal and impurity.

An examination of the coal sections illustrated on pages 47 to 52 shows that the thinner of these impure bands, those under ¼ or ⅜ inch in thickness, may easily become mixed with the finer coal and not be detected. The thicker ones must be removed at some stage of the preparation, since the appearance and consequently the sale of the coal is injured by their presence, perhaps more than the coal is actually deteriorated chemically. Unfortunately, many of the shale bands in Illinois coal are rather soft and tend to soften, peel, and disintegrate rapidly upon exposure to air and moisture. For this reason, their complete removal from the finer sizes of coal is a problem of some difficulty. In most cases, the purer the shale, the more it tends to soften and disintegrate. The floor shale "fireclay" tends to disintegrate much more rapidly than the interbedded or the roof shale. The miners of the state call these bands of impurities "blue band" (referring to the well known impurity of seam No. 6), "black band," "blackjack," "stone," "grit," "dirt," or "brash." Fig. 17 shows the blue band of Seam No. 6 in Franklin county. Lumps in which coal layers and shale bands are intimately mixed in the same piece are called "intermixed," or "intergrown" coal.

Group 3. Infiltrated or Subsequent Impurities.—This group includes the visible impurities in the coal which were introduced subse-

quent to the formation of the bed, such as pyrite, calcite, or gypsum.

Underground circulating waters contain considerable amounts of iron, lime, and gypsum salts in solution, which deposit or precipitate under favorable conditions. Such conditions are furnished by the reducing tendencies of the carbonaceous matter of the coal, and by the more porous layers of the seam which furnish easy channels of circulation for the solutions. The firmer bands of the seam tend to define

Fig. 18. Pyrite Lense in Illinois Coal.

and limit these channels. Having started deposition around some favorable nucleus, further deposition tends to enlarge the particle. The final result will be nodules, bands or lenses of pyrite (iron sulphide, FeS_2), containing if pure 46.6 per cent iron and 53.4 per cent sulphur. By the miners they are usually called "sulphur balls," "sulphur," "cat faces," "kidney sulphur," or "brasses." The nodules have an irregular maximum thickness of several inches. Less resistance usually has been offered to the growth of these masses along the bedding or lamination planes of the coal; for this reason sulphur bands

are horizontal in the bed, and may be either flat or slightly lenticular in shape. Often the bands are quite flat, are as much as one or two inches in vertical thickness, and may have a horizontal extent of many square feet. The lenses are sometimes 5 or 6 inches in vertical dimension and considerably greater in length along the bed. Like interbedded shale the pyrite is rarely pure in composition, an analysis of one lump showing 33.2 per cent of volatile matter and fixed carbon present. Occasionally, lumps of pyrite are seen, the forms of which suggest the replacement of bits of branches or other woody tissue. Fig. 18 illustrates the occurrence of pyrite lenses in Illinois coal.

The sulphur balls or bands, being brassy yellow in color and of high specific gravity (if pure from 4.9 to 5.1) are easily distinguishable by the miner, and are usually thrown into the gob. Frequently the pieces are more or less coated with adhering coal, and if missed by the miner are removed by hand picking in the tipple or on the railroad cars.

At one mine in the Standard (Belleville) district at which seam No. 6 is worked, enough of this lump pyrite is picked in a clean condition from the coal during screening and loading to justify saving and shipping the product to various chemical companies for use in the manufacture of sulphuric acid. At certain mines in the Danville district (seam No. 7), numerous large bands of pyrite are hand picked from the coal by the loaders, or in the tipple by pickers during screening and cleaning. Enough impure pyrite is secured from several mines to justify the erection and operation of a washing or jigging plant in which the raw pyrite or sulphur with its adhering bands or bunches of coal is crushed and washed. After the completion of this process the clean pyrite is shipped, and a quantity of fairly clean small coal is recovered as a by-product. In most other places in the state pyrite is justly regarded as a deleterious impurity, not only harming the appearance of the coal if not removed, but aiding materially through its combustion products FeS and FeO in the formation of clinker when the coal is consumed.

Information obtained by correspondence with various chemical companies indicates that the possible market for such a product as pyrite is limited since pyrite is used in quantities for the manufacture of sulphuric acid only when there is a scarcity of the usual and cheaper raw material—the sulphur in zinc blende. Any attempt on the part of Illinois coal operators to generalize the commercial production of pyrite as a by-product of washing or of picking belt, would result at present in a complete demoralization of the market and in a consequent lack of sale.

If all the sulphur in Illinois coals were in this lumpy form its removal would not present any serious difficulties since it can easily be removed from the smaller sizes by mechanical washing processes. Unfortunately, in many of the districts in this state the pyrite is found adhering to the coal in very thin leaves or plates, often several inches square and of almost infinitesimal thickness. These plates have

originated by reason of the incipient vertical cleavage in the coal, such planes of weakness having afforded opportunity for the deposition of pyrite in irregular thin plates. When the coal is mined, it breaks into lumps more or less along these cleavage lines, thus exposing to view the thin brassy plate of pyrite. By actual weight or percentage the amount of sulphur in such a coal may be small, even smaller than in coals which show no sulphur to the naked eye. Coal with these glistening films is at a disadvantage on the market since consumers believe that they are an indication of an inferior fuel.

Such pyrite is difficult to remove from the coal underground; in fact, so many lumps of coal may have one or more glistening sulphur faces that a clean separation would involve the waste of a large percentage of the coal. Unfortunately for miner and operator alike, every fresh break in the coal during preparation is likely to expose fresh brassy faces. Again, these thin films hang closely to the coal, and their removal by any process of breaking and picking involves the loss of much lump coal. Another factor is that coal dust easily sticks to and hides these faces, making them indistinguishable until subsequent drying and jarring again bring them to light.

At one mine as many as fifty tons of coal having blotches of this pyrite are picked out in the tipple each day in an effort to ship coal that looks well. This is about 2½ per cent of the daily production of the mine. The coal thus separated is crushed and used as second grade fuel. At another mine which makes a specialty of sized domestic grades, the egg and nut coal are wetted or rinsed by sprays of water before being hand picked, thus removing any adhering dust and bringing to light the pyrite films. This rejected coal is also used for purposes for which appearance is not a requisite.

The author has not observed any regularity by districts in the distribution of this leaf pyrite, every seam in Illinois containing some of it, at least locally. For instance, at one mine no leaf pyrite is found in seam No. 6, while at another mine only a few miles distant it is present in the same seam. This form of pyrite cannot usually be removed by mechanical washing, because it adds little to the weight of the individual piece of coal to which is it attached. In general, operators troubled by it, though at a disadvantage in the open and domestic markets, should have little difficulty meeting competition based on specifications since the trouble often looks worse than it really is.

The total sulphur content of Illinois coals ranges from 1 to 6 per cent, most of it being held in the forms previously noted.* The remainder, probably from ¼ to ¾ per cent of the coal, is in some not well understood chemical organic combination, probably with the hydrogen and carbon of the coal. Such organic sulphur supposedly is completely burned in the process of combustion. This form of sulphur is present in all coals, and is so intimately combined with the coal substances that it is not apparent to the eye and cannot be mechan-

*S. W. Parr, Bulletin 16, Illinois State Geological Survey, p. 226.

ically separated from it. For this reason, it may be classed with the ash of Group 1. The minute percentages of phosphorus present in coal may be classed with this sulphur. Such organic sulphur does not lessen the value of a coal for combustion.

Only those forms of sulphur, such as pyrite, which occur as a mineral in the coal and which after combustion leave an incombustible basic residue such as iron oxide (FeO), promote the formation of the ash slag known as clinker. However, a complete solution of the clinker problem involves a study of combustion temperature and conditions* and of the proportion in the ash of the basic compounds (iron or lime) to acid compounds† (silica and alumina), and of their relative sizes and admixture. Pyrite alone need not cause trouble.‡

Calcite ($CaCO_3$), and in smaller quantities, gypsum or calcium sulphate ($CaSO_4$. $2H_2O$), and still smaller amounts of other salts occur in much the same way as flake pyrite. Many Illinois coal beds contain these thin flakes of calcium sulphate and of gypsum in the small joints and cleavage planes of the freshly mined coal. If these minerals are present in considerable quantity their whitish color offers an easy means of detection, and removal may be made by hand picking. Seams Nos. 1 and 7 especially contain in places considerable amounts of these impurities. The presence of gypsum adds a percentage of sulphur to the coal. Whether or not these impurities may be removed by washing depends upon their amount, thickness, and tenacity to adhere to the coal. At only a few mines is attention paid to them during preparation, their presence usually being of chemical interest only.

Another probably subsequent impurity is the clay or shale which often fills small vertical fissures through, or in the top or bottom of the coal bed. These are locally called slips, horsebacks, or mud seams. In some districts, for instance around Springfield, these impurities occur frequently in seam No. 5, and are called "clay slips." They are probably caused by the soft floor or roof material having worked, under pressure, into local slips or faults in the coal bed. Local rolls in roof or floor, causing thinning of the seam, together with more or less intermixture of coal and shale often occur. The method of separation of any of these from the coal is similar to that of removing the shales as noted in Group 2. Many such disturbances are avoided in mining and they are not necessarily an impurity.

Niggerheads are sometimes present in or just above the coal bed. In the same way that pyrite through deposition forms sulphur balls, calcium carbonate, or iron carbonate, upon deposition around a favorable nucleus, may form oval concretions or niggerheads often several inches or even feet in diameter. These impurities, readily detected on account of their size and shape, can be easily removed by the miner when shot down with the coal in the mine.

Even the coal part of a seam is not a solid homogenous mass of

*S. W. Parr, Bulletin 16, Illinois State Geological Survey, p. 226.
†F. R. Wadleigh, Coal Age, June 22, 1912, p. 1206.
‡W. B. Phillips, Coal Age, July 27, 1912, p. 111.

pure shining coal. A casual inspection shows that the seam is built up of alternate bands or laminae of bright, shiny coal, and dull, lustreless coal. According to T. E. Savage,* a close examination of much of the Illinois coal shows that these alternating laminae are generally between 1/32 and ½ inch and often more in thickness, and that in places the dull laminae make up nearly one-half of the coal bed. Coal in which the bright bands predominate has in the small sizes a bright appearance approaching anthracite, popularly supposed to be necessary with a good coal.

A powdery dull thin band generally of paper thickness, called mother of coal, mineral charcoal, or carbonized wood, forms a distinct parting between many of these layers or laminae in Illinois coals. On exposure such a band leaves a slight smut when touched with the finger. If of any thickness, it dusts badly when the coal is mined and must in such cases be a definite factor in the formation of fine dust. Although somewhat unlike the other dull coal in appearance, this mother of coal is usually associated with and constitutes a part of such a band.

David White and R. Thiessen,† and James Lomax,‡ have studied microscopically the bright and dull bands in coal and their conclusions are that dull bands may be as truly coal substance as the bright bands with which they are associated. E. C. Jeffrey¶ came to the same conclusions concerning mother of coal or mineral charcoal. These statements do not apply to the definite bone or shale bands which, although dull in color, are entirely different in appearance. An analysis of the dull mother of coal layers from seam No. 6 in Williamson county§ showed them to contain the same amount of ash as the average coal from the seam. Several analyses made recently by M. L. Nebel,** however, showed in every case higher ash values in the dull bands than in the bright bands in the same lump of coal. A sample of mother of coal taken from a band ½ inch in thickness in seam No. 6, Williamson county, and analyzed under the direction of the writer, gave the following proximate analysis: Moisture (as received) 0.16 per cent; volatile matter 9.75 per cent; fixed carbon 87.47 per cent; ash 1.72 per cent; and sulphur 0.90 per cent. This indicates that mother of coal is a high grade coal of different composition than the rest of the seam. Since the presence of the dull laminae and the mother coal are specially prominent in Illinois coals and since coal of a dull appearance is at a disadvantage in the open market, further analytical work concerning the relative purity and composition of these bands is desirable.

*Journal of Geology, Vol. 22, No. 8, 1914.
†"The Origin of Coal," Bulletin 38, U. S. Bureau of Mines, pp. 29 and 64.
‡"Microscopic Examination of Coal," T. I. M. E., Vol. 42, p. 2.
¶Economic Geology, Vol. 9, No. 8, p. 734.
§T. E. Savage, Journal of Geology, Vol. 22, No. 8, 1914.
**Results of these analyses will appear in Bulletin 89 of the Engineering Experiment Station, University of Illinois.

Impurities Entering the Coal from Roof and Floor.—In a majority of the mines in Illinois the roof directly above the mineable coal is of a soft crumbly nature. In some of the thicker coals, as in seam No. 6 in the southern districts, this roof is protected by leaving the bench of top coal in place. In other districts the shale roof is only a few inches thick, and when it spalls off or is taken down, is found to be overlaid by a firm limestone or other resistant roof. In mines in which the coal is under about six feet in thickness and in which it is necessary to take out the whole seam, excessive use of powder weakens the roof, which frequently breaks off, sometimes in large slabs, but oftener in small scales. When this "drawslate" is removed some of it unavoidably becomes mixed with the coal. Seam No. 6 in the Danville district has a notably crumbly roof. This impurity is more likely to occur in solid shooting mines than in machine worked or in longwall mines. In the longwall field, however, the soapstone or slippy shale roof is weak and brittle, and falls unless closely propped. In the state as a whole there are conditions of roof and customs in the use of powder which tend to introduce portions of this roof into the coal as an impurity. As a rule, this material keeps its structure well enough to make possible its removal at the proper place by picking or by mechanical means unless extra breakage allows it to pass into the screenings.

The different seams in the state generally have a soft weak clay shale or fireclay bottom. Often this floor is extremely hard when first exposed and makes a suitable bottom from which to shovel the coal; in other places it is softer and mixes with the coal. Usually, after exposure for a few weeks to the dry atmospheric conditions in the mine, the top layers of such a floor crumble and become loose and dusty. Sometimes the floor expands, swells, and rises, thus increasing the probability of the presence of impurities in the coal should shoveling be necessary at such places. In most of the cases which come under the writer's observation, the lumps of bottom fireclay, although they may be hard and firm, disintegrate if placed in water even for five minutes. Under like conditions in the mine the bulk of this fireclay passes into the screenings during preparation. In parts of the longwall field hand pick undercutting is practiced in this under shale band, producing considerable amounts of fine shale, which become mixed with the coal and pass into the screenings. Under this condition a clean high grade lump coal and dirty screenings are produced. This practice is not uniform.

In the room and pillar mines floor impurities are generally introduced through three causes:

(1) Careless shoveling.

(2) Excessive disturbance of the floor by heavy shooting.

(3) Undercutting in the bottom clay instead of in the lower bench of coal.

Careless shoveling explains itself. Concerning the second cause, W. R. Coleman* says that long 6 to 7 feet holes in solid shooting mines in Illinois may pick up from 4 to 10 inches of bottom mud, which becomes mixed with the coal during shoveling into the mine car. Although these are extreme cases, they illustrate the possibility of the addition of considerable impurity if such conditions are not kept under control. Little difficulty should be experienced in undercutting if the bottom or floor is fairly level. In some places uneven floors formed by rolls and horsebacks may be cut into.

PART III. REMOVAL OF IMPURITIES.

Removal of Impurities Underground.—Most of the larger pieces of separable impurities which have been mined with the coal may be removed underground by hand sorting during loading, but the time spent in doing this reduces a miner's daily output of loaded cars; consequently, there is an inclination to minimize this work. The regulations pertaining to this subject, as a part of the evolution of present preparation practice, have been outlined in Chapter I, pp. 23 to 29.

The claim has often been made that the most efficient place for inspection of coal for impurities is underground during loading. However, as the miners usually work in pairs only and in separate rooms, continual underground inspection in the dim light is impossible. At several mines in which persistent bands of impurities occur, the assistant mine manager makes regular trips through the mine and by noting the size of the waste or rejected pile, or the character of the coal in a partly loaded car, he is able to estimate rather closely the percentage of rejected impurities. If he detects carelessness, word is sent to the regular coal inspector at the surface plant to watch closely for the cars of the miner in question. It is probable, however, owing to the multiple duties of the assistant mine manager and to the limited number of these officials allowable under the agreement between the operators and miners, that underground inspection is casual rather than systematic.

Once in the mine car further inspection is impossible until the coal is dumped or spread out on the screen in the tipple. One mine only was visited in which inspection was attempted in the loaded cars at the shaft bottom before hoisting. Since the natural tendency is to put clean coal on the top of the car, it is difficult to see how such inspection could be effective. At the particular mine noted, condemned cars were not hoisted into the tipple, but were set off at the surface landing and kept there for the inspection of the interested parties, and in full view of all men entering or leaving the cage. It was explained that the effect of this was noteworthy.

*Proc. 1st Annual Convention, International Railway Fuel Association, 1909, p. 22.

It is necessary to emphasize how closely all the impurities loaded with the coal are within the control of the miner at the face. If he takes pride in his trade, is careful, undercuts with skill, uses powder with judgment, and loads with care, a considerable part of preparation has been accomplished, and he has done his part towards making a good name in the markets for the mine in which he works.

Removal of Impurities in the Tipple.—At practically all mines in Illinois an attempt is made to remove the bulk of impurities loaded underground at one or more of the following places in the surface plant:

(a) In the tipple during screening.

(b) On picking bands or belts after screening.

(c) During the loading of the railroad cars underneath the tipple.

At one of these places a coal inspector, dock boss, or rock man is stationed, who not only watches for impurities and penalizes the miner according to the joint agreement (see page 24), but often has general charge over the men picking and loading the coal. Some companies place an inspector at each tipple, others employ a single inspector who covers two or more of their neighboring surface plants. Large companies frequently have one chief inspector who superintends dock bosses at the individual tipples. In all cases close inspection was reported beneficial in lowering the percentage of ash in the coal. Frequently, the buyer is notified if a railroad car which has been loaded contains defective coal. An economy of 6/10 of one per cent will pay for inspection at the mines.*

At a few tipples in the state the dock boss "rides" the screen; that is, he is seated on a stationary support just above the shaking screen, from which with perhaps one or two assistants, he can bend down and remove the impurities from time to time. If the screen is narrow (7 feet or less), the dock boss and his assistants may stand at the sides and remove impurities; in either case throwing the rejected material into chutes which lead to a refuse car or pile below. Such inspection is not very efficient in the dusty surroundings if a considerable amount of impurities is present, if a large tonnage (perhaps 2,000 tons or more) passes over the screens, or if wide screens (7 to 10 feet) are used. Another factor hindering the detection of impurities is the mixture of sizes from nut to the largest lumps, which must be examined at the same time. In such a mixture of sizes, lumps are bound to hide the impurities of nut size. This difficulty is overcome at several tipples by the use of a patented combined screen and picking table. The screen is horizontal and of sufficient length and movement to separate efficiently the sizes and allow picking on the screen under favorable conditions. This screen will be described later.

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*Eugene McAuliffe, Proc. 4th Annual Convention, Int. Ry. Fuel Assoc., p. 280.

The Black Diamond, under date of Feb. 1, 1913, stated that picking tables after screens were new in Illinois. The demand for cleaner coal forced Illinois operators, especially at the large mines, in which from 2,500 to 5,000 tons per day were handled in the tipple, to adopt these picking tables or belts as a method of removing the refuse. The picking tables (Fig. 19), are from 3 to 5 feet wide, often 30 feet long, 30 inches above the floor, and are endless belts either of rubber or of steel links covered with small over-lapping sections of steel plate. The tonnage handled is up to 1,000 tons per day per belt. These belts catch the coal as it falls through or leaves the screen and convey it horizon-

Fig. 19. Operation of a Picking Table.

tally at right angles to the screen. Each size of coal may have an individual belt or picking table. Thus the coal, spread out on the belt and traveling at a speed of from 30 to 60 feet per minute, passes before men or boy pickers who can easily pick out the refuse. Dirty coals, of course, require low speed and additional pickers. Speed depends also on the size, on the color, and on the shape of the impurities; that is, the ease with which they can be distinguished.

In general, picking tables, bands, or belts have the following advantages:

(1) Thorough inspection; all coal being spread out in fixed sizes and moving at a fixed rate.

(2) Decreased length of chutes and height necessary to feed certain tracks.

(3) Loading of coal in a constant stream with minimum velocity and breakage.

(4) Safety and effectiveness of pickers; they are no longer exposed to the dust on the screens or to danger in the loading cars.

(5) Combined with a movable loading boom, they reduce further the breakage and allow easy loading on cars of different heights. The only disadvantages are first cost and maintenance.

At one mine in which pyrite refuse is saved from the tables, six rock pickers, three on lump and three on egg, pick out 50 tons of pyrite per day, or about 8 tons per man per day. On account of the high percentage of refuse, stationary plows are fixed on the tables close above the belt, so that the moving coal may be turned over and thus expose to view any hidden impurity.

At other mines, including some of the largest, inspection and picking are carried on only when the railroad cars are being loaded with the screened coal. The stream of coal filling the car is closely watched by one or two men, who throw overboard from time to time the noticeable pieces of impurities. They often use large rakes to pull the impurities from under the falling stream of coal. In such cases picking is for appearance only, the actual amount of impurities removed being usually under 1 per cent of the size loaded. At some mines only the lump sizes are picked. At other mines the egg and nut sizes as well as lump sizes are picked. At these mines the dock boss usually watches the railroad cars and not the screens for impurities. Fig. 20 illustrates car picking on egg coal.

Car picking is sufficient if the amount of refuse is small or consists of only occasional and accidental pieces, possibly one or two tons per thousand tons loaded. In these cases it is possible to remove all noticeable impurities from the top of the car before shipping. Car picking is not sufficient to clean systematically a dirty coal. A piece of impurity even if detected cannot always be removed from a stream of lump coal falling and rolling into a car without exposing the workman to danger of injury from the large lumps. Consequently such impurities are buried rather than removed. The danger of being injured by the small egg and nut sizes is not so great. In one case as much as 1 per cent of the egg size is thrown out of the car during loading; however, the percentage is usually much smaller. The removal in this way, as refuse, of a noticeable percentage of the coal would probably be commercially impossible. Table 6 shows detail of picking practice at several mines chosen at random.

The chief difficulty with hand picking in any form is that of securing conscientious labor to do the work. At several of the tipples visited the laborers made little pretense at picking unless the dock boss was in constant attendance. Another feature is the amount of

good coal going to waste with the shale and sulphur. For instance, if an impure band ½ inch thick is noted in perhaps an 8-inch lump of

FIG. 20. HAND PICKING AN EGG COAL DURING LOADING.

coal, the whole is likely to be discarded. An examination of some of the refuse heaps to which the rejected lumps are hauled, revealed many tons of good coal. It is unfortunate that this waste may be

TABLE 6.

Details of Tipple Picking Practice in Illinois Mines.*

Mine No.	Commercial Designation District	Picking Lump Coal				Picking Egg Coal				Picking Nut Coal				Coal Inspector	Remarks
		Tons P'k'd per Day	How Picked	Tons Wasted per Day	No. of Men	Tons P'k'd per Day	How Picked	Tons Wasted per Day	No. of Men	Tons P'k'd per Day	How Picked	Tons Wasted per Day	No. of Men		
1	Saline county...	315	In R. R. cars only	2	1	...	Not picked.....	–	–	...	Not picked.....	–	–	Part time	
5	Franklin county.	800	In R. R. cars only	1-2	1	800	On picking belt...	8-12	2-3	...	Rescreened	–	–	Yes	
7	Franklin county.	600	In R. R. cars only	–	–	660	R. R. car.....	–	–	480	R. R. Car.....	–	–	Yes	Waste: 10 tons per day.
9	Williamson " .	900	In chute and car	7	2	420	On picking table..	3	3	...	Washed	–	–	Yes	8–10 trimmers and pickers.
11	Franklin county.	600	On picking table and loading boom	–	–	600	On picking table..	–	–	400	Rescreened	–	–	No	Total waste: 3 tons per day.
17	Sangamon " .		No picking except occasionally if coal is dirty.....			...	.....	–	–	...	.....	–	–	Yes	
24	Montgomery " .	450	On picking table and loading boom	11	2-3 boys	400	On picking table..	10	2 boys	...	Not picked.....	–	–	Yes	
32	Longwall field...	360	In R. R. cars....	2 3	1	360	Not picked.....	–	–	...	Not made.....	–	–	Yes	Occasional pieces of sulphur.
34	Standard district	650	On screens.....	5	2	650	On screens.....	5	1	390	On screens included in other sizes.	–	–	Yes	3 men pick
35	Standard district	1300	On screens.....	5	–	1300	On screens.....	5	–	...	Not made.....	–	–	Yes	2 men and inspector pick on screens.
40	Standard district	1000	In R. R. cars....	2-3	1	...	Not picked.....	–	–	165	Not picked.....	–	–	Yes	
42	Marion county..	550	On screen and table	–	–	440	On screen and table	–	–	440	On screen and table	–	–	Yes	12 pickers; 60 tons waste per day.
43	Marion county..	720	In R. R. cars....	1	1	...	Not picked.....	–	–	...	Not made.....	–	–	Yes	Inspector picks.
45	Perry county....	700	On screen and table	–	–	360	On picking table..	–	–	340	On screen and table	–	–	Yes	4 pickers; 6 tons waste per day.
47	Perry county....	700	In R. R. cars....	1	1	420	On picking table..	4	2	420	On picking table..	4	2	Yes	
48	Danville district	150	On picking tables.	–	3	600	On picking table.	–	3	...	No nut made.....	–	–	No	50 tons impurities per day.

*Mines were chosen at random.

a commercial necessity at the present time. At a few mines these lumps are pulled from the picking belt, the impure band split off, and the good coal returned to the belt. This operation is called "skinning" the coal.

Sizing should take place before the picking is begun. If picking is attempted on 1¼-inch lump coal the eye of the picker cannot readily detect the different sizes of impurities mixed with the different sizes of coal. The more uniform the size of the coal that passes the picker, the better the impurities can be removed. A recently built dock cleaning plant sizes the coal for picking purposes, and then reunites the sizes to meet the demands of the market.

Other important questions regarding picking are: How small sizes can be hand picked? How shall the screenings and small sizes be cleaned? Should they be hand picked or washed? In general, what are the limits of refuse today in a commercial Illinois coal?

In European bituminous coal fields hand picking has been practiced for many years. In comparing their practice with our own, it must be kept in mind that cheaper labor and higher priced coal allow a closer and a greater range of work than is possible here. Various authorities treat this problem as follows:

"The process of hand picking can be successfully applied to pieces of coal and shale of 1½ inches to 2 inches in diameter, but not to smaller pieces. Washing can be used from 3 inches to 1/20 inch. There is no known process of separating shale dust from coal dust."*

"Hand picking cannot economically be applied to coal less than 2 inches in diameter, except in exceptional cases as where coal is fairly clean or wages low, in which case picking takes place even at 1 inch. It is usually done by boys, as it requires alertness of hand and eye."†

"On the continent sizes over 4 to 5 centimeters (1.6 to 2 inches) are generally picked by women or boys, sizes smaller than these are frequently washed."‡

In many places in Illinois hand picking is practiced on the lump size only, in others the 3-inch egg is the lower limit. It is probable that unless the refuse in such sizes had some special value it would not pay to mine and hand pick coal in Illinois containing more than from 3 to 5 per cent of refuse in the large sizes. Fortunately, in most districts the bulk of refuse passes into the finer sizes.

At present in Illinois washing is practiced on 3 or 3½-inch coal as a maximum size.¶ This is partly on account of the mechanical difficulties encountered in building jigs for washing larger sizes. The average amount of impurities removed from the washed coals in 35 plants examined was 11 per cent, with a maximum of 36 per cent and

*W. Galloway, "Lectures on Mining." Subject 8, p. 2.
†W. S. Boulton, "Practical Coal Mining." Vol. 3, p. 315.
‡J. Callon, "Cours d'Exploitation des Mines." Texte 3, p. 152.
¶F. C. Lincoln, "Coal Washing in Illinois," Bul. No. 69, Engineering Experiment Station, University of Illinois.

a minimum of 5 per cent. It is difficult to estimate an exact limit for commercially profitable washing because of the varying character of the refuse and its degree of freedom from contained coal. On the whole 3½ inches is the maximum size, and about 7 per cent refuse, containing at least 60 per cent of ash or its equivalent, represents a possible minimum of removable impurities. Under European conditions this limit has been put at from 4 to 6 per cent impurities.* As a lower limit, coal under ¼-inch in size is benefited little by washing and under from 20 to 50 mesh probably not at all. Considerable doubt exists as to the proper method of handling the small sizes of Illinois coals on account of the frequent occurrence of a high percentage of ash. These sizes are discussed further under Sizing in Chapter IV.

At a considerable number of mines, especially in the southern part of the state where rescreening plants are used for the separation of nut coal and screenings, all sizes below from 2 to 3 inches are divided into as many as five distinct sizes. Most of the impurities generally stay with the finest of these sizes, leaving two or three of the largest sizes of nut coal practically as clean as the lump. At several mines one or two men were noted in these rescreeners picking the largest sizes of nut, perhaps from 3 to 1¾ inches in size. Although no figures of amounts picked were obtainable, it is evident that the large number of pieces of this size that must be picked out of the coal to produce a ton of refuse makes the problem of doubtful economic value. If two men are picking 100 tons per day of such nut, the extra cost is roughly five cents per ton. From the ash standpoint one man must pick one ton per day of this fine material to reduce the ash content one per cent. Does the comparatively small amount picked pay for the better appearance? This question must be solved for each coal and each market condition.

Impurity and Inspection Standards.—Coal inspection is becoming more rigid, and large users, such as railroads, frequently have their own inspectors, even at the mines of the producer. Much coal is inspected by the buyer by a hasty examination of the top of the railroad car; but to secure the best results, more strict examination, such as inspecting the inside of the load while it is being loaded is necessary. If hopper bottom railroad cars are used and delivery is made through these to a bin or to a stock pile, individual car inspection is difficult.

A set of impurity and inspection standards has been proposed for railroads using bituminous coal as follows:

"The seller further agrees that all coal delivered under this contract shall not contain more than....per cent removable noncombustible or nearly noncombustible impurities. The quality of coal furnished....shall be subject to the inspection of the buyer, and the buyer's inspector....has the right to reject any of said coal which.... does not conform to specifications, at whatever point the same may be

*Soc. Min. Ind., Vol. 17, p. 384.

TABLE 7.

MAXIMUM PERCENTAGE OR LIMITS OF REMOVABLE IMPURITIES PERMISSIBLE IN GRADES OF COAL FROM THE DISTRICTS AS LISTED.

Mining District	Lump		Egg		Mine Run	Egg Run†
	Size	Percentage Impurities Allowed	Size	Percentage Impurities Allowed	Percentage Impurities Allowed	Percentage Impurities Allowed
Canton, Ill.....	1¼" R*	2.0	1¼" x 8" R	3.0	2.0	...
Virden, Ill.....	1½" R	1.5	1½" x 6" R	2.5	1.5	...
Centralia, Ill.....	1½" R	1.5	1½" x 6" R	2.5	1.5	...
Herrin, Ill.....	2 " R	1.0	2 " x 6" R	1.5	1.0	...
Oskaloosa and Albia, Iowa....	1¼" bar	4.0	.....	...	...	...
Centerville, Iowa	1¼" bar	1.0	.....	...	1.5	...
Bevier, Mo.....	¾" bar	2.0	.....	...	3.0	...
Novinger, Mo....	⅞" bar	3.0	.....	...	3.0	...
Walsenburg, Colo.	.....	...	.....	...	2.0	...
LaFayette, Colo..	.....	...	.....	...	0.5	...
Cambria, Wy.....	¼" Sq.	4.0	.....	...	4.0	...
Sheridan, Wy....	4 " bar	0.2	2½" x 4" bar	0.2	0.5	0.7
Kirby, Wy.....	.....	...	.....	...	0.8	1.0

*R = Railroad.

†See p. 104.

found. The buyer's inspectors shall have access to the seller's tipple screens, scales, washer, and yards while the coal is being hoisted and prepared.

"All coal delivered....may be inspected at the mines by the.... (representative) of the buyer....such inspection and refusal to be final and conclusive."*

A further paragraph covers the discovery of inferior coal after shipment when the same was not inspected at the mines.

Besides individual contracts, the most specific standardization of Illinois and other middle west bituminous coals in regard to the percentage of noncombustible or nearly noncombustible impurities allowable has been used by several railroads drawing their supply from mines in the territory mentioned. Through the kindness of C. G. Hall, Secretary-Treasurer, Int. Rwy. Fuel Assoc., figures of the limits or removable impurities as allowed by these railroads have been secured (see Table 7).

Table 8 shows the amounts of different impurities, etc., which are allowed to remain in the coal in the standard preparation of Pennsylvania anthracite,† and is given for comparison with Table 7.

TABLE 8.

IMPURITIES ALLOWED WITH STANDARD ANTHRACITE PRACTICE.

Size of Coal	Allowable Percentage			
	Slate	Bone*	Of Next Size Larger	Of Next Size Smaller
Broken	1.0	2	—	20
Egg	2.0	2	5	50
Stove	2.5	4	5	50
Nut	4.0	5	10	15
Pea	8.0	5	5	15 Buckwheat 15 Rice
Buckwheat	10.0	—	8	15
Rice	15.0	—	8	15
Barley	15.0	—	8	25

†Bone equals product of between 40 to 55 per cent of carbon.

PART IV. BREAKAGE.

Bituminous coals vary from 0.5 to 2.0 on Mohr's scale of hardness.‡ They are, as a rule, decidedly brittle and friable, tending to break into more or less cubical blocks, depending on physical structure, such as frequency of bedding laminae or planes of stratification, and development of cleat or vertical cleavage. Cleat may be defined as

*Inspection Specifications, Proc. 5th Annual Convention, Int. Rwy. Fuel Assoc., 1913, p. 28.

†Paul Sterling, "Preparation of Anthracite." T. A. I. M. E., 1911, p. 757.

‡Henry Lewis, "The Dressing of Minerals." p. 8.

the tendency for the coal to break into more or less cubical blocks along vertical or highly inclined planes either parallel or normal to the face of the coal.

Natural Physical Factors Causing Breakage in Illinois Coal.—Illinois coal may be considered comparatively hard, rather porous, more or less brittle, and friable; tending to break or split easily along definite major bedding laminae 5 to 12 inches apart and somewhat less easily along small or minor bedding planes. Vertical cleavage, prominent in many bituminous coals, is generally lacking or is developed only incipiently. This incipient cleavage is more marked in one direction than in the other, the lump coal usually being rather smooth and regular on the breakage faces in two directions, but tending to be irregular on the third face. This incipient cleavage determines the lines on which the coal frequently breaks into small blocks on first being heated. This cracking, which is probably due to shrinkage caused by the heat expelling the water, is also often more strongly developed in one vertical plane than in the other.

Exceptions to this rule occur in the case of east and west cleavage in parts of seam No. 6 in Franklin county, and in that of a cleavage in seam No. 2 in Jackson county (Big Muddy district). Here the cleavage is developed sufficiently to influence the direction of mining, as the ease of mining when working along the face cleavage here, tends to produce a larger percentage of lump coal with less powder than in the majority of mines. In most districts, however, mining takes place with no thought of cleavage.

There is considerable difference in the natural tendency towards breakage of coal in the different seams and in the different districts in Illinois. The coals of seams No. 1 and No. 2 in the northern field are of average hardness (firmness or cohesion); an example appears in the coal from seam No. 2 in the La Salle district, which breaks into rectangular lumps capable of withstanding much handling before breakage.* The coal from seam No. 5 in the Springfield and Peoria districts is harder and firmer than the average. Seam No. 6 west of the DuQuoin anticline, although covering a large area, is generally regarded as somewhat friable, especially in the northeast and southwest portions and also in those mines just west of the anticline, in which it is close to the surface. Seam No. 6, east of the DuQuoin anticline, and in general wherever it is situated under several hundred feet of cover, consists of a hard firm coal, especially in the smaller sizes. This coal withstands without breakage handling and shipment, and therefore forms a good domestic fuel in markets in which clean coal and close sizing are prerequisites. The coal of seam No. 5 in Saline county is also of a firm texture and well adapted for rescreening. Seam No. 6 in the Danville district contains rather friable coal. Seam No. 2 (Big Muddy), partly by reason of the cleavage noted, produces blocky lumps of a bright lustre. The coal of seam No. 7, Danville district, on the whole

*T. A. I. M. E., Vol. 29, p. 187.

is softer and more brittle than the others, perhaps on account of its nearness to the surface. It cannot undergo transportation as well as the average.* It is interesting to note that in the deepest mine in the state, at Assumption, a coal is produced from seams Nos. 1 and 2, which commands a premium, partly on account of its blocky firm character.

Importance of the Breakage Problem.—Aside from the inherent friability or tendency of the coals to break, as just discussed, the mining and preparation, or in general, any moving, dropping, or handling of a brittle substance like Illinois coal results in more or less breakage with each operation. Breakage, therefore, commences with the first operations of mining, continues with each succeeding operation, and ceases only when the coal is consumed. This gradual breaking of such a material into smaller sizes due to handling, exposure, etc., is called degradation.

None of the problems confronting the Illinois mining industry today is more important commercially than that of breakage. Although this problem, as far as surface preparation is concerned, could be solved so that a considerable increase in value per ton of the whole product would result, yet it has been neglected in all except a few plants recently erected. Breakage constantly reduces the larger sizes to form the smaller. The effect of this sizing on the prices received for the coal by the operator is shown in Fig. 21, prepared from the figures appearing weekly in "Black Diamond" during the years 1913 and 1914 for the average circular price f. o. b. cars at the mine in Franklin county. Although the curves do not represent the true price received in individual contracts, yet taken over such a long period, they represent the average difference in value of the sizes produced. The egg and nut sizes, although ranging with the lump in circular price, are frequently unsalable as such, and must be thrown into the cheaper steam sizes. The average price of lump coal during this period was $1.50 per ton, and the average price of screenings $0.73 per ton. The mere question of size reduces value over 50 per cent. Thus it is advantageous to produce the free burning Illinois coal in as coarse lumps as possible.

If these figures represent the average difference in price between lump coal and screenings, each per cent of lump gained (or each per cent of breakage reduced) is worth 0.77 cents per ton to the operator. At a mine at which three thousand tons per day are produced this means $23.10 per day or nearly $600.00 per month saved.

In the tipple a long, high angle dump chute, a poorly designed weigh box, or a drop of several feet onto the screens may cause the breaking of several per cent of lump coal into the finer sizes. At some mines the sound of the coal when dumped from the car is audible above the rest of the surface noises at a distance of several hundred yards from the tipple. The louder the noise the greater the

*Eng. and Min. Jour., Vol. 63, p. 165.

breakage. In some cases the dumping takes place with such force that the coal actually seems to be thrown from the car through a distance of from twelve to fifteen feet into the bottom of the weigh box, in which it is more or less broken by the rest of the load falling upon it. While the great speed of such apparatus, in some cases three dumps per minute, is a notable achievement, good

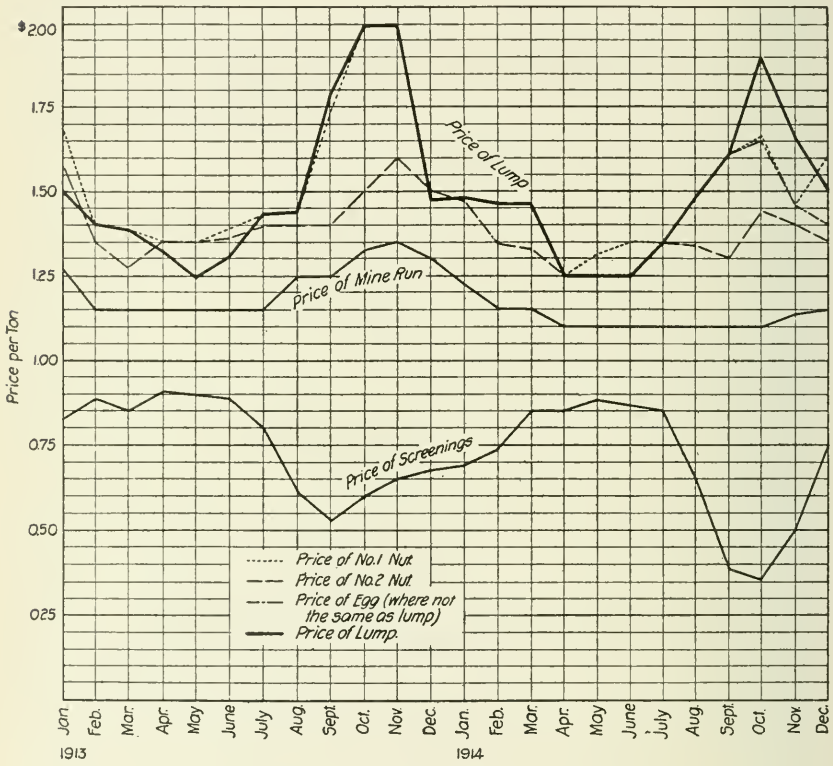

Fig. 21. Average Circular Price of the Different Sizes of Coal. Franklin County (1913-14).

engineering demands a closer study of the problem of decreasing the breakage during preparation.

Breakage or Degradation Standards.—The adoption of some standard of breakage for Illinois coal is desirable. Especially in making specifications in which close clean sizing is required it would be of advantage to have a standard allowable amount of breakage under fixed conditions with which each coal could be compared, in order that

it might be designated as harder or softer (more friable), or as containing more or less fines than the standard. If Illinois coal withstands transportation and handling better than competitive coal from other states, such a standard inserted into specifications, would emphasize a fact that is of great importance to retail dealers, especially to those to whom fine coal represents almost a dead loss. To show the difference in the friability of different bituminous coals, a certain bituminous coal in dock handling on lake shipments degraded from 30 to 35 per cent in two cases, while another bituminous coal under the same treatment degraded only from 12 to 14 per cent.*

While there has been much discussion concerning degradation on various coals no standard of degradation has been introduced into general practice. Attempts have been made by the U. S. Geological Survey† to fix such standard for the purpose of comparing the friability of coal briquets. For illustration, 50 lb. of briquets made from an Illinois coal were dropped five times through a distance of 6½ feet, and then screened on a one-inch wire screen; 35.5 per cent of the material passed through the screen. The briquets were then compared with others in this respect.

The U. S. Bureau of Mines‡ has adopted a similar standard for comparison of cokes. A fixed weight of coke is dropped 6 feet onto a cast iron plate. This operation is repeated four times. Then the sample is screened over a standard screen, and the percentage of fines is determined.

J. B. Porter¶ determines the comparative friability of fine bituminous coal and dust, by taking coal under ¼-inch screen in size and screening it through the following sieves: ¼-inch; ⅛-inch; 14-mesh; 24-mesh; 50-mesh; and 100-mesh, a screen ratio of practically 2. The percentage of each size is then plotted, and the assumption made that the several coals range in friability in the order of their percentage of fine material.

It is evident that in any set of breakage test on bituminous coal the large lumps of such a friable and non-homogenous material may break easily and the smaller sizes not so easily, or vice versa. The position of the coal in the seam, whether in the top, middle, or bottom bench, and the percentage of ash or other impurity in a particular piece have a decided effect on breakage. For these reasons any set of tests must be relative rather than absolute. Tests along the above lines are now being conducted in the Mining Laboratory of the University of Illinois to ascertain if such breakage standards are of any practical value.

Theoretical Considerations of Breakage.—Considered from a theoretical standpoint, breakage of coal takes place by splitting along

*Records of Interstate Joint Conference, Philadelphia, Pa., Feb. 10, 1914, p. 914.

†Bulletin No. 332, p. 44.

‡Technical Paper No. 50.

¶"Coals of Canada." Vol. 1, p. 195.

planes of minimum strength rather than by actual shearing or crushing of the coal substance itself. Thus the actual measure of the force in pounds per square inch required to crush or break coal is extremely variable. The application of Rittinger's theory* that the work necessary to break or crush minerals is proportional to the reduction in diameter, shows, for example, that three times as much power must be expended in breaking 6-inch cubes into 1½-inch cubes as in breaking 6-inch cubes into 3-inch cubes.

The kinetic energy developed in any lump of coal being moved; in other words, the force with which one piece strikes another and causes breakage, is directly proportional to its weight and to the square of its velocity, or if dropping, directly proportional to the distance through which it is dropped. In simpler language a piece thrown or moved with twice the speed of another, has four times as much power to break or to shatter. The practical effects of these laws will become evident when breakage in bins, in chutes, and in other surface plant devices is discussed. Since force is proportional to weight, the general effect of size or weight is that large pieces striking on their own ragged edges sustain breakage from a drop that will not affect the smaller sizes.

If coal is dropped piece by piece from a height of from 10 to 15 feet upon iron or steel it will show from three to four times as much breakage as if dropped onto wood.† It is claimed, in unloading a bin or a car, that the first drop only causes breakage and that after the pile below has started there is little breakage. Probably the breakage is greater in the dumping of large volumes than in the dumping of small volumes of coal. Still another point is the effect of squeezing and rubbing of the particles against one another in a bin from which coal is being drawn from the bottom. If other conditions are equal, the actual breakage is greater in handling or transporting sized coals than in handling or transporting run of mine coal because the fines in run of mine coal form a bed or nest upon which the large sharp pieces of lump may ride without breakage.

Data on Degradation of Bituminous Coals.—There is a practical limit to the freedom of a particular size of coal from smaller sizes, because reduction in the amount of smaller sizes beyond a certain point by rescreening means the making of an additional amount of the large coal into fines. At one mine it was reported that each rescreening of the coal meant 5 per cent was broken into small sizes and passed the screens. Concerning this point, "Experiments show that there is a limit beyond which there is no advantage in attempting to rescreen and further prepare anthracite and bituminous coal. In one instance in which a ton of hand picked coal was used the process of screening furnished 3 per cent undersize which went through the screen, while a larger per cent of undersize formed in the handling

*R. H. Richards, "Text Book of Ore Dressing. p. 167; also, H. von Rittenger, Lehrbuch der Aufbereitungskunde, p. 19.
†Black Diamond, August 29, 1914, p. 165.

still remained in the coarse coal. In repeating the operation, another 3 per cent of undersize was caught, while the (coarse) coal again held a proportion of undersize similar to that previously found. It is believed that the process might be repeated indefinitely without materially benefiting the grade of the coal."*

In one test made by a consumer in Chicago clean Illinois nut coal over ¾-inch screen was shoveled into a wheelbarrow, weighed, dumped, and then rescreened over the same screen. In one lot 4 per cent and in another lot 2 per cent degradation resulted from the shoveling and dumping.

At a mine in the central part of the state, the degradation of clean screened coal of from 3 to 6 inch size, which was rescreened after passing through a bin 20 feet in height was 5 per cent. In a similar case in the southern part of the state 2 per cent degradation was reported.

Another test, made on a car of carefully loaded domestic lump coal by removing clean lumps from the car with a fork, showed 5 per cent of smaller sizes left in the car. J. D. Rogers† states that a breakage of 5 tons per car, or 10 per cent, is common and is even greater if loading is carelessly done.

In elevating and passing an Arkansas bituminous coal through a 50-foot railroad coaling station the increase of slack was 25 per cent.‡ This coal is somewhat softer than Illinois coal.

An English test on bituminous coal falling 10 ft. showed a loss of 1s. 3d. per ton or 15.6 per cent on coal valued at 8s. per ton.¶

At a large Chicago retail yard a degradation of 7 cents per ton is allowed on all sizes of bituminous coal passing through the yard; on an estimated average value of $2.50 per ton this would be a loss of about 3 per cent. J. W. Hardy** records the breakage on bituminous coal as 4 per cent for every 10-foot vertical drop.

Transferring Coal in Railroad Cars.—Considerable discussion has taken place regarding the amount of degradation caused by the transferring of coal from one railroad car to another en route. H. C. McKinney in The American Coal Journal for Dec. 5, 1914, states that the loss of 25 cents per ton (at the point in question this amount represents possibly 10 per cent of the value) conceded by the railroads is insufficient. Since the practice of transferring coal from one car to another en route is common, more data on this point would be valuable.

The random references given show the present day practice in estimating the general breakage on Illinois and other bituminous coals. In every case dropping coal even once causes serious breakage. Considering the treatment that Illinois coal receives in the tipple in prepara-

*Eng. and Min. Jour., Feb. 16, 1907, p. 339.
†"Preparation of a Domestic Coal." Kentucky Min. Inst., December, 1912.
‡Proc. 5th Annual Convention, Int. Rwy. Fuel Assoc., p. 257.
¶Gillott on Kirkby Colliery. Inst. of C. E. of London, Vol. 127, 1897, p. 177.
**Black Diamond, Aug. 29, 1914, p. 165.

tion, it is doubtful, especially on the prepared sizes, if it pays to strive for tonnage records at the expense of extra breakage of the coal.

As anthracite is essentially a domestic coal prepared under standard practice, the results of tests made to determine its breakage are given for the purpose of comparison.

(1) Test on Dropping.* Sized anthracite dropped 7 feet onto iron plate gave 0.77 per cent breakage into finer sizes, and dropped 7 feet onto wood plate gave 0.34 per cent breakage into finer sizes. In other words, the breakage of anthracite upon striking iron is more than twice as great as upon striking a softer material like wood.

(2) Tests on Handling and Shipping.† The loss by breakage in storage and picking up averages 2 per cent, but varies from ½ per cent with the coal and with the amount of handling. At one plant in which coal is dropped into a hopper beneath the car, though the fall is small, the breakage measures ½ per cent. The breakage in shipping east without rescreening is from 2 to 3 per cent. In shipping west, in which case the anthracite is transferred and rescreened, breakage is from 8½ to 9 per cent. By breakage is meant the actual amount of all sizes smaller than the size specified.

(3) Breakage by Dropping.‡ A summary of tests made both by dropping carefully sized anthracite through measured distances and by dropping carload lots into pockets is given in Table 9.

TABLE 9.

Breakage of Anthracite by Dropping.

Size	Amount of Breakage into Smaller Prepared* Sizes	Amount of Breakage into Sizes Smaller than Nut	Total Amount of Breakage
Broken	3% plus 43/100D†	2% plus 17/100D	5% plus 6/10D
Egg	4% " 43/100D	2% " 17/100D	6% " 6/10D
Stove	2% " 33/100D	2% " 27/100D	4% " 6/10D
Nut	.....	4% " 40/100D	4% " 4/10D
Pea	.....	2% " 50/100D	2% " 5/10D
Buckwheat	.....	1% " 25/100D	1% " 25/100D

*Prepared sizes are broken, egg, stove, and nut.
†D equals drop of coal in feet.

Table 9 shows that not only is the percentage of degradation through fall for a hard coal like anthracite surprisingly large, but also that breakage increases with an increase in size and in height of drop. The breakage in an Illinois coal must be considerably greater.

*Black Diamond, July 12, 1913, p. 16.
†Mines and Minerals, Vol. 25, p. 23.
‡R. V. Norris, "The Storage of Anthracite." T. A. I. M. E., Vol. 42, p. 316.

General Analysis of the Breakage Problem.—In general during mining and preparation of a coal, breakage may occur in any of the following necessary operations:

- A. Breakage in the mine incident to.....
  - a. Undercutting.
    - 1. In longwall mines.
    - 2. Shooting after undercutting and solid shooting.
    - 3. Permissible explosives.
  - b. Snubbing and drilling.
  - c. Breaking the coal from the face.
  - d. Handling ..
    - 1. Loading.
    - 2. Haulage.
    - 3. Dumping into a skip at shaft bottom.
- B. Breakage in surface plants incident to...
  - e. Use of self-dumping cage.
  - f. Dumping into weigh box and onto the screen.
  - g. Screening.
  - h. Loading .....
    - 1. Passage over loading chutes, aprons, or booms.
    - 2. Loading and trimming railroad cars.
  - i. Rescreeners and bins.
  - j. Washing (not included in this bulletin).
- C. Breakage in transportation and rehandling (not discussed fully in this bulletin).

Breakage in the Mine.—Undercutting.—About 52 per cent of the tonnage in Illinois is produced in mines in which undercutting is practiced either by hand picks, by machines of the puncher type, or by electric chain machines. The last named type is the most common, and its use is increasing. Contrary to the practice in many bituminous fields, undercutting in Illinois takes place almost entirely in the coal; the exceptions being at several mines in the longwall field, in which hand picks are used to undercut in the clay bottom. At other scattered mines, the line of undercutting follows some thin dirt band in the lower part of the seam, and at others in which the floor rolls or is irregular, accidental undercutting of the floors takes place, in all of which cases the cuttings are high in ash or impurities. In general, however, the miners avoid working in the tough bottom clay. This leads to the reduction into fine sizes of a certain portion of any particular seam, the percentage depending on the height of the undercut; that is, on the particular machine or method used.

The cuttings obtained, called "bug dust" by the miner, are generally assumed to consist only of the finest sizes of coal, and on account of a high percentage of dangerous dust contained, are often loaded out before breaking down the face of the seam undercut. Therefore, cars of bug dust are generally at the foot of the shaft when hoisting begins in the morning; consequently, the first railroad cars of screen-

TABLE 10.

AMOUNTS AND SIZES OF CUTTINGS MADE BY VARIOUS MINING MACHINES.

1	2	3	4	5						6		7	8	9	10	11	12	13
				Screen Analysis Percentage						Chemical Analysis of Cuttings								
				a	b	c	d	e	f	a	b							
Mine	District	Seam Number	Kind of Undercutting	On 1"	On ½"	On ¼"	On 10 Mesh	On 100 Mesh	Throu 100 Mesh	Per Cent Ash	Per Cent Sulphur	Total Undercut During Test. Sq. ft.	Total Weight of Cuttings, lb.	Pounds of Cuttings per sq. ft. of Seam Undercut.	Total Per Cent of Cuttings if Seam Were 6 ft. Thick.	Per Cent of Cuttings Under 1¼" r'nd hole. i.e., 1¼" Screenings.	Weight of Cuttings per sq. ft. passing 1¼ in. Round Hole Screen.	Per Cent of 6 ft. Seam Made into Screenings by Hand or Machine Mining.
1	Northern Longwall	2	Hand	33.2	20.2	18.7	17.3	10.3	0.3	17.5	4.7	14	1180	84.3	17.3	66.8	56.3	11.6
2	Southern Room and Pillar	6	Electric	11.2	12.3	13.9	21.4	39.4	1.8	14.0	2.7	129	4075	31.6	6.5	88.8	28.0	5.8
3	Southern Room and Pillar	6	Ch. Mach.	3.7	15.2	23.7	34.5	22.1	0.8	10.2	0.9	166	5290	31.8	6.5	96.3	30.6	6.3
4	Southern Room and Pillar	6	Ch. Mach.	5.9	16.4	20.4	31.0	25.2	1.1	9.2	1.5	130	4393	33.8	6.9	94.1	31.8	6.5
5	Southern Room and Pillar	6	Ch. Mach.	6.6	19.4	22.8	25.6	25.2	0.4	13.0	1.2	94	2935	31.2	6.4	93.4	28.1	5.8
6	South-western Room and Pillar	6	Ch. Mach.	12.8	15.6	19.8	22.4	28.2	1.2	17.1	6.8	64	2150	33.6	6.9	87.2	29.3	6.0

7	Southwestern Room and Pillar	6	Air Puncher	37.2	13.5	13.8	17.8	17.7	0.0	26.5	5.1	55	3780	68.7	14.1	62.8	43.1	8.9
8	Danville Room and Pillar	7	Air Puncher	35.4	16.4	15.4	17.3	14.8	0.7	17.5	4.1	26.8	2750	103.0	21.2	64.6	66.5	13.7
9	Southwestern Room and Pillar	6	Air Puncher	45.1	13.2	13.5	15.6	12.0	0.6	27.6	6.7	87	11550	133.0	27.3	54.9	73.0	15.0
10	Southern Room and Pillar	6	Air Puncher	28.5	17.8	16.6	19.9	16.2	1.0	15.8	1.2	32.6	2250	69.0	14.2	71.5	49.3	10.1

ings loaded may contain more than the usual percentage of this fine coal, which is often of lower grade than the regular screenings.

There has been considerable discussion as to which type of machine makes the least cuttings, coarsest cuttings, lowest percentage of dangerous dust, etc. To throw light on these and other pertinent questions in connection with the use of machines in Illinois mines, H. H. Lauer of the Department of Mining Engineering of the University of Illinois, in connection with the Co-operative Investigation of Illinois mining conditions, made a series of ten tests at mines in which undercutting was variously practiced with hand picks, punchers, and electric chain machines of different types. In each test a room or a definite length of face in a mine was carefully cleaned, and then undercut by one of the above methods as in regular practice. The total amount of cuttings produced was carefully collected, weighed, and sampled. One sample was saved for analysis while another was passed through various standard screens, the object being to ascertain the percentages of the various sizes made, and in this way to compare the cuttings produced by the various methods of undercutting. These operations were all carried out underground at the face. The full results have not yet been published, but through the courtesy of the Co-operative Investigation the results regarding amounts and sizes of cuttings, summarized in Table 10, were secured.

The screens used in these tests were of square mesh; the openings, therefore, were slightly greater in area than the corresponding sizes of round holes, which are more convenient as a standard. A screen ratio or sieve scale of ½ inch was used, excepting with the smallest size of screen—100-mesh. The percentage of coarse coal on the 1-inch screen is high in mines in which pick mining or puncher machines were used. The very small percentage of cuttings through a 100-mesh screen is surprising. In other words, machine cuttings are more granular than they are commonly supposed to be, and the percentage of dangerous dust (at least through 100-mesh) is smaller than was anticipated.

Column 6, a and b, shows that, as a general rule, cuttings are of lower grade than the average face sample from the same seam (see Table 5). The presence of a much smaller amount of ash in the top benches of the seam at many mines in the state than in the lower benches influenced unfavorably the results given in Column 6. Column 7 gives the total square feet of the seam undercut during the test, and Column 8 the total weight of cuttings; consequently, dividing Column 8 by Column 7 gives Column 9, which shows the pounds of cuttings made per square foot of the seam undercut.

To standardize the test still further, the seams are assumed to be 6 feet thick, and Column 10 shows the total percentage of a seam made into machine cuttings on the basis of a six-foot seam. The

proper percentage to allow for a seam of any other thickness may be easily calculated.

If machine cuttings were large enough to pass into the lump sizes their quantity would be of minor importance. If a 1¼-inch round hole is taken as the standard size over which lump may be prepared (this size is nearly equivalent to a 1-inch square hole opening); Column 11 shows that considerable difference exists in the percentage of the cuttings which are fine enough to pass a 1¼-inch round hole; namely, to be screenings. The percentages in Column 11, recalculated on the basis of weight in Column 12, are reduced to the basis of a standard six-foot seam, and are given in Column 13. Percentages for thicker or thinner seams may be easily figured from this calculation. If there were no other breakage in coal mining and preparation, this percentage of the seam would represent the amount of 1¼-inch screenings made by undercutting.

The tonnage won after undercutting has in each case been assumed to be that part or block of the seam directly over the undercut. The frequent practice of boring holes deeper than the undercut increases the tonnage, and so decreases the percentage of the seam made into screenings by undercutting. The puncher machine and hand pick show more favorably in Column 13 than in Column 10; that is, while these machines make more cuttings than the electric chain machines, the sizes of their cuttings are coarser. An average of the electric chain machines shows 6.08 per cent screenings, and of punchers 11.92 per cent. The writer visited two mines of the same company within four miles of each other at which ½-inch bar screens of the same size were used. At one of these mines 11 per cent screenings was produced with electric mining machines; at the other, 17 per cent screenings was produced with punchers. These facts tend to confirm the reliability of the figures given in Table 10.

The agreement of the electric chain machines in Column 13 is remarkable, five machines in five different mines differing only 7/10 of 1 per cent in total cuttings when reduced to a common basis of measurement. Such a difference or even a greater one might depend upon the nature of the coal cut, upon the number of positions of bits used in the chain, upon the kind of bits; that is, chisel or pick point, and also upon whether the bits were sharp or dull. The height of the cutting opening made, or "kerf," also varies somewhat with different machines.

Excepting special manufacturers' tests, this is believed to represent the first work on this special problem carried out in the central bituminous field. In Mines and Minerals for March, 1908, p. 397, a table is given for similar work carried on in the Westmoreland mine, Pittsburg Seam, Pennsylvania. For purposes of comparison the results are given in Table 11.*

*See also G. S. Rice, "The Explosibility of Coal Dust." U. S. Bureau of Mines, Bul. No. 20, p. 35.

TABLE 11.

AMOUNTS AND SIZES OF CUTTINGS MADE BY VARIOUS MINING MACHINES IN THE PENNSYLVANIA BITUMINOUS DISTRICT.

Method	Total Cuttings		Through 40 Mesh	
	Pounds	Per Cent	Pounds	Per Cent
Puncher	3436	10.95	394	1.250
Chain Machine..	1836	5.86	155	0.494
Hand Pick.....	4533	14.45	128	0.408

The percentage of total cuttings here is also reduced to a six-foot seam as a basis of common measurement.

Considering the physical differences between the coals of the two fields and other possible conditions, as noted previously, the general agreement of these results with those of Table 9 is significant.

Snubbing and Drilling.—By snubbing is meant the practice common in some districts before shooting, of cutting a triangular section from 18 inches to 3 feet high from the lower face of the coal above the undercutting. If this is not done the shooting is likely to loosen the coal only enough to fill the space undercut and, unless excessive powder is used, does not break it sufficiently to allow easy loading. Proper snubbing causes the coal to roll and spread out in the room with a minimum of powder.

The amount of snubbings made is less than 3 per cent of the average seam, and since the work is often done with a hand pick and there are two free faces to break to, the relative sizes of the snubbings are large. No determinations of the sizes have been made. Since from only two to five 2-inch drill holes are bored to a depth of from 5 to 8 feet in a single face, the percentage of fine coal made in this way is too small to be considered. These cuttings, however, contain a considerable percentage of fine dust. One per cent or less screenings probably represents the fine coal made by snubbing and drilling.

Breaking the Coal from the Face.—By the method of applying roof pressure or wedging as in longwall mines, the coal breaks slowly and into blocks, governed by the resultant direction of pressure and by the larger incipient cleavage planes, and consequently loosens ready for loading with a minimum of breakage. At certain longwall mines as small an amount as 15 per cent screenings (1¼-inch round hole or ⅞-inch bar) has been reported after loading, hauling, hoisting, and dumping over the screen. "On the whole, the longwall field shows 15 to 20 per cent more lump coal over 1¼-inch screen as compared

with the rest of the state."* These figures agree with those gathered by the author at individual mines. Although the question is somewhat complicated by partial hand pick undercutting at some of the long-wall mines, the conclusion seems logical that the amount of screenings in the coal at the face need not be greater than from 5 to 10 per cent.

The amount of fines or breakage produced in shooting is largely under the control of the miner (Chapter I). Heavy charges of powder shatter the coal. No conclusive data on this subject are available, and it will be discussed by making a comparison of general results obtained from the practice of two forms of mining; namely, shooting after undercutting vs. solid shooting.

In the bituminous field of Arkansas the change at one mine from undercutting to solid shooting increased the slack coal 14 per cent and the amount of slate from 11 to 23 per cent in three years; in the state as a whole, the change to solid shooting increased the amount of slack coal 50 per cent.† It is also claimed that overshooting weakens the lump coal so that it readily slacks off on standing and is more easily broken by handling. A retail dealer stated that when a car of clean lump made with undercutting was unloaded at destination, from 3 to 5 per cent of degradation was left in the car, while under solid shooting the amount was as great as 15 per cent.‡ A. A. Steel states that Oklahoma has had like experience,¶ and other writers complain that heavy solid shooting not only increases the amount of fines but also jars the coal and breaks the grain, producing incipient shattering, and even though it may hold together until it passes over the screen, it disintegrates more easily afterwards.

A direct comparison of these methods has been made in several cases of Illinois. A. J. Moorshead, in a paper read before the 1913 meeting of the Illinois Mining Institute,** states that from long experience he believes Illinois coal mined by machine has from 3 to 10 per cent less screenings on the average than the same coal shot off the solid, but that some of the harder coal gives as few screenings when shot off the solid as when undercut. Also he states that the percentage of screenings made from the coal from seam No. 6 in Williamson county when shooting off the solid is larger than that made when undercutting is practiced, the difference being probably from 7 to 10 per cent. The difference varies with the manner of shooting. At a mine†† in one district in the state, where shooting off the solid is practiced 55 per cent lump over 1¼ inches is made, while at three mines where undercutting is practiced 70 per cent lump is made over the same size screen. A study was made of 100 typical mines throughout

*S. O. Andros, Co-op. Bul. No. 5, p. 40.
†A. H. Perdue, Proc. American Mining Congress, 1909 to 1911, p. 227.
‡J. E. Turney, Ibid., p. 233.
¶Proc. 3rd Annual Convention, Int. Rwy. Fuel Assoc., p. 56.
**From reprint in Colliery Engineer, 1914, p. 435.
††S. O. Andros, Co-op. Bul. No. 6, p. 23.

the state;* at 33 mines where shooting off the solid is practiced an average of 65 per cent of the coal produced is larger than 1¼ inches, while at 43 mines where undercutting is practiced 67 per cent of the coal is larger than this size.

A personal communication to H. H. Stoek gives the following test made at an unnamed Illinois mine:

TABLE 12.
COMPARISON OF SIZES PRODUCED, SOLID SHOOTING VS. UNDERCUTTING.

Test	Per Cent of Lump	Per Cent of Nut	Per Cent of Slack
Solid Shooting.....	35	30	35
Undercut by Electric Machine	60	20	20

Generally, the amount of screenings made by either process of shooting is measured by screening through the tipple screens. Since the amount of breakage in handling, in weighing, and in screening the coal, varies for different mines, a direct comparison of breakage by shooting in sometimes impossible. Figures gathered by the writer concerning these two classes of mines in the same districts, show in every case as great or a greater percentage of screenings at solid shooting mines than at undercutting mines. Of two neighboring mines in the southern part of the state, one at which solid shooting is used, 35 per cent of screenings through an 1¼-inch round hole are produced. At the other, where electric machines are used, the same percentage of 1½-inch round hole screenings are produced. At a third mine in this district, where electric machines are used, 33 per cent 1½-inch round hole screenings are produced.

Permissible Explosives.—Since the amount of permissible explosives used in the state in 1915 amounted to 1,342,334 pounds, their influence on the coal is of considerable importance. Concerning permissible explosives vs. black powder, S. O. Andros† doubts that permissibles make more slack if properly used.

J. J. Rutledge and Clarence Hall state, "Permissible explosives have come into use so recently that it is not easy to get reliable figures showing the increased proportion of fine coal they make as compared with black blasting powder. The estimates of this increase given by mine superintendents run from no increase to 10 per cent. Some superintendents maintain that although smaller lumps of coal may be made by using permissible explosives, yet changing from black blasting powder to these explosives does not increase the proportion of fine coal. Some persons state that the lumps of coal produced by using permissible explosives are not so easily broken up during trans-

*S. O. Andros, "Coal Mining in Illinois," Co-op. Bul. No. 13.
†Co-operative Bul. No. 8, p. 34.

portation or exposure to the air as are those made by using black blasting powder, whereas other persons maintain the reverse. However, if the coal is undercut or sheared and the blasting is done with judgment, the permissible explosives make as good coal as black blasting powder and at approximately the same cost."*

J. R. Fleming, Assistant Engineer of the U. S. Bureau of Mines, has gathered data for a bulletin on "Use of Explosives in Illinois with Special Reference to Permissibles." These data show that the percentages of fines made by the use of permissibles at several mines are less than when black blasting powder is used; in other cases little difference is noted, while in still others an increased amount of fines in recorded when permissibles are used. Following are summaries from several mines:

Mine No. 1. The extreme case reported in which permissibles made more fines than black blasting powder.

Size of Coal	Year 1910 Black Blasting Powder	Year 1912 Permissibles
6 in. lump	24.19 per cent	20.41 per cent
6 in. x 3 in. egg	19.28 per cent	21.29 per cent
3 in. x 2 in. nut	16.50 per cent	12.88 per cent
2 in. screenings	40.03 per cent	45.72 per cent
Total	100.00 per cent	100.00 per cent

Mine No. 2. A more favorable case.

Size of Coal	Black Blasting Powder	Permissibles
6 in. lump	15.2 per cent	13.5 per cent
1¼ in. x 6 in. egg	49.5 per cent	51.0 per cent
1¼ in. screenings	35.3 per cent	35.5 per cent
Total	100.0 per cent	100.0 per cent

Mine No. 3. A very favorable case.

Size of Coal	Black Blasting Powder	Permissibles
2 in. lump	47.4 per cent	50.0 per cent
2 in. screenings	52.6 per cent	50.0 per cent
Total	100.0 per cent	100.0 per cent

The conclusion is that if permissibles are used with judgment no material increase of fines results.

Handling the Coal.—Loading.—Loading the broken coal into the mine car in Illinois is done with a hand shovel. The only exception to this rule occurs in the case of two stripping mines at which the broken coal is loaded with small steam shovels. At only one mine, near La

*"The Use of Permissible Explosives." U. S. Bureau of Mines, Bul. No. 10, 1912.

Salle, is the seam inclined enough to necessitate the use of chutes or other rehandling devices. Breakage from this cause, which is often a serious question in other states in which the seams are highly inclined, is therefore at a minimum. Whether or not the miner loads his coal in the largest possible lumps or rebreaks it, either by more powder, by pick, or by the back of a shovel into sizes convenient for shoveling, is a somewhat disputed point. It is not uncommon, however, to see mine cars containing lumps possibly as large as 12 inches by 18 inches, 2½ feet long, and weighing 300 pounds; as large in fact as the two men in the room can lift into the car.

Haulage.—The coal, having been loaded into mine cars, is moved to the foot of the shaft by mule or electric locomotive, or both. At times the writer has heard complaint about breakage caused by such transit, especially if in making up a trip the locomotive bumps the loaded cars, jarring the contents considerably and even breaking the top load.

Probably the custom of excessive topping, or loading the coal to too great a height above the top of the car, is responsible for most of the breakage in haulage. Bumping easily knocks off lumps from a poorly topped car which fall on the roadway and become broken. At comparatively few mines in the state is the load on the car limited; these few, however, have a maximum limit. As an illustration, at one mine the car is supposed to hold 4,000 pounds and if any carload weighs over 4,500 pounds, the excess weight goes to the check weighman fund. Such a rule is intended to prevent loss and breakage caused by excessive topping of cars. It is probable that breakage from actual car transportation, whether below ground or in railroad cars, has been rather overestimated.

Breakage Through Dumping into a Skip at the Shaft Bottom.—At nearly all the mines in the state coal is hoisted to the surface and into the tipple in the mine cars. Attempts have been made to dump the coal from the mine car at the bottom of the shaft into skips which are hoisted into the tipple and there dumped automatically. A similar arrangement is common at ore mines and is used at some coal mines in the Appalachian districts in which the coal is used for coking, and in which extra breakage is a benefit.

When a mine in the southern part of this state abandoned this system for the ordinary self-dumping cage, an estimated gain of 10 per cent in the coarser sizes resulted. A mine in the longwall field using skips produces 15 per cent more screenings over the same size of screen than the average of six other mines in the district using cages. In Franklin county a mine operating 7-ton skips reports 6 per cent more 1¼-inch screenings than the average of thirteen other mines in the district, and only 12 per cent of 6-inch lump against an average of 22 per cent for the other mines mentioned which hoist the mine car in self-dumping cages.*

*S. O. Andros, Co-op. Bul. No. 8, p. 48.

If, however, as in one or two cases in the state, the output of a mine is used for special purposes not affected by the general commercial market requirements for large lumps, skips may be the best solution of the engineering problem of a small shaft and high tonnage requirement.

Breakage in Surface Plants.—Final preparation and sizing in the tipple are carried on under the advantages of daylight and easy access to all parts of the plant. Underground the coal is subjected to a series of movements causing breakage, some necessary and some unnecessary, but all more or less under the control of the operator and miner. In the surface plant the success attained depends largely upon the design and erection of the tipple building, chutes, screens, conveyors, loading devices, etc. The operator usually feels that this part of his mine plant is distinct from the rest, and its design and erection is usually entrusted to some engineering firm which makes a specialty of distinctive and often patented designs of headframes, weigh boxes, screens, chutes, and other equipment.

The general types of Illinois coal mine tipples or surface plants have been described in Chapter II. While special engineering features of tipples are reserved for future discussion, several features causing breakage should be mentioned at this point.

Breakage from Use of the Self-Dumping Cage.—In older designs, the cage, after being brought to a quick stop is tipped suddenly with considerable throw and jerk, throwing the coal out of the tipped car. At the required angle for dumping, the coal at the top end of the car is frequently from 6 to 8 feet above the dump shoe, thus giving a considerable momentum to the outgoing load. If the shape of the dump shoe and connecting chutes is carefully designed, the coal will slide out with a minimum of breakage and danger.

In the interests of safety as well as breakage more care should be taken to prevent the coal being dumped from missing its proper chute and falling either down the shaft or to the surface around the headframe. It is not unusual to see considerable coal falling when dumping takes place.

Dumping into Weigh Box and onto the Screen.—The coal on being dumped from the car slides down a short dump chute into the weigh box. In one of the older tipples a dump chute, 6 feet long and set at an angle of 75 degrees, gives an additional falling height to the coal of about 5 feet. At this mine 4 per cent more screenings are produced than at a neighboring one with a modern tipple. The increase in screenings can be attributed to such defects. The tipple breakage is even apparent to the eye.

The slope of the bottom of the type of weigh box, pan, hopper, or basket illustrated in Fig. 22 is usually 35 to 40 degrees, and since the upper end may be from 10 to 12 feet above the lower end, the distance moved and the speed acquired by the coal may cause considerable breakage before it comes to rest and is ready to be weighed.

This is evident from the flying bits of coal noticeable in the usual type. The weigh box or basket is usually made large enough to hold two or even three loads from the mine car, and although this means waste space, lost headroom, and chance for extra breakage if used for single weighings only, the large capacity allows the hoisting to proceed continuously even though the screens below may be stopped.

Weigh boxes with a more gentle slope or at least a changing and decreasing slope, which would bring the coal to rest without a shock and then would deliver it on the screens below in a more gradual manner, would be an improvement. Weigh boxes of this type have been

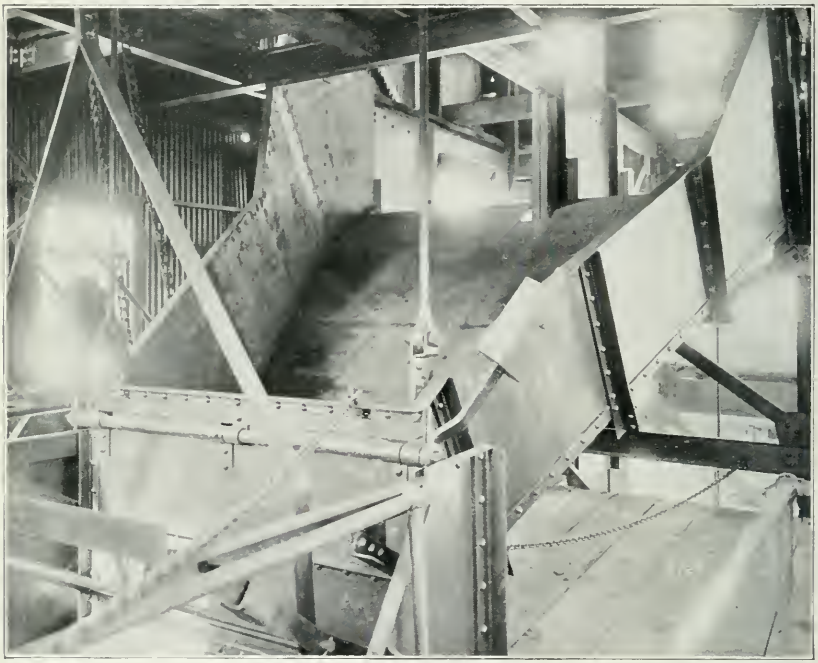

Fig. 22. Weigh Box.

designed, but complaint is made that they tend to decrease the speed of operation. A weigh box having a bottom sloping at a low enough angle to allow a car of dry lump to slide gently may not be steep enough to handle a car of moist or of fine coal, and if the angle is great enough to allow wet coal or slack to run, lump coal usually acquires considerable velocity in passing down the incline.

Use of Screen Feed Hoppers.—The custom of allowing coal from the weigh box to discharge, either at the end or bottom, direct to the shaking screen, is meeting with disfavor for the following reasons:

(1) A drop between the weigh box and screen is a factor in the amount of breakage; in certain tipples a drop of 3½ feet has been noted.

(2) Coal is weighed in from 2 to 5-ton lots, and when this amount is dumped suddenly on the upper screen, it throws an extra weight upon it, which seriously affects the balance of weight that should be maintained between the upper and lower screens, and gives additional vibration to the screen structure.

(3) The sudden rush of material from the weigh box frequently chokes the fine screens (which usually come first), and allows considerable fine coal to be carried into the coarser sizes, damaging their appearance.

(4) The sudden load thrown on the screen puts an undue strain on the driving belt, causing slipping unless it is laced to a considerable and often damaging tension. At one mine two engines are used to drive the screens, thus avoiding the dead center troubles and lessening the strains just mentioned. This scheme has not been adopted generally.

(5) Any stoppage of the screens for the purposes of repair or to shift railroad cars causes the cessation of hoisting after one or two dumps have filled the weigh box; that is, there is no storage capacity between weigh box and screen.

For these reasons, steel feed hoppers (Fig. 23), with sloping sides and holding from two to four mine cars of coal are, in many of the newer installations, placed under the weigh box and at the head of the screen. The bottoms of these hoppers have a reciprocating motion, due to an adjustable crank arm, the full stroke being from 8 to 12 inches, and the speed from 40 to 60 revolutions per minute. The lower part of the end of the box next to the screen is cut away, thus allowing coal to fall on the screen gently and regularly with each back stroke of the bottom. The partial load of coal usually in these hoppers reduces the breakage since a new load from the weigh basket falls gently onto coal instead of striking steel. If such a feed hopper cannot be used on account of a lack of sufficient headroom some form of conveyor with sides may accomplish the same purpose.*

The most serious objection to the use of a feed hopper is that when two or three mine cars of coal are in it at the same time, and the coal is being mixed and fed continuously onto the screen, it is impossible for a coal inspector to properly single out individual cars of dirty coal. Some companies claim that by good engineering design of the various tipple appliances these feed hoppers may be dispensed with.

Screen Breakage.—Gravity Bar Screens.—Usually there is a dead or blank plate at the top and bottom of the bar screen, the former to spread out the coal before screening, the latter to collect it into the loading chute to the railroad car. Since at certain times the coal

*Coal Age, Vol. 1, p. 1143.

may be damp or extra fine, or weather conditions unfavorable, a considerably greater slope is given the screens than is necessary to allow dry lump to screen by gravity. The result is that a lump attains considerable velocity on the screens and suffers breakage when allowed to shoot directly into a railroad car. It is difficult to see how a car of commercial lump of good size and appearance can be made under such conditions, unless a simple steel adjustable chute is installed

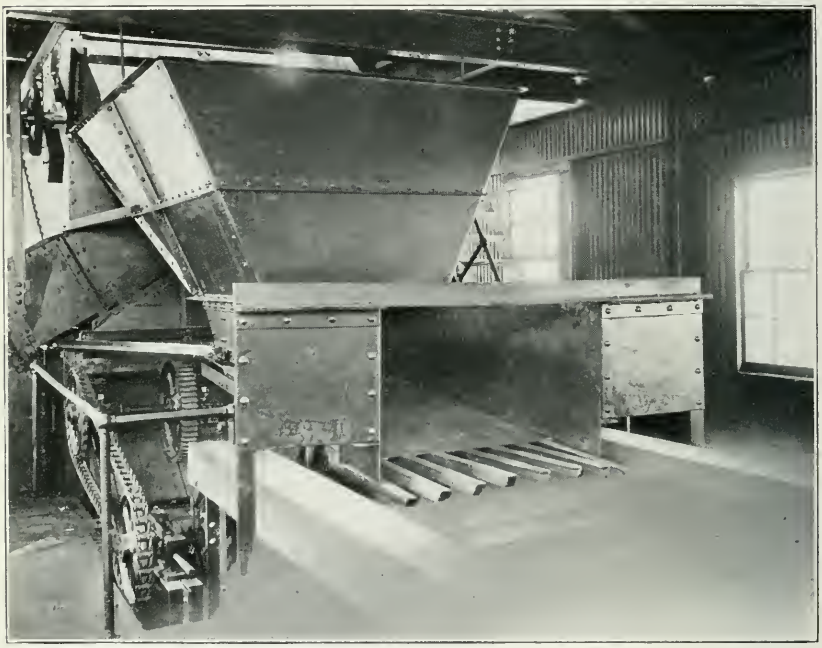

Fig. 23. A Steel Feed Hopper.

between bar screen and railroad car to slow up the coal and allow it to fall more gently into the car. The breakage is frequently increased by the chains, car wheels, or short logs of wood which are hung above and about halfway down the screen for the purpose of checking the flight of the coal or of turning the lumps in order to free them from adhering fines. Also lumps frequently become wedged in the bars, and upon being struck by succeeding lumps are broken and drop into the screenings. At many tipples at which bar screens are used to prepare coal for special purposes not in commercial competition with lump coal from adjoining mines, breakage is of secondary importance; the

bar screen being used to free the lump from the impurities in the slack rather than from the slack coal itself.

Shaking Screens.—The great variance in speed, in slope, and in size of holes, found in the different designs of shaker screens, cause differences in breakage and freedom of one size from another. In most designs the small holes come first, and often become clogged by sudden rushes of coal. The best results are obtained at the mines at which this fine screen is protected by a false one of larger opening placed about a foot above it, for the purpose of providing against overloading and crushing by the bulk of the lump coal. Where drops of more than one foot are allowed in passing from upper to lower screen some splintering of large lumps can generally be detected.

On a number of screens, lumps of coal fall into the larger screen holes and are not dislodged until worn or broken enough to pass into the undersize. This fault may be remedied by increasing the slope, speed, or length of throw of the screen, or by a combination of these points.

Revolving, Trommel, or Roller Screen Breakage.—On account of the breakage caused by roller screens they are not used in Illinois tipples for the screening of lump coal. They are, however, commonly used, especially in the southern part of the state, for producing the smaller sizes of coal, and indeed they were the only kind of screens used in rescreening plants till about 1910. Now, they are being replaced in the newest rescreeners by the Parrish screen or other approved shakers designed especially for small coal. Roller screened coal can often be distinguished by the rounded corners of the individual pieces, showing that attrition has taken place in the screen, with a consequent production of the dust sizes.

The severest criticism of the roller screen is that in shape and in general principle of action it is similar to the Bradford disintegrator, an efficient machine built for breaking coal. At the Joliet plant of the Illinois Steel Company a Bradford disintegrator 11 feet in diameter and 25 feet long with screen openings of 1⅝-inch diameter breaks Illinois lump coal so completely that less than 3 per cent oversize is discharged, and this is mostly hard refuse. Practically all the coal is broken during its passage through the screen. Although the diameter of the disintegrator is 5 feet greater than that of the ordinary roller screen, and although ribs are placed inside to lift the coal and secure extra height of drop, the general parallelism of action holds.

Breakage in Loading—Passage Over Loading Chutes, Aprons, or Booms.—In England the loading of clean lump coal is assured often by loading from the screen or belt by hand. Two men and a boy can load 60 tons per 8 hours in this way. Because of the price of labor and low value of the coal at Illinois mines coal must be loaded mechanically or automatically, even at the expense of slight breakage. Since

most tipple screens have considerable slope, inclined chutes are constructed to lower the various screened products into the railroad cars. Such chutes may be fixed, hinged, shaking, or movable, or may consist of a moving or travelling adjustable steel loading boom, belt or apron (Fig. 24), with which is often combined the picking belt (Fig. 19). Most of the tipples built within the last five years have been equipped for loading the lump and egg sizes of coal with these travelling adjustable loading booms, or with some other form of movable

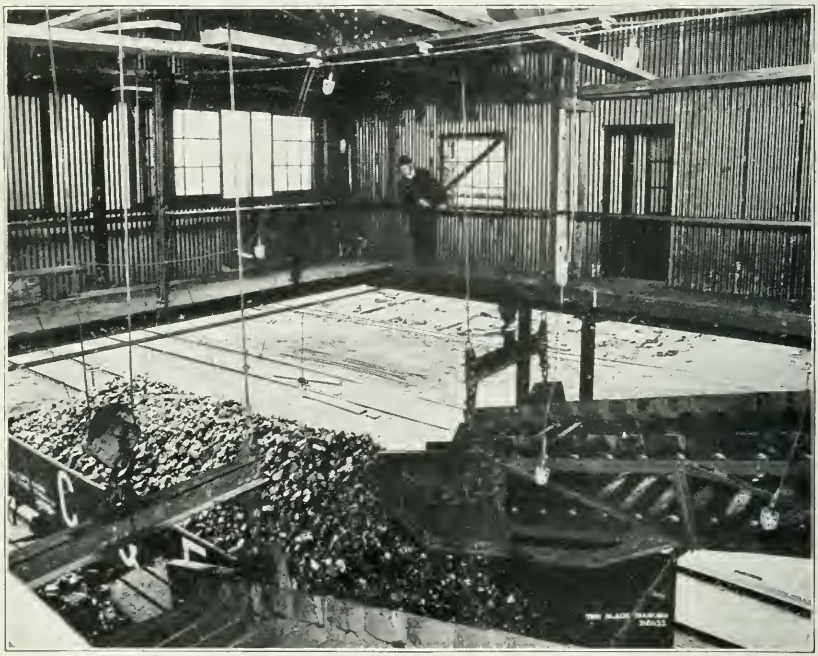

Fig. 24. Loading Boom.

loading chute which can be lowered into the empty car and raised again when that end of the car fills.

Loading chutes may lead directly into the car at right angles to the track or if they have bends or right angle turns or drops they may lead into the car in a direction parallel with its length. Chutes which discharge straight into the car at right angles to the track, unless flat and shaking, shoot the coal into the car. At one or two of the older mines in which these chutes are still in use, a sheet steel buffer is hung over the opposite side of the car in order to prevent spilling, and since the coal comes with considerable force, even trimming and hand

picking are rendered difficult. In a chute with curved bottom the lumps slide down only in the lowest part and one at a time, so that a swiftly moving piece may overtake and break the piece ahead. Otherwise, these chutes give good results, especially if bends are necessary, because they turn the direction of the coal gradually.

At most new installations the loading chutes are placed parallel with the track since from this position the coal can be loaded with less breakage, the chutes can be lowered into the cars, spilling of the coal over the sides of the car can be eliminated to a great extent, hand picking can be conducted efficiently and with greater safety, and the coal can be trimmed more uniformly. Whether or not the flow of coal in such a chute should be with or against the direction in which the loading car is moving is open to argument. Most chutes in Illinois are placed so that the coal moves in the same direction as the car. Advocates of this system claim:

(a) If movable loading booms are used, it is possible to get them well down into the empty car.

(b) Lumps can be examined and picked with safety.

Those in favor of the other system claim:

(a) More regular trimming is possible, especially if the cars are moved by hand or by gravity.

(b) Less breakage occurs, since the coal is always moving in the direction of the slope of the coal in the car. The only reference noted on this subject favors this design.*

Breakage in the loading of the smaller prepared sizes—egg and nut—is not so severe on account of their smaller size, lighter weight, and greater uniformity. In order to meet the demands of domestic trade, these sizes must be unusually free from degradation products. A number of mines have recognized this fact and have installed small degradation or secondary screens in the bottom of the loading chutes for these sizes. These screens, bar, round hole, wire screen hole, or lip, are usually stationary and form the bottom of the chute for their length. They usually have less than 1-inch openings, but they remove the last trace of accidental fines and allow only the clean, sized coal to enter the car. The undersize usually passes into the screenings car or to the boiler room.

The perforations in the lip screens (Fig. 25) are long in comparison with their width, and they become slightly wider at the lower or discharge end. A slight drop in the screen at the end of each series of slots allows wedged pieces to be released at these points. This screen also aids in removing small flat impurities which have been sized on the round hole shaker with the more cubical pieces of coal.

Loading and Trimming Railroad Cars.—If all railroad cars were of the same dimensions as regards height, it would be a simple matter to regulate a chute so that loading might take place with a minimum of breakage. Coal cars differ greatly in design, in height, and in

*A. J. Reef, Coal Age, Dec. 14, 1912.

capacity. They consist of three classes: (a) Gondola cars, with or without dumping devices. (b) Hopper bottom cars. (c) Box cars. The special loaders of the box cars will be treated later. Open gondolas and hoppers are the common classes, and at present range in capacity from 25 to 50 tons, and in height or clearance of sides above the rails they vary from about 7 to more than 11 feet in the newer "high sides" or "battleships," as they are often called at the mines. Such a range of clearance seriously affects the successful loading and trimming of the cars, especially if the chutes are not easily and quickly adjustable, since in "spotting" the cars one with steel sides is likely to be followed by a low wooden gondola. At some of the old mines, screens and chutes are so low that the highest cars cannot be loaded on the lump track. In such a case, they are usually switched under the screenings chute, where more headroom is available and breakage not so important. Also, by this practice, excessive shifting of the loading devices is avoided.

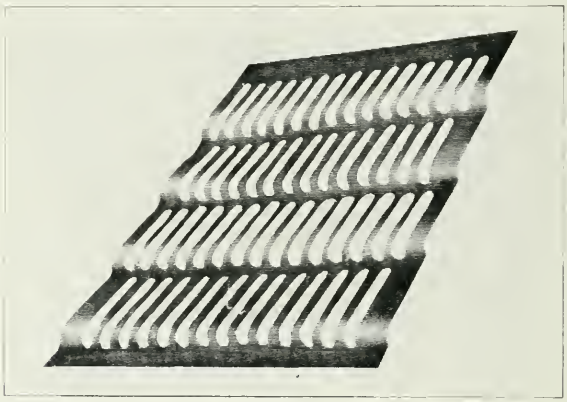

FIG. 25. LIP SCREEN (CUT LOANED BY CROSS ENGINEERING CO., CARBONDALE, PA.)

There is a wide difference in the inside height of coal cars. They range from 3 feet 6 inches in the smaller flat bottom gondolas to 9 feet in the deepest part of a modern steel hopper car. The difference in breakage in loading two such cars from a chute of the same height is evident. At one mine the loading chute had been raised to a height of about 14 feet above the tracks in order to load properly and trim a high side car. The next car was only 8 feet in height and, therefore, the coal dropped about 10 feet before striking the bottom of the car. Another measurement showed that the end of the lump coal chute had a clearance of 8 feet above the car being loaded at the time.

In general, car loading without breakage is an engineering problem which has been practically solved. Frequently the loaders do not take advantage of the adjustable features of the chutes by following the loading closely, and often no attempt whatever is made to use

them. In some cases bad balancing makes the shifting of the automatic loading devices laborious. The newer inclined loading belts or booms are generally controlled by small electric hoists which make adjustment easy.

When loading a car under the tipple, first the end should be filled to the full height of the car, and then the car gradually and slowly moved along, preferably only a few inches at a time. In this way the coal coming from the loading chute strikes near the top of the loaded pile, and after dropping a minimum distance, rolls gently down the slope of the pile into the car. Sometimes the cars are pinched by hand while being loaded, along a grade which has become somewhat irregular by reason of long service, while on other tracks the grade is sufficient to start the car on releasing the brakes; in either case, there is danger of excessive breakage from lumps dropping the full distance to the floor if the car is allowed to move too far. At the more modern and larger mines the cars are retarded and moved either by means of a car puller, which is simply a wire rope attached to a power driven drum, or by a patented car retarder, which consists of a rope attached to a small friction drum under control of an operator. This allows the car to be moved forward as desired and insures a uniform unbroken lump.

While it may be argued that car breakage as discussed has no effect on the percentage of sizes made, the following illustration gives an idea of its effect on the buyer. A large retail dealer gives an average figure of 5 per cent degradation on Illinois lump coal after careful loading, transporting to Chicago, and then unloading with a fork. Other cars, which he thinks must be carelessly loaded, frequently contain up to 15 per cent fines. A saving of 10 per cent breakage in carefully loaded coal is shown by these figures.

Box cars are loaded at many Illinois mines for the following reasons:

(1) General shortage of regular coal cars.

(2) To avoid wetting and possible freezing of the coal in winter.

(3) Prevention of theft in distant shipments.

(4) To prevent box cars returning empty to the states west of Illinois.

(5) Less shrinkage in weight of the coal en route.

(6) Railroads sometimes require box cars to be used for shipments to Texas in order that they may be available there for reshipping cotton.

According to J. J. Rutledge,* twenty years ago in Illinois it was common to see from six to twenty laborers shoveling and carrying lump coal from the center doors of the car and piling it in the ends. Since that time a number of mechanical devices have been perfected, called box car loaders, which load the coal evenly in the car, or where desired.

*Min. Mag., March, 1906.

A modern box car loader in the tipple should fulfill the following requirements: (a) A capacity equal to that of the shaker screen. (b) Ready adaption to the regular coal chutes. (c) Mobility.—Can be moved in and out of the cars quickly. (d) Freedom of injury to the car ends. (e) Freedom from breakage in conveying and depositing the lump coal in the ends of the cars. The first four requirements will come properly in the discussion of the engineering features of the tipple. The fifth feature, breakage, is being overcome in the newer designs.

At a number of the tipples the box car loaders, which had been installed several years previously and were of older designs, were rusty and unused. Inquiry revealed that the main reason for this was "too much breakage of coal," although it is fair to say that other reasons were, "we don't get the box cars we used to," and "the trade don't call for it loaded that way." Another objection to their use is that no inspection or picking is possible in the car. At many mines, however, the loaders are used constantly, and the fact that some of the newest plants have installed them is proof that they fill a need.

Much of the breakage in connection with a box car loader is due to sliding the coal from the shaker screen down the steep chute necessary to bring the lump coal to the box car loader. Mr. A. J. Reef describes the difficulty as follows: "It has been the universal practice to feed box car loaders over stationary chutes, and even when shaking screens are used, the oscillating portion is at such height as to load open cars, and for the loading of box cars, a long and rather steep chute is always employed. The coal necessarily attains such speed in passing through this latter that it is certain to splinter more or less on striking the loader."* Recent installations in Illinois have improved this obvious defect.

Railroad cars are usually loaded to the limit of 110 per cent of their rated capacity, and the railroads require loading at least to their rated capacity. Several devices have been put on the market to make the top or exposed portion of the carload of coal appear free from small coal. A well appearing and nicely trimmed car is attractive and calls the purchaser's attention to the fact that preparation is receiving due attention.†

The St. Louis and San Francisco Railroad Company has published a bulletin on the proper way to load a railroad car with coal.‡ This states that steel cars should not be loaded more than 4 inches above or below the top of the car against the sides and ends. The top load should not be over 26 inches high. The same rules apply to wooden cars, excepting that the top load should not be over 20 inches high. Cars are properly loaded according to the rules if they contain 10 per cent in excess of the marked capacity. This saves one car in ten, which is important when difficulty is experienced in get-

*Coal Age, Dec. 14, 1912, p. 830.
†B. S. Thym, Coal Age, Vol. 2, p. 248.
‡For abstract see Black Diamond, Dec. 6, 1914, p. 21.

ting cars spotted. Further, they note that the above advantage should more than pay for careful trimming. These rules also reduce loss and breakage to a minimum.

Rescreener and Bin Breakage.—These have been considered briefly under Date on Degradation of Bituminous Coal, p. 76. Further data concerning this subject are not available for this bulletin.

Attention has been called to every possibility of breakage because it is believed that the great care taken at the face to avoid the production of fines can be rendered useless by carelessness in handling below and especially above ground, from the time the coal is hoisted until it is shipped. A single illustration, chosen from a reliable source, shows the importance of modern engineering devices in the tipple in preventing breakage.

At a mine in southern Illinois a new tipple was recently installed in which particular attention was given to the question of breakage. Comparative data given in Table 13 show the percentages of sizes made in the old and new tipples, together with a probable realizable value per ton of coal produced. The prices per ton are an average of the circular prices for the Franklin county field for the years 1913 and 1914.

TABLE 13.

Relative Percentages and Prices Received for Coal with Old and New Tipple Equipment.

Sizes	Old Tipple Equipment			New Tipple Equipment		
	Percentage of Output	Value per Ton	Percentage times Value or Value of Each Size in a Ton	Percentage of Output	Value per Ton	Percentage times Value or Value of Each Size in a Ton
Screenings	37	$0.73	$0.2701	33	$0.73	$0.2409
Small Nut	6	1.38	0.0828	7	1.38	0.0966
2x3-in. Nut	12	1.50	0.1800	11	1.50	0.1650
3x6-in. Egg	24	1.47	0.3528	23	1.47	0.3381
6-in. Lump	21	1.50	0.3150	26	1.50	0.3900
Total	100	....	$1.2007	100	....	$1.2306

Without changing underground methods and by tipple improvements alone the value of the coal was increased 3 cents per ton. At a 2,000-ton mine this is an added income of sixty dollars per day; enough to yield considerable profit after covering the extra fixed charges. Another benefit not apparent from an inspection of the table is that with the new tipple the percentage of domestic sizes is somewhat less than with the old. Although about the same prices are quoted for these sizes as for lump, it is well known that they are at times a drug on the market and must be sold at a lower price.

CHAPTER IV.

Sizing and Sizes of Illinois Coal.

STATEMENT OF PROBLEM.

No feature of coal preparation in Illinois has caused so much discussion as the question of sizing coal in the mine tipple for the various markets. The tendency during the past few years undoubtedly has been towards a multiplication in the number of sizes with a consequent increase of mechanical equipment and preparation cost for each ton produced. The introduction of a new size for a particular purpose at one mine has forced producers at competing mines to introduce a like or even greater refinement in order to hold their trade. Such competition has been severe especially among the smaller mines at which the limited tonnage produced makes difficult the disposal of small amounts of special sizes made as a by-product during the production of the more salable special sizes. The domestic trade today demands the larger coal;* thus the considerable amount of fines produced in its preparation must be absorbed by the steam trade. At times this causes demoralization of the steam coal market.

The coal mining industry is seriously asking the question, "Are too many, enough, or too few sizes of coal being made in Illinois today?" A majority of the operators have expressed themselves in favor of a simplification of sizing and of sizes made, contending that (a) formerly the trade was satisfied with fewer sizes, (b) present sizing tends to cause undue loss through the production of considerable amounts of fines during screening and preparation, (c) at certain periods of the year such carefully prepared sizes are not in demand and must be sacrificed with the smaller and cheaper coal, and (d) consumers do not receive sufficient benefit from sized coals to warrant their paying the increased price per ton which careful preparation demands. They also claim that retail dealers do not like to handle so many sizes in their yards. Some even favor shipping run of mine coal only, since the lumps in such coal, having a cushion of fines to ride on, ship with minimum breakage. If sizes are needed for domestic trade, the retail yard could prepare them as needed in a perfect condition. This plan has been adopted at one Chicago coal yard.†

Others contend that (a) the present sizing practice is the legitimate evolution of an increased knowledge and consequent demand on the part of the consumer, (b) much capital expense has been incurred by mining companies in the erection of special screening and rescreening plants to meet this demand, and (c) having won satisfied customers in this way, a change would be folly. Still others contend that the benefits resulting from very closely sized coals are great, and

*Black Diamond, Oct. 10, 1914, p. 281.
†Black Diamond, March 8, 1913.

that even closer sizing is in demand. They favor the multiplication of sizes as fast as the market will absorb them.

Many of the arguments over the above questions fail to consider the basic question: namely, "Have the correct sizes been made?" Is not the present practice in sizing, although complex, based on custom and convenience, rather than on accurate knowledge of what sizes or range of sizes are most efficient for the consumer and consequently of greatest potential value to the producer?

Necessity for Close Sizing.—In discussing the question of sizing Illinois bituminous coal, care should be taken to differentiate it from Pennsylvania anthracite and from the Appalachian bituminous coal with which it competes.

No doubt, close sizing is necessary for the successful burning of anthracite coal. One of the best summaries on this question is given by J. Callon,* who says that Pennsylvania anthracite will not decrepitate enough in the fire to burn without sizing; that it cannot be in too large sizes because the cold mass would be in too great proportion to the incandescent surface; that all the fragments must be of nearly the same size so that all the particles will be under the action of the flame at the same time; that little pieces must not be so numerous as to fill up the space between the large; and that these little particles are useless because they do not coke. Moreover, he states that the hard nature of anthracite permits close sizing without excessive breakage.

In burning the friable Appalachian bituminous coal, a new set of conditions arises. The coal, whether large or small, when introduced into the fire, heats, gives off its volatile combustible matter, swells, becomes pasty, and fuses or cokes into a more or less coherent mass. In hand fired furnaces this coke must sometimes be broken with a slice bar in order to give the air free access to the fixed carbon or coke remaining, although the coke is in itself porous, and usually admits air, causing combustion to take place throughout the whole mass. Much of the value of close sizing is lost through this coking of the pieces, whether large or small.

Illinois coals are essentially free burning or non-coking in an open fire, are higher in volatile matter than either of the two coals just discussed, and in point of hardness lie between them. On account of its lack of coking properties it might be well to imitate the close sizing used with anthracite; at least to size enough to prevent fines from falling through the grate bars, although in some cases wetting of the coal is said to cause this fine material to adhere to the larger lumps long enough for it to be burned with success. On the other hand, its property of decrepitating readily in the fire into more or less rectangular blocks, allowing free admission of air would seem to refute

*Cours d'Exploitation des Mines, Texte 3, p. 153.

the general claim that increased furnace efficiency is a result of close preliminary sizing.

The following reasons have been advanced for closely sizing Illinois coal:*

(1) Increased furnace and boiler efficiency.
(2) Less loss of fuel in ash.
(3) Less smoke.
(4) Uniformity in quality better assured.
(5) Uniform combustion insuring great capacity.
(6) Less clinker and cinders.†
(7) Less draft needed.
(8) Longer life of grates.
(9) Less cleaning of fires and tubes.
(10) Greater efficiency of fireman, through handling evenly sized lumps.
(11) Furnace under better control; steam can be increased more rapidly.
(12) Sized coal may be stored with little danger of spontaneous combustion.
(13) Increased capacity of furnace through more rapid combustion.
(14) Even and regular air supply encircling each piece being burned.
(15) Sizing allows depth of fire to be adjusted so as to give each size its proper draft.
(16) Quicker fire on account of great number of surfaces in proportion to the weight immediately exposed to the fire.
(17) Less dirt and dust.

It is not province of this bulletin to discuss all of these reasons. Some which are true in the case of anthracite, coking bituminous, or even briquetted coal, are not necessarily true in the case of Illinois coal, or may be true when the coal is used for a particular purpose only. In general, the size of coal which can evaporate the most water for a dollar under the conditions imposed will ultimately determine which sizing practice will be adopted. Until tests have settled this point, a universally acceptable standardization, at least of steam coal, is not feasible.

Another view point is the following: "From an engineering standpoint, exact sizing is right for effective combustion, but consumers for whom sizing is done do not want to pay for it, therefore in a commercial sense sizing is wrong."‡

Sizes Prepared in Illinois.—In order to bring out present sizing practices in Illinois a list of the various sizes of coal prepared has been

*Partly after C. T. Malcolmson. "Briquetted Coal." Proc. 1st Annual Convent., Int. Rwy. Fuel Assoc., 1909.

†Bul. 325, U. S. G. S., p. 40, gives a table showing that size has no effect on clinkering.

‡Black Diamond, December 5, 1914, p. 455.

compiled and is given below, together with trade names and general use or significance of these sizes. The screen in each case is understood to be a standard round hole plate, unless otherwise stated.

(1) Run of Mine.—Coal sold as mined without preparation, and including all sizes mixed together.

(2) Lump Coal.—The larger sizes, from which the finer have been removed. Lump may be made by screening over any one of a variety of sizes of holes. In different parts of the state the size of perforation varies from ½ inch to 8 inches. A few sizes, however, are so common that special names have been given to the lump coal made over them. They are:

(3) Big Lump or Fancy Lump.—Made over a screen with 8-inch holes.

(4) Standard Lump or Chunk.—Made over a screen with 5-inch, 6-inch, or even 8-inch holes. This is the common large size made for the retail trade, and is a favorite, especially if coal must stand considerable handling before using, as in the northwestern trade. In one district a mixture of egg and lump mixed is called standard lump.

(5) Domestic Lump.—Usually a lump made over a screen having 3-inch or 4-inch holes; or it may be identical with (4).

(6) Steam Lump or Ordinary Lump.—Usually made over a screen with 1¼-inch or frequently 1½-inch or 2-inch holes; at some mines the holes may be 1⅛ inches or smaller. Bar screens are also used to prepare this product.

(7) Three Quarters Coal.—Lump and nut coal mixed. (A term adopted from the Pittsburgh district in which bar screens are used.)

(8) Modified Lump.—Made by taking only part of the screenings from run of mine coal. (More frequently used in the Kansas districts, and is made by taking 25 per cent screenings from run of mine containing about 40 per cent screenings.)

(9) Railroad Lump or Standard Railroad Lump.—About the same size as No. 6. In places this is dumped over a bar screen with 8 to 10-inch spaces for the purpose of reducing the largest lumps in order that they may be fired without the necessity of breaking. In this case it may be called Engine Coal or Locomotive Coal.

(10) Engine Coal or Locomotive Coal.—(See (9).) These terms may also refer to run of mine coal broken to pass bars from 8 to 10 inches apart, placed over the receiving hopper or pockets of the coaling station. Locomotives burn other sizes of coal and at least locally, the terms defined may refer to almost any size.

(11) Chute Coal.—A coal containing all sizes under about 6 inches, including screenings, which is frequently sold for local uses. Railroad Chute Coal may be any size used in locomotives.

(12) Local Trade or Wagon Coal.—At many of the mines situated in or near towns there is an auxiliary chute leading from below the weigh box to the ground at a place convenient for loading wagons. This chute usually contains a bar screen with as small as ¾-inch or as large as 2-inch spaces, over which this size is prepared.

(13) Egg Coal.—If screens with from 4 to 8-inch holes are used in preparing a lump, the largest of the sizes below this limit is called egg. Thus egg coal may be as small as 1¼-inch or as large as 8 inches in diameter. If a wide range is included in the egg size, as from 1¼ to 6 inches, it is frequently called Railroad Egg or Railroad Coal. The most common egg coal, however, is that made through a 6-inch and over a 3-inch hole. In retail trade this is sometimes called Furnace Coal.

(14) Egg Run.—Made by leaving all the smaller sizes in the egg coal. May be all coal passing a screen with 5, 6, or even 8-inch holes. Used mostly by locomotives.

(15) Nut Coal.—The sizes smaller than egg and from which the smallest coal has been removed. Nut is made as large as 3½ inches and as small as ¼ inch in diameter. The usual size, however, is through a 3-inch and over a 1¼-inch screen.

(16) Nut Run.—Includes nut coal and all smaller sizes. If nut run is rescreened, it may be divided into any combinations of five sizes, the usual dimensions of which are as follows:

(17) No. 1 Nut.—Through 3-inch and over 2-inch. (May be called Small Egg.)

(18) No. 2 Nut.—Through 2-inch and over 1¼-inch. (May be called Stove.)

(19) No. 3 Nut.—Through 1¼-inch and over ¾-inch. (May be called Chestnut.)

(20) No. 4 Nut.—Through ¾-inch and over ¼-inch. (May be called Pea, Buckshot, or the smaller sizes, Buckwheat.)

(21) No. 5 Nut.—Through ¼-inch and over 0-inch. (May be called Dust or Slack.)

Nut coal is frequently washed, in which case sizes may be prepared, ranging as follows:*

(22) No. 1 Extra Washed.—Always under 3¾-inches and always over 2½-inches.

(23) No. 1 Washed.—Always under 3½-inches and always over 1¾-inches.

(24) No. 2 Extra Washed.—Always under 2¼-inches and always over 1⅜-inches.

(25) No. 2 Washed.—Always under 2¼-inches and always over ⅞-inches.

(26) No. 3 Washed.—Always under 1½-inches and always over ⅝-inches.

(27) No. 4 Washed.—Always under ⅞-inch and always over 3/16-inch.

(28) No. 5 Washed.—Always under 7/16-inch and always over 0-inch.

If only two sizes are made at a washery, the larger is designated "washed nut" and the smaller "washed slack."

*F. C. Lincoln, "Coal Washing in Illinois." Bul. No. 69, Engineering Experiment Station, University of Illinois, p. 44.

(29) Pea Coal.—In Illinois this term refers to the intermediate sizes of nut. (See (20).) Coal ranging in any of the sizes between 2 inches and ¼ inch has been called pea. Generally this term refers to some size of coal made with less than 1¼ inch as a maximum dimension and with greater than ¼ inch as a minimum.

(30) Screenings or Raw Screenings.—The general term for the fines made in the tipple, if they are not rescreened, includes all sizes passing 2-inch holes or more generally 1¼-inch holes, although screenings are made with larger or smaller maximum limits than these. It is the standard chain grate stoker coal.

(31) Slack.—Refers to screenings from which the larger sizes have been taken. It may be from ¾ inch (plus or minus) to zero in size. The term is frequently used interchangeably with screenings.

(32) Coarse Slack or Stoker Coal.—A combination of Slack, Pea, and Nut.

(33) Fancy Screenings.—Refers to Pea and Slack sizes not separated, and is sometimes called Pea Run.

(34) Duff.—The smaller sizes of rescreened coal. It may be ½ inch (plus or minus) to zero, or may refer to the same sizes as Slack or as Dust Coal.

(35) Dust Coal.—Usually refers to the smallest coal. Thus No. 5 Nut (See (28)) of a size ¼ inch to zero may be called Dust Coal. This coal is also referred to as Culm.

(31), (34), and (35) are sometimes called Waste Coal, especially if they are so dirty that they have little market value.

(36) Pulverized or Powdered Coal.—Coal used in metallurgical furnaces or cement kilns is frequently ground before burning so that 92 per cent or more passes a 100-mesh screen. This is usually done at the plants in which it is used.*

Common terms used in connection with prepared Illinois coal, although not restricted to any special sizes are the following:

Secondary Coal.—Coal in general not of the first grade. It may be a product from the picking belts in the tipple or from the second compartment of a washing jig.

Bug Dust.—Machine cuttings.

Clean Coal.—A coal properly prepared without visible impurities; and not one free from small sizes, as sometimes thought.†

Dirty Coal.—A term ordinarily used when fines or degradation products are visible in the coal; should be used only to designate the presence of visible foreign matter or impurities.†

Conditions Affecting the Sizes Produced.—Considerable differences exist in the actual sizes of the coals just defined, due to differences in the screening surface and to the conditions under which screening is carried on. The screening surface may be composed of

*For a symposium on powdered fuel see: Journal American Society of Mechanical Engineers, October, 1914.

†Black Diamond, May, 11, 1912.

bars, of screen plate with round or oblong holes, or of wire screens with square or rectangular openings. The latter are used only on roller sceens in rescreening plants. Also, the steeper the angle at which the screen is placed, the smaller is the maximum piece which will just pass the holes.

Round and Square Hole Screens.—The round hole screen is by far the most common in Illinois, and is generally taken as a standard. No tests are available showing the width of bar screen or the diameter of square opening which will pass the same percentage of an Illinois coal as will a round hole of given diameter.

Tests on other bituminous coals show that a ⅝-inch bar screen will pass the same percentage of run of mine coal as a 1-inch round hole.* Another test shows that a square hole screen will pass as much material as a round hole screen with a diameter 1.23 times as large.† Thus, a 1-inch square hole may be equivalent to about a 1¼-inch round hole, and a 1¼-inch bar screen equivalent to a 2-inch round hole screen. The largest pieces made on a round hole screen are more nearly of uniform dimensions in two directions. For these reasons, if exact sizes are desired, specifications should state not only the sizes of coal desired, but also the kind of screen over which they are to be made, since an operator using a 1¼-inch bar screen produces a considerably coarser screenings than one using a 1¼-inch round hole screen.

Sizes in Competing Districts.—The market for Illinois coal covers at least parts of eighteen states. In different portions of the territory this coal comes into competition chiefly with the bituminous coal from West Virginia, Kentucky, Ohio, and Indiana on the east, and from Iowa, Missouri, Kansas, Oklahoma, etc., on the west. Multiplication of sizes in all of these districts has been rapid during the past few years, and many mines formerly producing only lump and screenings, now ship two or more prepared sizes.

Until recently the initial preparation of coal in much of the eastern territory mentioned was governed by an agreement between the coal operators and the United Mine Workers of America, which specified as follows:

"Screens hereby adopted for the State of Ohio, Western Pennsylvania, and the bituminous district of Indiana shall be uniform in size, six feet wide by twelve feet long, built of flat or Akron‡ shaped bar of not less than ⅝ of an inch surface with 1¼ inches between bars, free from obstructions, and that such screen will rest upon a sufficient number of bearings to hold the bars in proper position."

Frequently, the slack or screenings made through this standard screen are rescreened over a bar screen having ½ or ¾-inch spaces, making pea and slack. In other cases this second screening is over

*Henry Louis, "The Dressing of Mineral," p. 12.

†Eng. and Min. Jour., March 13, 1915, p. 493.

‡The name Akron bar refers to a bar of particular cross section, taking its name from the city of Akron, Ohio, where it originated, and not from the resemblance of the cross section of the bar to an acorn, as has been suggested.

a rotary screen with square or round holes. As has been stated, Ohio has recently passed to the mine run basis which will lead to a change in preparation and possibly to a multiplication of sizes to meet competing districts.

In districts of West Virginia in which coking coal is mined not so much attention is paid to sizing as in districts in which splint and gas coals are mined. Thus, in the New River field 3-inch bar lump, 1 or 1½-inch bar to 3-inch bar egg, and 1 or ½-inch bar screenings are prepared. From the Fairmont region of northern West Virginia the following sizes are shipped:*

(a) Lump.—Four inches, 3 inches, 2½ inches, 2 inches, 1½ inches, 1¼ inches, and ¾ inch.

(b) Egg and Nut.—The sizes depend upon the size of lump.

(c) Pea.—Through ¾-inch and over ½-inch.

(d) Slack.—Through ½-inch. This district has been sizing its coal closely since 1900. The recent tendency is towards a simplification in the number of sizes.

In Indiana 1¼-inch bar screenings are prepared which may be rescreened into pea or slack. The 1¼-inch bar lump is frequently made into nut, egg, and lump which competes with Illinois sizing.

In Kentucky the practice varies greatly, but in many of the newer tipples, especially in the eastern part of the state, and in several cases in the western part, shaker and other screens have been installed to prepare the domestic and other sizes common in Illinois.

In Iowa, Missouri, and Kansas, although payment to the miner is on the run of mine basis, preparation is generally more simple than in Illinois. One, two, or three sets of bar screens are often used, making at the most four sizes; through 1¼-inch, through 2½-inch, then through 4, 5, or 6-inch, and a large lump over this size. Round hole shaker screens, however, are not uncommon in these fields. For example, Kansas coal for general domestic trade is prepared over round hole shaker screens into 3-inch lump, 2-inch to 3-inch egg, ¾-inch to 2-inch nut, and a ¾-inch slack.

Oklahoma. (For Oklahoma sizes see p. 109.)

Standardization of Sizes.—Such a multitude of sizes and names of bituminous coal must be a cause of uncertainty, trouble, and annoyance to producer and consumer alike; to the producer in endeavoring to adjust his own screens to meet the extremely varying market, and to the consumer from lack of information regarding the nature and real worth of the many sizes. Aside from the question as to whether more or fewer sizes are advantageous, Illinois and the rest of the bituminous field are badly in need of some standardization of sizes.

In November, 1914, a radical change, proposing to do away with the many sizes now made and to return to the old methods of producing 1¼-inch screenings and 1¼-inch lump coal only, was advocated

*H. H. Stoek, "Mechanical Preparation of Coal." Proc. 2nd Annual Convention, Int. Rwy. Fuel Assoc., 1910.

in the present method of preparing bituminous coal at the mines in Illinois and in the eastern competing region. This plan contained a provision for producing a domestic nut size when necessary.* The plan was not to be adopted until accepted by enough operators to represent 90 per cent of the tonnage in the field.

Mr. H. C. Adams, the originator of the plan, states that Ohio, Indiana, West Virginia, and western Kentucky appeared to be favorable to the change, but that Illinois and the southeastern Kentucky did not give sufficient support to make it successful.† Figures given for Illinois show that operators representing about two-thirds of the tonnage were willing to standardize if competing districts would do likewise, and that over 14 per cent of the remainder represented mines not entering the general market. It is remarkable that such a percentage of the operators were willing to return to the simplest of all preparation,—two sizes, and of course run of mine. Further opposition to the plan was made by the railroads which in some cases were afraid the change would mean forced sizing in their own chutes, especially if they had been accustomed to burn lump.‡

Standard Practice in Various Districts.—Standardization has been accomplished in several districts, as is shown by the following:

TABLE 14.

COMMERCIAL SIZES OF ANTHRACITE.

Name of Size	Diameter of Ring		Uses
	Over inches	Through inches	
Lump	6½	...	Locomotive Steam Coal. (Very little now made.)
Steamboat	4½	6½	Blast Furnace Coal. Smith's Forge Coal. (Very little now made.)
Broken	3¼	4½	Domestic Furnace Coal.
Egg	$2\frac{5}{16}$	3¼	" " "
Stove	1⅝	$2\frac{5}{16}$	Domestic Range Coal.
Nut	$\frac{15}{16}$	1⅝	" " "
Pea	⅝	$\frac{15}{16}$	Domestic Furnace Coal.
Buckwheat	$\frac{7}{16}$	⅝	Steam Boiler Coal.
Rice	¼	$\frac{7}{16}$	" " "
Barley	$\frac{3}{32}$	¼	" " "

In 1916 the standard sizes of anthracite were changed and a reduction made in the number.¶

*The Coal Trade Journal, November 14, 1914.
†Black Diamond, February 20, 1915.
‡Coal Age, March 6, 1915.
¶Coal Age, p. 839, May 13, 1916.

TABLE 15.

STANDARD SIZES OF BITUMINOUS COAL. (OKLAHOMA.)

Name of Size	Diameter of Round Perforations in the Shaker Screen Used	
	Through, inches	Over, inches
Domestic Lump	...	2½
Screened Nut	2½	1¼
Chestnut or Pea	1¼	¾
Slack	¾	...

In the anthracite districts of Pennsylvania the commercial sizes given in Table 14 are produced,* and the standardized sizes adopted by the producers in the Oklahoma bituminous fields are given in Table 15. This standardization has been in force since about 1912, and is used at practically all mines in Oklahoma, irrespective of district or seam. Five sizes cover the entire range of these bituminous coals. They are probably somewhat softer than Illinois coals and, with the exception of one seam, are non-coking. It has proved satisfactory to both producers and consumers.

A committee on standard form of contract covering purchase of railroad fuel reported in favor of a standard for bituminous coal as follows:† (See Table 16.)

TABLE 16.

STANDARD SIZES OF BITUMINOUS COAL FOR RAILROADS.

Kind of Coal	Screen Made Over			Screen Made Through		
	Round Inches	Square Inches	Bar Inches	Round Inches	Square Inches	Bar Inches
Pea	...	⅜	...	...	1¼	... or
Pea	½	...	...	...	1¼	... or
Pea	...	¾	...	1¼	...	... or
Pea	½	...	...	2	...	...
Nut	1¼	...	...	2½	...	...
Egg	2½	...	...	7	...	... or
Egg	3½	...	...	6	...	... or
Lump	7	...	...	...	...	... or
Lump	...	...	4	...	...	...
Pea Run	...	...	...	1¼	...	...
Nut Run	...	...	...	2½	...	...
Egg Run	...	...	...	...	...	5 (not less than)
7″ Round Egg Run	...	...	...	7	...	...

*Paul Sterling, "The Preparation of Anthracite." T. A. I. M. E., Vol. 42, 1911, p. 264.

†Proc. 5th Annual Convent., Int. Rwy. Fuel Assoc., 1913, p. 31.

Mine Run Coal shall not contain more than 30 per cent screenings through 1-inch diamond bar screen, but shall contain more than 30 per cent of 5-inch bar screen lump or more than 40 per cent of 7-inch round hole screen lump.

The recommendations also included the use of relief screens to prevent overcrowding of the fine screens, and specified a minimum area for screens making the different sizes. Although favored by the convention in question the provisions have not been adopted by the railroads.

"In 1903 the operators in Williamson county, Illinois, adopted the following standard of sizes for washed coal. The numbers used refer to screens with round perforations."* This standardization has not been maintained in all cases.

Designation	Through	Over
No. 1	3 in.	1¾ in.
No. 2	1¾ in.	1 in.
No. 3	1 in.	¾ in.
No. 4	¾ in.	¼ in.
No. 5	¼ in.	.....

Standardization means little to the small consumer, whose aggregate consumption, however, forms a considerable percentage of the coal produced. The general ignorance on this subject is well brought out in a report of a committee of the Boston Chamber of Commerce on Buying and General Handling of Steam Coal (November, 1909), wherein it is stated that of 225 manufacturing firms using steam coal, 25 per cent did not even know the particular kind of coal they were using.

The recent trend of public opinion towards some form of standardization for coal is significant. In 1913 the United Improvement Association of Boston voted to present three bills to the legislature:

(1) To establish a standard of quality for coal sold for domestic purposes.

(2) To establish a standard for sizes of meshes in sieves used in screening coal sold for domestic purposes.

(3) To require that a list of prices and notices of change of the same be filed with the state police.

Requirements for Standardization.—In considering the standardization of Illinois coal several viewpoints may be taken: (1) That of the producer, who desires to make only the largest percentage of the sizes of greatest market value. (2) That of the dealer who must furnish the coal which customers demand and are accustomed to.

*C. S. McGovney, "Tests of Washed Grades of Illinois Coal." Bul. No. 39, Engineering Experiment Station, University of Illinois.

(3) That of the fuel agent, or purchaser for the larger consumers, who generally wants the most heat units for a dollar. (4) That of the consumer, each class of whom, for various reasons, demands a different fuel. It would appear that his viewpoint must be the final judgment as to what coal may be produced.

Consumers of Illinois coal may be subdivided into seven groups as follows:

(1) Railroads using coal for locomotives.

(2) Stationary power plants in which coal is used to produce steam.

(3) Domestic, retail, or household users.

(4) Large heating plants (as brick kilns and metallurgical plants).

(5) Those using pulverized coal. (Generally as a fuel for cement kilns and for certain metallurgical purposes.)

(6) Coke oven operators. (Ovens operated primarily for manufacture of metallurgical coke.)

(7) Gas Manufacturers using (a) gas producers for power purposes, and (b) retorts.

Many consumers in the last four classes obtain their coal from mines operated for their particular demand, or if buying in the general market they consume only a small fraction of the total production; therefore, they will not be given detailed consideration. Pulverization (5) has been referred to (p. 105). It is noteworthy that recently considerable Illinois coal has been used to produce metallurgical coke in the by-product ovens in northern Illinois and Indiana, 25 per cent and even more of the high volatile coal of Illinois having been mixed with low volatile eastern coals with success. Low ash and sulphur contents are the prerequisites for the coal used for this purpose, and since the coal is crushed to ¼ inch and smaller before using, size is of minor importance.

The first three classes noted—railroads, power plants, and domestic users—enter most strongly into the general market, the railroads alone buying 18 per cent of the total production of Illinois coal in 1914.* There is doubt that these three users, with their widely varying demands, can be satisfied with the same coal.

Railroad Fuel for Locomotive Use.—The railroads have and can use almost any size of fuel produced in Illinois today. The type of locomotive, class of service, grate and fire box design, draft and load (whether light or heavy) may render one size of fuel more efficient than another. Ease of firing, kind of fuel to which the fireman is accustomed, desire to assist a mine being served by the railroad, and relative price converted into ton miles under the conditions imposed, also influence the sizing or lack of sizing favored.

*Illinois Coal Report, 1914.

It has been stated that Illinois run of mine coal under present conditions is the most economical coal per ton mile obtainable.*

The sizes listed on p. 103 as railroad fuel are the ones frequently used, although a number of companies, considering the difference in price, prefer run of mine; and recently the automatic stoker, burning the lower priced screenings, has been successfully introduced for this work. Most authorities agree that the ideal fuel for hand fired locomotives would be one with lumps of a maximum size of from 3 to 5 inches,† which would allow the fireman to shovel without taking his time to break lumps; and of a minimum size of about ½ inch, which would lessen losses through the grates and prevent the draft drawing the fine fuel out of the stack. Large lumps generally necessitate breaking in the railroad chutes or coaling stations.

From the standpoint of absolute efficiency, a number of railroad tests made with run of mine vs. variously sized coals, do not show conclusively that it is cheaper to use a closely sized fuel for this work.

Domestic, Retail, or Household Fuel.—Individual fuel users burn only a few tons per year and usually buy the size of fuel recommended by their dealer as best suited to their needs. The standard domestic fuel, anthracite, is sized closely, and until recently most stoves and heating furnaces were designed for this fuel; consequently a domestic bituminous coal, to sell well, should resemble anthracite in appearance. Absence of dirt and dust are important factors which make closely sized fuel a favorite. In discussing this subject, J. D. Rogers states that in grading a bituminous coal for domestic use three points must be noted: (a) absence of slack in the prepared grades, (b) absence of impurities in all grades, and (c) uniformity of size of the smaller grades.‡ For interstate shipments designed for house heating furnaces, or for threshing engine boilers and similar uses, where the coal must be handled several times, often with excessive breakage, it seems essential that a large lump should be shipped. Many householders favor a sized coal because it is said to sustain better a mild, steady combustion.

Tests by J. M. Snodgrass¶ conducted for the Engineering Experiment Station of the University of Illinois, have shown that with Illinois coal as a fuel, water can be evaporated in house heating boilers at about 50 per cent of the fuel cost of anthracite, and at about 75 per cent of the fuel cost of Pocahontas coal or coke. The relative worth of the different sizes in use for this purpose is not discussed.

*Proc. International Rwy. Fuel Assoc., 2nd Annual Convention, p. 88.

†Proc. International Rwy. Fuel Assoc., 2nd Annual Convention, pp. 19 and 21.

‡"Preparation of a Domestic Coal." Kentucky Mining Institute, December, 1912.

¶Proc. Illinois Fuel Conference, Urbana, Ill., 1909.

Whatever the comparative effective heating power of unsized vs. sized coal for domestic trade, it is outweighed by appearance, freedom from dust and cleanliness in handling of the prepared sizes. These demands must be met by the producers furnishing this trade.

Stationary Power Plants Using Coal to Produce Steam.—The selection of coal for steam boilers depends on five things; namely, (a) relative price per ton, (b) total heating value, (c) relative percentage of the heating value that can be utilized in the boiler (d) maximum capacity which may be developed in the boiler, and (e) cost of handling different coals and the ashes produced by them.*

The results obtained by this selection depend in turn on: (a) the nature and condition of the coal, (b) character of the furnace, and (c) conditions of firing and furnace control.†

Effect of Size of Fuel.—The great number of variables thus introduced into any boiler test have tended somewhat to obscure the effects of sizing or of the range of sizes to be used under any particular condition. Regarding sizes of Illinois coal for a steam boiler fuel the following data are available.

"Tests made with different sizes were negative in results, and emphasize the statement—that the study of the effects of the various elements of size of coal has not been made in sufficient detail."‡

"The size of coal influences the capacity of any given equipment owing to its effect on the draft. . . . When dust and fine coal are fed into the furnace they either check the flow of air or are taken up by the draft and after being only partly burned are deposited back of the bridge walls. . . . Coal of uniform size forms the most satisfactory fuel, as it does not pack so closely as coal of different sizes mixed. The furnace design may be changed to suit the coal in view."¶ In the same bulletin (p. 6) it is stated that almost any fuel may be burned with reasonable efficiency in a properly designed apparatus.

L. P. Breckenridge states, "When coal fed into a furnace is fairly uniform in size it is much easier to burn it without smoke than when it is of different sizes. . . . Just to what extent it will pay to size coal for regular use is not yet clear."** Still another investigator states, "The size of the interstices between the coal particles increases with the size of the coal particles. Consequently with lower grades (meaning smaller sizes) of uniform coal a large proportion of the total air passes through the fuel bed. . . . For each size a depth of fuel bed will be found for which a minimum excess (of air) occurs."††

*Colliery Engineer, Vol. 19, p. 64.
†Colliery Engineer, Vol. 10, p. 114.
‡U. S. G. S., Bul. No. 325, p. 49.
¶U. S. G. S., Bul. No. 428, p. 8.
**"How to Burn Illinois Coal Without Smoke." University of Illinois, Engineering Experiment Station, Bul. No. 15, p. 43.
††C. S. McGovney, "Tests of Washed Grades of Illinois Coal." University of Illinois, Engineering Experiment Station, Bul. No. 39, p. 55.

In general none of the references quoted give information specific enough to aid the operator of a hand fired stationary grate power plant in choosing among a variety of roughly or more closely sized coal. Such a consumer demands a coal large enough to avoid losses through the grate and to burn freely under the weak draft conditions existing in these plants. The tendency of fines to produce smoke may be an added factor, although "any fuel may be burned economically and without smoke if it is mixed with the proper amount of air and at the proper temperature."*

The ordinary coal sold for this purpose is steam lump with more or less screenings removed as demanded. Certain types of automatic stokers also are run with such lump coal.

Use of Screenings.—Most of the automatic stokers in power plants using Illinois coal today are designed for burning the sizes of coal under two inches, usually raw screenings. A favorite type is the chain grate stoker. In 1914 Illinois mines produced 19,740,000 tons of screenings size, or 32.5 per cent of their coal output, and although considerable of this was rescreened, a larger part was burned in automatic stoker boiler plants for the generation of steam. At the power houses of one public service corporation in Chicago, over 1,500,000 tons of coal were burned in 1914. This was mostly Illinois screenings, the cost of which represented about three-fourths of the total cost of each kilowatt of electricity generated.

"During the past twenty years the price of screenings has risen from practically nothing to a figure that in 1913 threatened for a time to make possible the crushing of larger prepared sizes for use in this work."† Normally, however, screenings are of lower value. Considerable success has been attained by rescreening this raw product, the average circular price of the different sizes so produced being higher than that of the raw screenings. A market for these sizes must necessarily be limited and unable to absorb the major tonnage of the raw screenings, unless some such separation is proved to be of added benefit in those sizes used in automatic stoker steam plants.

Because of this great production and market any proposed standardization of coal sizes should take into account the following questions: What size of screenings, if any, is best adapted for chain grate work? Will 1¼-inch or 2-inch or even some larger or smaller sizes or range of sizes give the coal for the most economic combustion? Other factors being equal, 2-inch screenings bring a higher price than 1¼-inch. Is this justified? Another point concerns the occurrence, distribution, and effect of the ash and impurities in the various sizes of screenings. Unlike many eastern bituminous coals, Illinois screenings are uniformly higher in ash than the larger sizes of coal and are not available for coking purposes, which throws practically the entire output into the steam trade.

*University of Illinois, Engineering Experiment Station, Bul. No. 15, p. 16. p. 16.

†H. W. Weeks, Black Diamond, March 20, 1915, p. 233.

Mr. H. W. Weeks, in discussing the general subject, says of the standard 1¼-inch screenings, "Screenings of this size are not satisfactory when used on many different kinds of equipment, the nature and amount of the load having an important bearing on their efficient use."* On the whole he favors larger screenings than these, especially if heavy loads are frequent.

To show the general effect of size on the efficiency with which Illinois screenings can be burned, the following are quoted: "Coals ranging in size from ¼ inch to 1½ inch burn much more rapidly than either very small or very large sizes."† (They must therefore give a greater capacity for any given equipment.) "Many Illinois screenings contain 40 per cent or over of material under ¼ inch."‡ "A coal with evaporative efficiency of 66 per cent when 10 per cent of it is under ¼ inch diameter, will give an efficiency of only 62 per cent when 60 per cent of it is under ¼ inch in diameter."¶ "Small sizes of coal burned with less smoke than large sizes, but developed lower capacities;"§ however, "31 tests on coal with an average diameter of 0.39 inches give an average efficiency of 66.88 per cent and 53 tests on coal with an average diameter of 1.46 inches give an average of 65.97 per cent."**

A. P. Kratz made a number of boiler tests with 1¼-inch round hole Illinois screenings from which the following is abstracted.†† In test No. 6 using 1¼-inch screenings of which 19.6 per cent were smaller than ¼-inch round hole opening, the overall efficiency of the boiler was 68.09 per cent; while in test No. 8, with 50.5 per cent passing the ¼-inch screen, the overall efficiency was 61.08 per cent. He states (personal interview) that draft and thickness of fuel bed are factors in high efficiency as well as any particular sizing of the coal.

Recent specifications frequently limit the amount of the screenings under ¼ inch in diameter, one examined limiting the amount to 40 per cent.

Tests made by the Commonwealth Edison Company of Chicago in 1906 ‡‡ show that for their chain gate stoker work with a constant thickness of fire, maximum efficiency was obtained when using sized coal of an average diameter of ¾ inch, and that especially below this size efficiency decreases rapidly and is very low for screenings under ¼ inch in size. This view has been somewhat modified since

*"Influence of Size on the Cost of Making Steam." Black Diamond, March 20, 1915, p. 233.

†U. S. G. S., Bul. 325, p. 176.

‡U. S. G. S., Bul. 290 (Tables).

¶University of Illinois, Engineering Experiment Station, Circular No. 3, p. 37.

§U. S. G. S., Bul. 373, p. 10.

**U. S. G. S., Bul. 373, p. 148, Table 37.

††"A Study of Boiler Losses," University of Illinois, Engineering Experiment Station, Bul. No. 78.

‡‡W. L. Abbott, "Some Characteristics of Coal as Affecting Performance with Steam Boilers." J. W. S. E., Vol. 11, 1906.

that time. Recently the above company have conducted a great number of boiler tests with Illinois screenings to ascertain among other points the size or combination of sizes best adapted for their work. Through the courtesy of W. L. Abbott, Chief Operating Engineer of the company, the writer was able to examine the results of many of these tests, which throw considerable new light on the whole subject of screenings. They point towards the following conclusions:

(1) There is a great difference in ash content among the various raw screenings produced in Illinois today, car samples received showing a minimum of 10.1 per cent and a maximum of 29.7 per cent, with an average of 17.4 per cent for thirty-two samples examined (dry basis).

(2) Screenings made through the same size of screen and coming from different mines or even from the same mine at different times show great differences in the percentages of relatively coarse and fine coal contained. Certain cars showed as much as 50 per cent and others as little as 17.8 per cent of coal under ¼-inch round hole in size. A large percentage of this fine coal in screenings greatly lowers the efficiency of the boiler under test.

(3) The company has placed a maximum limit to the amount of screenings they will buy of 55 per cent under ¼ inch in size. With such material, combustion is slow because the air will not go through it. Most of the trouble comes from the part of this fine material which is smaller than ⅛ inch in size. There is no objection to pieces of ⅛ inch size.

(4) Three per cent moisture added to screenings containing high duff greatly aids combustion.

(5) By removing all material below ⅛ inch from an Illinois screenings in one test the engineers were able to raise the furnace temperature from 2,500° F. to 2,800° F.

(6) On the other hand, coal sized with an entire absence of the sizes under ¼ inch does not show increased efficiency, probably due to difficulty in securing uniform air admission. The engineers report crushing an egg coal to stoker sizes and obtaining indifferent results because of a lack of a certain necessary amount of fine material. Ten per cent of the material under ¼ inch probably represents roughly a minimum desirable.

(7) Screenings with a maximum size of 1 inch are as efficient for these conditions as larger sizes.

Briefly, the best screenings for these and similar conditions are those with a maximum size of 1 inch and containing a limited amount only (15 or 20 per cent) of sizes under ¼ inch. (It is evident that a sized coal would produce this amount of fines as a degradation product if it received two or three handlings before being burned.)

The effect of a variation of ash in the screenings is to reduce the capacity and efficiency of the particular size of coal slowly until 15

to 20 per cent is reached and then more rapidly, until with about 40 per cent ash the efficiency of that size of coal is zero.*

Other references state, "Ash in dry coal has little effect on efficiency up to 15 per cent, above this point efficiency drops rapidly;"† and "overall efficiency of coal decreases only 2 to 3 per cent until ash content rises above 20 per cent when efficiency falls rapidly. In general, ash increase of 1 per cent causes a drop in efficiency of about 2 per cent."‡ In other cases this was less. U. S. government specifications for coal also recognize the greater decrease in efficiency with an increase of ash than would be expected by the actual decrease in combustible matter shown by such added percentage of ash.¶

Tests Made with Illinois Screenings at the Mining Laboratory of the University of Illinois.—Believing that ash and possibly sulphur were unevenly distributed among the various small sizes of coal which compose Illinois screenings, and that the location of certain sizes containing uniformly a high percentage of ash might be of assistance in solving the problem of how to make screenings more valuable, tests were undertaken in the Mining Laboratory of the University of Illinois. It was thought that screening tests, followed by analysis of the separate sizes, might reveal sizes the removal of which might pay both operator and consumer if cost, freight, and comparative boiler efficiency were considered. An outline of these tests follows:

(1) Requests were sent to ten mines in the state, asking that a barrel of screenings be collected from under the tipple at random during loading and be shipped to the laboratory. The mines were chosen with a view of representing all the seams mined in the state and widely separated districts.

Such a sample in no way represents an average of the screenings being made at any of the mines, and does not reflect on or bring credit to any mine or district. It was believed, however, that such samples would give a preliminary guide as to the relative percentages present in the different sizes, how impurities occurred in the screenings, and a hint as to their possible removal.

(2) In any set of sizing tests a standard set of screens must be chosen. Unfortunately many sizing tests are made with no idea of uniformity in the succession of screens used, either regarding shape or relative sizes of the holes. The round hole screens were adopted in each case as conforming to the usual commercial conditions. A 1-inch diameter hole was taken as a standard, and with screens below this size, a ratio of ½ was used; thus the diameter of each hole is ½ the diameter of the hole of next greater size. The screens used were 1 inch, ½ inch, ¼ inch, and ⅛ inch in diameters. For sizes above an inch a 2-inch screen would be the next in the ratio; this was used when the size of the screenings tested warranted it, but

*W. L. Abbott, Jour. West. Soc. Eng., Vol. 11, 1906.
†U. S. G. S. Bul. 325, p. 176.
‡U. S. G. S., p. 38.
¶U. S. G. S. Bul. 339, p. 13.

since some of the screenings were of smaller size, as 1¼ inch, this size of intermediate screen was also used.

(3) Upon receipt, the whole barrel of screenings was carefully weighed, sampled, and a portion saved for analysis.

(4) Another portion was subjected to a float and sink test to determine the relative amounts of high grade and low grade coal present. In this test a solution was brought up to a specific gravity of 1.35 by the additions of zinc chloride. A representative lot of the screenings was stirred into the solution, the light weight purer coal

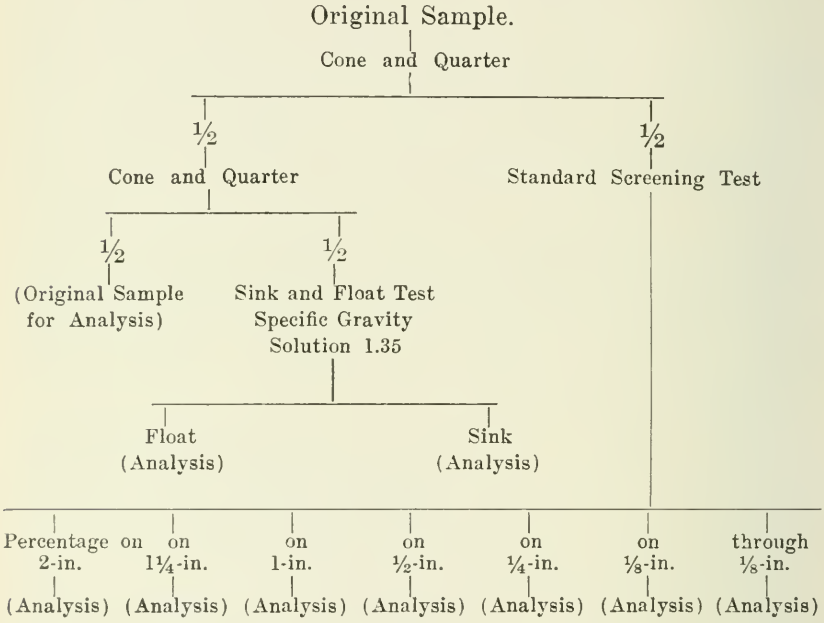

FIGURE 26. OUTLINE OF WORK. SCREENINGS INVESTIGATION.

floating, and the heavier bone coal and refuse sinking. The two products were carefully separated and analysed. The results appear in the tests under the heading "Float and Sink Tests." These results are relative only, as another standard of specific gravity of the test solution would produce different results.

(5) Half of the original sample was carefully screened in succession through the standard set of screens mentioned under (2), effort being made to prevent breakage.

Fig. 26 shows an outline of the work just described.

As received, the eleven samples varied in ash from 6.72 to 24.88 per cent and in sulphur from 0.90 per cent to 7.16 per cent. The float and sink tests show that the greatest part of the screenings is a rather pure coal of from 4.22 to 8.74 per cent ash and of low sulphur, and that on the average 23.2 per cent of the screenings is represented by a separable coal having an average ash content of 36 per cent and correspondingly high sulphur.

Test (1) was made with a 1-inch bar screenings, the other tests were upon screenings through round holes of the sizes indicated. When test (1) was subjected to screening on round hole screens, 23.5 per cent of the material was over 1 inch, showing the relatively large coal produced by bar screens. The presence of 1.47 per cent of coal on a 2-inch round hole in this test, and an ash content of 49.2 per cent and sulphur of 10.78 per cent show how these impurities, by breaking into flat pieces, pass a bar screen when they would not pass a round hole screen.

The largest amount of coal under ¼ inch was in test (4), the total being 30.1 per cent. The largest amount of dust under ⅛ inch occurred in test (5), 21 per cent, and this test had an average ash content of 17.81 per cent or 40 per cent more than the coarsest sizes in the same screenings.

In every test excepting (1) the largest sizes of the screenings show the least ash; this ash increases slightly until the dust coal under ⅛ inch is reached, when it shows in every case a decided increase, tending to confirm the opinion that the universal high ash content of this finest coal may be one cause of its low efficiency. In test (1) this fine material had an ash content of 40.9 per cent, enough to render it practically useless as an efficient fuel. Several others show striking increases measured on a percentage basis.

Contrary to the usual opinion, sulphur showed no increase in the fine sizes; a decrease even being noted in several cases. This tends towards the opinion that a large percentage of the separable sulphur is contained in the lump or ball form rather than in the brittle leaf form as is the case in many coals.

Regarding moisture, the results were irregular, and for this reason they are not included in the table. This may have been partly due to working with considerable quantities of the samples in a steam heated laboratory. It is significant, however, that in no case did the moisture vary greatly among the different sizes or show any tendency to increase in the finer sizes.

The percentage of the total ash was obtained by multiplying per cent weight by per cent ash. It shows the percentage of the total ash in the screenings in any particular size. As an illustration, in test (1) 18.7 per cent of the screenings which passed ⅛ inch, contained over 32 per cent of the total ash.

Calculated on an efficiency basis, if the screenings in tests (1), (2), and (11) were to stand any considerable freight rate, there is

TABLE 17.
Sizing Tests on Illinois Screenings.

No. of Test	Origin of Sample	Size of Coal	Outline of Work (See Fig. 26)	Original Sample	Float and Sink Tests		Sizing Tests — Sizes of Round Hole Screens Used						
					Float	Sink	On 2-in.	On 1¼-in.	On 1-in.	On ½-in.	On ¼-in.	On ⅛-in.	Through ⅛-in.
1	Seam No. 1 North	1-in. bar screenings	Per cent of weight	100.00	69.37	30.63	1.47	11.55	10.50	29.22	18.28	10.30	18.70
			" " " ash	24.88	8.72	46.53	49.20	19.17	16.15	16.10	21.21	25.70	49.90
			" " " total ash	100.00	29.75	70.25	3.01	9.44	7.21	20.02	16.50	11.27	32.55
			" " " sulphur	7.16	3.03	10.75	10.78	6.47	5.42	5.74	6.78	6.91	8.39
2	Seam No. 2 North	1⅛-in. round hole screenings	Per cent of weight	100.00	74.25	24.75	.....	.....	6.93	42.54	26.31	12.53	11.87
			" " " ash	19.95	6.62	46.41	.....	.....	14.78	16.18	18.25	20.88	29.21
			" " " total ash	100.00	30.00	70.00	.....	.....	5.43	36.62	25.50	13.90	18.55
			" " " sulphur	4.30	2.58	10.53	.....	.....	4.32	4.32	4.29	4.59	4.22
3	Seam No. 2 South	2-in. round hole screenings	Per cent of weight	100.00	92.50	7.50	5.82	23.60	10.25	25.90	14.85	7.63	12.05
			" " " ash	6.72	4.22	30.91	8.14	5.24	5.67	6.03	7.20	8.82	12.68
			" " " total ash	100.00	62.80	37.20	6.60	17.41	8.15	21.92	15.04	9.41	21.50
			" " " sulphur	0.90	0.60	6.19	3.48	0.97	0.89	1.03	1.15	1.23	1.85
4	Seam No. 5 Central	1¼-in. round hole screenings	Per cent of weight	100.00	73.07	26.93	.....	.....	11.11	36.50	22.30	13.00	17.10
			" " " ash	15.28	8.07	30.62	.....	.....	12.85	16.02	14.82	16.20	19.75
			" " " total ash	100.00	41.70	58.30	.....	.....	9.31	33.33	21.61	13.75	22.00
			" " " sulphur	5.08	3.18	7.42	.....	.....	4.92	4.56	4.86	5.60	4.17
5	Seam No. 5 South	1¼-in. round hole screenings	Per cent of weight	100.00	63.70	36.30	.....	.....	13.01	32.10	20.80	13.18	21.00
			" " " ash	14.73	6.60	31.03	.....	.....	12.70	14.65	15.65	16.52	17.81
			" " " total ash	100.00	27.89	72.11	.....	.....	10.62	30.26	20.99	14.03	24.08
			" " " sulphur	2.46	1.50	4.60	.....	.....	2.08	2.36	2.28	2.44	2.35
6	Seam No. 5 South	2-in. round hole screenings	Per cent of weight	100.00	31.20	18.80	.....	13.55	11.65	27.90	17.70	11.65	17.50
			" " " ash	11.28	6.86	26.79	.....	7.99	9.78	10.10	11.12	11.50	17.38
			" " " total ash	100.00	52.46	47.54	.....	9.48	10.00	24.75	17.32	11.79	26.75
			" " " sulphur	3.49	2.70	9.85	.....	3.28	3.92	4.23	4.47	4.69	4.84
7	Seam No. 6 Benton	2-in. round hole screenings	Per cent of weight	100.00	81.32	18.68	.....	14.50	15.35	38.52	15.40	6.45	9.84
			" " " ash	9.85	5.41	29.89	.....	10.08	9.85	9.25	9.10	9.25	12.40
			" " " total ash	100.00	52.46	47.54	.....	14.61	15.11	38.02	14.00	6.01	12.21
			" " " sulphur	1.63	1.15	4.73	.....	2.23	1.83	1.49	1.37	1.13	1.02
8	Seam No. 6 DuQuoin	2-in. round hole screenings	Per cent of weight	100.00	86.80	13.20	.....	19.75	13.50	23.50	15.68	9.93	17.76
			" " " ash	10.88	6.48	40.14	.....	9.15	9.35	9.87	11.00	12.60	16.00
			" " " total ash	100.00	51.51	48.49	.....	16.03	11.17	20.52	16.13	11.08	25.14
			" " " sulphur	1.70	1.41	2.90	.....	1.51	1.41	1.33	1.77	1.56	1.77

9	Seam No. 6 Livingston	2-in. round hole screenings	Per cent of weight	100.00	71.85	28.15	.....	18.75	11.10	27.18	15.88	10.53	16.60
			" " " ash	16.38	6.65	42.20	.....	13.80	13.31	14.95	17.92	19.01	22.45
			" " " total ash	100.00	28.74	71.26	.....	15.50	8.85	24.28	17.05	12.01	22.30
			" " " sulphur	4.42	1.59	9.50	.....	4.84	4.73	4.39	4.61	4.21	4.26
10	Seam No. 6 Nakomis	2-in. round hole screenings	Per cent of weight	100.00	74.10	25.90	.....	18.40	12.55	28.70	18.70	9.67	12.00
			" " " ash	16.18	7.08	30.64	.....	14.90	14.95	15.15	14.00	14.00	17.80
			" " " total ash	100.00	39.78	60.22	.....	18.16	12.47	28.85	7.38	9.02	14.19
			" " " sulphur	5.20	3.47	6.81	.....	5.65	5.16	5.02	4.93	5.21	4.58
11	Seam No. 7 Danville	1¼-in. round hole screenings	Per cent of weight	100.00	75.75	24.25	.....	.....	17.35	43.45	17.20	7.85	14.05
			" " " ash	15.65	8.74	40.62	.....	.....	13.95	14.80	15.50	17.50	26.35
			" " " total ash	100.00	31.46	68.54	.....	.....	14.62	38.80	16.00	8.35	22.35
			" " " sulphur	3.92	2.90	6.80	.....	.....	3.77	3.86	3.97	4.13	4.30

no doubt that the removal of the material under ⅛ inch would prove of benefit to producer and consumer alike.

Résumé.—This chapter leads to the conclusion that Illinois coal is badly in need of some standardization. As a possible guide, the essential differences of Illinois coal compared to certain others have been mentioned, and relative sizing has been discussed. It seems possible that the different markets, with their own needs, customs, and prejudices may always require particular coals. Furnace efficiency and relative price of any particular size will influence the final decision. It is to be regretted that in most boiler tests so many other variables have obscured the effect of sizing the various coals.

In regard to screenings, it has been shown that in the few tests available this important and often neglected product has a surprising individuality among its own sizes. So far as its standardization is concerned, further investigation is necessary before fixing a definite size or range of sizes. Its efficiency when sized may have a close relationship to the ash content of any sizes removed.

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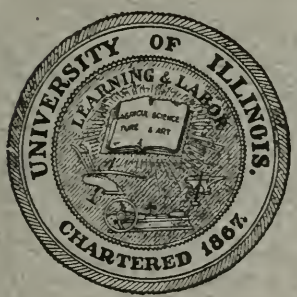
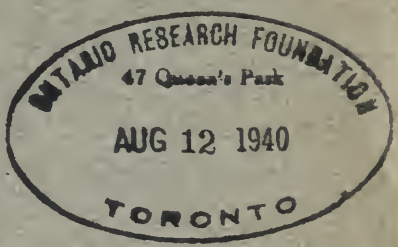
SPECIFIC GRAVITY STUDIES OF ILLINOIS COAL

BY

MERLE L. NEBEL





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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 89 JULY, 1916

SPECIFIC GRAVITY STUDIES OF ILLINOIS COAL

BY MERLE L. NEBEL,
RESEARCH FELLOW IN ENGINEERING EXPERIMENT STATION.

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SPECIFIC GRAVITY STUDIES OF ILLINOIS COAL

I. Introduction.

The specific gravity of coal, especially of high moisture bituminous coal, such as is found in Illinois, is not a fixed value. Coals of different types may have characteristically different specific gravities, and it is equally true that coals of the same type vary greatly in specific gravity.

The object of this bulletin is to present the results of a study of (1) the effect of moisture upon the specific gravity of coal, and (2) the methods of determining the specific gravity of coal. The commercial aspects of the problem are discussed, and values are given for the specific gravity of coal from many of the mining districts of the State of Illinois. The specific gravity of bright and dull coal is also considered.

The investigation was carried on under the direction of Professor H. H. Stoek of the University of Illinois, who suggested the problem and whose advice and criticism were of great value. Experiments were conducted in the laboratories of the Department of Mining Engineering of the University of Illinois, and field trips were made to the Vermilion county coal field, where samples of coal were collected from a number of mines. Other samples were obtained from various sources. The author is indebted to Professor E. A. Holbrook of the Department of Mining Engineering of the University of Illinois for assistance in securing samples of coal and for valuable suggestions and criticisms during the progress of the work; and also to Professor S. W. Parr of the Department of Chemistry of the University of Illinois, and Mr. Fred H. Kay, of the Illinois State Geological Survey, for advice.

II. Specific Gravity of Coal as Affected by Its Ash and Moisture Contents.

The chief factors which affect the specific gravity of coal, aside from difference in type, are (a) the amount of impurities, and (b) the amount of moisture present in the coal. A high-ash coal has a greater

specific gravity than a low-ash coal of the same type. In general, a wet coal has been found to have a greater specific gravity than a dry coal of the same type. However, the effect of moisture in varying proportions upon the specific gravity of coal is more or less complicated, and it is chiefly with this phase of the subject that the present bulletin deals. Though the effect of ash content has long been recognized, apparently very little study has been made of specific gravity under varying conditions of moisture content. This is probably because most of the coals of Europe and of the eastern United States which have been studied have a low moisture content, seldom over 3 or 4 per cent; whereas the coals of the Middle West may contain moisture as high as 15 or 20 per cent.

The ordinary method of determining the specific gravity of a solid, such as coal, is to weigh it in air, then immerse it in water and weigh it again, the specific gravity being equal to the ratio of the weight in air to the loss of weight in water. However, since coal is porous, probably most of the contained moisture is held mechanically in its pores. When coal is dried the moisture leaves the pores, which become filled with air; if the dry coal is placed in water, the air in the pores is displaced by the water, and the coal becomes saturated. Under such circumstances the length of time coal is immersed in water before it is weighed affects materially the value obtained for the specific gravity.

As early as 1892 Eckley B. Coxe recognized the effect of ash upon the specific gravity of coal. In his laboratory at Drifton, Pennsylvania, in which analyses of coal were made and boiler tests run, he required specific gravity determinations to be made of all coal samples. In his Presidential Address before the New England Cotton Manufacturers' Association in 1893, he said, "There seems to be no doubt that there is a close relation between the specific gravity of coal and its percentage of ash. . . . A careful study of a great number of analyses of coal and determinations of specific gravity has led us to believe that, although our experiments are not as yet absolutely conclusive, there is a strong probability that, for a given size of coal from the same colliery under ordinary circumstances, the determination of the specific gravity of an average sample will give very nearly the same percentage of ash as will be determined by analysis, although the relation may not be exactly the same for different mines or for different sizes of coal.

"If the specific gravity and percentage of ash, in any sample of coal below egg size, are known, the percentage of ash in any other sample of the same coal, and from the same colliery, can be satisfactorily de-

termined (we are inclined to think) from the specific gravity of that sample by the following formula:

$y' = y + (x' - x)a$, in which
x = the standard specific gravity,
y = the standard percentage of ash,
x' = the specific gravity of coal determined by our apparatus,
y' = the percentage of ash to be determined,
a' = a constant for coal from same mine.

"In the Lehigh region, for any size of coal, we find that, within what may be called the commercial limit of purity, an increase of 0.01 in the specific gravity corresponds to about the increase of 1½ per cent of ash; that is to say, that if coal of the specific gravity of 1.54 contained 8 per cent of ash, the same size of coal from the same mine when its specific gravity was 1.56 would contain twice 1½ per cent more ash, or 11 per cent."

Mr. Coxe evidently neglected the effect of moisture upon the specific gravity. This effect would not be great, however, in the case of the low-moisture anthracite with which he was chiefly concerned. M. S. Hachita* has suggested that if ordinary coal be considered a mixture of impurities with a pure coal substance, then the specific gravity of the mixture is equal to the sum of the specific gravities of each component multiplied by its percentage of the total mixture: that is,

$100\ gm = gpx + gi\ (100 - x)$, in which
gm = specific gravity of the mixture,
gp = specific gravity of the pure coal,
gi = specific gravity of the impurities,
x = per cent of pure coal in mixture,
$100 - x$ = per cent of impurities.

"Float and sink" tests of coal are very commonly made to determine the relation between the coal and the shale and other impurities which are mixed with it, and to study the possibilities of clean separation. Among the earliest of such tests in this country were those of Eckley B. Coxe, made in connection with the work on specific gravity previously referred to. The method in general use today is essentially the same as that used by him twenty-five years ago.

About 1910 the Department of Mines of Canada conducted an investigation of Canadian coals, including float and sink tests. Dr. J. B.

*Eng. and Min. Jour., Vol. 83. pp. 670-73.

Porter, who was in charge of the work, states that the specific gravity of pure bituminous coal is from 1.28 to 1.37.*

The Technologic Branch of the United States Geological Survey (now the United States Bureau of Mines), at the St. Louis Fuel Testing Plant in 1904, carried on float and sink tests, and with a Nicholson hydrometer determined the specific gravity of eighty-two samples of coal. The average value obtained for clean coal was 1.29 and for "average coal" was 1.34.†

III. Terminology of Specific Gravity.

The United States Bureau of Mines, recognizing that coal is a porous substance and that the pores of the coal may contain air or moisture or both, most of which can be easily removed, takes this factor into account by determining two values for the specific gravity; (a) the "apparent" and (b) the "true" or "real." These terms are defined as follows:‡

The "apparent" specific gravity is the specific gravity of the coal including the moisture or air contained in its pores. In determining this value no correction is made for moisture content, and care is taken that no air escapes from the pores during the weighing of the coal in water.

The "true" or "real" specific gravity is the specific gravity of the actual coal substance corrected for air and moisture content. In determining this value the weight of the moisture present is deducted from the weight of the coal and all air is removed from the pores of the coal before it is weighed in water. The methods employed by the Bureau in determining this value are described on page 38.

The true specific gravity of coal, as thus defined, may be compared to a coal analysis calculated to the "moisture-free" or "dry" basis. The true specific gravity is then merely the apparent specific gravity calculated to the dry basis.

Coal analyses are calculated to a dry basis because the moisture content of a given coal may vary greatly with differences in the atmospheric conditions to which it has been exposed. It is often desirable to compare analyses with respect to the thermal or other qualities of the coal, and in order to make such comparison accurate, the error introduced

*J. B. Porter and R. J. Durley, "An Investigation of the Coals of Canada," Vol. 1, p. 165, 1912.

†Bul. 323, U. S. Geological Survey.

‡Private communication from Dr. A. C. Fieldner, Chemist, U. S. Bureau of Mines.

by the variable moisture content must be excluded by calculating to the dry basis.

In order to obtain the true specific gravity it is necessary to know the moisture content of the coal; the specific gravity may then be calculated by the formula:

$$\text{Sp. Gr.} = \frac{W - W_m}{(W - W_m) - W'}, \text{ in which}$$

W = weight in air,

W' = weight in water,

m = percent of moisture.

Since W_m is equal to the weight of moisture present in a lump of coal of weight W, $(W - W_m)$ equals the weight of the actual coal substance excluding the moisture, and $(W - W_m) - W'$ equals the weight of the actual coal substance in water.

In Table I values of the true and apparent specific gravity of the same samples are compared for different contents of moisture. The true specific gravity is greater, a difference of from 0.10 to 0.21 being recorded.

TABLE 1.

TRUE AND APPARENT SPECIFIC GRAVITY FOR DIFFERENT MOISTURE CONTENT.

Sample No.	Moisture Per Cent.	Specific Gravity		Moisture Per Cent	Specific Gravity	
		Apparent	True		Apparent	True
138	4.04	1.23	1.35	4.91	1.26	1.38
141	3.30	1.25	1.35	5.06	1.23	1.37
144	3.07	1.24	1.34	5.64	1.24	1.34
147	3.26	1.24	1.40	5.55	1.22	1.40
150	3.79	1.22	1.38	5.01	1.15	1.36
154	4.85	1.21	1.34	5.48	1.24	1.36
156	4.51	1.24	1.37	4.94	1.22	1.36
159	4.31	1.21	1.35	4.62	1.26	1.38

The term "true" or "real" specific gravity is a rather unfortunate one, since its use implies that any other value for the specific gravity would be unreal or untrue. Such is not the case. Other values are as distinctly true or real as the value so termed, and each has a practical use, which may be even more important than that of the so-called true specific gravity. For this reason it is proposed that in this bulletin the term be replaced by the more logical term "dry specific gravity," which

is directly analogous to the "dry" values of fixed carbon, ash, or B. t. u., given in proximate analyses of coal.

In order to arrive at a more accurate basis for the comparison of the thermal quality of coals of the Illinois type, the idea of recalculation of analyses has been carried still further, and the "ash and moisture-free" or "unit-coal" basis has been developed by Parr.* To this end the effect of all of the impurities—the ash and sulphur, as well as the moisture—has been taken into account, and Parr has shown that the unit B. t. u., determined by such a calculation, is remarkably constant for the coal from a given bed over a considerable area.

Since the "unit-coal" basis of comparing analyses has been found so useful, and since the impurities in the coal affect its specific gravity as well as its other properties, the analogy might be carried still further and a "moisture-free" and "ash-free" or a "unit-coal" basis might be used for specific gravities. The specific gravity on such a basis would be called the "unit-coal" or simply "unit" specific gravity. Such a value might prove of considerable practical utility in comparing coals.

Since there is no term referring to proximate analyses of coal which corresponds to the term "apparent" specific gravity, and since it fills a very definite place, the term will be retained to designate the specific gravity of coal without any correction being made for variation in moisture or ash content.

The term "fresh" specific gravity is proposed to correspond to the "fresh" or "as-received" values of coal analyses. It designates the apparent specific gravity of the coal in its fresh condition; that is, in the condition in which it exists in the ground—saturated with moisture. It is a form of the apparent specific gravity, but it has a very distinct usage and is worthy of a distinct name.

Values of the true and fresh specific gravities of the same samples are compared in Table 2. In this case the true specific gravity is the greater by an amount varying from 0.02 to 0.07. The amount of variation in either case depends upon the moisture content of the coal.

*Bul. No. 16, Ill. State Geol. Survey, 1909, p. 212.
Bul. No. 29, Ill. State Geol. Survey, 1914, p. 40.

TABLE 2.
COMPARISON OF TRUE AND FRESH SPECIFIC GRAVITY.

Sample No.	Moisture Per Cent	Specific Gravity	
		Fresh	True
125	8.37	1.24	1.27
126	10.24	1.29	1.34
127	9.10	1.29	1.33
128	7.93	1.30	1.32
129	9.04	1.26	1.29
130	7.75	1.28	1.30
131	8.27	1.28	1.32
132	6.79	1.32	1.34
133	7.58	1.26	1.29
134	9.00	1.29	1.32
135	9.13	1.32	1.35
136	8.61	1.26	1.29
137	7.51	1.30	1.33
138	13.58	1.31	1.38
139	15.07	1 27	1.33
140	15.08	1.29	1.36
141	13.64	1.30	1.37
142	14.62	1.28	1.35
144	12.65	1.28	1.31
145	14.57	1.28	1.35
146	13.85	1.28	1.35
147	14.95	1.27	1.34
148	15.02	1.32	1.39
149	14.10	1.29	1.36
150	13.67	1.32	1.39
151	13.40	1.27	1.33
152	13.91	1.30	1.36
153	13.40	1.29	1.35
154	14.72	1.30	1.34
155	14.13	1.31	1.38
156	13.30	1.31	1.37
157	14.30	1.31	1.37
158	13.21	1.29	1.35
159	14.21	1.31	1.38
160	14.46	1.30	1.37
161	13.83	1.30	1.37

IV. LABORATORY STUDY OF SPECIFIC GRAVITY OF COAL.

After trying out a number of the well known methods of determining specific gravity, such as the hydrometer, pynenometer, and Jolly Balance, it was agreed that the Jolly Balance was the form of apparatus best adapted for carrying on the investigations contemplated in the present research. A discussion of the experiments carried out and a description of the various forms of apparatus used for determining specific gravity will be found in Appendix 1.

1. *Specific Gravity of Air-Dry Coal.*—In studying the specific gravity of air-dry coal it is necessary that all air be removed from the pores of the coal. The chemists of the United States Bureau of Mines accomplish this by boiling the coal about three hours under a partial vacuum obtained by the use of an aspirator.

When determining the specific gravity of air-dry coal by the Jolly Balance method, it was noted that very soon after the lumps were immersed in water, usually within a minute, bubbles of air began to rise through the water and that air continued to be given off for several hours. Two series of experiments were conducted to study (a) the effect of this expulsion of air on the specific gravity, and (b) the length of time required for the removal of all the air.

Experiment (a)—In the first experiment to determine the effect on specific gravity of expulsion of air the apparent specific gravity was determined for seventeen lumps from the same sample of coal, lumps which represented the average of the sample being selected. The lumps were immersed in water at room temperature for three days, and their weights in water were determined at the end of 1, 2, 3, 4, 5, 15, 20, 24, 48 and 72 hours. The specific gravity was determined in each case from the formula:

$$\text{Sp. Gr.} = \frac{Wa}{Wa - W}, \text{ in which}$$

Wa = weight in air,

W = weight in water.

The results are shown graphically by the curve in Fig. 1, which represents the average of the seventeen samples immersed for a period of 24 hours. The specific gravity increased in value as the air was expelled.

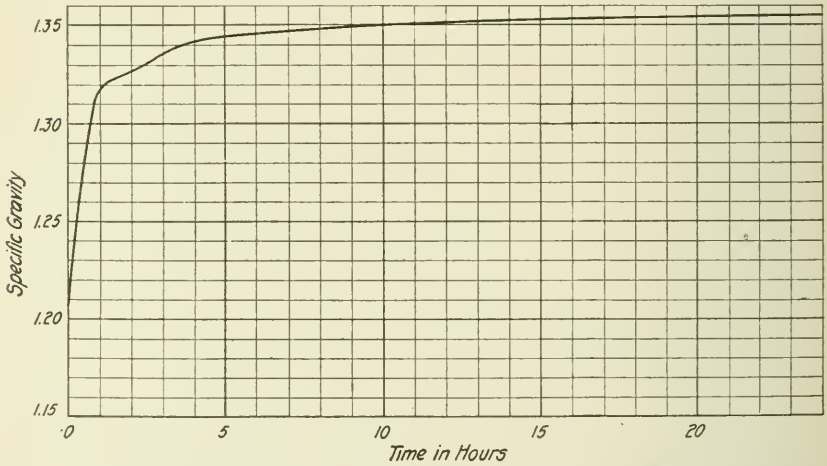

Fig. 1. Graph Showing Effect on Specific Gravity of Immersing Coal in Water Twenty-four Hours, Average of Seventeen Tests.

No appreciable change was noted after 15 hours, but by the end of 24 hours the action seemed to have been completed and all of the air to have been removed from the pores since no further increase was noted at the end of 48 and 72 hours.

Experiment (b)—As the greatest change took place during the first two hours, other experiments were conducted to determine the period of change. Three lumps were treated as before, but these were weighed at the end of 15, 30, 45, 60 and 120 minutes. The results are indicated graphically by the curve in Fig. 2, which represents the average values for

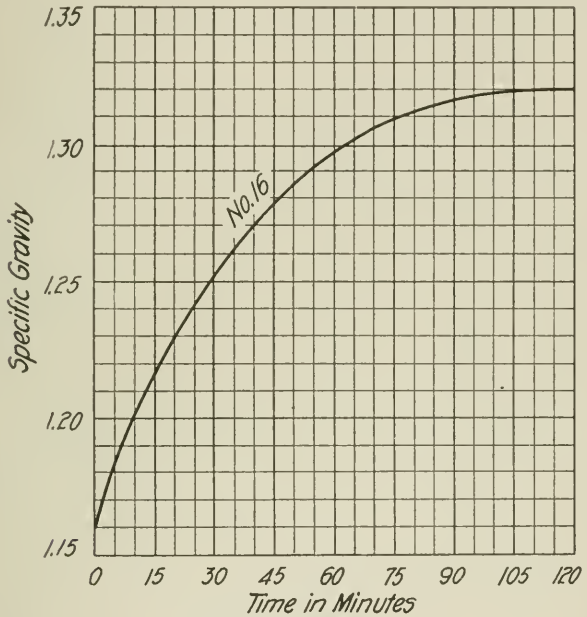

FIG. 2. GRAPH SHOWING CHANGE IN SPECIFIC GRAVITY OF COAL IMMERSED IN WATER AT ROOM TEMPERATURE, AVERAGE OF THREE TESTS.

the three lumps. These results show that the greater part of the change took place within one hour after immersion.

Other similar experiments were conducted in which two lumps of coal were used. Each lump was weighed in water at five-minute intervals for an hour, then at the end of two hours, and finally after twenty-four hours. The average results for the two lumps for the two-hour period are shown graphically by the curve in Fig. 3. There was no material change after two hours.

These two series of experiments, (a) and (b), indicate:

(1) That air-dry coal contains a considerable amount of air in its pores.

(2) That this air may be removed by immersing the coal in water, and that the specific gravity increases as the amount of included air decreases.

(3) That most of this air is displaced by water within about an hour after immersion.

(4) That this air is practically all removed at the end of twenty-four hours' immersion.

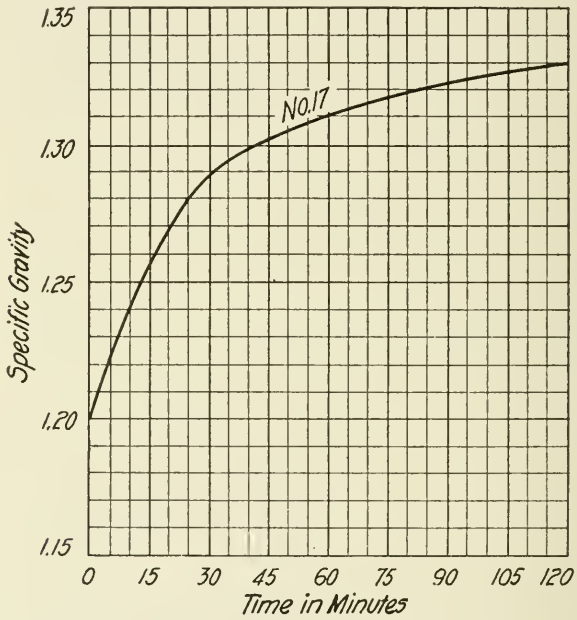

FIG. 3. GRAPH SHOWING CHANGE IN SPECIFIC GRAVITY OF COAL IMMERSED IN WATER AT ROOM TEMPERATURE, AVERAGE OF TWO TESTS.

In order to avoid delay in making specific gravity determinations it is desirable to remove the contained air as rapidly as possible. An increase in the rate at which the air is displaced may be accomplished in several ways:

(1) By increasing the temperature of the water in which the coal is immersed the density of the air in the pores of the coal is decreased, and consequently its tendency to rise is increased so that it will be displaced more rapidly by the water.

(2) By decreasing the pressure on the water and consequently on the air in the pores of the coal by placing it under a partial vacuum, the air expands and its density decreases, hence it is displaced more rapidly by the water.

(3) A combination of decrease of pressure and increase of temperature would have a double effect in increasing the rate at which the air is displaced by water. If this combination is effected by boiling under a vacuum, however, another factor must be considered; namely, the lowering of the boiling point of water with decrease in pressure. If the pressure is reduced to about 20 mm., water will boil at about room temperature, and the only effect accomplished is that due to decrease in pressure. It is probably that simple boiling under atmospheric pressure would have more effect.

Experiment (c)—To test the effect of boiling in hastening the displacement of the air, the specific gravity of each of a number of samples was determined, the weighed lumps of coal being boiled for about an hour and then weighed in water. The values are compared in Table 3 with the values obtained for samples of the same coal by immersing in water for twenty-four hours, each value representing the average of from six to eight determinations upon the same coal.

TABLE 3.

EFFECT OF BOILING UPON REMOVAL OF AIR.

Sample No.	Initial Sp. Gr.	After Boiling 1 Hour	Immersion for 24 Hours	Per Cent Air Removed by Boiling 1 Hour
9	1.26	1.34	1.34	100
15	1.19	1.31	1.32	92
42	1.28	1.32	1.32	100

Experiment (d)—In order to compare the relative effects of boiling and evacuating in hastening the displacement of the air the specific gravities of three samples were determined in duplicate, the weighed lumps of coal being evacuated in water for about an hour and then weighed in water. The evacuation was accomplished by means of an aspirator, and at the end of an hour the pressure was reduced to about 18 mm. so that the water was boiling at room temperature. The results are shown in Table 4, determinations being made in duplicate.

A comparison of the results given in Tables 3 and 4 shows that

Table 4.

Effect of Evacuation and Immersion Upon Removal of Air.

Sample No.	Initial Sp. Gr.	After Evacuating 1 Hour	Immersion for 24 Hours	Per Cent Air Removed by Evacuating
9	1.23	1 31	1.34	73
15	1.21	1.30	1.35	64
42	1.28	1.30	1.32	50

boiling the coal for one hour is much more effective than evacuating it for the same length of time.

2. *Specific Gravity of Fresh Coal.—Experiment (e)*—In order to study the specific gravity of fresh coal, samples were obtained from fresh faces in several mines in Vermilion county from Coal No. 7, and in Montgomery county, from Coal No. 6. Each sample contained about four pounds of coal, and was obtained by quartering down a large sample from a cut across the entire bed. The samples were sealed in air-tight cans and shipped to the laboratory in which the specific gravity of selected lumps was determined. Care was taken to select only those lumps which represented as nearly as possible the average of the entire sample. Lumps which contained pyrite or bands of shale were excluded, but both bright and dull coal were used in order to get a fair average.

The specific gravity of the fresh coal was determined with the Jolly Balance; the lumps were then boiled for an hour and the specific gravity again similarly determined. Instead of showing an increase in weight after boiling, as was characteristic of the air-dry coal previously tested, the lumps of fresh coal, with few exceptions, showed no increase (see Table 5). This indicates that the pores of fresh coal contain no air and are filled with moisture. Since this held true for coals which differed geologically and chemically, and since as-received analyses of coal always show a considerably higher content of moisture than air-dry or dry analyses, the evidence seems strong that Illinois coal in the fresh condition is saturated with moisture. It is probable that this moisture is held mechanically in the pores of the coal, since it is given off readily by drying upon exposure to the air, and since it can be taken up again in approximately the same amount if the coal is boiled in water for about one hour or simply immersed in water for a greater length of time, not exceeding twenty-four hours. If further investigation confirms these

results a method will have been established for comparing the specific gravities of different coals under standard conditions, in which the samples are boiled in water for an hour before determining the specific gravity. Although experiments (e) and (f) show the specific gravity thus determined to be slightly above the specific gravity of the fresh coal, it is much closer than the apparent specific gravity as usually obtained and near enough for all practical purposes.

TABLE 5.

SPECIFIC GRAVITY OF FRESH COAL.

Danville Field No. 7 Coal			Central Illinois Field No. 6 Coal		
Sample No.	Specific Gravity		Sample No.	Specific Gravity	
	Before Boiling	After Boiling		Before Boiling	After Boiling
125	1.24	1.24	138	1.31	1.31
126	1.29	1.29	139	1.27	1.27
127	1.29	1.29	140	1.29	1.29
128	1.29	1.30*	141	1.30	1.30
129	1.26	1.26	142	1.28	1.28
130	1.27	1.28*	144	1.28	1.28
131	1.28	1.28	145	1.28	1.28
132	1.32	1.32	146	1.28	1.28
133	1.26	1.26	147	1.27	1.27
134	1.29	1.29	148	1.32	1.32
135	1.32	1.32	149	1.28	1.29*
136	1.26	1.26	150	1.32	1.32
137	1.30	1.30	151	1.27	1.27
			152	1.29	1.30*
Average	1.28	1.28	153	1.29	1.29
			154	1.30	1.30
			155	1.31	1.31
			156	1.31	1.31
			157	1.31	1.31
			158	1.29	1.29
			159	1.31	1.31
			160	1.30	1.30
			161	1.30	1.30
			Average	1.29	1.29

*In one or two instances a slight increase in weight was noted, which may have been due to one or two causes; either the sample was taken from a portion of the bed in which the coal was not perfectly fresh, or the lumps had been accidentally permitted to dry a trifle before being used.

3. *Comparison of Specific Gravity of Fresh and Dry Coal.*—In order to study the effect of drying upon the specific gravity, portions of a number of the samples of fresh coal were placed in shallow containers and allowed to dry exposed to the air of the laboratory. The pieces of coal ranged in size from ½ in. to 2 in. in diameter. The air of the laboratory was at a fairly constant temperature of about 24°C.

Experiment (f)—The samples of Vermilion county coal were allowed to dry for sixty days, and at the end of that time lumps were selected and their specific gravities determined. In Table 6 the values are compared with those obtained for the fresh coal. They show that upon drying, the apparent specific gravity of the coal decreased considerably, but that the specific gravity obtained after boiling the air-dried coal showed an increase over the fresh coal. This variation is probably due to loss of moisture during the drying.

TABLE 6.

COMPARISON OF THE SPECIFIC GRAVITY OF FRESH AND AIR-DRY COAL FROM VERMILION COUNTY.

Sample No.	Fresh		Air-Dry for 60 Days	
	Before Boiling*	After Boiling	Before Boiling*	After Boiling
125	1.24	1.24	1.16	1.26
126	1.29	1.29	1 18	1.33
127	1.29	1.29	1.22	1.32
128	1.29	1.30	1 20	1.32
129	1.26	1.26	1.19	1.29
130	1.27	1.28	1 18	1.29
131	1.28	1.28	1.18	1.29
132	1.32	1.32	1.22	1.36
133	1.26	1.26	1.16	1.29
134	1.29	1.29	1.19	1.31
135	1.32	1.32	1.18	1.37
136	1.26	1.26	1.16	1.28
137	1.30	1.30	1.19	1.34
Average	1.28	1.28	1.19	1.31

*These values represent the apparent specific gravity of the coal.

Experiment (g)—The samples of coal from Montgomery county were dried in the same manner as those from Vermilion county, but lumps were selected and their specific gravity determined at the end of four weeks, and again at the end of five weeks. Moisture determinations were made each time in order that the relation between the loss of moisture and the variations in specific gravity might be studied quantitatively. In Table 7 the values obtained are compared with those obtained for fresh coal.

These values show that, as in the case of the Vermilion county coal, the specific gravity of the air-dry coal before boiling (the apparent specific gravity) is less than that of the fresh coal; also, that the specific gravity after boiling is more than that of the fresh coal. In addition they show that the coal lost from 9 to 10 per cent of moisture during the drying.

This loss of moisture explains the high values obtained for the specific gravity after boiling because, although the loss of moisture causes a decrease in the weight in air, the weight in water is approximately the same, whether the coal is fresh or dry. To determine the latter point the following experiment was carried out:

Experiment (*h*)—A number of samples of fresh coal were selected and their specific gravities determined. These samples were put aside and allowed to dry in the air for sixty days, and at the end of that time their specific gravities were again determined, each lump being weighed in

TABLE 7.

COMPARISON OF THE SPECIFIC GRAVITY AND MOISTURE CONTENT OF FRESH AND AIR-DRY COAL FROM MONTGOMERY COUNTY, ILLINOIS.

Sample No.	Fresh Coal			Air-Dry Coal					
				Four Weeks			Five Weeks		
	Moisture Per Cent	Specific Gravity		Moisture Per Cent	Specific Gravity		Moisture Per Cent	Specific Gravity	
		Before Boiling	After Boiling		Before Boiling	After Boiling		Before Boiling	After Boiling
138	13.58	1.31	1.31	4.04	1.23	1.33	4.91	1.26	1.37
141	13.64	1.30	1.30	3.30	1.25	1.33	5.06	1.23	1.34
144	12.65	1.38	1.28	3.07	1.24	1.33	5.64	1.24	1.31
147	14.95	1.27	1.27	3.26	1.24	1.37	5.55	1.22	1.36
150	13.67	1.32	1.32	3.79	1.22	1.36	5.01	1.21	1.34
154	14.72	1.30	1.30	4.85	1.21	1.32	5.48	1.24	1.33
156	13.30	1.31	1.31	4.51	1.24	1.34	4.94	1.22	1.34
159	14.21	1.31	1.31	4.31	1.21	1.33	4.62	1.26	1.36
Average	13.84	1.30	1.30	3.89	1.23	1.34	4.90	1.24	1.34

water before and after boiling. These weights, together with the corresponding specific gravities, are given in Table 8, which shows that the weight of the lumps in air decreased from 8 to 10 per cent after drying, while the weight of the lumps in water after boiling was approximately the same for the dry as for the fresh coal, the average difference for the thirteen samples being only 0.09. Thus the higher values for the specific gravity after boiling, given in Tables 6, 7, and 8, seem to be the natural consequence of the loss in moisture during the drying.

The amount of moisture lost when fresh coal is exposed to the air probably depends upon (a) the original moisture content, (b) the degree of fineness of the coal, (c) the humidity of the air in which the coal is exposed, and (d) the length of time the coal is exposed to the air. Table

7 shows that the moisture content was higher after five weeks' exposure than after four weeks. This increase may have been due to the fact that the air was more humid when the latter determinations were made, for the initial moisture and degree of fineness were about the same for all samples and the moisture determinations were made in the same manner, but the moisture in the air was not determined.

It was shown in Tables 6 and 7 that the apparent specific gravity of the coal decreased as the coal dried out. This might be explained by the loss in weight of the lumps due to the loss of moisture. If a 25-gram

TABLE 8.

THE EFFECT OF DRYING UPON WEIGHT OF LUMPS OF COAL IN AIR AND IN WATER.

Sample No.	Fresh Coal			Dry Coal				
					Before Boiling		After Boiling	
	Weight in Air	Weight in Water	Specific Gravity	Weight in Air	Weight in Water	Specific Gravity	Weight in Water	Specific Gravity
125	16.53	3.18	1.24	15.00	1.97	1.15	3.08	1.26
126	20.23	4.57	1.29	18.51	2.95	1.18	4.53	1.33
127	19.12	4.32	1.29	17.76	3.16	1.22	4.35	1.32
128	21.92	5.03	1.30	20.08	3.32	1.20	4.87	1.32
129	15.13	3.09	1.26	13.80	2.10	1.18	2.95	1.28
130	20.27	4.40	1.28	18.55	2.78	1.18	4.15	1.29
131	25.20	5.59	1.28	23.06	3.62	1.18	5.20	1.29
132	29.61	7.04	1.32	28.56	4.85	1.20	7.08	1.33
133	20.51	4.22	1.26	18.72	2.58	1.16	4.25	1.29
134	25.91	5.76	1.29	23.61	3.69	1.19	5.57	1.31
135	14.98	3.60	1.32	13.32	2.08	1.18	3.60	1.37
136	25.92	5.38	1.26	23.64	3.26	1.16	5.24	1.28
137	27.87	6.39	1.30	24.92	4.10	1.19	6.42	1.34
Average	21.78	4.81	1.28	19.96	3.11	1.18	4.72	1.31

lump of fresh coal weighs 5 grams in water, it displaces 25—5—20 grams of water, and its specific gravity is equal to $\frac{25}{25-5} = \frac{25}{20} = 1.25$.

If the same lump loses 10 per cent of its moisture when it dries out, its weight in air is decreased by 10 per cent and is then equal to 25 (1—0.10) = 22.5 grams. In determining the apparent specific gravity the lump is weighed before any water can enter its pores, so that it displaces the

same amount of water as before, or 20 grams. Then, its apparent specific gravity is equal to

$$\frac{22.5}{20} = 1.125.$$ Of, if

W = weight of fresh coal in air,
w = weight of fresh coal in water,
S_f = specific gravity of fresh coal,
L = per cent loss of moisture during drying, and
S_a = apparent specific gravity of air-dry coal,

then $S_f = \frac{W}{W - w}$, and $S_a = \frac{W - WL}{(W - L) - (w - L)} = \frac{W(1 - L)}{W - w}$

$= S_f (1 - L)$ (since $S_f = \frac{W}{W - w}$).

Therefore, if the loss in moisture alone causes the decrease in the apparent specific gravity, the value of the latter for air-dry coal can be easily calculated, knowing the loss of moisture and the specific gravity of the fresh coal. This is a simple relation and would prove of considerable practical value if it could be shown to accord with experimental facts. It does not seem to hold true, however, for the type of coal used in the present investigation. Values were calculated for a number of samples, according to the above formula, and compared by experiment: these are given in Table 9.

In every case except one (that of Sample 150), the calculated values are too low, or in other words the apparent specific gravity did not decrease with loss of moisture as much as it should if loss of moisture were the only factor affecting this decrease in specific gravity. Evidently, for this type of coal (Illinois bituminous) some other factor, or factors, partially counteracted the effect of drying on the apparent specific gravity.

One factor which might have an appreciable effect in this way is that of oxidation. A number of investigators have shown that if coal is exposed to the air for some time it absorbs oxygen, that oxidation takes place between the absorbed oxygen and some of the light hydrocarbons which are present in the coal, and that the oxidation is attended by an appreciable increase in weight. Sommermeier noted an increase in weight

from about 1 per cent to 2½ per cent for Illinois coals after from eight to thirteen months' exposure.* Parr states that one of the principal products of this oxidation is water, and that all coal samples prepared for laboratory work, even though kept in rubber-stoppered bottles, change in moisture content, so that after only a few weeks the moisture in the coal under these conditions will increase, sometimes to an extent of 1 per cent or 1½ per cent.† Hundreds of determinations have been made in his laboratory showing such increase in moisture.

In whatever form the light hydrocarbons existed in the coal, it is safe to conclude that if they were replaced by an oxidation product, such

TABLE 9.

COMPARISON OF CALCULATED AND EXPERIMENTAL VALUES OF THE APPARENT SPECIFIC GRAVITY.

Sample No.	Fresh Coal		Coal Dried 4 Weeks			Coal Dried 5 Weeks		
	Per Cent Moisture	Specific Gravity	Per Cent Moisture	Apparent Sp. Gr.		Per Cent Moisture	Apparent Sp. Gr.	
				Calculated	Experiment		Calculated	Experiment
138	13.58	1.31	4.04	1.18	1.23	4.91	1.20	1.26
141	13.64	1.30	3.30	1.16	1.25	5.06	1.19	1.23
144	12.65	1.28	3.07	1.16	1.24	5.64	1.18	1.24
147	14.95	1.27	3.26	1.12	1.24	5.55	1.15	1.22
150	13.67	1.32	3.79	1.19	1.22	5.01	1.20	1.15
154	14.72	1.30	4.85	1.16	1.21	5.48	1.17	1.24
156	13.30	1.31	4.51	1.19	1.24	4.94	1.20	1.22
159	14.21	1.31	4.31	1.18	1.21	4.62	1.19	1.26
	Average	1.30	Average	1.17	1.23	Average	1.19	1.23

as water, an increase in the specific gravity, as determined by the boiling or soaking process, would result.

4. *Special Method of Determining Fresh Specific Gravity.*—It has been shown that, in the case of the coal under consideration, the calculation of the specific gravity of coal in the fresh condition, based upon tests in the dry condition, or vice versa, is not feasible. It is not always possible to obtain fresh coal for specific gravity determinations, since such coal must be taken directly from unaltered portions of the coal bed. It would, however, be of considerable advantage to have some method of finding a value which would represent the specific gravity of coal in the

*Bul. 323, U. S. G. S., p. 22.
†Oral communication.

fresh condition, when only the air-dry coal or commercially-dry coal is available.

The experiments described previously suggested a simple method of determining the specific gravity of the fresh coal from samples of air-dry or commercial coal, such as can be easily obtained. It has been shown that if lumps of coal are soaked in water for about twenty-four hours they reabsorb about the same amount of water as they lose in drying, and their weight in water is nearly the same as when the coal was fresh. If the weight in water is the same as that of the fresh coal, the weight in air ought to be the same, if care is taken not to weigh any extra water which may cling to the surface of the coal. If both the weight in air and in water are the same for saturated dry coal as for fresh coal, then the specific gravities must be the same.

Experiment (i)—In order to ascertain whether the assumption made

TABLE 10.

COMPARISON OF FRESH SPECIFIC GRAVITY DETERMINED ON BOTH FRESH AND ON AIR-DRY COAL SATURATED.

Sample No.	Specific Gravity	
	Fresh Coal	Air-Dry Coal Saturated
140	1.29	1.30
141	1.30	1.30
144	1.28	1.29
147	1.27	1.29
150	1.32	1.32
154	1.30	1.31
156	1.31	1.31
161	1.30	1.32

in the preceding paragraph is true, eight or ten lumps of each of eight samples of dry coal were soaked in water until they were saturated (about twenty-four hours). At the end of that time they were taken out, the water adhering to the surface of each lump was removed with a cloth or with filter paper, and the lumps were exposed to the air for about five minutes. They were then weighed in air and in water and their specific gravity calculated. The values obtained in this way, as given in Table 10, checked closely with the values of the specific gravity previously determined for the same samples when the coal was fresh. Hence, if only air-dry coal is available, by this method the specific gravity of fresh coal can be determined to a degree of accuracy sufficient for all practical pur-

poses, and the extent to which the coal has dried out will probably not affect the accuracy of the results.

To determine whether or not the amount of water evaporated from the coal by leaving it exposed to the air before weighing would change the weight of the lump enough to affect its specific gravity appreciably, the following experiment was carried out:

Experiment (j)—Eight lumps of air-dry coal were soaked in water until they were saturated, and then were weighed in water. Each lump was dried only superficially, and was weighed before there was any chance for the moisture to evaporate to any extent. Then, each lump was allowed to dry for fifteen minutes, which gave ample opportunity for some of the moisture to evaporate, and was again weighed. Another weighing was made after the coal had dried for an hour. As shown by Table 11 there was no change in the specific gravity after drying for fifteen minutes, and in only one instance was there a change after an hour, and that was very small. Since in the general method outlined above the lumps were allowed to dry for only five minutes, it is evident that no error is introduced by water adhering to the surface of the lumps.

TABLE 11.

EFFECT UPON SPECIFIC GRAVITY OF DRYING MOIST LUMPS BEFORE WEIGHING.

Sample No.	Specific Gravity		
	At Initial Weighing	At End of 15 Minutes	At End of 1 Hour
1	1.33	1.33	1.33
2	1.35	1.35	1.36
3	1.30	1.30	1.30
4	1.30	1.30	1.30
5	1.35	1.35	1.35
6	1.29	1.29	1.29
7	1.30	1.30	1.30
8	1.29	1.29	1.29

5. *Specific Gravity of Bright and Dull Coal:*—Illinois coal is usually banded in appearance, consisting of alternate bright and dull layers. The coal material consists of organic and mineral matter, and the banded appearance is due to the alternation of layers that are an admixture of the organic and mineral matter in different proportions and in different ways.

The organic matter of coal may be divided into three substances:¶ (1) lignitoid or glance coal, (2) canneloid or matt coal, and (3) mother of coal or mineral charcoal. Jeffrey calls the bright layers "lignitoid coal," and considers that it is derived from woody material. He calls the dull layers "canneloid coal," and considers that it is derived largely from plant spores, together with fragments of leaves, stems, etc. Thiessen § speaks of the dull material as "debris," but his conclusions are similar to those of Jeffrey. These conclusions are based upon the results of refined microscopic study of thin sections of coal from which all mineral matter had been removed.

TABLE 12.

SPECIFIC GRAVITY OF BRIGHT AND DULL COAL.

Sample No.	Apparent Specific Gravity		Fresh Specific Gravity	
	Bright Coal	Dull Coal	Bright Coal	Dull Coal
401 a	1.26	1.31‡	1.29	1.35‡
b	1.27	1.31‡	1.29	1.35‡
c	....	1.56†	....	1.57†
402 a	1.30	1.42‡	1.33	1.45‡
b	1.26	1.42‡	1.29	1.45‡
c	1.25	.....	1.29	.....
403 a	1.15	1.26*	1.26	1.37*
b	1.19	1.22*	1.26	1.35*
c	1.21	1.25*	1.26	1.35*
d	....	1.26*	....	1.37*
e	....	1.19*	....	1.32*
404 a	1.22	1.39*	1.31	1.48*
b	1.19	1.24*	1.26	1.31*
405 a	1.26	1.65*	1.28	1.67*
b	1.26	1.57*	1.29	1.59*
c	1.26	.....	1.28	.....
d	1.26	.....	1.28	.....

* = Banded.
† = Shaly.
‡ = Homogeneous.

Coal which appears dull may, then, be dull because it is composed of the material referred to as canneloid coal, or as mother of coal. It may, however, appear dull because of the admixture of mineral matter, such as mud or shaly material, present either in intimate mixture or as thin layers or partings between layers of organic material. It is probable that the dull appearance of many Illinois coals is due in part to this admixture of shaly material, as well as to the presence of the canneloid

¶E. C. Jeffrey, Economic Geology, Vol. 9, No. 8, p. 741.
§R. Thiessen, Bul. No. 38, U. S. Bureau of Mines.

type of coal. The difference in specific gravity between bright and dull coal seems to indicate that this is true.

Experiment (k)—Specimens of bright and dull coal were selected at random from the same sample and their specific gravity determined. The bright specimens in each case appeared to be homogeneous with little or no admixture of dull or shaly material. The dull specimens in some cases appeared homogeneous, without distinct bright or shaly material; in other cases they were distinctly shaly; and in still other cases they appeared banded bright and dull, with a predominance of dull bands. In

Table 13.

Relation of Specific Gravity of Bright and Dull Coal to Ash and Moisture Content.

Sample No.		Fresh Sp. Gr.	Moisture (Air-dry)	Ash	Dry Ash
401	BC	1.29	5.83	2.77	2.95
	DC	1.35	3.82	37.15	38.62
	DS	1.57	2.06	36.65	37.42
402	BC	1.30	6.92	2.94	3.16
	DC	1.45	2.96	42.72	44.02
403	BC	1.26	5.79	2.70	2.86
	DB	1.35	2.99	11.32	11.67
404	BB	1.28	6.58	4.21	4.51
	DB	1.40	3.71	34.36	35.69
405	BC	1.28	5.96	2.65	2.82
	DB	1.63	2.78	27.51	28.29

BC = Bright, very clean coal.
DC = Dull coal, bony, but not distinctly shaly.
BB = Bright and banded in appearance with a few dull streaks.
DB = Dull and banded, with a few bright streaks.
DS = Dull and distinctly shaly.

every case the specific gravity of the dull coal was greater than that of the bright coal from the same sample. The results are shown in Table 12.

In order to study the effect of admixture of impurities in dull coal and, if possible, the relation between specific gravity and ash and moisture contents, determinations were made of the ash and moisture contents of the bright and dull lumps, of which the specific gravity was determined. The results of this study are shown in Table 13. The most important facts brought out by these data are:

(1) In every case the specific gravity of the dull coal was decidedly greater than that of the bright.

(2) The ash content of the dull coal was much greater than that of the bright coal.

(3) The moisture content of the bright coal was in every case greater than that of the dull coal, although all lumps had been subjected to about the same conditions.

These facts lead to the following conclusions:

(1) Admixture of impurities, especially of shaly material, is a common feature in the dull portions of Illinois coal.

(2) Bright coal is probably considerably more porous than dull coal, since in the experiments made it showed a uniformly higher moisture content, in spite of the fact that it had been exposed to the air for the same length of time and under the same conditions as the dull coal. This is probably due to the admixture of shale in the dull coal, since shale is known to have a very low porosity.

V. Field Study of Specific Gravity.

6. *Fresh Coal from Vermilion County.*—In order to determine the fresh specific gravity of Illinois coal on a more extensive scale, field trips were made to the Vermilion county coal field, near Danville, Illinois, and with a complete improved Jolly Balance equipment determinations of the fresh specific gravity were made at the mines, while the coal collected was perfectly fresh lumps ½ to 1 inch in diameter, being used in making the determinations.

Two coal beds of workable thickness, No. 6 and No. 7, of the Illinois State Geological Survey correlation, are mined in the district, and samples were collected from both. The coal is composed of alternate bright and dull bands of variable thickness, with numerous thin partings of shale, mineral charcoal or mother of coal, and pyrite. Some of these partings are quite persistent and divide the coal bed into benches which are known locally as the "top coal," "blacksmith coal," "bottom coal," etc.

Experiment (1)—In order to obtain a value for the specific gravity that would represent the whole coal bed with reasonable accuracy, it was found advisable to sample each bench, instead of sampling the bed as a whole; that is, lumps were collected from each bench appearing persistent over a considerable area. A value was thus obtained which was representative of the particular bench from which the sample was taken,

but not necessarily of the whole bed. The average value for the whole bed was found by multiplying the value for each bench by the thickness of that bench in inches, and dividing by the total thickness of the bed.

TABLE 14.

SPECIFIC GRAVITY OF COAL NO. 7 BY BENCHES.

Sample No.	Bench No. 1	Bench No. 2	Bench No. 3	Bench No. 4
230	1.24	1.33	1.25	1.29
231	1.24	1.29	1.26	1.29
232	1.24	1.28	1.27	1.26
233	1.24	1.32	1.26	1.28
234	1.24	1.30	1.26	1.35
235	1.24	1.27	1.27	1.28
236	1.24	....	1.27	1.31
237	1.24	....	1.27	1.31
238	1.24	1.31	1.28	1.32
239	1.25	1.34	1.26	1.30
240	1.25	1.31	1.25	1.28
241	1.25	1.27	1.25	1.34
242	1.24	1.26	1.26	1.30
243	1.25	1.30	1.25	1.36
244	1.23	1 30	1.26	....
245	1.25	....	1.28	....
246	1.24	....	1.31	1.29
247	1.26	1.26	1.26	1.34
248	1.27	1.25	1.28	1.27
249	1.26	1.32	1.25	1.26
250	1.26	1.25	1.26	1.31
251	1.28	1.31	1.33	1.27
252	1.25	1.28	1.26	1.26
253	1.28	1.30	1.29	1.27
254	1.26	1.31	1.30	....
255	1.27	1.32	1.27	1.30
256	1.29	1.28	1.26	1.29
257	1.25	1.27	1.28	....
258	1.22	1.29	1.27	....
259	1.29	1.29	1.27	....
260	1.26	1.27	1.26	....
261	1.25	1.26	1.28	....
262	1.27	1.29	1.27	....
263	1.27	1.27	1.27	....
264	1.27	1.31	1.27	....
265	1.30	1.26	....	....
Average	1.26	1.29	1.27	1 30

Bench No. 1 = Top Coal.
Bench No. 2 = Middle Bench.
Bench No. 3 = "Blacksmith Coal" (local name).
Bench No. 4 = Bottom Coal.

A large number of lumps were selected from different points along each bench and at a number of localities in the district, and the specific gravity of each of these lumps was determined (see Table 14). The coal from different benches showed an appreciable difference in specific gravity, although the value from each bench was fairly constant.

Samples from different benches were obtained from thirteen different mines in the district, and the specific gravities determined imme-

diately by means of the portable Jolly Balance. The average value of each bench and for the entire bed was calculated for each sample by the method described, and the results for the bed are given in Table 15.

TABLE 15.

FRESH SPECIFIC GRAVITY OF DANVILLE COAL.

Coal No. 7		Coal No. 6	
Sample No	Specific Gravity	Sample No.	Specific Gravity
201	1.27	207	1.29
202	1.29	208	1.28
203	1.27	209	1.29
204	1.29	210	1.27
205	1.29	211	1.27
206	1.27	212	1.28
213	1.27		
214	1.28	Average	1.28
215	1.29		
216	1.28		
217	1.28		
218	1.28		
219	2.26		
220	1.27		
221	1.27		
222	1.27		
223	1.28		
225	1.28		
226	1.29		
227	1.29		
228	1.28		
229	1.29		
230	1.29		
231	1.28		
232	1.28		
233	1.27		
Average	1.27		

7. *Coal From Other Districts.*—It was not found possible to collect samples of fresh coal from various parts of the state; therefore, in order to cover the ground thoroughly, samples of coal in varying stages of dryness were obtained from practically every district in the state.

The apparent specific gravity of each of these samples was determined by the Jolly Balance method. The values obtained for the different samples vary considerably, probably due to the difference in the length of time which elapsed between the mining of the coal and determination of its specific gravity. Also, the fresh specific gravity of each of these samples was determined by the approximate method described on pages 20 and 21, and the values obtained are given in Table 16.

Table 16.

Specific Gravity of Various Illinois Coals.

Sample No.	County	Coal Bed No.	Specific Gravity		County Average Fresh Spec. Grav.*
			Apparent	Fresh*	
301	Christian	6	1.24	1.30	
302	"	6	1.21	1.29	
303	"	6	1.24	1.34	
304	"	6	1.20	1.29	1.31—Christian
305	Franklin	6	1.25	1.30	
306	"	6	1.22	1.30	
307	"	6	1.26	1.30	
308	"	6	1.28	1.31	1.30—Franklin
309	Grundy	2	1.23	1.34	
310	"	2	1.24	1.34	
311	"	2	1.19	1.35	
312	"	2	1.18	1.29	1.33—Grundy
313	Jackson	2	1.26	1.31	
314	"	2	1.28	1.32	
315	"	2	1.27	1.34	
316	"	2	1.25	1.29	1.32—Jackson
317	La Salle	2	1.14	1.26	
318	"	2	1.21	1.32	
319	"	2	1.12	1.26	
320	"	2	1.17	1.28	
321	"	2	1.12	1.29	
322	"	2	1.13	1.28	
323	"	2	1.17	1.30	
324	"	2	1.16	1.29	1.28—La Salle
325	Logan	5	1.21	1.33	
326	"	5	1.18	1.30	
327	"	5	1.20	1.32	
328	"	5	1.17	1.28	1.31—Logan
329	Madison	6	1.22	1.28	
330	"	6	1.25	1.32	
331	"	6	1.22	1.29	1.30—Madison
332	Marshall	2	1.23	1.29	
333	"	2	1.18	1.27	
334	"	2	1.16	1.29	
335	"	2	1.17	1.28	1.28—Marshall
336	Mercer	1	1.23	1.29	
337	"	1	1.21	1.30	
338	"	1	1.22	1.31	
339	"	1	1.22	1.30	1.30—Mercer
340	Montgomery	6	1.25	1.31	
341	"	6	1.28	1.33	
342	"	6	1.29	1.29	
343	"	6	1.25	1.30	
344	"	6	1.24	1.29	
345	"	6	1.15	1.29	
346	"	6	1.19	1.30	
347	"	6	1.17	1.28	
348	"	6	1.15	1.30	
349	"	6	1.20	1.29	
350	"	6	1.19	1.31	
351	"	6	1.20	1.28	
352	"	6	1.22	1.29	
353	"	6	1.24	1.30	1.30—Montgomery
354	Perry	6	1.29	1.34	
355	"	6	1.26	1.31	
356	"	6	1.27	1.32	
357	"	6	1.24	1.35	1.33—Perry
358	Sangamon	5	1.08	1.27	
359	"	5	1.20	1.30	
360	"	5	1.14	1.28	
361	"	5	1.18	1.26	
362	"	5	1.17	1.30	
363	"	5	1.21	1.32	
364	"	5	1.20	1.30	
365	"	5	1.20	1.29	
366	"	5	1.22	1.31	
367	"	5	1.23	1.34	
368	"	5	1.21	1.30	1.30—Sangamon
369	Williamson	6	1.25	1.29	
370	"	6	1.29	1.33	
371	"	6	1 27	1.30	
372	"	6	1.29	1.32	1.31—Williamson

*Fresh specific gravity as determined on dry coal by the approximate method described on page 20.

VI. Practical Uses of the Specific Gravity of Coal.

There are several practical uses to which a knowledge of the specific gravity of coal may be applied, chief among which are the following:

(1) The calculation of the tonnage of coal in the ground over a given area.

(2) The calculation of the tonnage of quantities of broken coal, such as the coal stored in large bins or in stock piles.

(3) The comparison of coals by the rapid estimation of the ash and moisture content.

(4) The determination of the adaptability of coal to the removal of its impurities by washing processes.

8. *Tonnage of Coal in the Ground.*—It is often desirable to estimate the number of tons of coal in a given area underlaid by coal beds of a known or to be determined thickness. The volume of solid coal can be easily calculated, knowing the thickness of the beds and the area they cover; and the tonnage may then be found by multiplying the volume in cubic feet by the weight per cubic foot, and dividing by 2,000.

$$\text{Tonnage} = \frac{V \times W}{2{,}000}, \text{ in which}$$

V = volume in cubic feet, and
W = weight per cubic foot.

Since the weight per cubic foot of coal is equal to the specific gravity of the coal multiplied by the weight of a cubic foot of water (62.5 pounds), the formula becomes

$$\text{Tonnage} = \frac{62.5\ V\ S}{2{,}000} \quad (S = \text{specific gravity}).$$

It is necessary to know the specific gravity of the coal with reasonable accuracy if the estimate of tonnage is to have any value. An error of only a small amount in the specific gravity might cause the amount of coal recovered from a given area to fall millions of tons short of the estimated recovery. This fact was recognized by the Commission appointed to investigate the waste in coal mining in Pennsylvania in 1890. This Commission in its report stated that " . . . a variation of 1 per cent

in the specific gravity would reduce the total number of tons of coal in the ground 195,000,000."*

The importance of selecting the proper value for the specific gravity is evident when it is remembered that the specific gravity of any coal may be represented by a number of widely differing values, each perfectly correct for the condition which it represents. For example, the apparent specific gravity of a fresh coal may be 1.33, while after the coal has dried out it may be only 1.20. This difference of 0.13 in the specific gravity makes a difference of about 1,020 tons per acre for a bed of coal six feet thick, or over 650,000 tons for an area of only one square mile. This difference is far too great to be neglected, and it is therefore necessary to select the proper value for the specific gravity.

In selecting this value it should be remembered that the fresh specific gravity represents the condition in which the coal exists in the ground; the apparent specific gravity represents any condition of the coal at any stage in the drying process; while the dry and unit values represent only theoretical conditions that are never attained practically, and therefore, can be neglected in this connection.

The most practical value for use in the calculation of tonnage in the ground is one that represents the condition in which the coal exists when it is marketed, since the calculated weight will correspond to the weight upon which the price received for the coal is based. In the state of Illinois, coal that is marketed is usually settled for on the basis of weights at the mine. The coal is weighed as soon as it is placed in the railroad car ready for shipment. Only a short time elapses between the mining of the coal and weighing it, and therefore, the coal when weighed is practically fresh. It is not exposed to the air long enough after being mined to lose sufficient moisture to affect its weight appreciably. Since the coal is sold on a practically fresh basis, all calculations of tonnage in the ground should be based upon the fresh specific gravity of the coal.

9. *Tonnage of Broken Coal.*—In calculating the tonnage of a known volume of broken coal it is necessary to know not only the specific gravity of the coal, but also the amount of void spaces, or the amount of increased volume occupied by the coal when it is broken. For bituminous coal, such as that found in Illinois, this increase in volume probably varies from 60 per cent for large sizes to about 80 per cent for small sizes. Assuming

*Report of Penn. Coal Waste Commission, p. 11, 1893.

that the coal in question is in small sizes, the tonnage can be calculated by the formula:

$$\text{Tonnage} = \frac{62.5\ V\ S}{1.80}, \text{ in which}$$

V = volume in cubic feet, and
S = specific gravity.

Again, it is necessary to know which value of the specific gravity to use. In such a case, the manner in which the coal has been stored will affect its moisture content and consequently its specific gravity. Three cases should be considered; (a) when the coal has been stored under water, (b) when it has been stored in a tight bin, and (c) when it has been stored in an exposed stock pile.

(a) *Under Water Storage.*—Coal that is stored under water undergoes practically no deterioration, and its condition is practically the same after a long period of time as when freshly mined. Its specific gravity should not change, and consequently in calculating the tonnage of a given volume of coal stored under water the fresh value of the specific gravity should be used.

(b) *Closed Bin Storage.*—Coal that is stored in a closed bin should lose its moisture very slowly, but after a considerable length of time, for instance, several months or years, a large proportion of its contained moisture would probably be lost, especially if the coal were handled much during the process of transferring it to the bin. Under such conditions it would probably be necessary to obtain samples of the coal and actually determine its apparent specific gravity in order to obtain a reliable value upon which to base any calculations of tonnage.

(c) *Storage in Exposed Piles.*—Coal that is stored in exposed piles will lose moisture in dry weather and gain moisture in wet weather, and its specific gravity will vary accordingly. It is probable also that the specific gravity will not be the same on the surface of the pile as in the center or at the base, because evaporation of the moisture takes place more rapidly at the exterior. Under such conditions it would be almost impossible to estimate with any degree of precision the average specific gravity of the coal from any known values. Samples should be selected from different depths in the pile and the apparent specific gravity determined. On such an average value the estimate of tonnage should be based.

10. *Comparison of Coals.*—If the specific gravity of two coals of the same type and of the same moisture content, such as two bituminous coals from different parts of the same bed or even from different beds are compared, it is probable that the one having the higher specific gravity has also the higher ash content. They can be more conveniently compared if the moisture is entirely eliminated by calculating the dry specific gravity. By comparing the dry specific gravities of different coals of the same type a great deal might be learned about the other properties of the coal, especially the ash content. Since the heat value varies inversely as the ash content for a given coal, some idea might also be obtained as to the relative heat values of the two coals. Such a comparison could not be made between two types of coal such as a bituminous and an anthracite, or a bituminous and a lignite, but for the same types of coal valuable comparisons could be made. By eliminating all the impurities and using unit value of the specific gravity it might even be possible to draw valuable conclusions as to the relative heat values of different coals. Such a possibility is only suggested, but it may be worth further consideration.

11. *Specific Gravity and Coal Washing.*—Before an attempt is made to remove the impurities from coal by washing, the properties of both the coal and its impurities should be investigated to determine whether the coal is adaptable to washing, and which sizes give the best results. The common method of testing is the "float and sink" test, by means of which the impurities are removed from the crushed coal by immersing it in a heavy solution, such as a solution of calcium chloride or of zinc chloride, in which the impurities sink and the clean coal floats and can be skimmed away. The object of washing the coal is twofold; first, to reduce the ash below an arbitrary maximum, which depends upon the purposes for which the product is to be used, and secondly, to reduce the sulphur content to a minimum, especially if the product is to be utilized in manufacturing metallurgical coke. By testing with solutions of different specific gravity, one can be selected which will separate float coal with a little less than the maximum allowable ash. If the sulphur in the float coal has at the same time been reduced to a minimum, the coal is suitable for the manufacture of metallurgical coke, otherwise it is not. However clean the float coal may be, the sink material must contain only a small amount of coal if the separation is to be a commercial success. If the sink contains much coal the loss in the tailings will reduce the profits realized by the increased price of the clean coal until the washing is no longer economical. The lighter the coal and the heavier

the impurities, the cleaner will be the separation. If float and sink tests are carefully made by a standard method of procedure they give valuable comparative data.

Specific gravity tests may be used to advantage in determining the adaptability of coal to washing. They may be made either on average samples from the float coal and the impurities derived from float and sink tests, or they may be made directly upon the coal and upon the shale, which constitutes its chief impurity. All washing methods for the separation of coal from its impurities depend upon differences in specific gravity. The trough, washers, and the grading boxes depend upon the free settling ratio,* and the tub washers, jigs, and bumping tables upon the hindered settling ratio between the coal and its impurities.

Whether the washing takes place under free or hindered settling conditions, the greater the settling ratio the greater will be the range of sizes allowable in the coal and the cleaner will be the separation for a given size.

It has been shown that the apparent specific gravity of the coal itself decreases as the coal dries out. Values as low as 1.12 were obtained for very dry coal by the author, and values of 1.18 or 1.20 are quite common; whereas the apparent specific gravity of the same coal when fresh is about 1.30 or even more. As the specific gravity of the coal decreases its hindered settling ratio increases and the coal becomes more suited to washing, provided the specific gravity of the impurities remains unchanged. Pyrite and shale, the chief impurities, with the exception of fire-clay, are practically nonporous, and their saturation or nonsaturation with water should not affect their specific gravity. Fire-clay breaks up upon coming into contact with water, and in washing operations it is carried away with the water and does not enter into the process, except perhaps to slightly increase the specific gravity of the water. One example will suffice to illustrate the effect upon the specific gravity of drying the coal. Assuming hindered conditions with a quicksand having a specific gravity of 1.10, with impurities having a specific gravity of 2.40 and with fresh coal having a specific gravity of 1.30, the settling ratio becomes:

$$\frac{D}{D'} = \frac{2.40 - 1.10}{1.30 - 1.10} = \frac{1.3}{0.2} = 6.5.$$

*For a discussion of free and hindered settling ratios, see Bul. No. 69, University of Illinois Engineering Experiment Station, "Coal Washing in Illinois," by F. C. Lincoln.

If the coal is air-dry, with a specific gravity of 1.20,

$$\frac{D}{D'} = \frac{2.40 - 1.10}{1.20 - 1.10} = \frac{1.3}{0.1} = 13.$$

The hindered settling ratio $\frac{D}{D'}$ for the air-dry coal is just twice that for the same coal when fresh, and theoretically the coal when air-dry can be washed with a range of sizes twice as great as when fresh; or with a much cleaner separation for the same range of sizes. Consequently from this viewpoint it would seem advisable:

(1) Not to wash fresh coal, in other words, not to wash coal at the mine; the time required to ship coal to a distant point permits a certain amount of drying to take place. If the question should come up as to whether one large washery should be established near the market and all coal shipped from a number of mines to it, or a number of washeries should be built at the different mines and the product shipped to the market, a consideration of the relation between moisture content and the cleanness of separation would favor the central washery plan.

(2) Not to feed wet coal to the jigs or to other washing machines. The coal should be as dry as possible when it reaches the medium in which the separation of clean coal and impurities takes place. When washing fine sizes of coal it may be necessary to feed it wet in order to keep the coal from matting and floating over the jigs without being wetted. In such cases the wetting should be delayed as long as possible so that the coal will not remain wet long before being fed to the jigs.

These conclusions are based upon theoretical considerations of the results of study of the specific gravity of coal, and have little practical bearing.

VII. Summary.

(1) In general, no fixed value, such as can be given for mineral substances of definite chemical composition, can be given for the specific gravity of coal. Different types of coal, such as anthracite and bituminous, have different specific gravities, and even coals of the same general type may have quite different specific gravities, due to differences in content of ash and moisture.

(2) Fresh coal of the Illinois type is probably saturated with moisture, most of which is held mechanically in the pores of the coal.

(3) The specific gravity of fresh coals of the same general types should be about constant if the content of ash is about the same. The specific gravity of coal in the fresh condition has been called the "fresh" specific gravity.

(4) If coal has dried to any extent its specific gravity will vary, depending upon the amount of moisture lost. As the moisture content decreases the specific gravity should decrease (the ash content remaining the same), but not necessarily in a direct ratio. The specific gravity uncorrected for moisture or ash content has been called the "apparent" specific gravity.

(5) By calculating values of specific gravity to exclude the effect of moisture, values may be obtained which have been called the "real" or "true" specific gravity, and which might well be called the "dry" specific gravity to correspond with analyses of coal which have been calculated to the "dry" or "moisture-free" basis. By excluding both ash and moisture, values might be obtained which could be referred to as the "unit" specific gravity, to correspond with the unit coal B. t. u of Parr.

(6) By saturating air-dry coal with moisture before determining its specific gravity, a value can be obtained which represents quite closely the "fresh" value of the specific gravity, or the specific gravity of the coal in its fresh condition.

(7) The fresh specific gravity of coal from different counties in Illinois was found to vary from 1.28 to 1.33, with an average of between 1.30 and 1.31. These values, excepting those for Vermilion and Montgomery counties, were obtained from air-dry coal by the approximate method described on page 20 of this bulletin.

(8) In computing tonnages it is suggested that (a) for coal in the ground, the fresh specific gravity is the proper value to use, (b) for broken coal stored under water, the fresh specific gravity should be used, and (c) for broken coal stored in bins or in open piles the apparent specific gravity should be found for the average condition of the coal at the time the computations are made, and that value used.

(9) In coal washing operations the coal theoretically should be as dry as feasible when fed to the washing machines, since the settling ratios are considerably greater for dry than for wet coal, and dry coal can consequently be washed more efficiently.

VIII. Appendix A.—Methods of Determining Specific Gravity of Coal.

12. *General Methods for Solids.*—A number of methods have been devised for determining the specific gravity of solids, involving various methods of procedure and requiring various kinds of apparatus. Most of these methods are too well known to need description. They are as follows:

(a) Hydrostatic Balance Method.
(b) Jolly Balance Method.
(c) Pycnometer Method.
(d) Heavy Solution Method.

A few modifications of the Hydrostatic and Jolly Balances have been devised which permit especially rapid manipulation, but since these are not particularly applicable to the determination of the specific gravity of coal, a mere reference to them will suffice.*

13. *Special Methods for Coal and Coke Determinations.*—A number of modifications of the general methods have been used for the determination of the specific gravity of coal.

(a) *Ordinary Pycnometer Method of the U. S. Bureau of Mines.*†—"To determine the true specific gravity of coal and coke substance, the procedure is as follows: Approximately 3.5 grams of the 60-mesh coal or coke is weighed and introduced into a 50-c.c. pycnometer with about 30 c.c. of distilled water. In order to avoid loss of particles of the sample during boiling, a one-bulb 6-inch drying tube, *a* (Fig. 4), is connected with the pycnometer by means of a small piece of pure gum tubing or a rubber cork, *c*. The other end of the drying tube is connected with the aspirator. Suction is applied and the contents of the flask are gently boiled on the water bath *d* under partial vacuum for three hours in order to expel all air from the sample. The pycnometer is then detached, almost filled with boiled and cooled water, allowed to cool to the temperature of the balance room, stoppered, and weighed. The temperature of the contents of the pycnometer is taken immediately after

*M. von Schwartz, "Two New Types of Balance for Density Determinations," Munich Centr. Min. Geol., 1913, 565-570.

*Franz Toula, "A Quick Working Hydrostatic Balance," Min. Pet. Mitth., 26, 233-237.

*A. H. Sabin, "A Specific Gravity Balance for Solids," Orig. Com. 8th Inter. Cong. Appl. Chem., I, 441-443.

*W. Bahrdt, "Measurement of the Density of Solid Bodies," Z. Physik Chem., Unterricht, 26, 6-7; Chem. Zentr., 1913, I, 1390.

†Technical Paper No. 8, U. S. Bureau of Mines, 1914, "Methods of Analyzing Coal and Coke," by F. M. Stanton and A. C. Fieldner.

weighing. Each pycnometer is accurately calibrated and a table is constructed giving its capacity in grams of water at different temperatures.

"The true specific gravity is determined by use of the following formula:

$$\text{True specific gravity} = \frac{W}{W-(W'-P)}, \text{ in which}$$

W = weight in grams of dry coke = weight in grams of sample minus its moisture content.

W' = weight in grams of pycnometer plus dry coke, plus water to fill.

P = weight in grams of pycnometer plus water to fill."

(b) *Special Method of the U. S. Bureau of Mines.**—"The Hogarth flask recommended by Blair† for the determination of the specific gravity

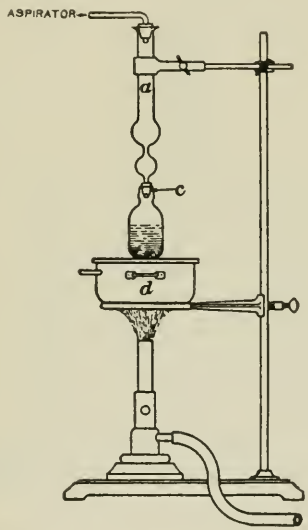

FIG. 4. ARRANGEMENT OF PYCNOMETER, U. S. BUREAU OF MINES.

of iron ores is more convenient and accurate for routine determinations of the specific gravity of coal or coke substances than is the ordinary pycnometer described in the preceding method. With the ordinary pycnometer it is difficult to insert the stopper without catching some floating particles between the stopper and neck.

"With the Hogarth flask is no such difficulty. The method of determination with the Hogarth flask is as follows:

*Technical Paper 8, U. S. Bureau of Mines, 1914.

†A. A. Blair, "The Chemical Analysis of Iron," 7th ed., 1908, p. 278.

"A 10-gram portion of the 60-mesh coal or coke is weighed and carefully introduced into the weighed flask (Fig. 5) with enough distilled water to fill the flask half full. The capacity of the Hogarth flasks obtained on the market varies from 100 to 125 c.c. The flask is then placed on a small electric hot plate in a 10-inch vacuum desiccator. The desiccator is evacuated by means of an aspirator or air pump. A current sufficient to keep the water boiling is passed through the hot plate. With an efficient vacuum pump all the air is expelled in 30 minutes. The flask is then removed from the desiccator, filled to the tubulure with recently boiled and cooled distilled water, and the stopper inserted. It is advisable to apply a thin film of vaseline to the stopper to prevent leakage.

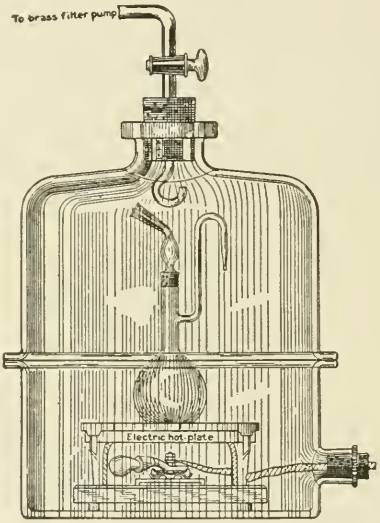

FIG. 5. HOGARTH FLASK METHOD.

"After the flask has been cooled to about 25° C. in a water thermostat, distilled water that has been cooled in the same thermostat is drawn through the tubulure until the water level is slightly above the mark on the capillary of the stopper. This may be done without removing the flask from the thermostat by inserting the end of the tubulure in a small beaker of water and applying a slight suction on the stopper. The flask should remain in the thermostat until the temperature of contents is exactly 25° C. The water level is adjusted to the mark on the capillary by touching a piece of filter paper to the end of the tubulure or by

drawing in a little water. The flask is then removed from the thermostat, wiped dry, and weighed. The true specific gravity is calculated as in the preceding method. (See formula, p. 39.) The value for P is obtained by filling the flask with boiled water, cooling, and weighing, as described above.

"By this method no difficulty is experienced in duplicating the figures for specific gravity to two decimal places."

(c) *Blakely and Chance Method.**—Messrs. Blakely and Chance, formerly chemists for the Philadelphia & Reading Coal & Iron Co., have described a method which they used for specific gravity determinations of coal. A representative sample of coal is reduced to 100 grams and placed in a 250 c.c. volumetric flask which has been previously calibrated. Then 100 c.c. of water is run in, suction applied, and the flask shaken. When the air-bubbles cease to rise, the suction is stopped and water run in from a burette up to the 250 c.c. mark.

$$\text{Specific gravity} = \frac{100}{V_0 - V_1}, \text{ in which}$$

V_0 = volume of the flask in cubic centimeters,

V_1 = number of cubic centimeters of water added after the evacuation.

By using a table of reciprocals calculation is facilitated, since the specific gravity equals 100 times the reciprocal of $V_0 - V_1$.

(d) *Brinsmaid's Method.*—Mr. William Brinsmaid developed in the laboratory of Professor S. W. Parr of the University of Illinois a volumetric method for specific gravity determinations. The apparatus consists of a pint fruit jar and an inverted glass funnel of the same diameter. (Fig. 6.) The rim of the funnel and the top of the jar are ground to insure a close joint. The center of the cap of the jar is cut out and the cap is placed over the funnel and screwed down to hold it firmly in place.

The volume of the two-part flask, as it may be considered, is determined by running in water from a burette up to a mark on the stem of the funnel. To make room for the lump of coal of which specific gravity is to be determined, enough water is removed through a pipette inserted in the funnel. The water is placed in a beaker and saved. Then the funnel is taken away and the lump of coal, previously weighed, is placed in the jar. The funnel is replaced, and water run in from the

*Mines and Minerals, Vol. 31, p. 499, March, 1911.

supply previously removed until the flask is again full up to the mark, the air-bubbles adhering to the coal being removed by shaking. Then the remaining water of the supply taken out at first is the water dis-

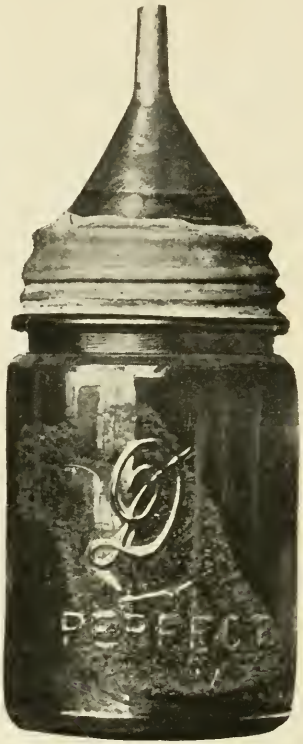

FIG. 6. PARR-BRINSMAID APPARATUS.

placed by the coal and has the same volume as the lump of coal. Its weight is determined from the following equation:

$$\text{Sp. Gr.} = \frac{Wc}{W}, \text{ in which}$$

Wc = weight of the coal,

W = weight of the water displaced.

(e) *Nicholson Hydrometer Method of the U. S. Bureau of Mines.**—"The apparatus used for determination of the apparent specific gravity (of both coal and coke) consists of a galvanized-iron cylinder (Fig. 7),

*Technical Paper No. 8, U. S. Bureau of Mines, 1914, p. 39.

which is filled with water to the water line, as indicated in the figure. In the cylinder is immersed a hydrometer made of brass. On the top of the hydrometer are two pans. The upper one is used for weights and the lower for the sample. Below the air buoy is a brass cage perforated with many holes to allow the air to escape when the instrument is immersed. The cage carries the sample when it is weighed under water. The method of determining the apparent specific gravity is as

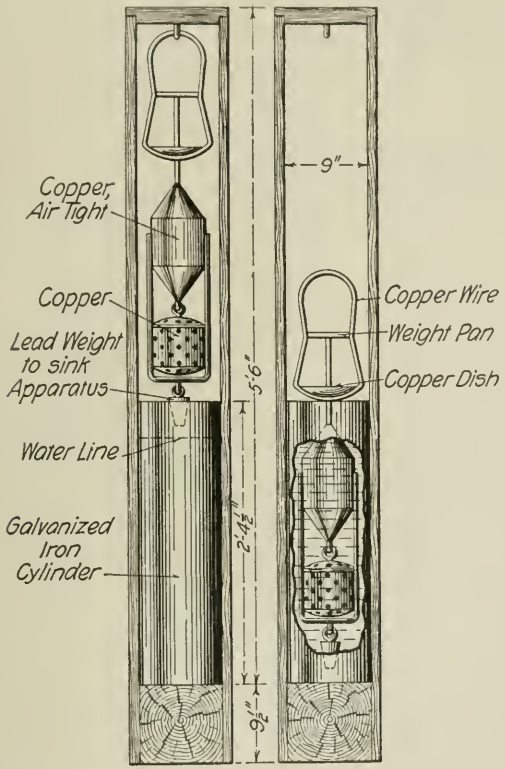

Fig. 7. Nicholson Hydrometer for Coal and Coke.

follows: Brass weights are placed on the upper pan until the hydrometer sinks to a mark on the stem between the copper pan and the buoy. The total weight required is recorded. The weights are removed, and about 500 grams of the sample in lump form (about 1½ to 2-inch cubes) are placed in the copper dish. Brass weights are then added until the hydrometer sinks to the mark on the stem. The difference in the weights

used gives the weight of the sample in air. The sample is then carefully transferred to the brass cage below the buoy. The weights on the upper pan are now adjusted until the instrument again sinks to the mark on the stem. The weight required to sink the hydrometer to the mark with no sample on the upper pan nor in the brass cage minus the weight required to sink it to the mark with the sample immersed in the cage equals the weight of the coke in water. Then:

If the weight of the sample in air $= x$
and the weight of the sample in water $= y$,

$$\text{the apparent specific gravity} = \frac{x}{x - y}$$

and

$$100\,x\,\frac{\text{apparent specific gravity}}{\text{True specific gravity}} = \text{percentage of volume of coke substance.}$$

Also,

100 — percentage by volume of coke substance = percentage by volume of cell space.

"In making apparent specific gravity determinations of coke the sample should preferably be in lumps of nearly the same size and shape. When the sample is immersed, the hydrometer should be moved rapidly up and down in the water a number of times in order to remove air bubbles. Since coke samples are porous, they take up water rapidly and should not be allowed to remain in contact with water more than five minutes during a determination. By observing the above-mentioned precautions satisfactory results can be obtained. All samples should be thoroughly dried before specific determinations are made."

(f) *Coxe's Beam Balance Method.*—In 1893 Eckley B. Coxe* described a method for the determination of the specific gravity of broken coal in comparatively large quantities, as used in his laboratory at Drifton, Pennsylvania. The apparatus (Fig. 8) consisted of a Fairbanks market beam scale and ordinary sheet-iron bucket, a cylindrical tin pan about 14 inches in diameter and 7 inches deep, and an ordinary washtub. The weighing beam was supported by a crane attached to a heavy post, and the apparatus was hung on the hook provided for suspending material to be weighed. A yoke was also attached to this hook, and the pan was suspended from it by means of wires and immersed in water in the tub. The coal of which the specific gravity was to be determined

*Presidential Address, New England Cotton Manufacturers' Association, 1893.

(about 20 lb.), after having been freed from dust and air-dried, was placed in the bucket, and its weight obtained by means of the poise and a suitable rider on the scale beam. It was then poured into the pan under water, and after it had been stirred in order to remove all air, the weight was obtained as before, and the specific gravity calculated in the usual manner.

(g) *Spring Balance Method.*—Mr. M. S. Hachita* has suggested the use of a large spring balance for obtaining the specific gravity of large sizes of coal. It could be used as well for large quantities of small

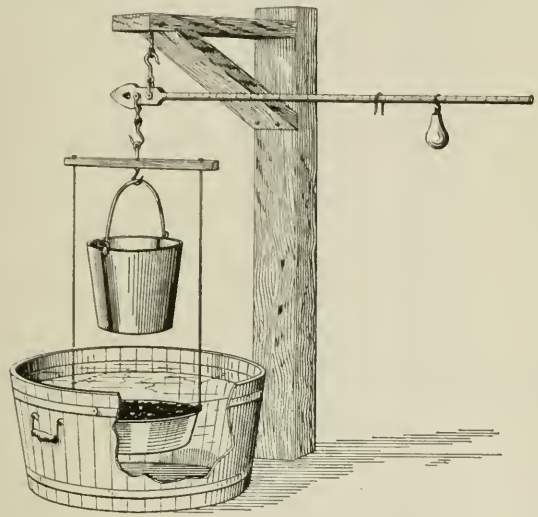

FIG. 8. COXE BEAM BALANCE.

sizes, and exactly in the manner described by Mr. Coxe for his beam balance.

(h) *Jolly Balance Method.*—The Jolly balance (Fig. 9) consists of a long delicate spiral spring suspended from an arm attached to a support *S* which can be moved vertically. This support is in the form of a tube graduated in centimeters and millimeters, and it moves inside of another tube to which a vernier is attached. Two pans are hung from the lower end of the spring, the lower one when in use being immersed in water up to a mark on the wire which holds it. The vertical support is placed so that its zero mark coincides with the zero of its vernier, then a movable mark on the stationary support is made to bisect the marks

*Mines and Minerals, Vol. 31, p. 499.

between the spring and the pans. These adjustments being made, the balance is ready for use.

A lump of coal is placed upon the upper pan, and the spring is stretched by raising the vertical support until a balance is reached. The amount the spring is stretched is read off by means of the vernier from the graduated scale of the vertical support. The lump of coal is then transferred to the lower pan and is immersed in water. The stretching

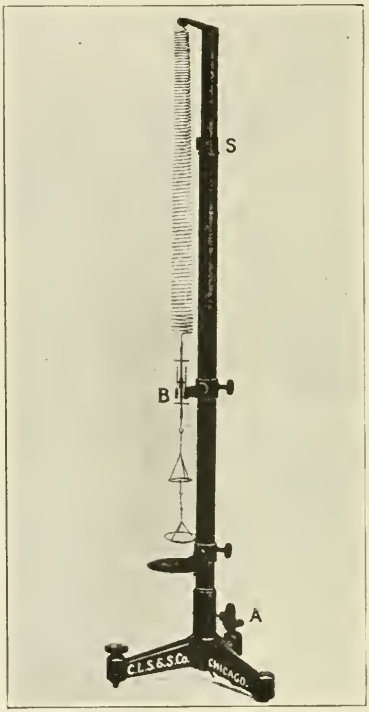

Fig. 9. Jolly Balance.

of the spring is again recorded, and the specific gravity calculated by the formula:

$$\text{Sp. Gr.} = \frac{Wa}{Wa - A}, \text{ in which}$$

Wa = wt. in air, and

W = wt. in water.

Since the specific gravity is merely a ratio, the stretch of the spring may be used directly for the values Wa and W.

This method of determination may be used directly to obtain the apparent specific gravity. The manipulation of the balance is quite simple, and its operation is very rapid. If the actual weight in grams is desired, the balance may be calibrated with a set of weights. The calibration of the balance used is indicated in Table No. 17.

TABLE 17.
CALIBRATION OF SPRING BALANCE

Grams	Balance Reading	Value of One Space in Grams	Value of One Gram in Spaces
1	3.03	0.330	3.03
2	5.97	0.335	2.99
3	8.90	0.337	2.97
4	11.82	0.338	2.95
5	14.74	0.339	2.95
6	17.67	0.340	2.94
7	20.60	0.340	2.94
8	23.50	0.340	2.94
9	26.40	0.341	2.93
10	29.28	0.342	2.93
11	32.19	0.342	2.93
12	35.10	0.342	2.92
13	37.98	0.342	2.92
14	40.87	0.342	2.92
15	43.75	0.343	2.92

14. *Selection of a Method.*—A method for the determination of the specific gravity of coal should combine as far as practicable accuracy, rapidity of operation, and simplicity of apparatus, and any such method should be adapted to the particular type of work for which it is to be used. If especially accurate results are desired and if time is no object, a method similar to one of those in use at the U. S. Bureau of Mines for true specific gravity determinations might be employed. When rapidity is desired and approximate results are sufficient, as in most commercial work, a more rapid method is better. If the determinations must be made in the field and the apparatus moved about from place to place, a method should be employed which requires only a simple instrument, and one that is easily portable, such as a Nicholson hydrometer or one of the simpler balances. If large lumps or large quantities of material are handled, a method like Coxe's Beam Balance method, or the Spring Balance method would be suitable. If the lumps are of moderate sizes the Nicholson hydrometer, or a method such as that suggested by Prof. Parr would serve the purpose. If only small sizes of coal are available, a pycnometer method should be used for accurate determinations, and a method such as the Jolly Balance method for the less accurate work.

Since the pycnometer methods of the Bureau of Mines are somewhat complicated, the author tried a more simple pycnometer method for purposes of comparison with the Jolly Balance. The specific gravity of a number of samples of dry coal was determined in the following way:

Pycnometers having a volume of 10 c.c. were weighed full of water at room temperature, a portion of the water was removed, and a one-gram sample of 20-mesh coal was placed in each. They were again filled with water, stoppered, and boiled in a beaker of water for about an hour, in order to remove all air from the coal. After they had cooled to room temperature they were filled to the mark with water and weighed. The specific gravity was computed by the formula:

$$\text{Sp. Gr.} = \frac{Wc}{(Wc + W) - W'}, \text{ in which}$$

Wc = weight of coal in air,

W = weight of pycnometer when full of water,

W' = weight of pycnometer when containing coal and filled with water, and

$(Wc + W) - W'$ = weight of water displayed by the coal.

The calculations were simple, as $Wc = 1$, W is a constant, and the specific gravity is the reciprocal of a constant minus W'.

This method was found unsatisfactory. Fine material was easily lost during the process, and it was difficult to remove all the air from the coal.

For purposes of comparison with the pycnometer method just described, the specific gravity of the same samples of coal was determined by the standard Jolly Balance method, the lumps being boiled in water for one hour and then cooled to room temperature before being weighed in water. The values obtained by the two methods are given in Table 18, and an examination of these data shows that the pycnometer method gave lower values than the Jolly Balance method. Evidently, in the latter case all the air was not removed from the pores of the coal before it was weighed in water.

15. *Method Adopted.*—The Jolly Balance method was found to be much more satisfactory, and it was adopted as the method for obtaining the specific gravity determinations made in connection with the preparation of this bulletin. The apparatus is quite simple, it requires lumps of a convenient size and only a small sample of coal, is rapid in operation, only a few seconds being required to make a weighing, and is sufficiently accurate for ordinary work. The chief objection to it is

that the lumps used for the determination are selected arbitrarily, which makes possible the admission of personal errors, since the operator may reject portions of the sample which contain pyrite or shale partings, and select only lumps which represent the average of the sample. In the investigation here described lumps weighing from 5 to 20 grams were used.

16. *Selection of Coal Sample.*—The effect of ash upon the specific

Table 18.

Comparison of Jolly Balance and Pycnometer Methods.

Sample No.	Specific Gravity	
	Jolly Balance	Pycnometer
1	1.28	1.26
2	1.26	1.27
3	1.28	1.27
4	1.33	1.27
5	1.31	1.32
6	1.33	1.26
7	1.33	1.27
8	1.35	1.29
9	1.33	1.29
10	1.32	1.26
11	1.29	1.26
12	1.32	1.27
13	1.29	1.25
14	1.28	1.28
15	1.31	1.25
16	1.31	1.31
17	1.31	1.31
18	1.31	1.28
19	1.32	1.29
20	1.39	1.29
21	1.35	1.28
22	1.36	1.31
23	1.37	1.32
Average	1.32	1.28

gravity has long been recognized. Dull coal may have a higher ash content than bright, glossy coal, and consequently a higher specific gravity. This condition should be taken into consideration, and lumps should be selected so that the average result will represent dull and bright coal in their proper proportions in the sample. Results which show abnormally high values are probably due to the fact that certain lumps contain an abnormal amount of shale or pyrite, and such results should be rejected in the general average, unless a large number of lumps are averaged.

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

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